Matrix and Vector

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1   Calculation of the squared Euclidean norm

From StackExchange.

1.1   Question

$$\|\mathbf{x} - \alpha\|^2 - \|\mathbf{x} - \beta\|^2$$
$$= \|\mathbf{x}\|\|\mathbf{x}\| - 2\|\alpha\|\|\mathbf{x}\| + \|\alpha\|\|\alpha\| - \|\mathbf{x}\|\|\mathbf{x}\| + 2\|\beta\|\|\mathbf{x}\| - \|\beta\|\|\beta\|$$
$$= \alpha^T \alpha - \beta^T \beta + 2(\sqrt{\beta \cdot \beta} - \sqrt{\alpha \cdot \alpha})\|\mathbf{x}\|$$
$$= \alpha^T \alpha - \beta^T \beta + 2(\sqrt{\beta \cdot \beta} - \sqrt{\alpha \cdot \alpha})(\sqrt{\mathbf{x} \cdot \mathbf{x}}),$$

where I used the fact that
$$\|a\|\|a\| = \sqrt{a \cdot a}\sqrt{a \cdot a} = \sqrt{a^T a}\sqrt{a^T a} = a^T a.$$

However, the article gives
$$2(\beta - \alpha)^T \mathbf{x} + \alpha^T \alpha - \beta^T \beta$$

1.2   Answer

Your transition from the first line to the second is incorrect. We should have

$$\|x - \alpha\|^2 - \|x - \beta\|^2$$
$$(x - \alpha)^T (x - \alpha) - (x - \beta)^T (x - \beta)$$
$$= \|x\|^2 + \|\alpha\|^2 - \|x\|^2 - \|\beta\|^2 - x^T \alpha - \alpha^T x + x^T \beta + \beta^T x$$
$$= \|\alpha\|^2 - \|\beta\|^2 - 2\alpha^T x + 2\beta^T x$$
$$= \alpha^T \alpha - \beta^T \beta + 2(\beta - \alpha)^T x$$

which is the desired result.

1.3 Caution

1.3.1

For any vectors u, v, $u^T v = v^T u$.

1.3.2

$$(x - \alpha)^T (x - \alpha) = x^T x - x^T \alpha - \alpha^T x + \alpha^T \alpha$$

2 General answer of $\left(\mathbf{x}_n - \boldsymbol{\mu}_k\right)^\top \boldsymbol{\Lambda}_k \left(\mathbf{x}_n - \boldsymbol{\mu}_k\right)$

From StackExchange.

2.1 Question

Can we say this generally?

$$\left(\mathbf{x}_n - \boldsymbol{\mu}_k\right)^\top \boldsymbol{\Lambda}_k \left(\mathbf{x}_n - \boldsymbol{\mu}_k\right) = \mathbf{x}_n^\top \boldsymbol{\Lambda}_k \mathbf{x}_n - 2\boldsymbol{\mu}_k^\top \boldsymbol{\Lambda}_k \mathbf{x}_n + \boldsymbol{\mu}_k^\top \boldsymbol{\Lambda}_k \boldsymbol{\mu}_k$$

Or is this the case when $\mathbf{x}_n$ comes from normal distribution, $\mathcal{N}(\mathbf{x}_n | \boldsymbol{\mu}_k, \Lambda_k)$?

I'm a bit confused because I know following equation is generally correct.

$$\left(\mathbf{x}_n - \boldsymbol{\alpha}\right)^T \left(\mathbf{x}_n - \boldsymbol{\alpha}\right) = \mathbf{x}_n^T \mathbf{x}_n - \mathbf{x}_n^T \boldsymbol{\alpha} - \boldsymbol{\alpha}^T \mathbf{x}_n + \boldsymbol{\alpha}^T \boldsymbol{\alpha}$$

2.2 Answer

Your statement is true as long as the $\boldsymbol{\Lambda}_n$ matrix is symmetric.

Expand the product and you'll get:

$$\left(\mathbf{x}_n - \mu_n\right)^T \boldsymbol{\Lambda}_n \left(\mathbf{x}_n - \mu_n\right) = \mathbf{x}_n^T \boldsymbol{\Lambda}_n \mathbf{x}_n - \mathbf{x}_n^T \boldsymbol{\Lambda}_n \mu_n - \mu_n^T \boldsymbol{\Lambda}_n \mathbf{x}_n + \mu_n^T \boldsymbol{\Lambda}_n \mu_n$$

The cross terms are equal only if the lambda matrix is symmetric.