3) $f(x) = \begin{cases} x & , \quad 0 < x \leq \pi \\ 2\pi - x & , \quad \pi \leq x < 2\pi \end{cases}$

$\rightarrow f(x) = a_0 + \sum a_n \cos nx + \sum b_n \sin nx \text{ .... } (1)$

$a_0 = \frac{\pi}{2}$

$a_n = \begin{cases} 0 & , \text{ if } n \text{ is even} \\ \frac{-4}{\pi n^2} & \text{ if } n \text{ is odd.} \end{cases}$

$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx \, dx$

$= \frac{1}{\pi} \left\{ \int_0^{\pi} x \sin nx \, dx + \int_{\pi}^{2\pi} (2\pi - x) \sin nx \, dx \right\}$

$= \frac{1}{\pi} \left[\left\{ x \left(\frac{-\cos nx}{n} \right) - 1 \left(\frac{-\sin nx}{n^2} \right) \right\}_0^{\pi} + \left\{ (2\pi - x) \left(\frac{-\cos nx}{n} \right) - (-1) \left(\frac{-\sin nx}{n^2} \right) \right\}_{\pi}^{2\pi} \right]$

$= \frac{1}{\pi} \left[\left\{ \left(\frac{-\pi(-1)^n}{n} + 0 \right) - (0+0) \right\} + \left\{ (0+0) - \left(\frac{-\pi(-1)^n}{n} - 0 \right) \right\} \right]$

$= \frac{1}{\pi}(0)$

$\boxed{b_n = 0}$

Putting above values in ①

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left\{\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \dots\right\}$$

By Parseval's Identity.

$$\frac{1}{2\pi}\int_0^{2\pi}\{f(x)\}^2\,dx = a_0^2 + \frac{1}{2}\Sigma(a_n^2+b_n^2)$$

Consider,

$$\frac{1}{2\pi}\int_0^{2\pi}\{f(x)\}^2dx = \frac{1}{2\pi}\left[\int_0^{\pi}x^2dx + \int_{\pi}^{2\pi}(2\pi-x)^2dx\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{x^3}{3}\right)_0^{\pi} + \left[\frac{(2\pi-x)^3}{-3}\right]_{\pi}^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left\{\frac{\pi^3}{3} - \frac{1}{3}\left[0-\pi^3\right]\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{\pi^3}{3}+\frac{\pi^3}{3}\right\}$$

$$= \frac{2\pi^2}{3}\times\frac{1}{2} = \frac{\pi^3}{3}$$

Thus,

$$\frac{2\pi^2}{3} = \left(\frac{\pi}{2}\right)^2 + \frac{1}{2}\cdot\frac{16}{\pi^2}\left\{\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\dots\right\}$$

$$\therefore \frac{2\pi^2}{3} - \frac{\pi^2}{4} = \frac{8}{\pi^2}\left\{\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\dots\right\}$$

$$\frac{5\pi^2}{12} = \frac{8}{\pi^2}\left\{\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\dots\right\}$$

$$\frac{\pi^4}{96} = \frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\dots$$

4) ⇒ Find the Fourier Expansion of $f(x) = \sqrt{1-\cos x}$ in $(0, 2\pi)$. Hence deduce that $\frac{1}{2} = \sum_{n=1}^{\infty}\frac{1}{4n^2-1}$

→ Let,

$$\sqrt{1-\cos x} = \sqrt{2}\sin\frac{x}{2} = a_0 + \Sigma a_n\cos nx + \Sigma b_n\sin nx \quad \dots ①$$

Now,

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi}f(x)\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\sqrt{2}\sin\frac{x}{2}\,dx$$

$$= \frac{\sqrt{2}}{2\pi}\left\{-2\cos\frac{x}{2}\right\}_0^{2\pi}$$

$$= -\frac{\sqrt{2}}{\pi}\{-1-1\} = \frac{2\sqrt{2}}{\pi}$$

$$\therefore \boxed{a_0 = \frac{2\sqrt{2}}{\pi}}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi}f(x)\cdot\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\sqrt{2}\sin\frac{x}{2}\cos nx\,dx$$

$$= \frac{\sqrt{2}}{2\pi} \int_0^{2\pi} \left\{ \sin\left(n+\frac{1}{2}\right)x - \sin\left(n-\frac{1}{2}\right)x \right\} dx$$

$$= \frac{1}{\sqrt{2}\pi} \int_0^{2\pi} \left\{ \sin\left(\frac{2n+1}{2}\right)x - \sin\left(\frac{2n-1}{2}\right)x \right\} dx$$

$$= \frac{1}{\sqrt{2}\pi} \left\{ \frac{-2}{2n+1} \cos\left(\frac{2n+1}{2}\right)x + \frac{2}{2n-1} \cos\left(\frac{2n-1}{2}\right) \right\}_0^{2\pi}$$

$$= \frac{\sqrt{2}}{\pi} \left\{ \frac{1}{2n+1} - \frac{1}{2n-1} + \frac{1}{2n+1} - \frac{1}{2n-1} \right\}$$

$$= \frac{2\sqrt{2}}{\pi} \left\{ \frac{1}{2n+1} - \frac{1}{2n-1} \right\}$$

$$= \frac{2\sqrt{2}}{\pi} \left\{ \frac{2n-1-2n-1}{(2n+1)(2n-1)} \right\}$$

$$= \frac{2\sqrt{2}}{\pi} \left\{ \frac{-2}{4n^2-1} \right\}$$

$$= \frac{-4\sqrt{2}}{\pi(4n^2-1)}$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} \sqrt{2} \sin \frac{x}{2} \sin nx \, dx$$

$$= \frac{\sqrt{2}}{2\pi} \int_0^{2\pi} \left(2 \sin nx . \sin \frac{x}{2} \right) dx$$

$$= \frac{1}{\sqrt{2}\pi} \int_0^{2\pi} \left\{ \cos\left(n-\frac{1}{2}\right)x - \cos\left(n+\frac{1}{2}\right)x \right\} dx$$

$$= \frac{1}{\sqrt{2}\pi} \left\{ \frac{2}{2n-1} \sin\left(\frac{2n-1}{2}\right)x - \frac{2}{2n+1} \sin\left(\frac{2n+1}{2}\right)x \right\}_0^{2\pi}$$

$$= \frac{1}{\sqrt{2}\pi} \{ 0 - 0 \} = \frac{1}{\sqrt{2}\pi} (0) = 0.$$

$$\boxed{b_n = 0}$$

Putting above values in (1)

$$\sqrt{1-\cos x} = \frac{2\sqrt{2}}{\pi} + \sum_{n=1}^{\infty} \frac{-4\sqrt{2}}{\pi(4n^2-1)} \cos nx$$

Put $x = 0$

$$\therefore \quad 0 = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi} \sum_{n=1}^{\infty} \frac{1}{4n^2-1}$$

$$\therefore \quad \frac{4\cancel{\sqrt{2}}}{\cancel{\pi}} \sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{2\cancel{\sqrt{2}}}{\cancel{\pi}}$$

$$\therefore \quad 2 \sum_{n=1}^{\infty} \frac{1}{4n^2-1} = 1$$

$$\boxed{\therefore \quad \frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{4n^2-1}}$$

⑤ $f(x) = \begin{cases} -\pi & , -\pi < x < 0 \\ x & , 0 < x < \pi \end{cases}$

State the value of $f(x)$ at $x=0$ & hence deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{3\pi^2}{8}$

→ Let,

$f(x) = a_0 + \sum a_n \cos nx + \sum b_n \sin nx \ldots$ ①

Now,

$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx$

$= \frac{1}{2\pi} \left\{ \int_{\pi}^{0} -\pi\,dx + \int_{0}^{\pi} x\,dx \right\}$

$= \frac{1}{2\pi} \left\{ -\pi (x)_{-\pi}^{0} + \left(\frac{x^2}{2}\right)_0^{\pi} \right\}$

$= \frac{1}{2\pi} \left\{ (-\pi)(0+\pi) + \left(\frac{\pi^2}{2} - 0\right) \right\}$

$= \frac{1}{2\pi} \left\{ -\pi^2 + \frac{\pi^2}{2} \right\}$

$a = \frac{-\pi}{4}$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) . \cos nx\,dx$

$= \frac{1}{\pi} \left\{ \int_{-\pi}^{0} -\pi \cos nx\,dx + \int_{0}^{\pi} x \cos nx\,dx \right\}$

$= \frac{1}{\pi} \left[-\pi \left(\frac{\sin nx}{n}\right)_{-\pi}^{0} + \left\{ x\left(\frac{\sin nx}{n}\right) - 1\left(\frac{\cos nx}{n^2}\right) \right\}_0^{\pi} \right]$

$= \frac{1}{\pi} \left\{ -\pi(0-0) + \left(0 + \frac{(-1)^n}{n^2} - 0 - \frac{1}{n^2}\right) \right\}$

$= \frac{1}{\pi n^2} \left\{ (-1)^n - 1 \right\}$

$a_n = \begin{cases} 0 & , \text{if } n \text{ is even} \\ \frac{-2}{\pi n^2} & , \text{if } n \text{ is odd} \end{cases}$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\,dx$

$= \frac{1}{\pi} \left\{ \int_{-\pi}^{0} -\pi \sin nx\,dx + \int_{0}^{\pi} x \sin nx\,dx \right\}$

$= \frac{1}{\pi} \left\{ -\pi \left(-\frac{\cos nx}{n}\right)_{-\pi}^{0} + \left\{ x\left(\frac{-\cos nx}{n}\right) - (1).\left(\frac{-\sin nx}{n^2}\right) \right\}_0^{\pi} \right\}$

$= \frac{1}{\pi} \left\{ -\pi \left(-\frac{1}{n} + \frac{(-1)^n}{n}\right) + \left(\frac{-\pi(-1)^n}{n} + 0 + 0 + 0\right) \right\}$

$= \frac{1}{n} - \frac{2(-1)^n}{n}$

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$$b_n = \frac{1}{n}\{1-2(-1)^n\}$$

Putting above values in ①,

$$f(x) = \frac{-\pi}{4} - \frac{2}{\pi}\left\{\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \dots\right\} + \sum_{n=1}^{\infty}\frac{1}{n}\{1-2(-1)^n\}\sin nx$$

Now,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} -\pi = -\pi$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = 0$$

$$f(0) = \frac{1}{2}\left\{\lim_{x\to 0^-} f(x) + \lim_{x\to 0^+} f(x)\right\}$$

$$= \frac{1}{2}(-\pi + 0) = \frac{-\pi}{2}$$

Put $x = 0$

$$f(0) = \frac{-\pi}{4} - \frac{2}{\pi}\left\{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right\}$$

$$\frac{-\pi}{2} = \frac{-\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

$f(x)$	E	O	E	O
$g(x)$	E	O	O	E
$f(x).g(x)$	E	E	O	O

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Function $f(x)$ is said to be even if

$$f(-x) = f(x)$$

eg:- $f(x) = \cos x$

Function $f(x)$ is said to be odd if

$$f(-x) = -f(x)$$

eg $f(x) = \sin x$

⇒ In case of even function:-

$$a_0 = \frac{1}{l}\int_0^l f(x)\,dx,$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$$

$$b_n = 0$$

⇒ In case of odd function:-

$a_0 = 0$,

$a_n = 0$,

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$$

Note:-

$f(x)$	E	O	O	E
$g(x)$	E	O	E	O
$f(x).g(x)$	E	E	O	O

⇒ Obtain Fourier series for

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$$

Deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

→ Here,

$$f(-x) = \begin{cases} 1 - \frac{2x}{\pi}, & -\pi \le -x \le 0 \\ 1 + \frac{2x}{\pi}, & 0 \le -x \le \pi \end{cases}$$

$$= \begin{cases} 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \\ 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \end{cases}$$

$$= f(x).$$

$\therefore f(x)$ is even function

$\therefore b_n = 0$

Let,

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx \quad \ldots (1)$$

$$a_0 = \frac{1}{\pi}\int_0^\pi f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^\pi \left(1 - \frac{2x}{\pi}\right) dx$$

$$= \frac{1}{\pi}\left\{x - \frac{x^2}{\pi}\right\}_0^\pi$$

$$= \frac{1}{\pi}\left\{\pi - \pi - 0\right\}$$

$$a_n = \frac{2}{\pi}\int_0^\pi f(x) \cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^\pi \left(1 - \frac{2x}{\pi}\right) \cos nx\,dx$$

$$= \frac{2}{\pi}\left\{\left(1 - \frac{2x}{\pi}\right)\left(\frac{\sin nx}{n}\right) - \left(\frac{-2}{\pi}\right)\left(\frac{-\cos nx}{n^2}\right)\right\}_0^\pi$$

$$= \frac{2}{\pi}\left\{\left(0 - \frac{2(-1)^n}{\pi n^2}\right) - \left(0 - \frac{2}{\pi n^2}\right)\right\}$$

$$= \frac{4}{\pi^2 n^2}\left\{1 - (-1)^n\right\}$$

$$\therefore a_n = \begin{cases} 0, & \text{if } n \text{ is even} \\ \frac{8}{\pi^2 n^2}, & \text{if } n \text{ is odd.} \end{cases}$$

putting above values in (1)

$$f(x) = \frac{8}{\pi^2}\left\{\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \ldots\right\}$$

Put $x = 0$

$$F(0) = \frac{8}{\pi^2}\left\{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right\}$$

$$1 = \frac{8}{\pi^2}\left\{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right\}$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

COMPLEX VARIABLES

⇒ Analytic Function:-

* A function $f(z) = u+iv$ is said to be analytic at z_0 if it is differentiable at z_0 and some neighbourhood of z_0

Condition for function $f(z) = u+iv$ to be analytic is.

i) u_x, u_y, v_x, v_y are continuous fns.

ii) $\left.\begin{array}{l} u_x = v_y \\ u_y = -v_x \end{array}\right\}$ Cauchy-Riemann condition (CR equations)

Note

1) If $f(z)$ is analytic, then
$f'(z) = u_x + i v_x$.

2) If $f(z)$ is analytic fn in z, then it can be differentiated & integrated in usual manner.

* CR equation in polar form:-

$f(z) = u(r,\theta) + i v(r,\theta)$, then CR eqn,

$$u_r = \frac{1}{r} v_\theta$$

$$u_\theta = -r v_r$$

* A function $\phi(x,y)$ is said to be Harmonic if it satisfies Laplace's equation.

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0.$$

* If $f(z)$ be an ~~harmo~~ analytic fn, then u & v are harmonic functions called Harmonic conjugates.

Laplace eqn in polar form:-

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0.$$

Eg 8) If $f(z)$ & $\overline{f(z)}$ are both analytic, prove that $f(z)$ is constant.

Let $f(z) = u+iv$ & $\overline{f(z)} = u-iv$.

$\because f(z)$ is analytic,

$u_x = v_y$ & $u_y = -v_x$ (CR eqn)

$\because \overline{f(z)}$ is also analytic

$u_x = -v_y$ & $u_y = -(-v_x) = v_x$ (CReqn)

Adding $u_x = v_y$ & $u_x = -v_y$

$\therefore 2u_x = 0$

$\therefore u_x = 0$

$v_y = 0$

Adding $u_y = -v_x$ & $u_y = v_x$

$\therefore 2u_y = 0$

$\therefore u_y = 0$

$\therefore v_x = 0$

$\because u_x = 0$ and $u_y = 0$, u = a constant.

Also $v_x = 0$ and $v_y = 0$, $\therefore v$ = a constant.

→ Method of Finding Analytic function.

Steps:- (Either u or v will given)

1) Find u_x, u_y or v_x, v_y
2) Use $f'(z) = u_x + iv_x$
3) Apply Milne - Thompson's Method, ie put $x = z$, $y = 0$.
4) Integrate.

→ To verify whether fn is harmonic.

Condition is $u_{xx} + u_{yy} = 0$ or $v_{xx} + v_{yy} = 0$.

→ To find orthogonal trajectory of family of curves, ① find analytic fn. ② find real & imaginary part ③ Orthogonal trajectory to u = real part, ~~orth~~ is v = imaginary part.

* * * * *

⇒ MAPPING

Imp $w = u+iv$ $z = x+iy$

Eg:- Determine the 'D' region in w-plane corresponding to the region D in the z-plane given by $x=0$, $y=0$, $x=1$, $y=1$ under transformation $w = z+(2-i)$

→ We have $w = z+(2-i)$

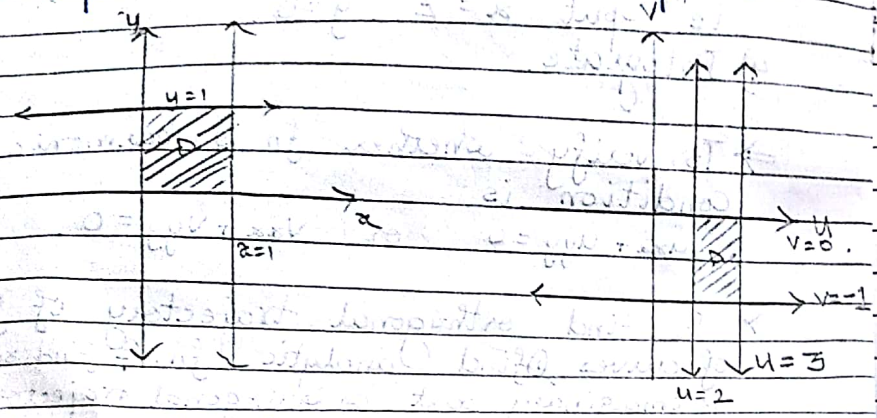
$= x+iy+2-i = (x+2)+i(y-1)$

$w = u+iv$

$\therefore u = x+2$, $v = y-1$

$\therefore$ When $x = 0$, then $u = 2$
When $y = 0$, then $v = -1$
When $x = 1$, then $u = 3$
When $y = 1$, then $v = 0$.

$\therefore$ z-plane   w-plane



→ **Bilinear Transformation:-**

A transformation of the type $w = \dfrac{az+b}{cz+d}$
where a, b, c, d are complex constants & $ad - cb \neq 0$, is called a Bilinear Transformation.

→ Cross Ratio:-

$$\frac{z_1 - z_2}{z_2 - z_3} \cdot \frac{z_3 - z_4}{z_4 - z_1}$$ is called cross ratio.

→ Preservation of Cross Ratio property:

$$\left(\frac{z_1 - z_2}{z_2 - z_3}\right)\left(\frac{z_3 - z_4}{z_4 - z_1}\right) = \left(\frac{w_1 - w_2}{w_2 - w_3}\right) \cdot \left(\frac{w_3 - w_4}{w_4 - w_1}\right)$$

→ Find the fixed points of bilinear transformation.

1) $w = \dfrac{z-1}{z+1}$

Put $w = z$

$$z = \frac{z-1}{z+1}$$
$$z(z+1) = z - 1$$
$$z^2 + z = z - 1$$
$$z^2 = -1$$
$$z = i, i$$

→ Find the bilinear transformation under which the points $z = 1, i, -1$ are mapped onto points $w = 0, 1, \infty$. Further show that under this transformation the unit circle in w-plane ($|w| = 1$) is mapped onto straight line in z-plane. Write name of this line.

Soln:- Let $w = \dfrac{az+b}{cz+d}$ ... ①

Put $z = 1$ & $w = 0$

$$\therefore 0 = \frac{a+b}{c+d}$$
$$\therefore a + b = 0 \quad \ldots ①$$

Put $z = i$ & $w = 1$

$$1 = \frac{ai+b}{ci+d}$$
$$ai + b = ci + d$$
$\therefore$ ~~a = a & b = d~~ ... ②

$\therefore \cancel{a = a = -b = d}$

Put $z = -1$, $w = \infty$

$\therefore \infty = \frac{a(-1)+b}{c(-1)+d}$

$\therefore -c+d = 0$

$\therefore c = d$ ... (4)

Put $a = -b$, $c = d$ in (3)

$\therefore id + d = -ib + b$

$d(1+i) = b(1-i)$

$d = \left(\frac{1-i}{1+i}\right) b$

$d = \frac{(1-i)^2}{1^2+1} b = \frac{(1-i)^2 b}{2}$

$\therefore d = \left(\frac{1-2i+i^2}{2}\right) b$

$d = \left(\frac{1-2i-1}{2}\right) b = -ib$

$\therefore c = d = -ib$.

$\therefore w = \frac{az+b}{cz+d} = \frac{-bz+b}{-ibz-ib}$

$= \frac{-b(z-1)}{-ib(z+1)}$

$= \frac{+(z-1)}{i(z+1)}$

$w = \frac{z-1}{iz+i}$

Now, $|w| = 1$

$\left|\frac{z-1}{iz+i}\right| = 1$

$\frac{|z-1|}{|i(z+1)|} = 1$

$\therefore \frac{|z-1|}{|z+1|} = 1$

$|z-1| = |z+1|$

$|z-1| = |x+iy+1|$

$|x+iy-1| = |x+iy+1|$

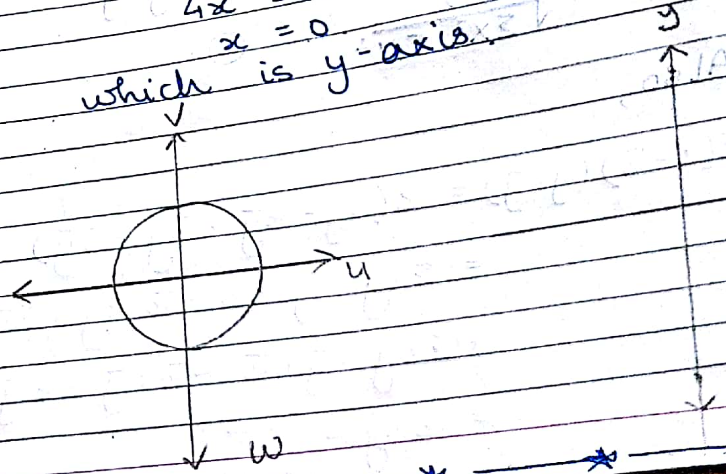
$\sqrt{(x-1)^2+y^2} = \sqrt{(x+1)^2+y^2}$

$x^2-2x+1+y^2 = x^2+2x+1+y^2$

$4x = 0$

$x = 0$

which is y-axis.



CORRELATION, REGRESSION & CURVE FITTING

1) Karl-Pearson's Co-efficient of Correlation (Denoted by 'r'):-

$$r = \frac{cov(x,y)}{\sigma_x \sigma_y} \quad \ldots (1)$$

where $\sigma_x^2 = \frac{\Sigma(x-\bar{x})^2}{n}$

$\sigma_y^2 = \frac{\Sigma(y-\bar{y})^2}{n}$ are std deviations in x & y

$$cov(x,y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$$

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 \; \Sigma(y-\bar{y})^2}} \quad (2)$$

Take $X = x-\bar{x}$, $Y = y-\bar{y}$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 \cdot \Sigma Y^2}} \quad (3)$$

Also,

$$\Sigma(x-\bar{x})(y-\bar{y}) = \Sigma(xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y})$$

$$= \Sigma xy - \bar{y}\Sigma x - \bar{x}\Sigma y + \bar{x}\bar{y}\Sigma 1$$

$$= \Sigma xy - \bar{y}n\bar{x} - \cancel{\bar{x}n\bar{y}} + \cancel{\bar{x}\bar{y}n}$$

$$= \Sigma xy - n\bar{x}\bar{y}$$

$$\therefore r = \frac{\Sigma xy - n\bar{x}\bar{y}}{\sqrt{(\Sigma x^2 - n\bar{x}^2)\cdot(\Sigma y^2 - n\bar{y}^2)}} \quad (4)$$

$$r = \frac{\Sigma xy - n\bar{x}\bar{y}}{\sqrt{\left(\Sigma x^2 - \frac{1}{n}(\Sigma x)^2\right)\left(\Sigma y^2 - \frac{1}{n}(\Sigma y)^2\right)}} \quad (5)$$

Results:-

1) $-1 \le r \le 1$
2) $r = \pm 1$ → Perfect Correlation.
3) $r = 0$ → No relation.

Q) Calculate Karl-Pearson's Co-eff.

X : 28, 45, 40, 38, 35, 33, 40, 32, 36, 33

Y : 23, 34, 33, 34, 30, 26, 28, 31, 36, 35

→ Here, $n = 10$

$\bar{X} = \frac{\Sigma x}{n} = \frac{360}{10} = 36$

$\bar{Y} = \frac{\Sigma Y}{n} = \frac{310}{10} = 31$

X	Y	$x = \bar{X} - X$	$y = \bar{Y} - Y$	x^2	y^2	xy

$\Sigma x^2 =$ $\quad \Sigma y^2 =$ $\quad \Sigma xy =$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 \cdot \Sigma Y^2}} =$$

For assumed mean

$$r = \frac{\sum dx\,dy - \frac{1}{n}(\sum dx)(\sum dy)}{\sqrt{\left[\sum dx^2 - \frac{1}{n}(\sum dx)^2\right]\left[\sum dy^2 - \frac{1}{n}(\sum dy)^2\right]}}$$

2) Spearman's Rank Co-efficient (Denoted by R).

$$R = 1 - \frac{6\sum D^2}{n^3 - n}$$

where $D = R_1 - R_2$

If ranks are repeating

$$R = 1 - \frac{6}{n^3 - n}\left\{\sum D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) \dots\right\}$$

—*—*—*—*—*—*—

⇒ REGRESSION :-

→ Line of Regression of Y on X

$y = a + bx$ OR $y - \bar{y} = b_{yx}(x - \bar{x})$

→ Line of Regression of X on Y

$x = a + by$ OR $x - \bar{x} = b_{xy}(y - \bar{y})$

Note:- 1) Line of Regression always passes through the mean point.

2) The slopes b_{yx} & b_{xy} are called regression coefficients.

$$b_{xy} = r\frac{\sigma x}{\sigma y}$$

$$b_{yx} = r\frac{\sigma y}{\sigma x} \qquad b_{xy} = \frac{\sum(x-\bar{x})(y-\bar{y})}{n\sigma y^2}$$

$$b_{yx} = \frac{\sum(x-\bar{x})(y-\bar{y})}{n\sigma x^2} \qquad b_{yx} = \frac{\sum XY}{\sum Y^2}$$

$$b_{yx} = \frac{\sum XY}{\sum X^2}$$

$$\tan\theta = \frac{1-r^2}{r}\left(\frac{\sigma x\,\sigma y}{\sigma x^2 + \sigma y^2}\right)$$

Note:-

1) $r^2 = b_{xy}\cdot b_{yx}$

- Regression co-efficient must be of same sign.

⇒ CURVE FITTING

1) Fitting a straight line:-

Let $y = a + bx$

∴ Normal equations,

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

Similarly for $x = a + by$

$$\sum x = na + b\sum y$$

$$\sum xy = a\sum y + b\sum y^2$$

2) Fitting a parabolic curve

$$\sum y = na + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

~~For assumed mean.~~

$$\cancel{r = \frac{\sum dx\, dy - \frac{1}{n}\sum dx \sum dy}{\sqrt{\left(\sum dx^2 - \frac{1}{n}(\sum dx)^2\right)\left(\sum dy^2 - \frac{1}{n}(\sum dy)^2\right)}}}$$

Z - TRANSFORM.

$$F(Z) = Z\{f(k)\} = \sum_{k=-\infty}^{\infty} f(k) . z^{-k}$$

$$\therefore \quad f(k) = Z^{-1}\{F(Z)\}$$