Problem Set 1

Problem 1

(a)

$$\exists x, S(x) \wedge A(x)$$

(b)

$$\forall x, T(x) \wedge A(x)$$

Solution:

$$\forall x \in X : T(x) \wedge S(x) \Rightarrow A(x)$$

(c)

$$\neg\left(\exists x, T(x) \wedge (\neg A(x))\right)$$

Solution:

$$\neg \exists x \in X : T(x) \wedge (\neg A(x))$$

(d)

$$\exists x, y, z, x \neq y, y \neq z, x \neq z, T(u) \wedge (\neg S(u)), u \in \{x, y, z\}$$

Solution:

$\exists x, y, z \in X : (\neg E(x, y) \wedge \neg E(y, z) \wedge \neg E(x, z)) \wedge T(x) \wedge \neg S(x) \wedge T(y) \wedge \neg S(y) \wedge T(z) \wedge \neg S(z)$

Problem 2

(a)

P	Q	R	$\neg(P \lor (Q \land R))$	$(\neg P) \land (\neg Q \lor \neg R)$
T	T	T	F	F
T	T	F	F	F
T	F	T	F	F
F	T	T	F	F
T	F	F	F	F
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

Prove.

(b)

P	Q	R	$\neg(P \land (Q \lor R))$	$\neg P \lor (\neg Q \lor \neg R)$
T	T	T	F	F
T	T	F	F	T
T	F	T	F	T
F	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

Disprove.

Problem 3

(a)

(1)

$$A \land B = \neg(A \barwedge B)$$

(2)

$$A \lor B = (\neg A) \barwedge (\neg B)$$

(3)

$$A \Rightarrow B = A \barwedge (A \barwedge B)$$

Solution:

$$\begin{aligned} A \Rightarrow B &= ((\neg A) \vee B) \\ &= \neg (A \wedge (\neg B)) \\ &= A \barwedge (\neg B) \end{aligned}$$

(b)

$$\neg A = A \barwedge A$$

(c)

$$\begin{aligned} \text{true} &= A \barwedge (A \barwedge A) \\ \text{false} &= (A \barwedge (A \barwedge A)) \barwedge (A \barwedge (A \barwedge A)) \end{aligned}$$

Problem 4

1. $12 \Rightarrow 4, 4, 4 \Rightarrow 4$.
2. $4 \Rightarrow 2, 2 \Rightarrow 2$.
3. $2 \Rightarrow 1, 1 \Rightarrow 1$.

Problem 5

Prove: If $r^{1/5}$ is rational, r is rational.

$$r^{1/5} = a/b,$$

where a, b are integers. We have

$$r = \left(r^{1/5}\right)^5 = a^5/b^5.$$

Problem 6

w	x	y	$w^2 + x^2 + y^2 = z^2$	z
odd	odd	odd	$(2i+1)^2 + (2j+1)^2 + (2k+1)^2$ $= 4\left(i^2 + j^2 + k^2 + i + j + k\right) + 3$	odd
even	odd	odd	$(2i)^2 + (2j+1)^2 + (2k+1)^2$ $= 4\left(i^2 + j^2 + k^2 + j + k\right) + 2$ $=$ even $+$ odd $+$ odd	odd
odd	even	odd	-	odd
odd	odd	even	-	odd
odd	even	even	$(2i+1)^2 + (2j)^2 + (2k)^k$ $= 4\left(i^2 + j^2 + k^2 + i\right) + 1$	odd
even	odd	even	-	odd
even	even	odd	-	odd
even	even	even	$(2i)^2 + (2j)^2 + (2k)^2$ $= 4\left(i^2 + j^2 + k^2\right)$	even