Semester : SE - Sem III Subject : Discrete Structure & Graph Theory Academic Year: 2022 - 2023

Module 1

Discrete structure - Discrete structure consists of a set of elements that define certain operations. e.g. discrete structures include integer arithmetic, monoids, semigroups, lattices, graphs, rings etc.

Applications of discrete structure in computer sci?
⇒ It can be used to study and describe objects and problems in computer science such as computer algorithms, programming languages, cryptography, automated theorem proving and software development.

A discrete structure is defined by a set of axioms.
Axiom : It is a statement or proposition which is regarded as being established, accepted true.

Calculus : It is a set of rules for calculating with symbols. —

Logic :- Logic helps organize code and enables programs to run more efficiently when implemented properly.

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A well organized code helps the readability of the code. Without properly applied logic, programs cannot produce the expected output.

* Propositional Logic -

Proposition or statement is a declarative sentence which is either true or false, but not both.

e.g. $3 + 5 = 10$ false.
Four is even true.

* Logical operations -

It is possible to form new and complicated statements from the primary statements by using certain connecting words, with whose usage, we are already familiar in the English language. These statements are called as compound statements.

① Negation -

The negation of a statement is formed either by using 'NOT' at a proper place or by prefixing the statement with the phrase.

if p is statement then negation of p = ~p

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e.g. if p = "I am going for a walk",
then ~p = "I am not going for a walk"
or It is not the case that I am going for a walk.

(2) Conjunction (AND) -
If p and q are statements then
'p AND q' is called as conjunction of p & q
& is denoted by $p \wedge q$.

ex (1) p = The sun is shining
q = The birds are singing.
Then $p \wedge q \Rightarrow$ The sun is shining and the birds are singing.

ex. (2) p = 2 is a prime no.
q = John is an intelligent boy.
$p \wedge q \Rightarrow$ 2 is a prime no. and John is an intelligent boy

ex. (3) Translate into symbolic form statement
'We watch television while we have dinner'.
Let, p = We watch television
q = we have dinner.
So given statement is in the symbolic form is $p \wedge q$.

inclusive or = $\vee$
exclusive or = $\overline{\vee}$

(2) Disjunction (OR) -

If p and q are statements, then the compound statement 'p or q' is called as the disjunction of p and q, is denoted by $p \vee q$.

ex There is an error in the program or the data is wrong.
Let p = There is an error in the program
q = The data is wrong
Then, $p \vee q$ ⇒ There is an error in the program or data is wrong.

ex Either I will read a book or go to sleep
Let p = I will read a book
q = I will go to sleep
Then $p \vee q$ ⇒ I will read a book or go to sleep

ex Let p = It is raining today
q = Olivia is an intelligent girl.
Then $p \vee q$ ⇒ It is raining today or Olivia is an intelligent girl.

ex I shall watch 'TV news' or I shall watch 'Discovery'
Let p = I shall watch 'TV news'
q = I shall watch 'Discovery'
∴ $p \overline{\vee} q$

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(4) Conditional (if ... then) -

If p and q are statements, the compound statements 'If p then q', denoted by $p \to q$ is called as conditional statement.

ex. Let p : Peter works hard.
q : Peter will pass the exam.
Then, $p \to q$: If Peter works hard, then he will the exam.

(*) If $p \to q$ is the conditional statement then
i) $q \to p$ is called as converse.
ii) $\sim p \to \sim q$ is called as inverse.
iii) $\sim q \to \sim p$ is called as contrapositive.

Exercise :-

ex. i) Give the converse and contrapositive of the conditional: If it rains, then I carry ~~can distribute~~ an umbrella.

$\Rightarrow$ Let p = It rains
q = I carry an umbrella.

$\therefore$ converse of $p \to q$ is, $q \to p$: if I carry an umbrella, then it rains.

& contrapositive of $p \to q$ is, $\sim q \to \sim p$: If I do not carry an umbrella, then it does not rain.

(5) Biconditional (if and only if) -

If p and q are statements, the compound statement 'p if and only if q' denoted by $p \leftrightarrow q$ is called as biconditional statement.

if and only if = iff.

e.g. i) An integer is even if and only if it is divisible by 2.

ii) Two lines are parallel if and only if they have the same slope.

ex. (preprositional form/ statement form).

1. Raju is poor but happy.
2. Raju is neither rich nor happy.
3. Raju is ~~a lost~~ either rich or happy.
4. Raju is poor or else he is rich & happily

⇒ 1. $\sim p \wedge q$
2. $\sim p \wedge \sim q$
3. $p \vee \sim q$
4. $\sim p \vee (p \wedge \sim q)$

IMP

ex. Write the logical negation of the following statements in the symbolic form.

1) John is intelligent and rich.

⇒ 1) John is intelligent but not rich.

2) John is either intelligent or rich.

Ex. Show that $p \to q$ and $\sim q \to \sim p$ are logically equivalent.

Note :- An Implication and its contra-positive are logically equivalent)

$$(p \to q) \equiv (\sim q \to \sim p)$$

Truth table -

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T
				①	②

by both the columns ① and ②, we can say they have identical truth values. Hence $p \to q$ and $\sim q \to \sim p$ are logically identical or equivalent.

e.g. Show that, $q \to p$ and $\sim p \to \sim q$ are equivalent

$$(q \to p) \equiv (\sim p \to \sim q)$$

ex. $(p \equiv q) \equiv (p \to q) \wedge (q \to p)$

ex. Prove that $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are logically equivalent

* Negation of compound statement -

(1) ⇒ Negation of conjunction -

A conjunction asserts that both of its conjuncts are true. its negation will mean that at least one of them is false.

e.g. Mr. John passed in English (p) and Logic (q).

∴ we can say that

$\sim p$: Mr. John did not pass in English OR

$\sim q$: Mr. John did not pass in Logic

so. $\sim(p \wedge q) \equiv \sim p \vee \sim q$

∴ $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

(2) Negation of disjunction -

A disjunction asserts that at least one of the disjuncts is true. Its negation therefore will mean that both are false.

e.g. Mr. John passed in English or he passed in Logic.

p : Mr. John passed in English

q : ~~Mr.~~ he ~~John~~ passed in Logic

$\sim p$: Mr. John did not pass in English AND

$\sim q$: ~~Mr. Jo~~ he did not pass in Logic

so. $\sim(p \vee q) \equiv \sim p \wedge \sim q$

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③ Negation of negation. – $\sim\sim p \equiv p$

e.g. It is not true that Newton did not develop calculus

Newton develop calculus

④ Implication as disjunction –

$\therefore S \rightarrow R \equiv \sim S \vee R$

⑤ Implication & Bi conditional in terms of conjunction, disjunction and negation.

if $p \rightarrow q$ and $q \rightarrow p$ then we can say that $p \leftrightarrow q$

$\therefore (p \rightarrow q) \wedge (q \rightarrow p)$ is $p \equiv q$. — ①

but we have $p \rightarrow q$ means $\sim p \vee q$ — ②

$q \rightarrow p$ means $\sim q \vee p$ — ③

Hence. $(p \rightarrow q) \wedge (q \rightarrow p)$ means $(\sim p \vee q) \wedge (\sim q \vee p)$

So that, from eq^n ①, ② & ③

$(p \equiv q) \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ — ④

⑥ Negation of an implication – $\sim(p \rightarrow q) \equiv p \wedge \sim q$

from eq ②

$\sim(p \rightarrow q) = \sim(\sim p \vee q)$

$\sim(p \rightarrow q) = \sim\sim p \wedge \sim q$

$\sim(p \rightarrow q) = p \wedge \sim q$ — ⑤ hence proved.

7) Negation of Biconditional -

Biconditional $p \equiv q$ is equivalent to

$(p \to q) \wedge (q \to p)$

$(\sim p \vee q)$ and $(\sim q \vee p)$

$\sim(p \equiv q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

... by eq^n 4 & 5

* Statement Forms :-

If we have two statements p and q. If p and q are any two unknown statement then we can denote a disjunction by $p \vee q$.

A statement form is a sequence of symbols containing statement variables such that when statement variables are replaced by statements the result is a statement.

3 categories of statement form.

i) Tautology .... forms are always true.

ii) Contradiction ... -||- always false.

iii) Contingency ... may be true or false

e.g. Using following statements, write :
p : I will study discrete structures.
q : I will go to a movie.
r : I am in a good mood.
write follo. statements in symbolic form.

i) If I am not in a good mood then I will go to a movie.
$\Rightarrow \sim r \rightarrow q$

ii) I will not go to a movie and I will study discrete structures.
$\Rightarrow$ ~~$\sim q \rightarrow$~~ $\sim q \wedge p$

iii) I will go to a movie only if I will not study discrete structure.
$\sim p \rightarrow q$

iv) If I will not study discrete structures, then I am not in a good mood.
$\sim p \rightarrow \sim r$.

ex. (2) Let p = Rajan is rich
q = Rajan is happy
write the following.

i) Rajan is poor and happy $\Rightarrow \sim p \wedge q$

ii) Rajan is neither rich nor happy
$\Rightarrow \sim p \wedge \sim q$

iii) Rajan is either rich or unhappy.
$\Rightarrow p \vee \sim q$

iv) Rajan is poor or else he is rich and unhappy.
$\Rightarrow \sim p \vee (p \wedge \sim q)$.

* Tautology, Contradiction & Contingency -

① Tautology => A statement form which is always true for all substitution instances is called a tautology (the same meaning) or said to be tautologous.

②

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★ Laws of Logic -

1. Idempotent laws -
$p \vee p \equiv p$
$p \wedge p \equiv p$

2. Commutative Laws -
$p \vee q \equiv q \vee p$
$p \wedge q \equiv q \wedge p$

3. Associative Laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

4. Identity Laws
$p \wedge true \equiv p$
$p \vee false \equiv p$

5. Domination Laws -
$p \vee true \equiv true$
$p \wedge false \equiv false$

6. Distributive Laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

7. Complement Laws
$p \vee \sim p \equiv true$
$p \wedge \sim p \equiv false$

8. Double Negation
$\sim(\sim p) \equiv p$
$\sim f = t$
$\sim t = f$

9. De Morgans Laws
$\sim(p \vee q) \equiv \sim p \wedge \sim q$
$\sim(p \wedge q) \equiv \sim p \vee \sim q$

10. Absorption Law
$p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$

* Quantifier -

Not all statements are propositions, since they do not having any truth value.

So if values are assigned to those statements/variables, each of them becomes either true or false.

e.g. x is tall and handsome.
$x + 3 = 5$
$x + y \geq 10$

These statements can be converted into:

Varun is tall and handsome. - true - (1)
$2 + 3 = 5$ - true - (2)
$2 + 5 \geq 10$ - false - (3)

Predicate : An assertion that contains one or more variables is called as a predicate. its truth value is predicated after assigned truth values to its variables.

A predicate P containing n variables $x_1, x_2, \ldots x_n$ is called as n-place predicate.

ex. 1 & 2 are one-place predicate.
3 is ~~to~~ 2-place predicate.

If we want to specify the variables in proad
$P(x_1, x_2, \ldots x_n)$: predicate P containing variables cat

e.g. x is a city in India '
It is denoted by P(x).
$x + y \geq z$ is denoted by $P(x, y, z)$
The values which the variables may assume constitute a collection or set called as the Universe of discource.
A predicate becomes a proposition only when all its variables are bound.

ex.
i) $P(x): x + 3 = 5$
Universe of discourse = ~~all~~ set of all integers
if $x = -1$ then stmt is false
if $x = 2$ we get proposition.

ii) $P(x, y): x + y = 10$
Universe of discourse is set of all integers
e.g. we set $x = -1$
we get the one-~~var~~ place predicate
$P(1, y): 1 + y = 10$
$y = 10 - 1 = 9$
so we obtain proposition $P(1, 9)$ which is true.

This method of binding individual variables in a predicate is by Quantification of variable.

There are 2 types of quantification
- Universal ($\forall$)
- Existential. ($\exists$)

① Universal quantifiers –
e.g. Let $P(x)$ be a predicate $x \geq 0$; where x is any positive integer.
then the proposition
$\forall x (P(x))$ is true.
if x is any float value.
$\forall x (P(x))$ is a false.

② Existential Quantifier –
e.g. Suppose for the predicate $P(x)$, $\forall x P(x)$ is false but there exist atleast one value of x for which $P(x)$ is true. So we can say
$\exists x P(x)$ means there exist x for $P(x)$ is true.

ex. $\exists x P(x)$

$\forall y \exists x P(x, y)$

$\exists x \exists y P(x, y)$

$\forall x \forall y P(x, y)$

Q.9 Negation of a quantified statements

~~$\exists x$~~ $\forall x\, P(x)$ is logically equivalent to
$\exists x\, [\sim P(x)]$

ex. on quantifier -

① All the invited guests were present for the dinner.

$\forall x\, P(x) \equiv \exists x\, (\sim P(x))$

x = guest is searching

② "There is a student ~~is~~ ~~stu~~ in the class, who is not familier with C prog.

$\Rightarrow \forall_x P(x)$

$\exists x\, (\sim P(x))$

③ ~~For all, x, $x \leq 5$~~ or $x \geq 5$

~~let $P(x)$: $x < 5$~~

~~$Q(x)$: $x \geq 5$~~

~~$\therefore \forall x$~~

There exists an x such that $x < 4$.

$P(x) : x < 4$.

$\Rightarrow \exists x\, P(x)$

(4) For every number x there is a number y such that $y = x+1$.
There are/is a no. y such that, for every number x, $y = x+1$.

$\Rightarrow \forall x \exists y \, P(x,y)$
$\exists y \forall x \, P(x,y)$

(5) Write in symbolic form.
i) All students have taken a course in communication skills.
ii) There is a girl student in the class who is also a sports person.
iii) Some students are intelligent, but not hard working.

$\Rightarrow$ i) $P(x)$: student have taken a course in CS.
$\exists x P(x)$

ii) $P(x) = x$ is girl student
$Q(x) = x$ is a girl
$R(x) = x$ is a sports person.
$\exists x [P(x) \wedge Q(x) \wedge R(x)]$

iii) There exists an x such that x is intelligent but x is not hard working.
$P(x) = x$ is intelligent
$Q(x) = x$ is not hard working
$\exists x [P(x) \wedge \sim Q(x)]$

(b) i) $\forall x \exists y$

e.g. irreflexivity to be

* Normal Forms - 2 types
1. Disjunctive NF
2. Conjunctive NF.

⇒ A statement which consists of a disjunction of fundamental conjuction is called as disjunctive normal form.

e.g. $(p \wedge q) \vee \sim q$

i.e. $(\sim p \wedge q) \vee (p \wedge q) \vee q$

e.g. obtain DNF of the $(p \to q) \wedge (\sim p \wedge q)$

⇒ $(p \to q) \wedge (\sim p \wedge q)$

$\equiv (\sim p \vee q) \wedge (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$

$\equiv (\sim p \wedge q) \vee (q \wedge \sim p)$

Mathematical Induction:-

The proof of mathematical induction consists of three parts.

Let $P(n)$ denote a statement (a formula or a theorem) associated with $n = 1, 2, 3 \ldots$ any natural number.

step 1: (Verification)

We first verify that $P(n)$ i.e., the theorem is true for n equal to one i.e. we can get on the first step of the ladder.

step 2: (Induction property): Assuming $P(n)$ i.e. the theorem to be true for some value of $n = k$, we prove by some known method that it is true for $n = k+1$. i.e. we can get from k^{th} step to $(k+1)^{th}$ step of the ladder.

step 3: (conclusion). The result is true for any value of $n \in N$.

ex. (1) Prove by mathematical induction $n < 2^n$ for all positive integers n.

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step 1 - Put $n = 1, 2$ we see that

$1 < 2^1 = 2$ — (1) $P(1)$

$2 < 2^2 = 4$ (2) $P(2)$

$\therefore$ The result is true for $n = 1, 2$

$P(1)$, $P(2)$ are true.

step 2 - Assume that the result is true for $n = k$ i.e.

assume that $P(k)$ is true.

$\therefore k < 2^k$

now multiply both sides by 2

$2k < 2^k \cdot 2$

$\therefore 2k < 2^{k+1}$

But $k+1 < 2k$ ... $2k < 2^{k+1}$

$\therefore k+1 < 2^{k+1}$

The result is true for $n = k+1$

step 3 - from step 1 and 2 we conclude ~~that first n positive integers is equal to $\frac{n}{2}(n+1)$ i.e. proved.~~

that the result is true for any positive integer.

ex. 2 Prove that the sum of the first n positive integers is equal to $\frac{n}{2}(n+1)$

prove that $p(n) = 1+2+3+\cdots n = \frac{n}{2}(n+1)$

$\Rightarrow$ step 1. put $n = 1, \cancel{2}$ we see that.

$P(1) = 1 = \frac{1}{2}(1+1)$

put $n = 2$

$P(2) = 1+2 = 3 = \frac{2}{2}(2+1)$

$\therefore$ The result is true for $n=1$ and $n=2$

$P(1)$ and $P(2)$ are true.

step 2. Assume the result to be true for $n = k$ i.e. $P(k)$

$$\underbrace{P(k) = 1+2+3+\cdots+k}_{LHS} = \underbrace{\frac{k}{2}(k+1)}_{RHS} \quad \text{— eq (1)}$$

we have to prove that

$$P(k+1) = 1+2+3+\cdots+(k+1) = \frac{(k+1)}{2}(k+1+1) \quad \text{— (2)}$$

Adding next +ve integer $k+1$ to LHS of eq (1)

LHS of $P(k+1) = 1+2+3+\ldots+k+(k+1)$

$= \frac{k}{2}(k+1) + (k+1)$ — by eq (1)

take $(k+1)$ common

$(k+1)\left[\frac{k}{2}+1\right] = \frac{(k+1)(k+2)}{2}$

$=$ RHS of eq (2) Hence proved

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(*) Examples on Mathematical Induction –

(1) $1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

i) Basis of induction

For $n=1$, $1^2 = \frac{1(1+1)(2(1)+1)}{6}$

$1 = \frac{1(2)(3)}{6} = \frac{6}{6}$

$\boxed{1=1}$ hence $P(1) = \text{true}$.

ii) Induction step.

For $n=k$,

$1^2+2^2+3^2+\dots+k^2 = \frac{k(k+1)(2k+1)}{6}$ —— (i)

(this is induction hypothesis)

Now $n=k+1$

$1^2+2^2+3^2+\dots+k^2+(k+1)^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$ —— (ii)

Using induction hypothesis (i)
put eq^n (1) in eq^n (2)

So we have,

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\therefore (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right] = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\therefore (k+1)\left[\frac{k(2k+1)+6k+6}{6}\right] = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$(k+1)\left[\frac{2k^2+k+6k+6}{6}\right] = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$(k+1)\left[\frac{2k^2+7k+6}{6}\right] = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$(k+1)\left[\frac{(2k+3)(k+2)}{6}\right] = \frac{(k+1)(k+2)(2k+3)}{6}$$

LHS = RHS hence $P(k+1)$ true.

$\therefore P(n)$ is true for all n using principle of mathematical induction.

(2) Prove that $\frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \frac{1}{7\cdot10} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$

$\Rightarrow$ Let $P(n) = \frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \frac{1}{7\cdot10} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$

i) Basis of induction.

for $n=1$ $P(1) = \frac{1}{1\cdot4} = \frac{1}{4}$ hence $P(1)$ true.

ii) Induction step -

Assume $n=k$.

$$P(k) = \frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \frac{1}{7\cdot10} + \cdots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1} \quad -(1)$$

~~pt~~ now we have.

$n = k+1$

$$P(k+1) = \frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \frac{1}{7\cdot10} + \cdots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3(k+1)-2)(3(k+1)+1)} = \frac{k+1}{3(k+1)+1} \quad -(2)$$

put eq^n (1) in eq^2 (2)

using induction hypothesis.

$$\frac{k}{3k+1} + \frac{1}{(3k+3-2)(3k+3+1)} = \frac{k+1}{3(k+1)+1}$$

$$\frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)} = \frac{k+1}{3(k+1)+1}$$

$$\therefore \frac{(3k^2+4k)+1}{(3k+1)(3k+4)} = \frac{k+1}{3(k+1)+1}$$

$$\frac{\cancel{(3k+1)}(k+1)}{\cancel{(3k+1)}(3k+4)} = \frac{k+1}{3(k+1)+1}$$

$$\frac{k+1}{3k+4} = \frac{k+1}{3(k+1)+1}$$

hence $P(k+1)$ is also true.

by principle of mathematical induction

Induction step,

$n = k$

$P(k) = 1+5+9+\dots+(4k-3) = k(2k-1)$ — (1)

$n = k+1$

$P(k+1) = 1+5+9+\dots+(4k-3)+(4(k+1)-3) = (k+1)(2(k+1)-1)$ — (2)

by using mathematical induction.

put eq^n (1) in eq^n (2)

$k(2k-1)+(4(k+1)-3) = (k+1)(2k+1)$

$2k^2-k+4k+4-3 = (k+1)(2k+1)$

$2k^2+3k+1 = (k+1)(2k+1)$

$(k+1)(2k+1) = (k+1)(2k+1)$ [factor]

LHS = RHS

hence proved $P(k+1)$ is also true.

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ex. (5) show that

$2+5+8+\dots+(3n-1) = \frac{n(3n+1)}{2}$

Let $P(n) = 2+5+8+\dots+(3n-1) = \frac{n(3n+1)}{2}$

Basis of induction.

$n=1$, $P(1) = 2 = \frac{1(3+1)}{2} = \frac{4}{2} = 2$

$n=2$, $P(2) = 2+5 = 2(6+1)/2 = 14/2 = 7$

ex (3) Show that $1+3+5+\dots+(2n-1) = n^2$ using mathematical induction.

i) let $p(1) = 1 = 1^2 = 1$

$n=2$ $p(2) = 1+3 = 4$

for $n=1$ and $n=2$ true.

ii) now $n=k$.

$p(k) = 1+3+5+\dots+(2k-1) = k^2$ — (1)

put $n = k+1$

$p(k+1) = 1+3+5+\dots+(2(k+1)-1) = (k+1)^2$ — (2)

using induction hypothesis.

put eq^n (1) in eq^n (2) we have.

$k^2+(2(k+1)-1) = (k+1)^2$

$k^2+2k+2-1 = (k+1)^2$

$k^2+2k+1 = (k+1)^2$

hence LHS = RHS proved by principle of mathematical induction.

ex. (4) Use mathematical induction to show that,

$1+5+9+\dots+(4n-3) = n(2n-1)$

$\Rightarrow$ Let $p(n) = 1+5+9+\dots+(4n-3) = n(2n-1)$

$n=1$, $p(1) = 1 = 1(2(1)-1) = 1$

$n=2$ $p(2) = 1+5 = 2(2(2)-1)$

$= 2(3)$

$= 6$

Induction step,

$n = k$,

$$P(k) = 2+5+8+\dots+(3k-1) = \frac{k(3k+1)}{2} \quad (1)$$

$n = k+1$, $P(k+1) = 2+5+8+\dots+(3k-1)+(3(k+1)-1) = \frac{(k+1)(3(k+1)+1)}{2}$ (2)

put eq^n (1) in eq^n (2)

$$\frac{k(3k+1)}{2} + (3(k+1)-1) = \frac{(k+1)(3(k+1)+1)}{2}$$

$$\frac{k(3k+1)}{2} + (3k+2) = \frac{(k+1)(3k+4)}{2}$$

$$\frac{k(3k+1)+2(3k+2)}{2} = \frac{(k+1)(3k+4)}{2}$$

$$\frac{3k^2+k+6k+4}{2} = \frac{(k+1)(3k+4)}{2}$$

$$\frac{3k^2+7k+4}{2} = \frac{(k+1)(3k+4)}{2}$$

$$\frac{3k^2+3k+4k+4}{2} = \frac{(k+1)(3k+4)}{2}$$

$$\frac{3k(k+1)+4(k+1)}{2} = \frac{(k+1)(3k+4)}{2}$$

$$\frac{(k+1)(3k+4)}{2} = \frac{(k+1)(3k+4)}{2}$$

LHS = RHS

hence proved $P(k+1)$ is also true.

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* Practice

Prove that, by mathematical induction.

(1) $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n! = (n+1)! - 1$

where n is positive integer.

(2) $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \dots + \frac{1}{(2n-1)(2n+2)} = \frac{n}{2n+1}, \; n \geq 1$.

(3) $n^4 - 4n^2$ is divisible by 3 for all $n \geq 2$.

(4) $n^2 + 2n$ is divisible by 8 for all $n \geq 1$

(5) Prove that $n^2 + n$ is an even number, for every natural number n.

(6) The sum of the cubes of three consecutive integers is divisible by 9.

$\therefore (n-1)^3 + n^3 + (n+1)^3$ is divisible by 9.

(7) $2^n > n^3, \; n \geq 10$.

(8) for any positive integer $n > 1$.

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}$