KKT conditions for constrained problem

Problem:

$$\begin{aligned} &\min_{x} f(x) \\ \text{subject to } & c_E(x) = 0 \\ & c_I(x) - s = 0 \\ & s \geq 0 \end{aligned}$$

Log-barrier method problem with slack variables S:

$$\begin{aligned} &\min_{x,s} f(x) - \lambda \sum_{i=1}^{m} \log s_i \\ & c_E(x) = 0 \\ & c_I(x) - s = 0 \end{aligned}$$

Note: $s_i > 0$ is satisfied automatically due to logarithm.

Perturbed KKT conditions for log-barrier:

$$\begin{aligned} & \nabla f(x) - A_E^T(x)\, y - A_I^T(x)\, z = 0 \\ & -\lambda S^{-1} e + z = 0 \text{ or } -\lambda e + Sz = 0 \\ & c_E(x) = 0 \\ & c_I(x) - s = 0 \end{aligned}$$

Note: $\lambda e = -Sz$

My problem:

$$\begin{aligned} &\min_{\tau} \iint -sgn(v)\, u\, f_Y(u, v; \tau)\ du\, dv \\ \text{subject to } & \kappa - \iint \operatorname{sgn}(v)\, w\, f_Z(w, v; \tau)\ dw\, dv \geq 0 \end{aligned}$$

Log-barrier formation:

$$\begin{aligned} &\min_{\tau,s} \iint -sgn(\nu)\, u f_Y(u, \nu; \tau)\ du\, dv - \lambda \log s_1 \\ \text{subject to } & \kappa - \iint \operatorname{sgn}(\nu)\, w f_z(w, v; \tau)\ dw\, dv - s_1 = 0 \end{aligned}$$

Note: $s_1 > 0$ is satisfied automatically due to logarithm again.

Perturbed KKT conditions:

$\exists \lambda, z_1, s_1$, such that

$$\iint -sgn\left(\nu\right) u \nabla_{\tau} f_Y \left(u, \nu; \tau\right) dud\nu - z_1 \left\{ \kappa - \iint sgn\left(\nu\right) \omega \nabla_{\tau} f_Z \left(\omega, \nu; \tau\right) dwd\nu \right\} = 0$$

$$\kappa - \iint sgn\left(\nu\right) w \nabla_{\tau} f_Z \left(\omega, \nu; \tau\right) dwd\nu - s_1 = 0$$

$$-\lambda / s_1 + z_1 = 0$$

From the last equation, we have $s_1 = \lambda / z_1 > 0$