	$0°$	$30°(\pi/6)$	$45°(\pi/4)$	$60°(\pi/3)$	$90°(\pi/2)$	$180°(\pi)$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\sin\theta = \cos(90-\theta)$

$\cos\theta = \sin(90-\theta)$

$\tan\theta = \cot(90-\theta)$

$\cot\theta = \tan(90-\theta)$

$\sec\theta = \operatorname{cosec}(90-\theta)$

$\operatorname{cosec}\theta = \sec(90-\theta)$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

Differentiation	Integration
① $\frac{d}{dx}(x) = 1$	$\int 1\,dx = x + c$
② $\frac{d}{dx}(ax) = a$	$\int a\,dx = ax + c$
③ $\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
④ $\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\,dx = -\cos x + c$

(5) $\frac{d}{dx}(\sin x) = \cos x$ $\qquad$ $\int \cos x \, dx = \sin x + C$

(6) $\frac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\int \sec^2 x \, dx = \tan x + C$

(7) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ $\qquad$ $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

(8) $\frac{d}{dx}(\sec x) = \sec x \tan x$ $\qquad$ $\int \sec x (\tan x) \, dx = \sec x + C$

(9) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$ $\qquad$ $\int \operatorname{cosec} x \cdot \cot x \, dx = -\operatorname{cosec} x + C$

(10) $\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\qquad$ $\int \frac{1}{x} dx = \ln|x| + C$

(11) $\frac{d}{dx}(e^x) = e^x$ $\qquad$ $\int e^x dx = e^x + C$

(12) $\frac{d}{dx}(a^x) = (\ln a)\, a^x$ $\qquad$ $\int a^x dx = \frac{a^x}{\ln a} + C$

(13) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1} x + C$

(14) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ $\qquad$ $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

(15) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ $\qquad$ $\int \frac{1}{|x|\sqrt{x^2-1}} = \sec^{-1} x + C$

---

(16)

$$\int e^{ax} \sin bx \, dx = e^{ax}\left[\frac{a \sin bx - b \cos bx}{a^2+b^2}\right] + C$$

$$\int e^{ax} \cos bx \, dx = e^{ax}\left[\frac{a \cos bx + b \sin bx}{a^2+b^2}\right] + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \sin^{-1}\frac{x}{a}\right] + C$$

$$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right| + C$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$$

$$\cancel{\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C}$$

$$\cancel{\int \frac{1}{x^2+a^2}}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$$

*

$$\int e^x\left[f(x)+f'(x)\right]dx = f(x)e^x + C$$

$\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

$\sin(A+B) - \sin(A-B) = 2\cos A \sin B$

$\cos(A+B) + \cos(A-B) = 2\cos A \cos B$

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$\sin 2x = 2\sin x \cos x \cancel{= 1+\cos^2 \cos^2 x - \sin^2 x}$

$\cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1 = \cos^2 x - \sin^2 x$

$$\tan m = \frac{2\tan x}{1-\tan^2 x}$$

STATISTICS

(1) $$\sigma_y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$$

$$cov(x,y) = \frac{1}{n}\Sigma(x-\bar{x})(y-\bar{y})$$

$$r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y}$$

(2) Karl Pearson's Coefficient of Correlation (r) i.e.

$$r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} = \frac{E(XY) - E(X).E(Y)}{\sqrt{E(X^2) - [E(X)]^2}\sqrt{E(Y^2) - [E(Y)]^2}}$$

$$= \frac{\frac{1}{n}\Sigma x_i y_i - \frac{1}{n}\Sigma x_i . \frac{1}{n}\Sigma y_i}{\sqrt{\frac{1}{n}\Sigma x_i^2 - \left(\frac{1}{n}\Sigma x_i\right)^2}\sqrt{\frac{1}{n}\Sigma y_i^2 - \left(\frac{1}{n}\Sigma y_i\right)^2}}$$

→ r always lies b/w -1 and 1
→ $r=1$ then its perfect correlation
→ $r>0$ then its +ve correlation / Direct Correlation
→ $r<0$ then its -ve correlation / Ind ~~d~~ verse Correlation
→ $r=0$ → no correlation.

(3) Spearman's Rank Correlation / Rank Correlation (ρ)

(1) If ranks aren't repeated:

$$\rho = 1 - \frac{6\Sigma d_i^2}{n(n^2-1)}$$

(ii) If ranks are repeated,

$$\rho = 1 - \frac{6\left[\Sigma d_i^2 + \frac{1}{12}\left(m_1(m_1^2-1) + m_2(m_2^2-1)\right) + \dots\right]}{n(n^2-1)}$$

④ Regression

(i) Regression line of Y on X:

$$Y - \bar{Y} = b_{yx}(X - \bar{X})$$

$$Y - \bar{Y} = r \cdot \frac{\sigma_y}{\sigma_x}(X - \bar{X})$$

$$\sigma_x = \sqrt{E(x^2) - [E(x)]^2}$$

$$\sigma_y = \sqrt{E(y^2) - [E(y)]^2}$$

(ii) Regression line of X on Y:

$$X - \bar{X} = b_{xy}(Y - \bar{Y})$$

$$X - \bar{X} = r \cdot \frac{\sigma_x}{\sigma_y}(Y - \bar{Y})$$

→ $b_{xy} * b_{yx} = r^2$ ⇒ b_{xy} & b_{yx} should have same sign.

→ $b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}$ ⇒ r & b_{xy} should have same sign.

→ S.D (σ) = $\sqrt{\text{variance}}$

⑤ Fitting of straight line / First Degree Curve

$$y = ax + b$$

$$\Sigma y = a\Sigma x + b \cdot n$$

$$\Sigma xy = a\Sigma x^2 + b\Sigma x$$

2 eqn

$X = x - \bar{x}$

when years are given.

⑥ Fitting of Parabola / Second Degree Curve.

$y = ax^2 + bx + c$ — 3 eqns

$\Sigma y = a\Sigma x^2 + b\Sigma x + c \cdot n$

$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$

$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$

Mean
$\bar{x} = \frac{\Sigma x_i}{n}$

LAPLACE THEOREM

① Generally, $\int_0^\infty e^{-st} \cdot f(t) \cdot dt = L[f(t)]$

② $\int uv = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots$

③ $\int e^{ax} \cos bx \, dx = e^{ax}\left[\frac{a\cos bx + b\sin bx}{a^2+b^2}\right]$

$\int e^{ax} \sin bx \, dx = e^{ax}\left[\frac{a\sin bx - b\cos bx}{a^2+b^2}\right]$

④ $L[e^{at}] = \frac{1}{s-a}$

⑤ $L[e^{-at}] = \frac{1}{s+a}$

⑥ $L[1] = \frac{1}{s}$

⑦ $L[\sin at] = \frac{a}{s^2+a^2}$

⑧ $L[\cos at] = \frac{s}{s^2+a^2}$

⑨ $L[\sinh at] = \frac{a}{s^2-a^2}$

⑩ $L[\cosh at] = \frac{s}{s^2-a^2}$

★ ⑪ $L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$

⑫ $L[\mathrm{erf}(\sqrt{t})] = \frac{1}{s\sqrt{s+1}}$

⑬ $L[k] = \frac{k}{s}$

⑭ $L[c^{at}] = \frac{1}{s - a\log c}$

⑮ $L[c^{-at}] = \frac{1}{s + a\log c}$

(16) Pascal's Triangle

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \longrightarrow (a+b)^0 \\
 & & & & 1 & & 1 & & & & \longrightarrow (a+b)^1 \\
 & & & 1 & & 2 & & 1 & & & \longrightarrow (a+b)^2 \\
 & & 1 & & 3 & & 3 & & 1 & & \longrightarrow (a+b)^3 \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \longrightarrow (a+b)^4 \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 \longrightarrow (a+b)^5
\end{array}
$$

$$(a \pm b)^n = a^n \pm {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 \pm \ldots b^n$$

(17) $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ $\qquad \sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

(18) $\cosh\theta = \dfrac{e^{\theta} + e^{-\theta}}{2}$ $\qquad \sinh\theta = \dfrac{e^{\theta} - e^{-\theta}}{2}$

(19) $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$

(20) $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \ldots$ (odd)

(21) $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \ldots$ (even)

(22) Change of Scale Property.

$$L[f(at)] = \frac{1}{a}\,\phi\left(\frac{s}{a}\right)$$

(23) First shifting Property.

$$L[e^{-at} f(t)] = (s+a)$$

$$L[e^{at} f(t)] = (s-a)$$

(24) Second Shifting Theorem

$$L[f(t)] = e^{-as}\phi(s)$$

(25) Effect of Multiplication by t:

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \cdot \phi(s)$$

(26) Effect of Division by t:

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty \phi(s).ds$$

(27) Laplace Transform of Integration:

$$L\left[\int_0^t f(u).du\right] = \frac{1}{s}\phi(s)$$

$$L\left[\int_0^t\int_0^t \dots \int_0^t f(u)\,(du)^n\right] = \frac{1}{s^n}\phi(s).$$

(28) Evaluation of Integration

To evaluate $\int_0^\infty e^{-at}.f(t).dt$

(i) First calculate $L[f(t)] = \phi(s)$

(ii) Put the value of $L[f(t)] = \int_0^\infty e^{-st}.f(t)dt = \phi(s)$

(iii) $\therefore \int_0^\infty e^{-st}.f(t).dt = \phi(s)$

then put $s = a$

$$\int_0^\infty e^{-at} f(t)dt = \phi(a).$$

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INVERSE LAPLACE THEOREM.

1. $L^{-1}\left(\frac{1}{s}\right) = 1$
2. $L^{-1}\left[\frac{1}{s-a}\right] = e^{at}$
3. $L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$
4. $L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{\sin at}{a}$
5. $L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$
6. $L^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{\sinh at}{a}$
7. $L^{-1}\left[\frac{s}{s^2-a^2}\right] = \cosh at$
8. $L^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{\Gamma n}$
9. Using first shifting Theorem,

$$L^{-1}[\phi(s+a)] = e^{-at} L^{-1}[\phi(s)]$$

$$L^{-1}[\phi(s-a)] = e^{at} L^{-1}[\phi(s)]$$

10. Partial Fraction

$$\phi(s) = \frac{F(s)}{(s+a)(s+b)} = \frac{A}{s+a} + \frac{B}{s+b}$$

$$\phi(s) = \frac{F(s)}{(s+a)(s+b)^2} = \frac{A}{s+a} + \cancel{\frac{Bs+C}{(s+b)^2}} \; \frac{B}{s+b} + \frac{C}{(s+b)^2}$$

$$\phi(s) = \frac{F(s)}{(s+a)(s^2+b^2)} = \frac{A}{s+a} + \frac{Bs+C}{s^2+b^2}$$

11. Convolution Theorem,

$L[f(t)] = \phi(s)$ $\quad$ $L[g(t)] = \psi(s)$

$\therefore f(t) = L^{-1}[\phi(s)]$ $\quad$ $g(t) = L^{-1}[\psi(s)]$

$$L^{-1}[\phi(s)\cdot\psi(s)] = \int_0^t f(u)\cdot g(t-u)\,du$$

(12) Use of Differentiation.

$$L^{-1}[\phi(s)] = \frac{-1}{t} L^{-1}[\phi'(s)]$$

$$\because L[t.f(t)] = -\phi'(s)$$

$$\therefore L^{-1}[\phi'(s)] = \frac{-1}{t} f(t) = \frac{-1}{t} L^{-1}[\phi(s)].$$

NOTE : (i) Use this property only for these funt^n where ILT of $\phi(s)$ isn't easy to find but ILT of $\phi'(s)$ is easy to find

(ii) Generally we use above for logarithmic & Inverse function [log, $\tan^{-1}$, $\cot^{-1}$, etc].

FOURIER SERIES.

(1) If n is an integer then,

(i) $\cos n\pi = (-1)^n$ (ii) $\cos 2n\pi = 1$

$\sin n\pi = 0$ $\sin 2n\pi = 0$

(iii) $\cos(n\pm1)\pi = \cos n\pi \cos\pi \mp \sin n\pi . \sin\pi = -\cos n\pi$

(iv) $\sin(n\pm1)\pi = \sin n\pi \cos\pi \pm \cos n\pi \sin\pi = 0$

(v) $\cos(2n+1)\frac{\pi}{2} = 0$ (vi) $\sin(2n+1)\frac{\pi}{2} = (-1)^n$

(2) Let m, n be any integers, $m, n \neq 0$ then.

Case I $m \neq n$

$$\int_0^{c+2\pi} \cos mx . \cos nx \, dx = 0$$

$$\int_0^{2\pi} \sin mx . \sin nx \, dx = 0$$

$$\int_c^{c+2\pi} \sin mx . \cos nx \, dx = 0 \quad , \forall m, n$$

Case II $m=n$

$$\int_c^{c+2\pi} \cos mx . \cos nx \, dx = \int_c^{c+2\pi} \cos^2 mx \, dx = \pi$$

$$\int_c^{c+2\pi} \sin mx . \sin nx \, dx = \int_c^{c+2\pi} \sin^2 mx \, dx = \pi$$

Case III $\int_0^{c+2\pi} \cos mx \, dx = 0$ & $\int_0^{c+2\pi} \sin mx \, dx = 0$

(3) A function $f(x)$ is even when $f(-x) = f(x)$
eg: $\cos x, \sec x, x^2, x^4$, etc

(4) A function $f(x)$ is odd when $f(-x) = -f(x)$
eg: $\sin x$, $\operatorname{cosec} x$, x, x^3, etc.

(5) A function can neither be odd nor even
eg:- e^x, 10^x, $x^2 - x$, etc.

(6) $\int_{-a}^{a} f(x) dx = 2\int_0^a f(x) dx$ — f is even

$= 0$ — f is odd

(7) Product of even functions = even

(8) Product of odd functions = even

(9) Product of odd & even = odd

(10) $(c, c+2l)$ ~~$f(x) = \frac{a_0}{2} +$~~

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

where $a_0 = \frac{1}{L} \int_{0}^{C+2L} f(x)\,dx$

$$a_n = \frac{1}{L} \int_{C}^{C+2L} f(x) \cos\left(\frac{n\pi x}{L}\right) \cdot dx$$

$$b_n = \frac{1}{L} \int_{C}^{C+2L} f(x) \cdot \sin\left(\frac{n\pi x}{L}\right) dx$$

→ One can derive similar formulae for intervals,
$(C, C+2\pi)$, $(0, 2\pi)$, $(-L, L)$, $(-\pi, \pi)$

(ii) If f is even $(-L, L)$

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x)\,dx \quad \therefore \boxed{a_0 = \frac{2}{L} \int_{-L}^{L} f(x)\,dx \quad \text{-- (even)}}$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx \quad \therefore a_n = \frac{2}{L} \int_{-L}^{L} f(x) \cos\left[\frac{n\pi x}{L}\right] dx$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx = 0$$

--- Product of even & odd is odd.
f is even & $\sin$ is odd.

$$\therefore \boxed{\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) dx}$$

(12) f is odd.

$a_n = 0$. $\quad a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = 0$ — f is odd.

$$b_n = \frac{2}{L}\int_0^L f(x)\cdot\sin\left(\frac{n\pi x}{L}\right)dx$$ ($\because$ f is odd & sin is odd $\therefore$ product is even).

$\therefore$ ~~but~~ when f is odd

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)}$$

Similarly, for $(-\pi, \pi)$ replace L by π

(13) Half Range Series. → $(0, L)$ defined

(i) Cosine Series.

$b_n = 0$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right).$$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx \quad \& \quad a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$\boxed{\therefore \frac{2}{L}\int_0^L [f(x)]^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2}$$

(ii) Sine Series $\quad a_0 = 0$ & $a_n = 0$

$$\therefore f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx.$$

$$\boxed{\therefore \frac{2}{L}\int_0^L [f(x)]^2\,dx = \sum_{n=1}^{\infty} b_n^2}$$

COMPLEX VARIABLES

(1) $f(z) = u + iv$

(2) When $f(z)$ = analytic then it satisfies Cauchy - Riemann (CR) equations.

i.e $U_x = V_y$ and $U_y = -V_x$

(3) CR eqns in polar coordinates.

$x = r\cos\theta$ $\quad$ $y = r\sin\theta$

$z = x + iy = r\cos\theta + ir\sin\theta = r.e^{i\theta}$.

Let $f(z) = u + iv$

$\therefore f(z) = u + iv = f(r.e^{i\theta})$ —— (1)

Diff^n above paritially w.r.t r.

$$\therefore \frac{\partial u}{\partial r} + i.\frac{\partial v}{\partial r} = f'(r.e^{i\theta}).e^{i\theta} \quad \text{—— (2)}$$

Diff^n (1) partially w.r.t θ

$$\therefore \frac{\partial u}{\partial \theta} + i.\frac{\partial v}{\partial \theta} = f'(r.e^{i\theta})re^{i\theta}.i$$

$$= -r\frac{\partial v}{\partial r} + i.r\frac{\partial u}{\partial r}$$

Equating real & Imaginary parts

$$\therefore \frac{\partial u}{\partial \theta} = -r.\frac{\partial v}{\partial r} \quad \& \quad \frac{\partial v}{\partial \theta} = r.\frac{\partial u}{\partial r}. \quad \text{or.}$$

$$\boxed{\frac{\partial u}{\partial r} = \frac{1}{r}.\frac{\partial v}{\partial \theta}} \quad \& \quad \boxed{\frac{\partial u}{\partial \theta} = -r.\frac{\partial v}{\partial r}}$$

④ Using Milne Thompson method.

(i) To find imaginary part.

→ from u, find u_x & u_y

→ Put $\phi_1(x,y) = u_x$ and $\phi_2(x,y) = u_y$

→ Put $x = z$ and $y = 0$. Find $\phi_1(z,0)$ & $\phi_2(z,0)$

→ Then $f'(z) = \phi_1(z,0) - i\,\phi_2(z,0)$.

~~Integrating~~ → $f(z) = \int f'(z)\,dz + C$.

★ $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ → Harmonic function.

(ii) To find Real part

→ from v, find v_x and v_y

→ Put $\phi_1 = v_y$ and $\phi_2 = v_x$

→ Put $x = z$ and $y = 0$. Find $\phi_1(z,0)$ & $\phi_2(z,0)$

→ Then $f'(z) = \phi_1(z,0) + i\,\phi_2(z,0)$

→ $f(z) = \int f'(z)\,dz + C$.

PROBABILITY

(1) Random Variable.

$\sum_x P_x = 1$

x				
$P(X=x)$		k	$2k$	

To find k

$\ldots + \ldots 2k + k \ldots = 1$

$\Rightarrow k = \text{=}.$

(2) Distribution function / Cumulative Distribution function (c.d.f)

~~$f(x) = P(X \leq$~~

$$f(x) = P(X \leq x) = \sum_{x_i \leq x} P_i$$

$P_i \rightarrow$ Probability mass funct^n of X

where $P_i = P(X = x_i)$

$$f(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx.$$

$f(x) \rightarrow$ Probability density function of X

(3) Expectation of Random Variable.

(i) for discrete random variable

$$E(X) = P_1 x_1 + P_2 x_2 + \ldots P_n x_n$$

$$E(X) = \sum P_i x_i \quad \text{where } \sum P_i = 1$$

(ii) For continuous random variable.

$$E(X) = \int_{-\infty}^{\infty} f(x) \cdot X \, dx \qquad \int_{-\infty}^{\infty} f(x)\,dx = 1$$

NOTE Expectation of const. is const.

(i) $E(c) = \sum p_i c = c \quad (\because \sum p_i = 1)$

(ii) $E(c) = \int_{-\infty}^{\infty} c\, f(x)\,dx = c \cdot \int_{-\infty}^{\infty} f(x)\,dx = c \quad \left(\because \int_{-\infty}^{\infty} f(x)\,dx = 1\right)$

Expectation also known as Expected value

OR = +
AND = ×

④ Variance

$$var(x) = \sigma^2 \cancel{= E(x^2) [E(x)]^2}$$

$$= E(x^2) - [E(x)]^2$$

⑤ Standard Deviation $(\sigma) = \sqrt{\text{Variance}}$

⑥ Mean $= E(x)$

Variance $= E(x^2) - [E(x)]^2$

↳ find mean and $E(x^2)$

$$\boxed{E(x) = \int_{-\infty}^{\infty} x f(x)\,dx} \qquad \boxed{E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx}$$

⑦ Bayes Theorem

$$P(A_i|B) = \frac{P(B|A_i)\,P(A_i)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \cdots}$$

⑧ Conditional Probability

$$P(A|M) = \frac{P(A \cap M)}{P(M)}, \quad P(M) > 0$$