Repeated Games →

Some Relevant Questions?

→ what happens when players interact again & again in a strategic setting?

→ can cooperation be sustained b/w two players interacting strategically in a prisoner's dillema like situation?

→ what is the role of reputation & punishment in strategic interactions?

→ Sometimes we exhibit 'tit-for-tat' response. Is it a good strategy?

- In a repeated game, a game say (G) is played multiple times.

- Stage Game → A single play of game (G)
- Each occurence of G is called an iteration or a round.

→ Two kinds of Repeated games
  - Finitely Repeated
  - Infinitely Repeated

Finitely Repeated → Games with finite & known no. of Repetations.

Infinitely Repeated → Games ~~with~~ that continue for ever or games that ends at a random, unknown time.

→ Assumption: Players observe and remember the outcome of all previous stage games.

→ For every different observation of the outcome of stage games, players could have a different response.

Example: Repeated Prisoner's Dillema

		P2 Quiet	P2 Fink
P1	Quiet	2, 2	0, 3
P1	Fink	3, 0	1, 1

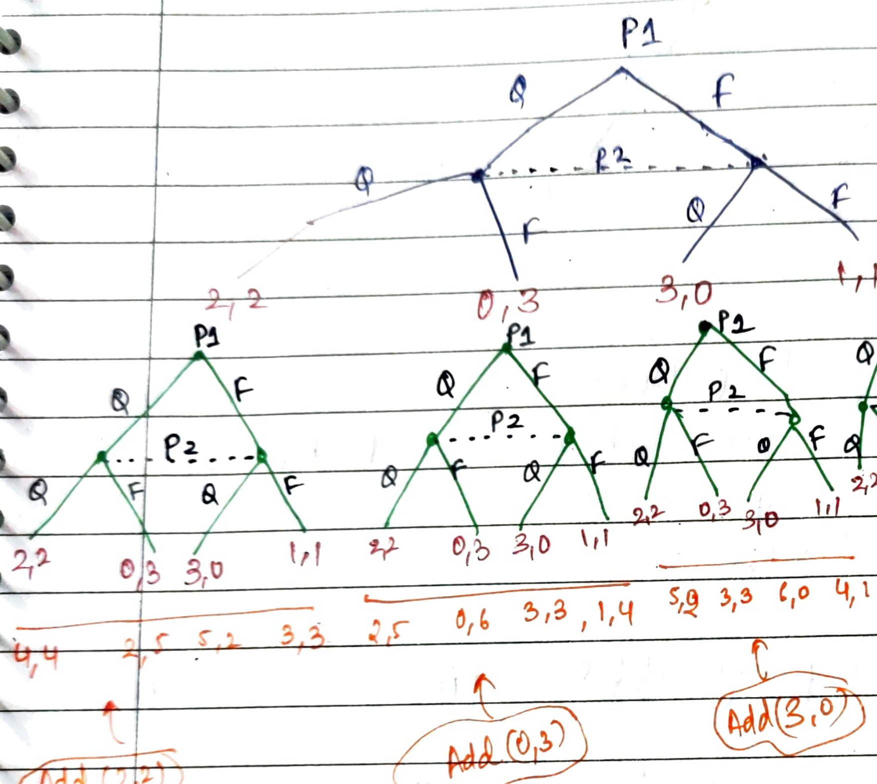
⇒ NE = (Fink, Fink)

→ Here only one pure strategy NE at (Fink, Fink)

→ If game is repeated T times {T is finite}
- → How should we write the payoffs?
- → Are there any strategies possible that would sustain (Quiet, Quiet) as the equilibrium strategy in at least some of the iteration of the game?

First, Lets take twice Repeated Prisoner's Dillema:
- 2 players play prisoner's dillema twice.
- Before the second stage game, each knows the outcome of the first stage game.
- Assumptions: Payoff is the sum of earnings on the two stages.

How to solve it? using Extensive form representation.



P1 can play Q or F.
P2 doesn't know what P1 has played, so P2 can also play Q or F.

this is end of stage 1, where both players know the outcome of stage 1.

After this the players realize the total payoff (sum of earning of two stages)

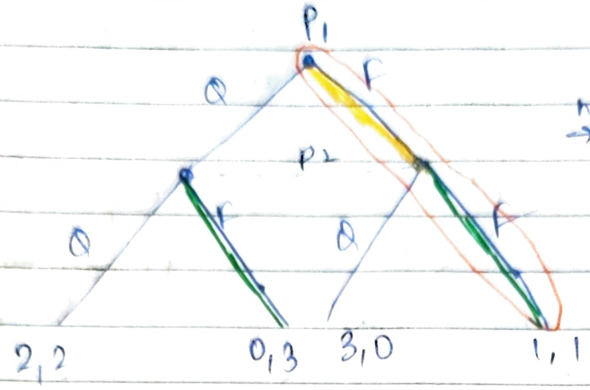
		P2 Q	P2 F
P1	Q	4,4	2,$\boxed{5}$
	F	$\boxed{5}$,2	$\boxed{3}$,$\boxed{3}$

(F, F) is Dominant strategy Here.

	P2 Q	P2 F
P1: Q	2,5	0,$\boxed{6}$
F	$\boxed{3}$,3	$\boxed{1}$,$\boxed{4}$

(F, F) is dominant strategy Here

Similarly.
In all the subgames of stage 2, (F, F) is dominant strategy.



Backward Induction →

P2 always choose F (more payoff)

now → P1 can choose (Q, F) or (F, F)

0 1

more payoff

So, If we repeat prisoners dillema twice, nothing changes, players always choose to Fink. ∴ For repeating this game twice, we don't get any cooperation.

lets say they play the game 1000 (finite no) times.

Stage 1
|
Stage 2
:
Stage 1000

So, In the last stage they are going to play the prisoner's dillema's & would always defect.

when they are playing second last stage, they both know that in the last stage, they both are going to defect, nothing changes, so in the 2nd last round also they would defect, & if we take this reasoning back and comeback to the first stage, there also player would defect. So we don't get any cooperation if we repead it for FINITE no of times.

Infinitely Repeated games →

→ Two kinds of models
- → Games that repeated forever
- → Games that may end after each repetition with some positive probability.

→ why do we need the concept of infinitely repeated games? cause sometimes we don't know how many time the game will be repeated, so we need a notion to represent such situations.

→ How to write the payoffs? Adding up doesn't make Sense, as it would lead to comparing infinities.

⇒ Add up the discounted sum.

* cooperation can often be sustained as NE in infinitely repeated games.

Infinitely Repeated Prisoner's Dillema

P1 \ P2	Q	F
Q	2, 2	0, 3
F	3, 0	1, 1

→ one equillibrium in infinite repeated games: both players always play Fink.

→ Are there any other equillibriums?

* The key element is that a players can adopt contingent strategies, like "If you "misbehave" on this round of the game, then I will respond on the next round.

→ This option remains available if there is some chance ~~coat~~ there will be a next round.

Trigger Strategy →

- If someone doesn't cooperate or "misbehaves" in one round of the game, this player can be punished in the next rounds.
- If the threat of punishment is sufficient to change the players strategies then we have found a new equillibrium termed as trigger strategy equillibrium.

In a trigger strategy :-

→ Begin by cooperating:
→ cooperate as long as the rivals do
→ upon observing a defection:
→ immediately revert to a period of punishment of specified length in which everyone plays non-cooperatively.

Two extreme Trigger Strategies :

1) Tit for Tat

→ whatever a player does in this round of the game, the other player will do the same in the next round.

→ the simplest form of trigger strategy.

→ Most forgiving / Shortest memory – If someone misbehav for one period, he gets punished for only one period. and immedietly they can go back to cooperation.

→ Propostional, credible but lack deterrence.

2) Grim Trigger strategy:

→ punish the player forever for his "misbehaviour"

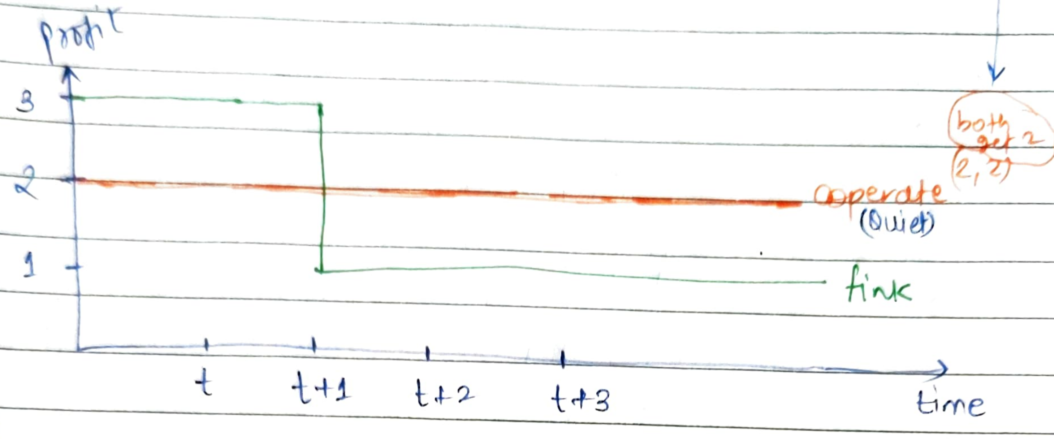
→ Least forgiving / Longest Memory

→ Adequaute deterrence but lack credibility.

(In prison's dillema, punishing others means, punishing yourself also, so it is less credible. but it has adequate deterrence in a sense that it deter the player for his misbehaviour continuosly).

If they Both cooperate

Payoff stream (Grim Trigger Strategies) →



p2

	Q	F
Q	2,2	0,3
F	3,0	1,1

If player 1 keeps on cooperating (quiet) & player 2 ~~defects~~ (fink), then p2 gets 3 & p1 get 0 but this is grim trigger, so p1 would now punish him forever & now p1 is going to play ~~def~~ fink forever → so in this case both players are always going to get 1, 1 cause now p2 knows that if he cooperates (Quiet), he gets $\boxed{0}$ pay off, so p2 also fink {1 payoff}.

cooperate – 2, 2, 2, 2, – – } which stream has higher value?
fink – 3, 1, 1, 1, – –

To calculate, we are going to use notion of discount.

{Money in your pocket right now is more valuable than the money you are promised to get after one year.}

Discount factor (δ) and summation →

lets say there is a summation, that we have

$$S = 1 + \delta \cdot 1 + \delta^2 \cdot 1 + \delta^3 \cdot 1 + \cdots$$

1: earning in this period

$\delta \cdot 1$: 1 is earning in next period but discounted by δ, so $\delta * 1$

$\delta^2 \cdot 1$: for next period, discounting for 2 periods, so $\delta^2 * 1$

$\delta^3 \cdot 1$: discounting for 3 periods

lets say this summation is called S.

discount factor value between 0 & 1.

$$\boxed{0 < \delta < 1}$$

If we multiply S by δ,

$$S = 1 + \cancel{\delta \cdot 1} + \cancel{\delta^2 \cdot 1} + \cancel{\delta^3 \cdot 1} + \cdots$$
$$\delta S = \cancel{\delta \cdot 1} + \cancel{\delta^2 \cdot 1} + \cancel{\delta^3 \cdot 1} + \delta^4 \cdot 1 + \cdots$$

$$S - \delta S = 1$$
$$\Rightarrow \boxed{S = \frac{1}{1-\delta}}$$

lets say, we only have t periods, then

$$S = 1 + \cancel{\delta \cdot 1} + \cancel{\delta^2 \cdot 1} + \cancel{\delta^3 \cdot 1} + \cdots + \cancel{\delta^{t-1} \cdot 1}$$
$$- \delta S = \cancel{\delta \cdot 1} + \cancel{\delta^2 \cdot 1} + \cancel{\delta^3 \cdot 1} + \cdots + \delta^{t-1} \cdot 1 + \delta^t \cdot 1$$

$$1 - \delta S = 1 + \delta^t$$

$$\boxed{S = \frac{1-\delta^t}{1-\delta}}$$

calculas for Grim trigger strategy –

A person would not like to deviate if

2, 2, 2, 2, ... > 3, 1, 1, 1, ...
(stream of payoff of 2) (stream of payoff of 3 & then 1 in all periods)

→ If we take discounted sum,

* If we sum stream of 1 for infinite time, we get $S = \frac{1}{1-\delta}$

So for 2, 2, 2, ...

$$\boxed{S = \frac{2}{1-\delta}}$$

$$S = 2 + \delta \cdot 2 + \delta^2 \cdot 2 + \delta^3 \cdot 2 + \cdots$$
$$\delta S = \delta \cdot 2 + \delta^2 \cdot 2 + \delta^3 \cdot 2 + \delta^4 \cdot 2 + \cdots$$

$$S - \delta S = 2$$

$$\boxed{S = \frac{2}{1-\delta}}$$

~~for 3, 1, 1, 1, ... =~~

for 3, 1, 1, 1, ...

$$S = 3 + \delta + \delta^2 + \delta^3 + \cdots$$
$$= 3 + \delta(1 + \delta + \delta^2 + \cdots)$$
$$3 + \delta\left(\frac{1}{1-\delta}\right)$$

$$\boxed{S = 3 + \frac{\delta}{1-\delta}}$$

So, see if,

$$\frac{2}{1-\delta} > 3 + \frac{\delta}{1-\delta}$$

$$\frac{2}{\cancel{1-\delta}} > \frac{3(1-\delta) + \delta}{\cancel{1-\delta}}$$

$$2 > 3 - 3\delta + \delta$$

$$2\delta > 1$$

$$\delta > 1/2$$

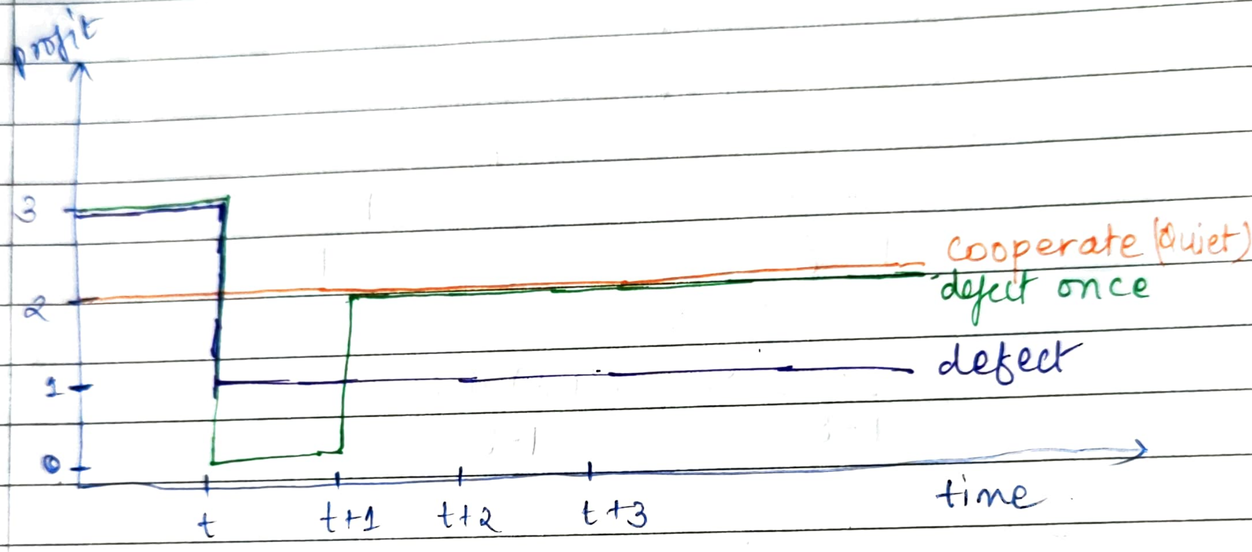
So, In this case, grim trigger strategy sustains if δ is atleast as large as half.

Sustainability: The minimum discount rate required to sustain the collusive outcome depends on the payoff structure.

→ Greater Relative profits from cheating → Need larger discount rate.

→ Smaller Relative Profits from cheating → Need smaller discount rate.

Payoff stream (Tit for Tat) →



	Q	F
Q	2,2	0,3
F	3,0	1,1

Here, cooperation would sustain if stream of

2, 2, 2, 2, 2 - - > 3, 1, 2, 2, 2 - - -

same ——— same → so can ignore.

$2 + 2\delta > 3 + \delta$

$\boxed{\delta > 1}$, not possible, so its difficult to sustain co-operation under tit-for-tat.