MARKOV REGIME SWITCHING MODEL (MRS):

The Markov Regime switching Model (MRS) is a statistical model that allows for dynamic changes in the behaviour of a time series by switching between different regimes (states), where each regime has different statistical properties (mean, variance etc). It is widely used in finance and economics:

For example, in finance you have two Regime:

Regime 1 (Bull Market): High returns, low volatility.

Regime 2 (Bear Market): Low or negative returns, high volatility.

A Markov process determines the probability of switching between these regimes.

(2) Mathematical Representation:

Let Y_t be the observed time series (eg. stock returns), and S_t be the unobservable regime (state) at time t. The model is:

$$\boxed{Y_t = \mu_{S_t} + \sigma_{S_t} * \epsilon_t}$$

where:

$S_t \rightarrow$ Regime (state) at time t, which follows a Markov chain.

$\mu_{S_t} \rightarrow$ Mean return in regime S_t.

$\sigma_{S_t} \rightarrow$ Volatility in regime S_t.

$\epsilon_t \sim N(0,1)$ is a standard normal shock.

(3) Markov Transition Matrix:

The probability of switching between regimes is governed by a transition matrix P.

$$\boxed{P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}}$$

where,

P_{11} = Probability of staying in Regime 1.

P_{12} = Probability of switching from Regime 1 to Regime 2.

P_{21} = Probability of switching from Regime 2 to Regime 1.

P_{22} = Probability of staying in Regime 2.

Each row must sum to 1:

$$P_{11} + P_{12} = 1 \quad , \quad P_{21} + P_{22} = 1$$

Example:-

A stock follows two regimes with different return behaviours:

(1). Bull Market (Regime 1):

Mean Return = 5%

Standard Deviation = 2%

(2) Bear Market (Regime 2)

Mean Return = -2%

Standard Deviation = 3%

The markov transition probability matrix is $P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$

$$\pi_1 = 1 - \pi_2.$$

$$= 1 - 0.4286.$$

$$\boxed{\pi_1 = 0.5714}$$

Interpretation of data:-

Thus, in the long run:

* 57.14% of the time, the stock is in a bull market.
* 42.86% of the time, the stock is in a bear market.

(2) Simulate a 5-Period Market Model:

Given:-

Regime 1 (Bull Market) : $\mu_1 = 5\%$, $\sigma_1 = 2\%$

Regime 2 (Bear Market) : $\mu_2 = -2\%$, $\sigma_2 = 3\%$.

$\epsilon_t = (-0.5, 1.2, -1.8, 0.9, -0.7)$

Now, compute the returns:

t	S_t (Regime)	shock ϵ_t.	$Y_t = \mu_{S_t} + \sigma_{S_t}\epsilon_t$
1	1 (Start in Bull)	-0.5	$5+2(0.5) = 4\%$
2.	1 (Stays in Bull)	1.2	$5+2(1.2) = 7.4\%$
3.	2 (Shift to Bear)	-1.8	$-2+3(-1.8) = -7.4\%$
4.	1 (Shift to Bull)	0.9	$5+2(0.9) = 6.8\%$
5.	1 (Stay in ~~Bear~~ Bull)	-0.7	$5+2(-0.7) = 3.6\%$

↳ Can be assumed.

The stock starts in Bull state and gives 4% return.

Semester : VIII | Subject : AIFB | Academic Year: 2024-25

Find the following :-

(1) Compute steady-state probabilities for each regime.

(2) Simulate a 5-period sequence of stock returns mathematically. Assume shocks ϵ_t are : (-0.5, 1.2, -1.8, 0.9, -0.7

Solution:

Step 1: Compute steady-state Probabilities:

The steady state probabilities (π_1, π_2) satisfy:

$$\pi_1 = 0.7\pi_1 + 0.4\pi_2 \quad -①$$
$$\pi_2 = 0.3\pi_1 + 0.6\pi_2 \quad -②$$

From the transition matrix, coloumnwise data

Since $\pi_1 + \pi_2 = 1$, solve:

From ①.

$$\pi_1(1-0.7) = 0.4\pi_2.$$

$$\pi_1(0.3) = 0.4\pi_2. \qquad [\pi_1 = 1-\pi_2].$$

$$(1-\pi_2)(0.3) = 0.4\pi_2.$$

$$0.3 - 0.3\pi_2 = 0.4\pi_2.$$

$$0.3 = 0.4\pi_2 + 0.3\pi_2 = 0.7\pi_2$$

$$\pi_2 = \frac{0.3}{0.7}$$

$$\boxed{\pi_2 = 0.4286}$$

When it stays in bull state there is 7.4% of return. Then when it shifts to bear state it gives negative return of -7.4%.
Again when it shifts to Bull state it gives a return of 6.8%.
When it stays in Bull state (5th transition) with a shock of -0.7, it gives 3.6% return

This is Markov Regime Switching Model (MRS) which allows us to understand the dynamic changes in the behaviour by switching between different regimes.

BAYESIAN REASONING:-

Bayesian reasoning is a probabilistic approach to inference, where we update our beleifs based on new evidence using Baye's Theorem. Bayesian reasoning is widely used in finance for risk assesment, portfolio optimization, asset pricing, and fraud detection. It allows investors and analysts to update their beleifs about financial markets as new information becomes available.