Descriptive Statistics

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Outline

1 Descriptive statistics of an univariate sample
- Motivation
- Initial step
- Histograms of "Stable" samples
- Single mode: central tendency
- Dispersion: Variability around the central tendency
- Going further
- Summarizing a distribution

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Motivation

We have set up a world where we keep collecting data, huge amount of data...

Sweet, what knowledge can we exctract from such data? How do we summarize a data set?

With a few numbers, some graphics? How? Why is this difficult?

> *There are three kinds of lies: lies, damned lies and statistics*
>
> – Mark Twain's Autobiography
>
> *Statistical thinking will one day be as necessary for efficient citizenship as the ability to read or write*
>
> – Attributed to H. G. Wells
>
> *The only statistics you can trust are those you falsified yourself*
>
> – Winston Churchill

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I just got new Tees!

- A series of measurements (one value per measurement)
- Nature of the measurements
  - Factors (nominal data)

```
 [1] Red   Red   Black Green Blue  Black White Black Blue
[10] White Black White Red   Black Black Red   Red   Black
[19] Black Black
Levels: Black Blue Green Red White
```

  - Ordered factors (ordinal data)

```
 [1] XL M  S  XL M  M  M  XL M  L  M  L  M  M  M  L  M
[18] M  XL M
Levels: S < M < L < XL
```

  - Numbers (e.g., price, duration, . . .) (numerical data)

```
 [1]  9.1  4.7  9.5 13.6 15.7  8.7  9.2  4.7 11.4  8.1
[11] 11.4 12.1 13.1  8.2 11.5  4.8  7.6  7.4  2.8 10.1
```

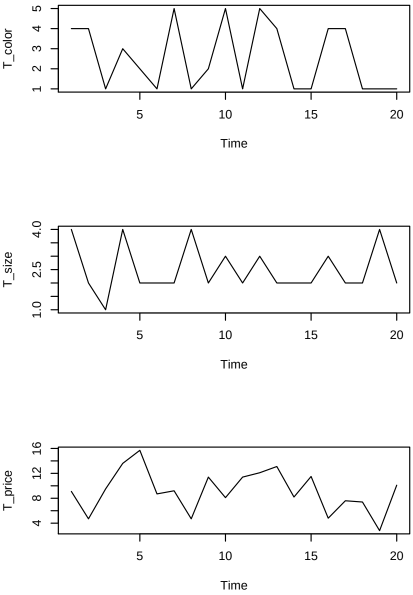
```
str(T_size); # May want to use the str function
```

```
Ord.factor w/ 4 levels "S"<"M"<"L"<"XL": 4 2 1 4 2 2 2 4 2 3 ...
```

Are these sample "structured"?

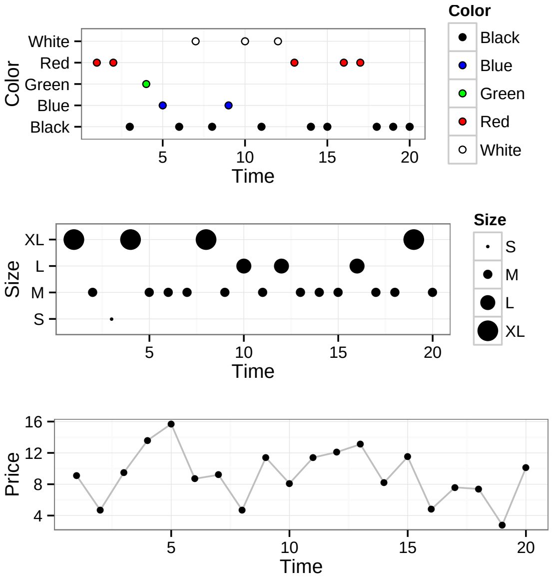
Use `plot.ts` (for time series)

```
par(mfrow=c(3,1));
plot.ts(T_color,xy.lines=F);
plot.ts(T_size,xy.lines=F);
plot.ts(T_price,xy.lines=F);
```

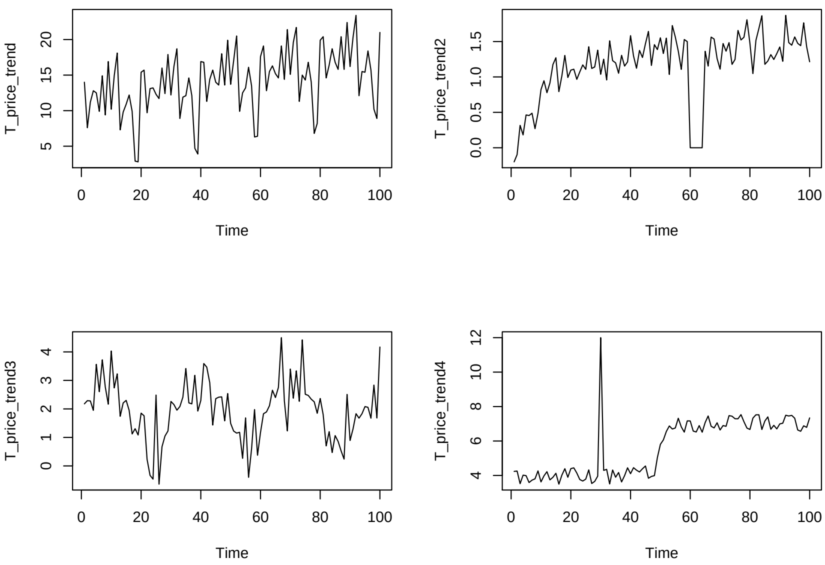


Are these sample "structured"?

Fancier output can be built using ggplot2



There could indeed be "trends"



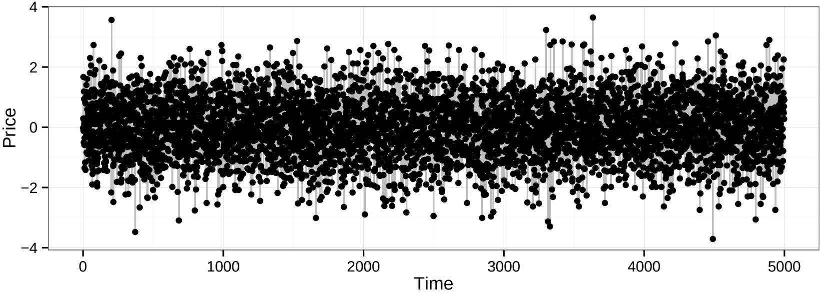
What should we look for?

- Structured/unstructured
- Trend, evolution
- Localization/order of magnitude
- Outliers, aberrant values

This preliminary study will:

- guide your analysis
- provide feedback on your experimental setup

This may be harder to do than it looks...



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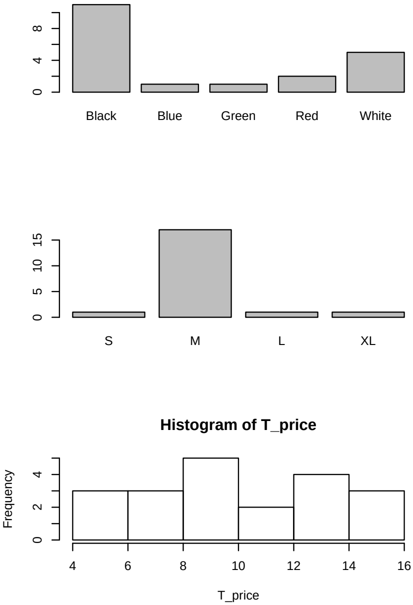
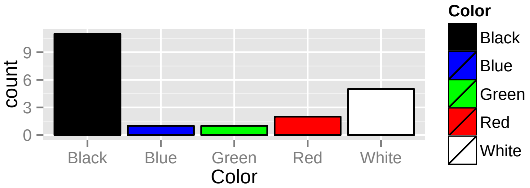
Bar charts vs. Histograms

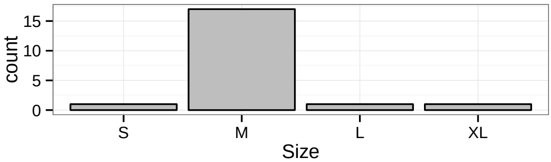
```
par(mfrow=c(3,1));
plot(T_color,xy.lines=F);
plot(T_size,xy.lines=F);
hist(T_price,xy.lines=F);
```

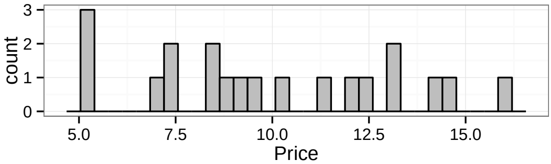


Bar charts vs. Histograms

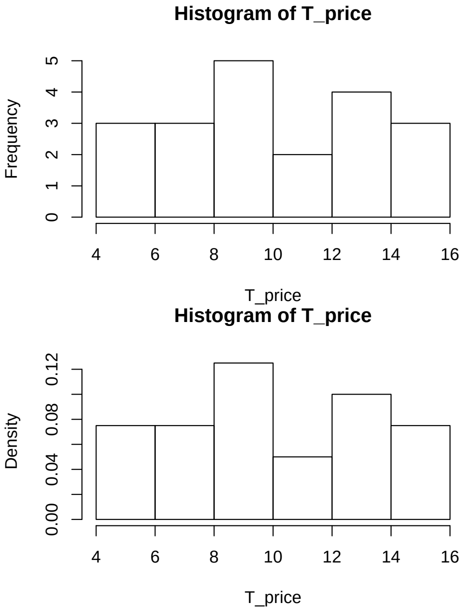
Again, fancier output can be built using ggplot2

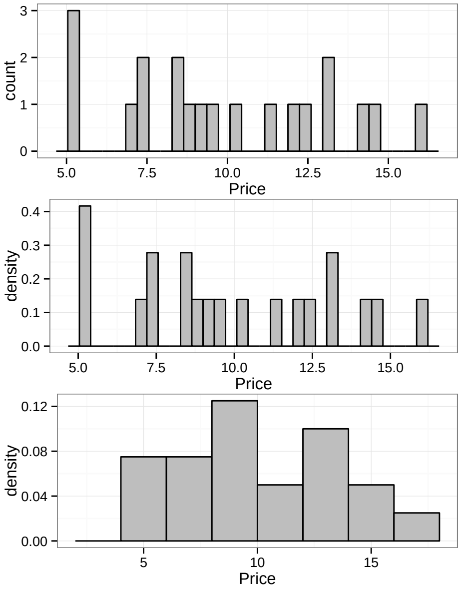






Wait, why are these histograms so different?





Beware of histograms

Rather indicate density than count

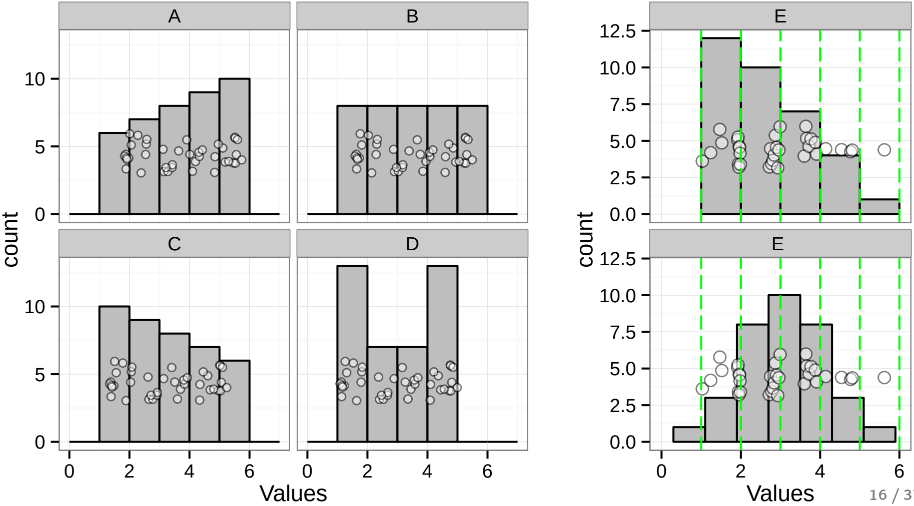
How many bins? Which binwidth?

- `ggplot` defaults to $k = 30$ bins of width $h = \texttt{range}/30$ ☹
- Square-root choice: $k = \sqrt{n}$ (Excel, ☹)
- Sturges: $k = \lceil \log_2 n + 1 \rceil$ (default for `hist` in R)
- Rice: $k = \lceil 2n^{1/3} \rceil$
- Scott: $k = \left\lceil \frac{\max x - \min x}{h} \right\rceil$, where: $h = \frac{3.5\hat{\sigma}}{n^{1/3}}$ (equivalent to Rice under some conditions)
- ...

Beware of Histograms

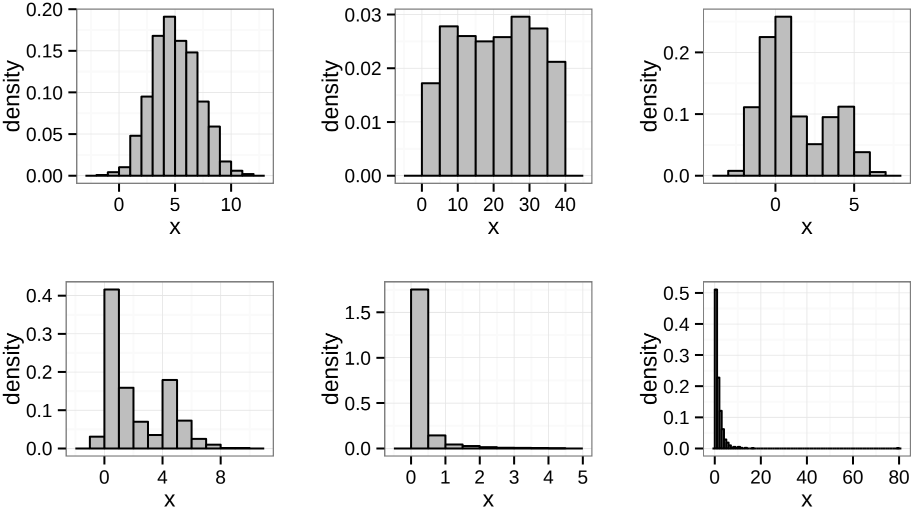
At which value should the bin start?

- In most cases, the binning is aligned on human readable values, which can create nasty artifacts (nice illustration from *stackexchange*)



What should we look for?

Shape: flat? symmetrical? multi-modal? Play with `binwidth` (and `origin` if you have few samples) to uncover the full story behind your data...



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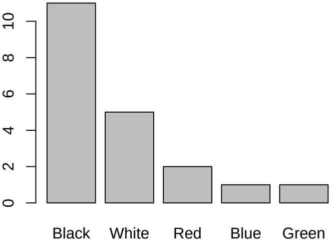
Nominal Values

- What is the mode (most frequent value)?
- Sort values according to their frequency...

```
summary(T_color)
```

```
Black  Blue Green   Red White
   11     1     1     2     5
```



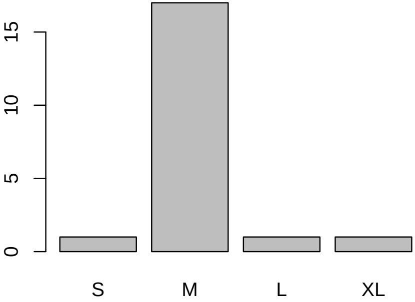

```
col_freq=table(T_color);
T_color <- factor(T_color,
    levels = names(col_freq[order(col_freq, decreasing = TRUE)]));
plot(T_color);
```

Ordinal Values

- What is the mode (most frequent value)?

```
summary(T_size)
```

```
S  M  L XL
1 17  1  1
```



- May still want to sort values according to their frequency...
- Median: not implemented in standard R for ordinal values, as it's not well defined

```
median(T_size)
library(DescTools)
median(T_size) # :(
```

```
Error in median.default(T_size) : requires numerical data
[1] NA
```

Numerical Values

```
str(T_price);
```

```
num [1:20] 14.5 13.1 9.3 6.9 8.6 7.2 7.3 12.4 13.1 16 ...
```

```
summary(T_price);
```

```
 Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
5.200   7.275   9.500   9.960  12.580  16.000
```

- min, max, median in R
- Median: 50% of values are smaller than 9.5
  (a possible measure of central tendency)

Numerical Values

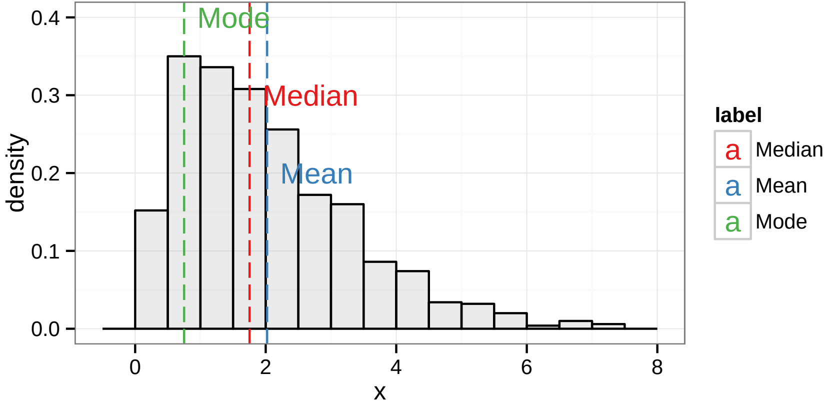
The mode and the median are measures of central tendency (typical value)

- Note: There may be several modes and it depends on binning...

There is also the (arithmetic) mean: $A = \overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$

```
mean(T_price)
```

```
[1] 9.96
```



Things to know about the mean

- This measure is sensitive to "outliers".
  - One aberrant (say very large) value will drag the mean to the right while it would not change the median
- The key question is what makes sense?
  - Your favorite pair has been added a +20% mark-up in August but you have a -20% discount as a regular customer. Is the price the same?
    - No, you actually saved 4% of the original price ($1.2 \times .8 = .96$).
  - You drove half the way at 50mph and half of the way at 100mph. Did you drive on average at 75mph?
    - Obviously not...
  - Although you can compute the average of gains/loss, it is not at all what you would consider as the average gain.
  - May want to consider the geometric or the harmonic mean...

$$G = \sqrt[n]{\prod_{i=1}^{N} x_i} \text{ or } H = \frac{1}{\frac{1}{N}\sum_{i=1}^{N} \frac{1}{x_i}}$$

What should I look for?

- If the distribution is unimodal and symmetrical, then

$$\text{mean} = \text{mode} = \text{median}$$

- Depending on the problem, one or the other may be more relevant
- Anyway, reporting such measure with no indication about variability is generally useless

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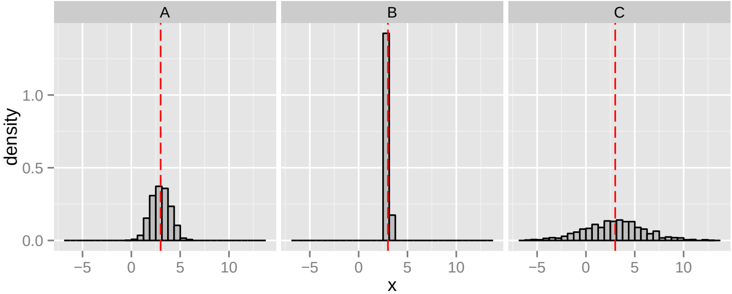
Variance

We expect most values to be "around" the mean



Departure from the mean:

- Mean absolute deviation: $\frac{1}{N}\sum_{i=1}^{N}|x_i - A|$
  - Rarely used
- Variance: $V = \frac{1}{N}\sum_{i=1}^{N}(x_i - A)^2$
  - only positive values and gives more importance to large deviations ☺
  - not homogeneous to the mean (units) ☹
- Standard deviation: $SD = \sqrt{V}$

Quantile

```
quantile(T_price,c(.05,.25,.5,.75,.95))
```

```
    5%    25%    50%    75%    95%
 4.605  7.550  9.150 11.425 13.705
```

Inter-Quantile Range:

- Inter-quartile range: $IQR = Q_{75} - Q_{25}$
- But other values are possible, e.g., $Q_{95} - Q_5$
- Range: $\max - \min$ (may grow unbounded)
    - $\rightsquigarrow$ quite difficult to use

What about nominal or ordinal values?

There is for example the notion of Entropy: how many bits are required to encode the sample?
Say there is a fraction f_v of items with value v.

$$H = -\sum_{v \in V} f_v \log_2(f_v)$$

$-(x+y)\log_2(x+y) < -x\log_2(x) - y\log_2(y)$ so the smaller the entropy, the more condensed/predictable the sample distribution

- $H([0,1,0,0]) = 0$
- $H([.25,.25,.25,.25]) = 2$
- $H([1/n,\ldots,1/n]) = \log_2(n)$ so you generally normalize H by $\log_2(n)$

This notion can be extended to numerical values (but the computation is complex as it depends on the binning...)

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Skewness

Remember the mean and the variance:

- $A = \overline{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$
- $V = \frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})^2$

Could we measure the asymmetry of the samples around the mean?

- Proposal 1: $\frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})$ (always 0… ☹)
- Proposal 2: $\frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})^3$ (not well normalized… ☹)

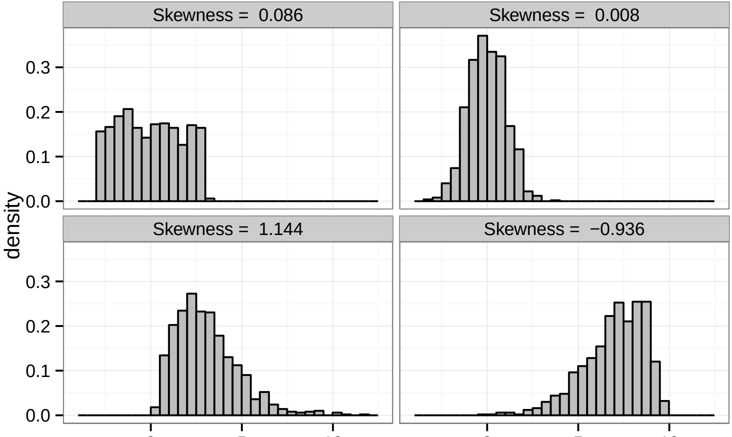
$$S = \frac{\frac{1}{n} \sum_{i=1}^{n} (x_i - \overline{x})^3}{\underbrace{\left[\frac{1}{n} \sum_{i=1}^{n} (x_i - \overline{x})^2 \right]}_{\text{variance}}^{3/2}}$$

Skewness

Could we illustrate this a bit?

```r
library(moments)
skewness(runif(1000))
```

```
[1] 0.04626483
```



Kurtosis

- peakedness (width of peak), tail weight, lack of shoulders...
- measure infrequent extreme deviations, as opposed to frequent modestly sized deviations

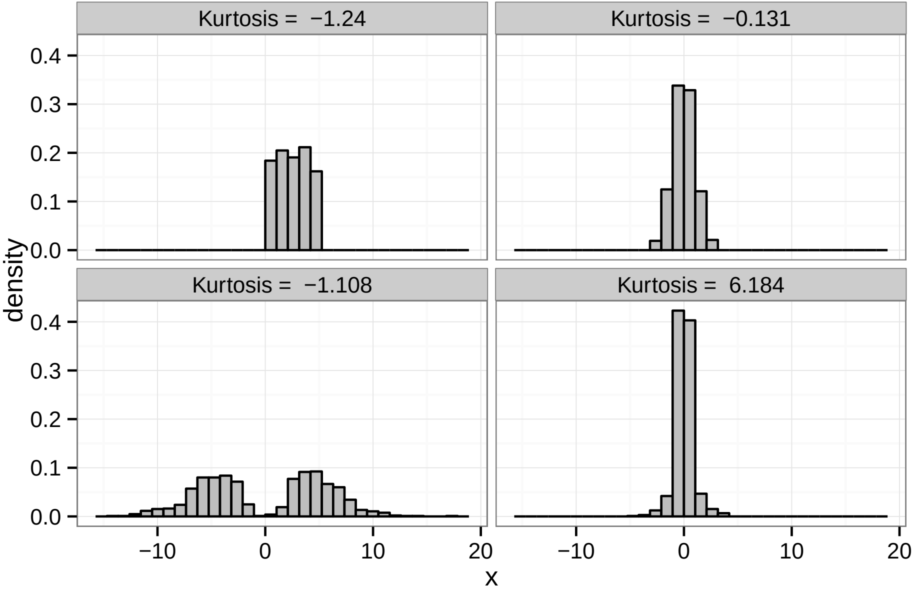
$$K = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^4}{\underbrace{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2\right]}_{\text{variance}}^2} - 3$$

The -3 is here so that normal distribution have a Kurtosis of 0

```r
library(moments)
x = rnorm(1000) ; var(x);
kurtosis(x)-3
```

```
[1] 1.039743
[1] 0.01825114
```

Kurtosis



Kurtosis = −1.24
Kurtosis = −0.131
Kurtosis = −1.108
Kurtosis = 6.184
density
0.0
0.1
0.2
0.3
0.4
−10
0
10
20
x

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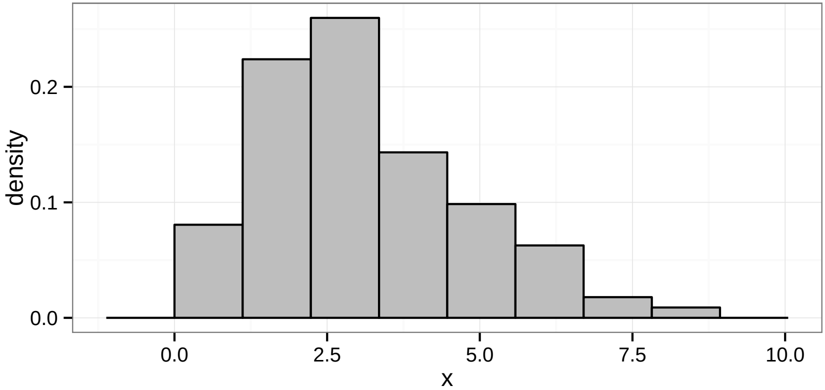
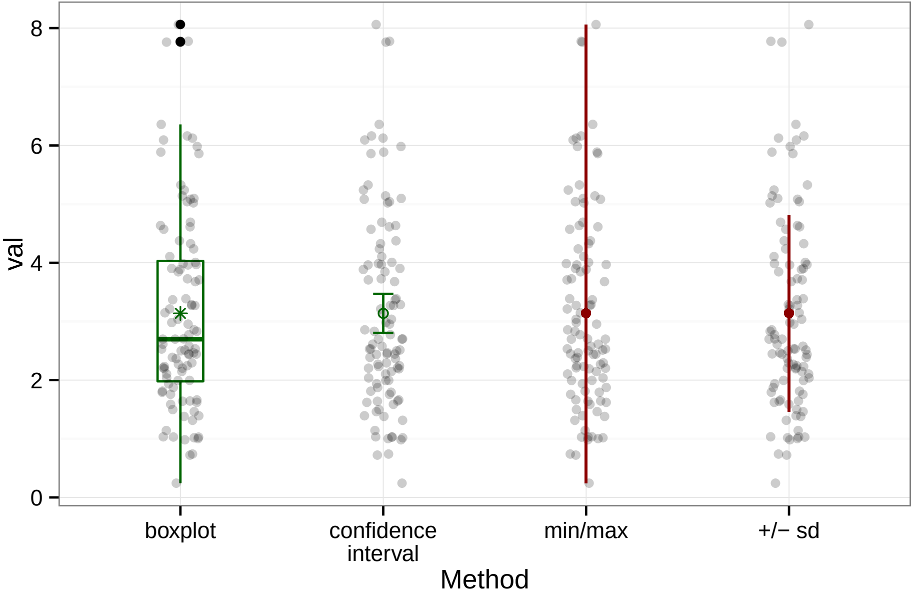
Classical information



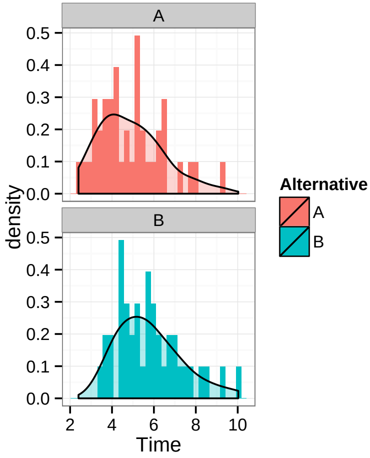

```r
summary(x)
var(x)
```

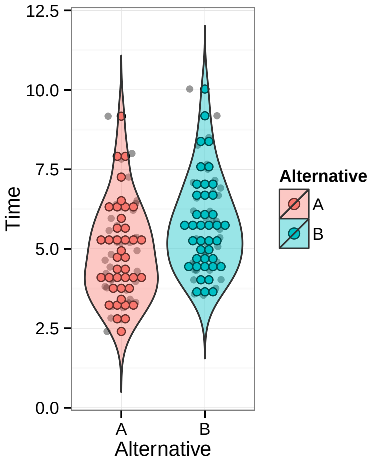
```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.4065  1.8430  2.5020  2.8660  3.6310  7.0220
[1] 2.117541
```

Good and bad summaries



Be careful with fancy plots you do not fully understand!





The average human has one breast and one testicle

– Des McHale

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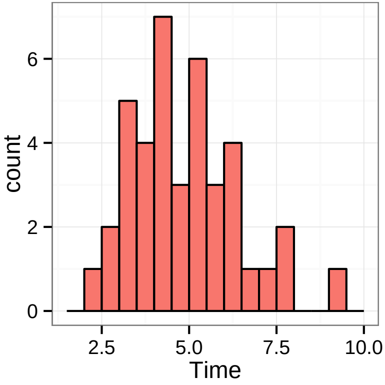

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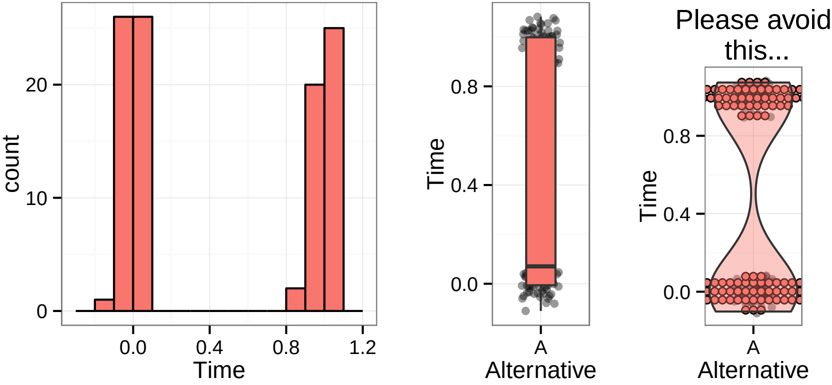

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