Numerical Dispersion Compensation for FDTD via Deep Learning

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***Abstract*—A hybrid deep neural network consisting of convolutional layers and LSTM structures is developed for compensating for the numerical dispersion error in FDTD. The proposed network is designed for the wideband computation of S-parameters of planar microwave circuits at coarse meshes. Without dispersion compensation, the accuracy of these computations is severely compromised. We show that deep learning can restore the accuracy to the level obtained by dense mesh simulations.**

***Index Terms*—Finite-difference time-domain (FDTD) method, numerical dispersion error, deep neural network.**

I. Introduction

The remarkable advances in the available computational power over the past few years, and those anticipated to come, have propelled machine learning algorithms, some developed decades ago, to the forefront of research interest in a wide and diverse range of fields: from medicine to autonomous vehicles and robotics. As the interest in this area deepens, new algorithmic and theoretical developments are reported and additional applications are explored. This trend has been extended to computational science and engineering, with machine-learning algorithms designed to either improve [1] or even replace [2], [3] existing numerical solvers for linear and nonlinear problems.

In this work, we follow the former route to explore machine learning algorithms for numerical dispersion compensation in the Finite-Difference Time-Domain (FDTD) method. A hybrid deep neural network is trained with FDTD simulations of varying cell size, with the goal of "learning" the pattern of numerical dispersion errors by comparing solutions of various layouts of planar microwave circuits at coarse and dense grids. Hence, our training data include not only a wide collection of geometries, but also meshes of variable density for each problem. We present a thorough analysis of the structure of the proposed network and its error performance as a function of the training data. We evaluate its ability to act as a numerical dispersion compensation engine: one that can predict the result of an FDTD simulation in a dense mesh from the results of a coarse mesh simulation.

In terms of training, the network has two parts of data as input: first, a Convolutional Neural Network (CNN) extracts the global information from the layouts of circuits. Then, a Recurrent Neural Network (RNN), with Long Short Term Memory (LSTM) structure, analyzes the S-parameters from the coarse mesh simulation. The outputs from the CNN and RNN are used to infer the dense mesh simulation results through a Fully-Connected Neural Network (FCNN). We use a wide range of planar, two-port circuits, including filters and matching circuits to verify the performance of our proposed network.

II. Numerical Dispersion and Proposed Method

In the FDTD method, the centered difference approximations of the differential operators in Maxwell's equations introduce well known numerical dispersion errors [4]. Management of these errors requires that the Yee cell size be kept electrically small and typically smaller than one-tenth of a wavelength. This limitation has been addressed with various dispersion compensation methods [5] and with high-order methods.

In this paper, we compensate for the numerical dispersion errors in coarse mesh FDTD simulations, via a hybrid neural network with convolutional layers and LSTM structure. It is observed that the S-parameters of a coarse mesh simulation and dense mesh simulation share some common features. Since RNNs, especially with LSTM structure, have good performance in dealing with contextual sequence problems like speech recognition, language modeling and translation [6], we apply the LSTM networks in this work. Besides, a CNN is used to process the simulated geometries. The structure of this hybrid network is shown in Fig. 1. CNN extracts the features of the input geometries (metallization patterns printed on a dielectric substrate), along with the substrate height and permittivity. The RNN learns from the coarse mesh FDTD simulation. The outputs of the RNN and the CNN are used to infer the dense mesh simulation results from several FCNNs.

During the training process, we study the function and usefulness of the CNN through a comparative experiment. The same data set is trained twice, with CNN and without CNN. Fig. 2 shows how the network converges, as the training proceeds. From this comparison, we can see that the CNN can improve the performance of the network significantly.

III. Numerical Results

In this study, the FDTD method is applied to generate ground-truth data. The S-parameters are extracted by FFT as in [7]. For the dense mesh simulation, the number of cells per minimum wavelength (at 20 GHz) varies from 16.32 to 25.81. For the coarse mesh simulation, the number of cells

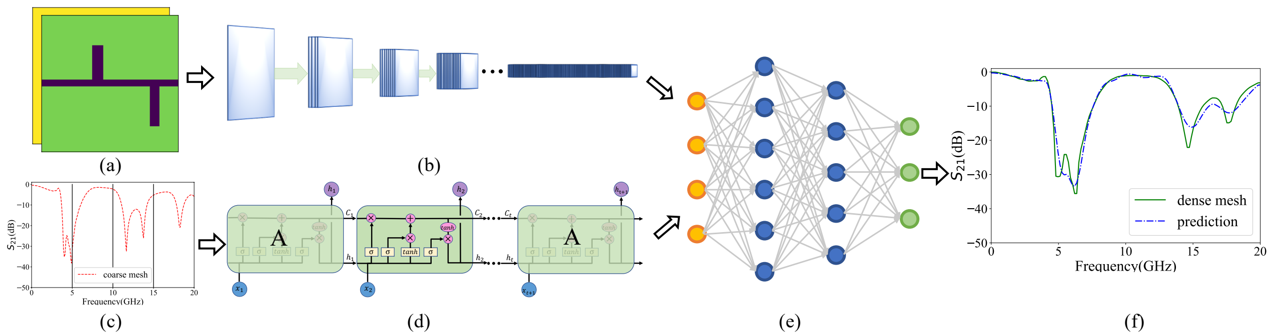

Fig. 1: The input, output and structure of the proposed hybrid network. (a) Input layout and substrate properties. (b) CNN. (c) Input: S-parameters computed by coarse meshsimulation. (d) RNN. (e) FCNN. (f) Output: dispersion compensated S-parameters.

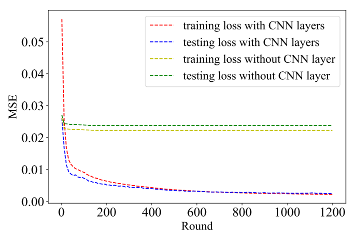

Fig. 2: The comparison of how the network converges, with and without CNN.

per minimum wavelength (also at 20 GHz) varies from 5.44 to 8.60. The thickness of the substrate varies from 0.05 to 0.1 of the minimum wavelength. Besides, the relative dielectric constant of the substrate varies from 2 to 5.

The results from the dense mesh simulation are used as ground truth in this work. The network inputs are layouts of microwave circuits and the coarse mesh simulation results. The geometries under test include various multi-stub filters, stepped filters and radial stubs. Each input layout is represented by two arrays, one for the metallization area on the substrate, and the other for the thickness. There are 10000 sets of data for each category. 80% of the data are used for training, and the other is used for testing. Fig. 3 shows several results from the testing set.

The Mean Square Error (MSE) is 0.00218 and 0.00230 for the training set and testing set, respectively. For the coarse mesh simulation, the MSE is 0.0313. Therefore, the trained network can decrease the numerical dispersion error by one order of magnitude, in terms of the MSE. The improvement is also evident in the plots of Fig. 3.

IV. CONCLUSION

We studied the feasibility of applying deep neural networks to compensate for the numerical dispersion error in FDTD simulations. To that end, a hybrid neural network with convolutional layers and LSTM structure was successfully used to compensate for the numerical dispersion error in coarse mesh FDTD simulations of planar, two-port circuits. This approach can be extended to general electromagnetic geometries.

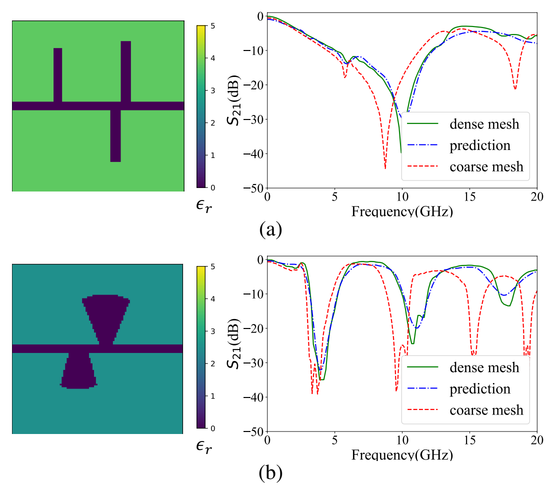

Fig. 3: Two cases from the testing set for the proposed FDTD dispersion compensation. The relative substrate thickness for them are (a) $h/\lambda = 0.1$, (b) $h/\lambda = 0.08$.

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