Matrix and Vector

Contents

1 Calculation of the squared Euclidean norm

From StackExchange.

1.1 Question

$$
\begin{aligned}
&\|\mathbf{x}-\alpha\|^2-\|\mathbf{x}-\beta\|^2\\
&=\|\mathbf{x}\|\|\mathbf{x}\|-2\|\alpha\|\|\mathbf{x}\|+\|\alpha\|\|\alpha\|-\|\mathbf{x}\|\|\mathbf{x}\|+2\|\beta\|\|\mathbf{x}\|-\|\beta\|\|\beta\|\\
&=\alpha^T\alpha-\beta^T\beta+2(\sqrt{\beta\cdot\beta}-\sqrt{\alpha\cdot\alpha})\|\mathbf{x}\|\\
&=\alpha^T\alpha-\beta^T\beta+2(\sqrt{\beta\cdot\beta}-\sqrt{\alpha\cdot\alpha})(\sqrt{\mathbf{x}\cdot\mathbf{x}}),
\end{aligned}
$$

where I used the fact that

$$\|a\|\|a\|=\sqrt{a\cdot a}\sqrt{a\cdot a}=\sqrt{a^Ta}\sqrt{a^Ta}=a^Ta.$$

However, the article gives

$$2(\beta-\alpha)^T\mathbf{x}+\alpha^T\alpha-\beta^T\beta$$

1.2 Answer

Your transition from the first line to the second is incorrect. We should have

$$
\begin{aligned}
&\|x-\alpha\|^2-\|x-\beta\|^2\\
&(x-\alpha)^T(x-\alpha)-(x-\beta)^T(x-\beta)\\
&=\|x\|^2+\|\alpha\|^2-\|x\|^2-\|\beta\|^2-x^T\alpha-\alpha^Tx+x^T\beta+\beta^Tx\\
&=\|\alpha\|^2-\|\beta\|^2-2\alpha^Tx+2\beta^Tx\\
&=\alpha^T\alpha-\beta^T\beta+2(\beta-\alpha)^Tx
\end{aligned}
$$

which is the desired result.

1.3 Caution

1.3.1

For any vectors u, v, $u^T v = v^T u$.

1.3.2

$$(x - \alpha)^T (x - \alpha) = x^T x - x^T \alpha - \alpha^T x + \alpha^T \alpha$$