D.H.B

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Nash Equilibrium

Hermione

	Left	Right
Up	4 8	7 4
Down	6 3	9 2

Mr. Darcy

Liz	Left	Right
Up	5 9	2 6
Down	3 5	4 7

Nash eqbm: "No regrets",
Best Response to best response.

BOS Game :- Battle of Sexes.

Wife

Husband	B	O
B	2,1	0 0
O	0 0	1,2

Prisoner's Dilemma.

	cooperate	defect
cooperate	1 1	3 0
defect	0 3	2 2

Stag hunt :-

1) Two hunters go out to catch meat
2) There are two rabbits in the range and one stag. The hunters can each bring the equipment necessary to catch one type of animal.
3) The stag has more meat than the rabbits combined, but both hunters must chase the stag to catch it.
4) Rabbit hunters can catch all of their prey by themselves.

Player 2

		Stag	Rabbit
player1	Stag	3 , 3	0 , 2
	Rabbit	2 , 0	1 , 1

NE :- A game is said to be in Nash Equilibrium when no player has incentive to deviate from his strategy all other players are playing
→ that is, everyone is playing their own "best response". to what all others are doing
* Atleast one Nash Equilibrium exists in all finite game.

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Matching Pennies.

		Player 1 Heads	Player 1 Tails
Player 2	Heads	3 , −3	−2 , 2
	Tails	−2 , 2	1 , −1

20 rounds.

Mixed Strategy Nash Equilibrium

1) If no equilibrium exists in pure strategies one, must exist in mixed strategies.
2) A mixed strategies is a probability distribution over two or more pure strategies
   - That is the players choose randomly among their options in equilibrium.
   - In equilibrium, each player's probability distribution makes all other's indifferent between their pure strategies.

Mixed Strategy Nash Equilibrium

Prob.	Person1 ↓ / Person2 →	R (Prob q)	L $(1-q)$
1	R	0, 1	3, 0
$(1-p)$	L	4, 0	0, 2

Player's 1's choice of p:

$\pi_2(R) \rightarrow \cancel{(1,0)}\ p^*(1) + (1-p)^*(0) = p$

$\pi_2(L) \rightarrow p(0) + (1-p)(2) = 2-2p$

$p = 2-2p$

$3p = 2$

$p = \frac{2}{3}$

Player's 2's choice of q :-

$\pi_1(R) \rightarrow q(0) + (1-q)(3) = 3-3q$

$\pi_1(L) \rightarrow q(4) + (1-q)(0) = 4q$

$3-3q = 4q$

$3 = 7q \quad \therefore \boxed{q = \frac{3}{7}}$

Cournot Model of oligopoly

↳ Augustine Cournot → French Researcher economist

Book → Researches into the Mathematical Principles of the Theory of Wealth.

→ Micro economics

→ Cournot competition / duopoly.

Imperfect market, two firms compete with each other and their product is identical and cost funcⁿ is also identical.

Cartell → X

Two firms are different and will compete with each other.

→ Sale mineral water, ~~the~~ it is homogeneous. i.e same product.

Assumptions:-

i) Two firms and then economist generalized it to many or few.

ii) Homogeneous (mineral water)

iii) Linear Demand Curve.

iv) For sake of simplicity, cost of production is zero

v) Cost and technology both are identical.

Sale < Demand then Profit is high.

* Cournot said that o/p is an adjusting variable. producer adjust his o/p believing that the other continue to produce same output as he is doing
* Base is production.

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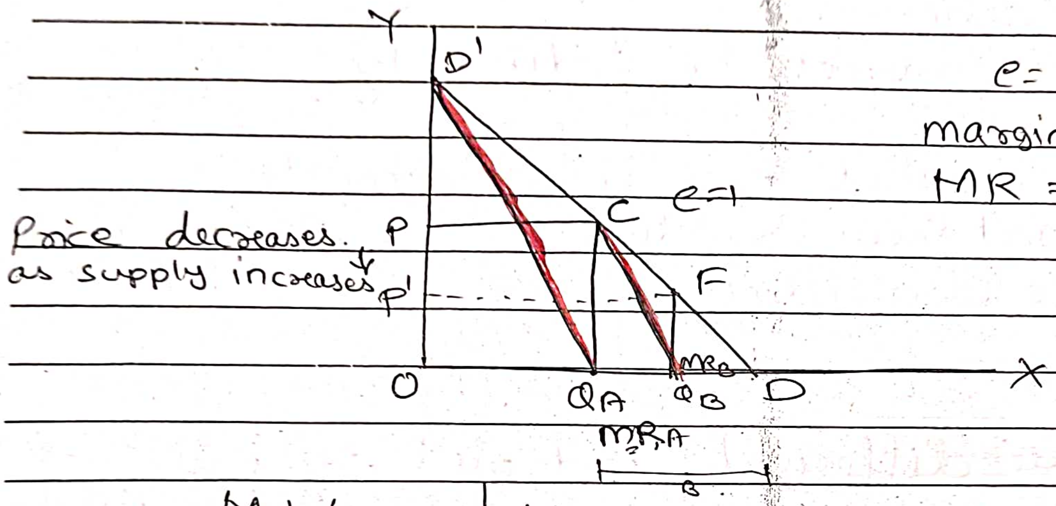
vi) Each firm chooses its output to maximize its own profit and takes the o/p of the rival firm as given (assume)

vii) They can only change prodⁿ and not change price of product as market decides it.



e = elasticity unit.
marginal revenue = cost
MR = MC

MW_A	MW_B
$1 \times \frac{1}{2} = \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
$\frac{1}{2}(1-\frac{1}{4}) = \frac{3}{8}$	$\frac{1}{2}(1-\frac{3}{8}) = \frac{5}{16}$
$\frac{1}{2}(1-\frac{5}{16}) = \frac{11}{32}$	$\frac{1}{2}(1-\frac{11}{32}) = \frac{21}{64}$
$\frac{1}{2} \times (1-\frac{21}{64}) = \frac{43}{128}$	$\frac{1}{2}(1-\frac{43}{128}) = \frac{85}{256}$
$\frac{1}{3}$	$\frac{1}{3}$

No. of firms = share + 1

$$\frac{Q}{n+1}$$

Q = market size.
n = no. of firms.
Q = 1, if n = 5 then $\frac{1}{5+1}$

$\frac{1}{3}$ } open and

Demand will remain in market price will start incr.

...ack to the
do not produce any output and then sell his product whatever price it can brings too. (9)
... says that first producer will fix the price and then decide the production based on that price. (how much to produce)

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Bertrand Duopoly model of oligopoly

↳ Joseph Louis Francis Bertrand
→ 1883, critized Cournot's model.
→ Bertrand Paradox.
→ Strong competition, 2 firms, (multiple firms)

* adjusting variable is price and not the o/p production

* We do not want to know the entire market.
* Price can only be reduced upto some limit. i.e $P = MC$ (Marginal Cost) otherwise it will be a loss.

Assumptions :-
i) Two firms
ii) Homogeneous product (perfect substitute)
iii) Technology is same for Production
iv) Identical cost (Both MC and Average cost)
v) Set price simultaneously to maximize profit.
vi) Unlimited productive capacity.

P = Price

$P_1 = f(P_2)$
Q = Market Quantity (full) $= Q_1 + Q_2$

$P_1 < P_2$ ...... $Q_1 = Q$ (since price of P_1 is less than Firm 2).
$Q_2 = 0$

if $P_1 > P_2$ ..... $Q_1 = 0$ & $Q_2 = Q_1$ (since price is more)

$P_1 = P_2$ ..... $Q_1 = \frac{Q}{2}$, $Q_2 = \frac{Q}{2}$

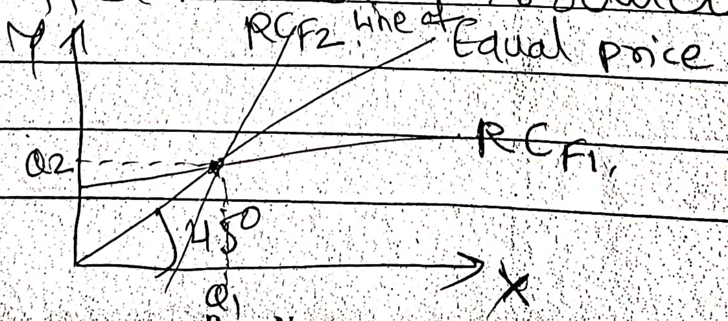
This is same as Nash Equilibrium.
Price can be reduced upto MC.

$Q = Q_1 + Q_2$ $\qquad$ $MC = 10$.
$P = 60 - 2Q$ $\qquad$ $AC = 10$.
$10 = 60 - 2Q$ $\qquad$ $P = 10$.
$2Q = 60 - 10$ $\therefore 2Q = 50$
$\therefore Q = 25$.

$P_1 = P_2$. $\frac{Q}{2} = \frac{25}{2} = 12.5$

$\pi = Q(P - ATC)$ $\qquad$ ATC - Average Tota
$= 25(10 - 10)$
$= 25(0) = \underline{0}$

Differentiated Product.



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Solving sum:-

Bertrand Competition.
- 2 firms.
- Identical goods.

Market Demand.
$Q = 15 - 3P$
$MC = 2$ $\quad P = MC$

$P = MC$.
$\therefore P = 2$.
$Q = 15 - 3(2) = 15 - 6 = 9$
$\therefore Q = 9$.

$MC_1 = 10$ $\quad MC_2 = 20$.
$P_1 = 10$ $\quad P_2 = 20$
$\therefore Q_1 = Q$ $\quad Q_2 = 0$
$\pi_1 = 0$.

Firm 1 can take entire market if
$P = 19.99 < MC_2$
then $Q_1 = Q$ $\quad Q_2 = 0$
$19.99 = 60 - 2Q$
$2Q = 40.01$
$Q = 20.005$
$\pi_1 = 20.005(19.99 - ATC) = 20.005(19.99 - 10)$.
$= 199.85$
$\pi_2 = 0$.

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* Simple First-Price Auction :-

Person 1 : $28
Person 2 : $24

	Bid $10	Bid $20
Bid $10	$9, $7	$0, $4
Bid $20	$8, $0	$4, $2

Similar to stag hunt.

Assumptions :-

1. Bidder's secretly submit one bid.
2. Bid's must be in $10 increments.
3. Bid's cannot be $0.
4. Highest bidder wins and pays their bid.
5. If both bid same price then they have 50-50 probability of winning.

We should not bid more than 20 $ bcoz players think the actual price is 24 $ & $28 so, lower end $20. If $30 is bid then they have to pay extra i.e negative pay-offs.

If you bid \$20 and opponent bids \$10 then you win the bid and get a profit of \$8

If you bid \$10 and opponent bid \$20 then opponent wins the bid and get a profit of \$4

In other two cases, it is a little complicated as they result in tie. In such a case you the auction awards the books to both of you with 50% probability.

If both of you bid \$10, then you receive \$18 as profit and opponent receives \$14 as profit. But 50% ~~profit~~ probability so profit will also be reduced to 50%. So you will receive \$9 and opponent will get \$7

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Electoral Competition

In this example,
We will be trying to look at how people vote, how no. of candidates will be decided, what are the agenda said by the candidates if they want to win the election or if they have some ideological persuasions. So, what will be the agenda that will be said by the candidates who will win in a particular equilibrium situation, so these are some of the points and if suppose there are some costs involved in standing in the election; if you want to be a candidate you have to bare some cost, then does it hamper the election process thus the no. of people who are running for the election does that go down. So there are these important issues which we want to address in a very elementary manner. So the framework that we have is the following :-

There is a continuum of numbers. Each number represents the favorite position of at least one vote. So, I have this real line, for example, in this real line if I picked up any point then

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this point represents a number. This number will be the favourite position of atleast one voter. We can imagine that each point on this real line is basically corresponding to one voter. There may be multiple voters, but atleast one voter will be there. When we say favourite position, we mean that the preference, the political preferences of the voters can be represented in a uni-dimensional scale, this is an simplified assumption. Becoz a person's political preference might be multi-dimensional having 2 or 3 or more set of points or numbers. And these points will be the representative of the voter. When we talk about political issues we talk about leftist and rightist. So at back of our mind we have a uni-dimensional scale, some persons is preferring some policy which are to the left which means may be in this line you are going to this direction, if we ~~are preferring~~ say rightist then that point will lie on right.

* Strategic games in which players may randomize.

→ Each player is given VNM preferences about lotteries over the set of action profiles.

→ A strategic game consists of
a) a set of players
b) for each player, a set of actions
c) for each player preferences regarding lotteries over action profiles that may be represented by the expected value of a payoff funcⁿ over action profiles.

A two-player strategic game with VNM preferences in which each player has finitely many actions may be presented. The table looks same as it did before, though the interpretation of the nos. in the boxes is different. Previously these values were the payoff funcⁿs that represent the player's preference over deterministic outcomes. Here they are the values of payoff funcⁿ whose expected values represent

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the player's preferences over lotteries.
Eg. Prisoner's Dilemma
Best outcome → she chooses F and other player chooses Q then
worst outcome → she chooses Q and other player chooses F
(Q,Q) and (F,F)

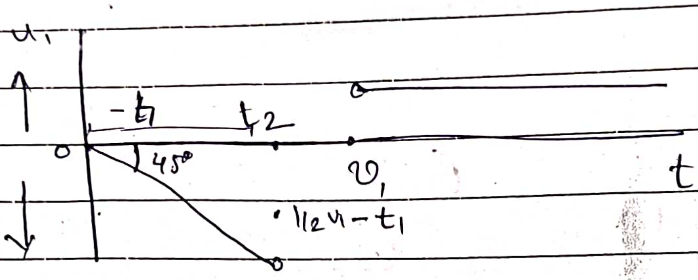
	Q	F
Q	2,2	0,3
F	3,0	1,1

	Q	F
Q	3,3	0,4
F	4,0	1,1

* Mixed Strategy Nash Equilibrium. (pg 105)
* Dominated Actions
(pg 117)

War of Attrition :-

if $t_2 < v_1$



Let $v_1 = 10$

& $t_2 = 8$

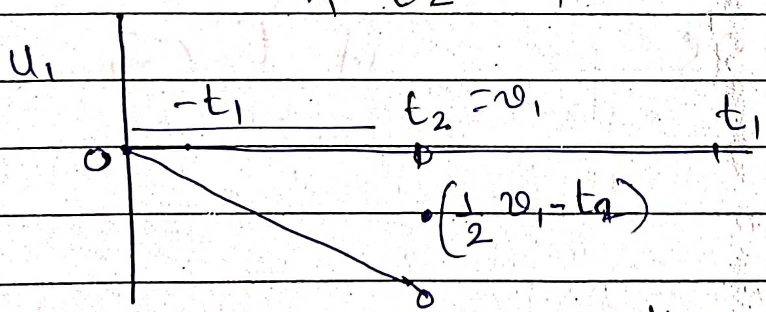
$u_1 = \frac{10}{2} - 8 = 5-8$

$= -3$

$v_1 \leq 2t_2$ so we get 0 payoff.
and if it is greater then we get positive pay
$\Rightarrow t_1 > t_2$.

i.e. if $t_2 < v_1$ & $v_1 \leq 2t_2$
then $t_1 > t_2$

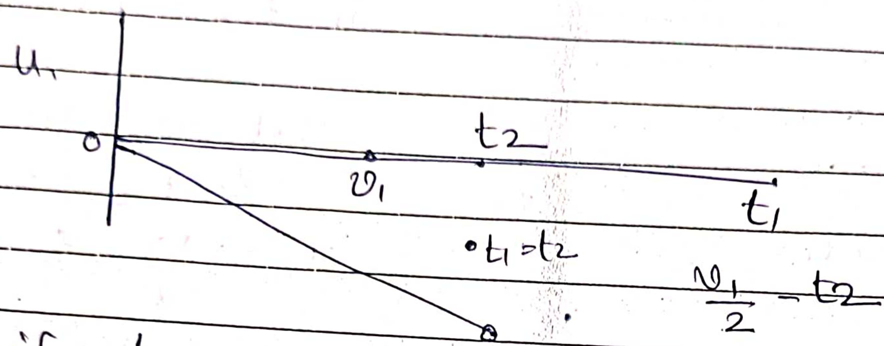
* if $t_2 = v_1$



$u_1 = v_1 - t_2 = 0$.

if $t_2 = v_1$
either $t_1 = 0$ or $t_1 > t_2$

if $t_2 > v_1$



if $t_1 > t_2$, then he will get negative payoff
if $t_2 > v_1$
then $t_1 = 0$

Best Response function

$$B_1(t_2) = \begin{cases} \{t_1 : t_1 > t_2\} & \text{if } t_2 < v_1 \\ \{t_1 : t_1 = 0 \text{ or } t_1 > t_2\} & \text{if } t_2 = v_1 \\ \{t_1 : t_1 = 0\} & \text{if } t_2 > v_1 \end{cases}$$