M4 All Required Formulas

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A) Complex Integration :

$$z = x + iy \quad , \quad z = re^{i\theta}$$

$$\bar{z} = x - iy = re^{-i\theta}$$

$$|z| = \sqrt{x^2 + y^2} = |x + iy| = r$$

Steps for Complex Integration :
1) Equation
2) Derivative
3) $f(z)$
4) dz
5) Integration.

Euler's Formula :

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

B) Cauchy's Theorem :

1) **Non Repeated Pole :**

pole

$$\int_C \frac{f(z)}{(z - z_0)^1} dz = 2\pi i \, f(z_0)$$

2) **Repeated Pole :**

pole

$$\int_C \frac{f(z)}{(z - z_0)^n} dz = \frac{2\pi i}{(n-1)!} f^{n-1}(z_0)$$

Circle

Standard Circle → only $|z|$

General Circle → $|z - \text{something}|$

$$\frac{d}{dx} f(x)^n = n\, f(x)^{n-1} \frac{d}{dx} f(x)$$

c) Residue :

1) Non Repeated Pole :

Residue at $\Big|_{z = z_0}$ = $\lim_{z \to z_0}$ ① Transfer multiply by $(z - z_0) \times F(z)$

2) Repeated Pole :

$$\text{Residue at } \Big|_{z = z_0} = \frac{1}{(n-1)!} \lim_{z \to z_0} \frac{d^{n-1}}{dz^{n-1}} (z - z_0)^n \times F(z)$$

3) Cauchy's Residue Theorem :

$$I = 2\pi i \left[\text{Sum of Residues} \right]$$

D) Taylor's and Laurents Series :

i) Standard Expansions :

1) $\sin x = +\dfrac{x^1}{1!} - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dots$

2) $\cos x = +\dfrac{x^0}{0!} - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dots$

3) $\sinh x = +\dfrac{x}{1!} + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \dots$

4) $\cosh x = 1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \dots$

5) $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \dfrac{x^5}{5!}$

6) $e^{-x} = 1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \dfrac{x^4}{4!} - \dfrac{x^5}{5!}$

ii) Binomial Expansion :

$-x-$ 1) $(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$

$+-$ 2) $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \dots$

3) $(1-x)^{-n} = 1 + nx + \dfrac{n(n+1)x^2}{2!} \dots$

4) $(1+x)^{-n} = 1 - nx + \dfrac{n(n+1)x^2}{2!} \dots$

E) Poisson Distribution :

1) $P(X=x) = \dfrac{e^{-m}\, m^{x}}{x!}$

2) Mean
Variance $\Big\}$ m OR (np)

Final Answer

Find Probability — $= p(x)$

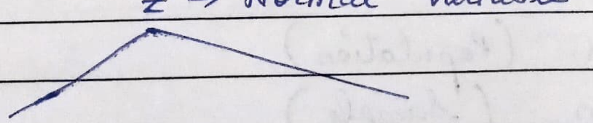
Find Number — $= N \times p(x)$
where N = large sample.

F) Normal Distribution :

μ = Mean = Expectation
σ = Standard Deviation.

- If variance is given then $\sigma = \sqrt{\text{variance}}$.

z → Normal Variable



$$z = \frac{x - \mu}{\sigma} \qquad\qquad z = \frac{x - \mu}{\sigma / \sqrt{n}}$$

use this formula when "Central Limit Theorem" is given.

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6) Sampling Theory:

i) Large Sample Test (n > 30)

1) Interval Estimation

Single Sample

$$\bar{x} \pm Z_\alpha \hat{\sigma}_x$$

$$\hat{\sigma}_x = \frac{\sigma}{\sqrt{n}}$$

Two Samples (Standard Error)

$$|\bar{x}_1 - \bar{x}_2| \pm Z_\alpha \cdot S.E$$

$$S.E. = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

2) Testing of Claims

Single Sample

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Two Sample

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{S.E.}$$

- μ = Mean (Population)
- $\bar{x}$ = Mean (Sample)
- σ = Standard Deviation $= \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$

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α: Flexibility / Tolerance / Level of Significance

$\alpha = 1\% = 2.58$

$\alpha = 5\% = 1.96$

$\alpha = 10\% = 1.64$

ii) Small Sample Test $(n < 30)$

1) Interval Estimation

Single Sample

$$\bar{x} \pm t \cdot \hat{\sigma}_x$$

$$\hat{\sigma}_x = \frac{\sigma}{\sqrt{n-1}}$$

Two Samples

$$|\bar{X}_1 - \bar{X}_2| \pm t \cdot S.E.$$

$$S.E. = Sp \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$Sp = \sqrt{\frac{n_1 \sigma_1^2 + n_2 \sigma_2^2}{n_1 + n_2 - 2}}$$

(standard proportion)

2) Testing of Claims

Single Sample

$$t = \frac{\bar{X} - \mu}{\sigma / \sqrt{n-1}}$$

Two Samples

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S.E.}$$

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3) Raw Data :

i) $\bar{X} = A + \frac{\Sigma d}{n}$ | A = Assumed Mean. d = X − A

ii) $\Sigma (x - \bar{x})^2 = \Sigma d^2 - \frac{(\Sigma d)^2}{n}$ [Sum of squares of deviation from mean.]

iii) $\sigma = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$

4) Degree of Freedom (choice)

Single Sample

$df = n - 1$

Two Samples

$df = n_1 + n_2 - 2$

iii) χ^2 − Test (Chi-square test)

Formula: $\chi^2 = \frac{\Sigma (O - E)^2}{E}$

where O: Observed Frequency
E: Expected Frequency.

Expected Frequency calculation for

Raw Data

$E = \frac{\Sigma f}{n}$

Tabular Data

$E = \frac{R_T \times C_T}{F_T}$

where R_T = Row Total, C_T = Column Total,
F_T = Final Total.

df

Raw Data	Tabular Data
$df = n-1$	$df = (r-1)(c-1)$

r = Row
c = Column.

4) **Z - Transform :**

Definition : If $f(k)$ be any sequence then,

$$Z\left\{f(k)\right\} = \sum_{k=-\infty}^{\infty} f(k)\, z^{-k} = F(Z)$$

- $1 + x + x^2 + x^3 + \dots = \dfrac{1}{1 - \text{second term}} = \dfrac{1}{1-x}$
  $= (1-x)^{-1}$
- $1 + \dfrac{x^1}{1!} + \dfrac{x^2}{2!} + \dots = e^{\text{second term}} = e^x$
- $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$ • $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$
- $\sinh\theta = \dfrac{e^{\theta} - e^{-\theta}}{2}$ • $\cosh\theta = \dfrac{e^{\theta} + e^{-\theta}}{2}$

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- **Standard Functions :**

1) $$Z\left\{\sin\alpha k\right\} = \frac{z\cdot\sin\alpha}{z^2 - 2z\cdot\cos\alpha + 1}$$

2) $$Z\left\{\cos\alpha k\right\} = \frac{z^2 - z\cdot\cos\alpha}{z^2 - 2z\cdot\cos\alpha + 1}$$

3) $$Z\left\{\sinh\alpha k\right\} = \frac{z\cdot\sinh\alpha}{z^2 - 2z\cdot\cosh\alpha + 1}$$

4) $$Z\left\{\cosh\alpha k\right\} = \frac{z^2 - z\cdot\cosh\alpha}{z^2 - 2z\cdot\cosh\alpha + 1}$$

5) $$Z\left\{U(k)\right\} = \frac{z}{z-1}$$

- **Compound Angles :**

1) $\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$
2) $\sinh(A+B) = \sinh A\cdot\cosh B + \cosh A\cdot\sinh B$
3) $\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$
4) $\cosh(A+B) = \cosh A\cdot\cosh B + \sinh A\cdot\sinh B.$

- **Properties :**

1) **Change of Scale Property :**

If $Z\left\{f(k)\right\} = F(z)$

then $Z\left\{c^k f(k)\right\} = F(z/c)$

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2) **Convolution :**

If $f(k)$ and $g(k)$ be the sequence -

$$Z\left\{ f(k) * g(k) \right\} = F(Z) * G(Z)$$

3) **Shifting Property :**

If $Z\left\{ f(k) \right\} = F(Z)$

then $Z\left\{ f(k+n) \right\} = Z^{n} \, Z\left\{ f(k) \right\}$

$$Z\left\{ f(k-n) \right\} = Z^{-n} \, Z\left\{ f(k) \right\}$$

4) **Multiplication by k :**

If $Z\left\{ f(k) \right\} = F(Z)$

then $Z\left\{ k \, f(k) \right\} = \left(-Z \frac{d}{dZ} \right) F(Z)$

$$Z\left\{ k^{2} f(k) \right\} = \left(-Z \frac{d}{dZ} \right)\left(-Z \frac{d}{dZ} \right) F(Z)$$

I) **Inverse Z - Transform :**

$$Z^{-1}\left\{ F(Z) \right\} = f(k)$$

$= \text{coeff of } Z^{-k}$

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J) Matrices :

- **Characteristic Equation :**

If 3 x 3 matrix,

$\lambda^3 - (\text{sum of diagonal elements})\lambda^2 + (\text{sum of minors of diagonal elements})\lambda - |A| = 0$

- **Diagonal Matrix :**

$D = M^{-1} A M$

M = Model Matrix | Transforming Matrix | Diagonalizing matrix.

$M = [X_1 \ X_2 \ X_3]$ where X_1, X_2, X_3 are the Eigen vectors.

- Matrix is diagonalizable if -
  1) All eigen values are different.
  2) If eigen values are repeated, then check for AM and GM. If AM = GM then matrix is diagonalizable.

AM = Algebraic Multiplicity.

GM = Geometric Multiplicity.

K) NLPP

1) **Optimization with One Equality :**

$$x_0$$

Minima	Maxima
$- \ - \ - \ -$	$+ \ - \ + \ -$
$\Delta_3 \ \Delta_4$	$\Delta_3 \ \Delta_4$

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Determinant

$$\Delta_{n+1} = \Delta_{3+1} = \Delta_4 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{cccc} & x_1 & x_2 & x_3 \\ \hline 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \\ \partial h/\partial x_1 & \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \frac{\partial^2 L}{\partial x_1 \partial x_3} \\ \partial h/\partial x_2 & \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \frac{\partial^2 L}{\partial x_2 \partial x_3} \\ \partial h/\partial x_3 & \frac{\partial^2 L}{\partial x_3 \partial x_1} & \frac{\partial^2 L}{\partial x_3 \partial x_2} & \frac{\partial^2 L}{\partial x_3^2} \end{array}$$

$\uparrow$ (no of variable)

4×4

2) Optimization with Two Equality:

x_0

Minima — +ve

Maxima — −ve

Bordered Hessian Matrix:

$$H^B_0 = \left[\begin{array}{c|c} 0 & P \\ \hline P' & Q \end{array}\right]$$

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(Null Matrix)

$$P = \begin{array}{c} \\ h_1 \\ h_2 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \frac{\partial h_1}{\partial x_3} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \frac{\partial h_2}{\partial x_3} \end{bmatrix} \end{array}$$

$$
\begin{array}{cccc}
 & x_1 & x_2 & x_3 \\
\end{array}
$$

$$
P = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}
\begin{bmatrix}
\frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \frac{\partial^2 L}{\partial x_1 \partial x_3} \\
\frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \frac{\partial^2 L}{\partial x_2 \partial x_3} \\
\frac{\partial^2 L}{\partial x_3 \partial x_1} & \frac{\partial^2 L}{\partial x_3 \partial x_2} & \frac{\partial^2 L}{\partial x_3^2}
\end{bmatrix}
$$

3) Kuhn - Tucker Conditions

1) $\frac{\partial L}{\partial x_1} = 0$

2) $\frac{\partial L}{\partial x_2} = 0$

3) $h(x_1, x_2) \leq 0$ ← Judge

4) $\lambda h(x_1, x_2) = 0$ ← Decision maker

5) $x_1, x_2 \geq 0$