

KKT conditions for constrained problem

Problem:

$$\begin{aligned} \min_x & f(x) \\ \text{subject to } & c_E(x) = 0 \\ & c_I(x) - s = 0 \\ & s \geq 0 \end{aligned}$$

Log-barrier method problem with slack variables S :

$$\begin{aligned} \min_{x,s} & f(x) - \lambda \sum_{i=1}^m \log s_i \\ & c_E(x) = 0 \\ & c_I(x) - s = 0 \end{aligned}$$

Note: $s_i > 0$ is satisfied automatically due to logarithm.

Perturbed KKT conditions for log-barrier:

$$\begin{aligned} \nabla f(x) - A_E^T(x) y - A_I^T(x) z &= 0 \\ -\lambda S^{-1} e + z &= 0 \text{ or } -\lambda e + Sz = 0 \\ c_E(x) &= 0 \\ c_I(x) - s &= 0 \end{aligned}$$

Note: $\lambda e = -Sz$

My problem:

$$\begin{aligned} \min_{\tau} & \iint -\text{sgn}(v) u f_Y(u, v; \tau) du dv \\ \text{subject to } & \kappa - \iint \text{sgn}(v) w f_Z(w, v; \tau) dw dv \geq 0 \end{aligned}$$

Log-barrier formation:

$$\begin{aligned} \min_{\tau,s} & \iint -\text{sgn}(v) u f_Y(u, v; \tau) du dv - \lambda \log s_1 \\ \text{subject to } & \kappa - \iint \text{sgn}(v) w f_Z(w, v; \tau) dw dv - s_1 = 0 \end{aligned}$$

Note: $s_1 > 0$ is satisfied automatically due to logarithm again.

Perturbed KKT conditions:

$\exists \lambda, z_1, s_1$, such that

$$\iint -\operatorname{sgn}(\nu) u \nabla_{\tau} f_Y(u, \nu; \tau) du d\nu - z_1 \left\{ \kappa - \iint \operatorname{sgn}(\nu) \omega \nabla_{\tau} f_Z(\omega, \nu; \tau) d\omega d\nu \right\} = 0$$

$$\kappa - \iint \operatorname{sgn}(\nu) w \nabla_{\tau} f_Z(\omega, \nu; \tau) d\omega d\nu - s_1 = 0$$

$$-\lambda/s_1 + z_1 = 0$$

From the last equation, we have $s_1 = \lambda/z_1 > 0$