

Matrix and Vector

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1 Calculation of the squared Euclidean norm

From StackExchange.

1.1 Question

$$\begin{aligned}\|\mathbf{x} - \alpha\|^2 - \|\mathbf{x} - \beta\|^2 &= \|\mathbf{x}\|\|\mathbf{x}\| - 2\|\alpha\|\|\mathbf{x}\| + \|\alpha\|\|\alpha\| - \|\mathbf{x}\|\|\mathbf{x}\| + 2\|\beta\|\|\mathbf{x}\| - \|\beta\|\|\beta\| \\ &= \alpha^T \alpha - \beta^T \beta + 2(\sqrt{\beta \cdot \beta} - \sqrt{\alpha \cdot \alpha})\|\mathbf{x}\| \\ &= \alpha^T \alpha - \beta^T \beta + 2(\sqrt{\beta \cdot \beta} - \sqrt{\alpha \cdot \alpha})(\sqrt{\mathbf{x} \cdot \mathbf{x}}),\end{aligned}$$

where I used the fact that

$$\|a\|\|a\| = \sqrt{a \cdot a} \sqrt{a \cdot a} = \sqrt{a^T a} \sqrt{a^T a} = a^T a.$$

However, the article gives

$$2(\beta - \alpha)^T \mathbf{x} + \alpha^T \alpha - \beta^T \beta$$

1.2 Answer

Your transition from the first line to the second is incorrect. We should have

$$\begin{aligned}\|x - \alpha\|^2 - \|x - \beta\|^2 &= (x - \alpha)^T (x - \alpha) - (x - \beta)^T (x - \beta) \\ &= \|x\|^2 + \|\alpha\|^2 - \|x\|^2 - \|\beta\|^2 - x^T \alpha - \alpha^T x + x^T \beta + \beta^T x \\ &= \|\alpha\|^2 - \|\beta\|^2 - 2\alpha^T x + 2\beta^T x \\ &= \alpha^T \alpha - \beta^T \beta + 2(\beta - \alpha)^T x\end{aligned}$$

which is the desired result.

1.3 Caution

1.3.1

For any vectors u, v , $u^T v = v^T u$.

1.3.2

$$(x - \alpha)^T(x - \alpha) = x^T x + -x^T \alpha - \alpha^T x + \alpha^T \alpha$$