

The manufacturer of certain make of electric bulbs claims that his bulbs have a mean life of 25 months with a standard deviation of 5 months. A random sample of six such bulbs gave the following values

Life of months	$x - \bar{x}$	$(x - \bar{x})^2$
24	1	1
26	3	9
30	7	49
20	-3	9
60	-3	9
18	-5	25

$$\sum (x - \bar{x})^2 = 102$$

Can you regard the producer's claim to be valid at 1% level of significance.

$$\bar{x} = 23$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{102}{5}} = \sqrt{20.4} = 4.516$$

$$t = \frac{|\bar{x} - u|}{s} \sqrt{n}$$

$$t = \frac{|23 - 25|}{4.516} \sqrt{6}$$

$$t = \frac{2}{4.516} \sqrt{6}$$

$$t = \frac{4.8989}{4.516}$$

$$t = 1.084$$

$$\text{Calculated } t = 1.084$$

$$\text{Degree of freedom} = n-1 = 6-1 = 5$$

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Degree of freedom = 5 (Column wise)

level of significance 1%. (Row wise)

4.032

$t_{cal} < t_{table}$

then we Accept the hypothesis

let us take the hypothesis that there is no significant difference in the mean life of bulbs in the sample and that of the population

Conclusion: calculated t value: less than table t value
 \therefore Hypothesis accepted.

Therefore the producer's claim is not valid at 1% level of significance

$\bar{x} =$

2. Random Sample Size 16 has a mean of 53
 the sum of the squares of the deviations taken
 from mean is 135. $\sum(x_i - \bar{x})^2 = 135$
 Can this sample be regarded as taken from the
 Population having a mean of 56 $\mu = 56$.
 Obtained 95% and 99% confidence limits of the
 mean of the population

$$5\% \text{ LOS} \quad 1\% \text{ LOS}$$

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{135}{16-1}} = \sqrt{\frac{135}{15}} = 3$$

$$s = 3$$

$$t = |\bar{x} - \mu| / \sqrt{s}$$

$$t = |53 - 56| / \sqrt{15}$$

$$t = \frac{3}{\sqrt{15}} = \frac{3}{\sqrt{15}} = \frac{3}{3} = 1$$

$$t_{cal} = 4 \quad \text{DOF} = 15$$

$$\text{LOS} = 5\% \text{ or } 0.05$$

$$t_{tab} = 2.131$$

$$\text{DOF, LOS} = 1\% \text{ or } 0.01$$

$$t_{tab} = 2.947$$

$$t_{cal} > t_{tab}$$

\therefore Reject the Hypothesis
 (Not accept)

For 95%.

$$53 + \frac{3}{\sqrt{16}} 2.131 \quad 53 - \frac{3}{\sqrt{16}} 2.131$$

$$53 + \frac{3}{4} 2.131 \quad 53 - \frac{3}{4} 2.131$$

$$54.59825 \quad 51.40175$$

$$51.5 + 0.52$$

For 99%.

$$53 + \frac{3}{\sqrt{16}} 2.947 \quad 53 - \frac{3}{\sqrt{16}} 2.947$$

$$53 + \frac{3}{4} 2.947 \quad 53 - \frac{3}{4} 2.947$$

$$55.24025 \quad 50.78975$$

$$50.8 + 0.55.2$$

Chi-Square Test. (χ^2 test)

Web Testing often goes beyond A/B Testing and tests multiple treatment at once

The Chi Square Test is used with count data to test how well it fits some expected distribution

In an anti-malarial campaign in a certain area Quinine was administered to 1624 persons out of total population of 6496. The number of fever cases is shown below:

Treatment	Fever	No Fever	Total
Quinine	40	1584	1624
No Quinine	440	4432	4872
Total	480	6016	6496

Discuss the usefulness of Quinine in checking malaria.

Let us take the hypothesis that Quinine is not effective in checking malaria.

To determine the value of χ^2 steps:

- Calculated the Expected frequencies

$$E = \frac{RT \times CT}{N}$$

RT is Row Total for Row containing the cell

CT is Column Total for Column containing the cell

N is the total no. of observations

- Take the difference between the observed and Expected frequencies and obtained the square of differences

- Divide the values of observed - Expected squares obtained in Step 2 by the respected expected frequency. and obtain the total. This gives the values of Chi-square which can range from 0 to ∞ .

If $\chi^2 = 0$ it means that observed and expected frequencies completely coincide.

The greater the discrepancy b/w observed and expected values the greater will be value of χ^2

Conclusion:

The calculated value of χ^2 is compared with the table value for the given degrees of freedom at a certain Level of Significance. If at a stated level (5%) the calculated value of χ^2 is more than the table value of χ^2 the difference b/w expected and observation is considered to be significant. On the other hand, calculated value of χ^2 is less than the table value the difference b/w expected and observation is not considered as significant.

$$F = \frac{1624 \times 480}{6496}$$

$$E = 120$$

$$\chi^2 \text{ test} = \sum \left[\frac{(O-E)^2}{E} \right]$$

$$E = \frac{6016 \times 4872}{6496}$$

$$E = 4512$$

$$DOF = (r-1)(c-1) \text{ don't include total}$$

$$(2-1)(2-1)$$

$$DOF = 1$$

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$$E = \frac{480 \times 4872}{6496}$$

$$E = 360$$

$$E = \frac{6016 \times 1624}{6496}$$

$$E = 1504$$

Expected Frequency

120	1504	1624
360	4512	4872
480	6016	6496

O	E	$\frac{(O-E)^2}{E}$
40	120	53.33
440	360	17.77
1584	1504	4.255
4432	4512	1.4184
		76.7734

$$LOS = 5\% \quad DOF = 1$$

$$\chi^2 = 30.841$$

$$\chi^2_{cal} > \chi^2_{tab}$$

Hypothesis is rejected.
It is significant.

From the data given below about the treatment of 250 patients suffering from the disease State whether the new treatment is superior to the conventional treatment

Treatment	Favourable	Non Favourable	Total
New Treatment	140	30	170
Conventional Treatment	60	70	80
Total	200	50	250

Hypothesis: There is no significant difference b/w the new and conventional treatment.

$$E = \frac{170 \times 200}{250} = 136$$

$$E = \frac{170 \times 50}{250} = 34$$

$$E = \frac{200 \times 80}{250} = 64$$

$$E = \frac{50 \times 80}{250} = 16$$

O	E	$(O-E)$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
---	---	---------	-----------	---------------------

140	136	4	16	0.1176
30	34	-4	16	0.4705
60	64	-4	16	0.25
20	16	4	16	1

$$\sum \frac{(O-E)^2}{E}$$

$$= 1.8381$$

$$DOF = (r-1)(c-1)$$

$$= (2-1)(2-1) = 1$$

DOF = 1 because only one test is done

At 5% level of significance, $\chi^2_{tab} = 3.841$

$$\chi^2_{cal} < \chi^2_{tab}$$

Hypothesis is Accepted

2m IP-2

on the other end If the calculated value is less than the table value the difference is not significant

One Way Classification Model

Sources of Variation	Sum of squares	Degree of freedom (v)	Mean square	Variance ratio F
Between Samples	SSC	$v_1 = C - 1$	$MSC = \frac{SSC}{v_1}$	
within samples	SSE	$v_2 = n - C$	$MSW = \frac{SSE}{v_2}$	$F = \frac{MSC}{MSW}$
Total	SST	$n - 1$		

Total = sum of squares of variations

SSC = Sum of squares between samples (columns)

SSE = Sum of squares within samples (rows)

MSC = mean sum of squares between samples

MSW = mean sum of squares within samples

To assess the significance of possible variation in performance in a certain test between the different schools of a city, a common test was given to a

number of students taken at random from the senior 5th class of each of the 4 schools concerned. The test results are given below make an analysis of variance of data

A	B	C	D
8	12	18	13
10	11	12	9
12	9	16	12
8	14	6	16
7	4	8	15

$$\text{Avg} = 9 \quad \text{Avg} = 10 \quad \text{Avg} = 12 \quad \text{Avg} = 13$$

$$45 + 50 + 60 + 65$$

$$\frac{4}{55} \\ 225$$

$$\frac{4}{55}$$

$$\frac{9+10+12+13}{4} = \frac{44}{4} = 11$$

A B C D

$$(x_i - \bar{x})^2$$

$(9-11)^2$	4	$(10-11)^2$	1	$(12-11)^2$	1	$(13-11)^2$	4
$(9-11)^2$	4	$(10-11)^2$	1	$(12-11)^2$	1	$(13-11)^2$	4
$(9-11)^2$	4	$(10-11)^2$	1	$(12-11)^2$	1	$(13-11)^2$	4
$(9-11)^2$	4	$(10-11)^2$	1	$(12-11)^2$	1	$(13-11)^2$	4
$(9-11)^2$	4	$(10-11)^2$	1	$(12-11)^2$	1	$(13-11)^2$	4

20 5 5 20

$$SSC = 50$$

Between samples

SSE

with n samples

	A	B	C	D
	$(x - \bar{x}_1)^2$	$(x - \bar{x}_2)^2$	$(x - \bar{x}_3)^2$	$(x - \bar{x}_4)^2$
1	4	36	0	
1	1	0	16	
9	1	16	1	
1	16	36	9	
4	36	16	4	
16	58	104	30	

$$16 + 58 + 104 + 30$$

$$SSE = 208$$

One Way classification model

Source of variation	Sum of squares	Degrees of freedom (v)	mean square	Variance
Between sample	50	$v_1 = 4 - 1 = 3$	$MSB = \frac{SSB}{v_1} = \frac{50}{3} = 16.7$	$F = \frac{MSB}{MSW} = \frac{16.7}{1.28} = 13$
within samples	208	$v_2 = 20 - 4 = 16$	$MSW = \frac{SSW}{v_2} = \frac{208}{16} = 13$	
Total.	258	19		

3.2389

n Numerator dof = 3 row

m Denominator dof = 16 Column

$$1.28 < 3.24$$

5% LOS

Conclusion

The calculated value of F is less than the table value of F and hence the difference in the mean value is not of the samples is not significant i.e. the samples could have come from the same universe

Shootout method:

The 3 samples below have been obtained from normal populations with equal variances.

Test the hypothesis that the sample means are equal

	Sample 1 (S_1^2)	Sample 2 (S_2^2)	Sample 3 (S_3^2)
8	64	7	49
10	100	5	25
7	49	10	100
14	196	9	81
11	121	9	81
50	530	40	336
Σ	114	14	12
	530	40	336
	150	14	12
	530	40	336
	150	14	12

The table value of F at 5% LOS for $v_1 = 2$ and $v_2 = 12$ is 3.88

$$\text{Correction Factor} = \frac{T^2}{N} = \frac{(150)^2}{15} = \frac{22500}{15} = 1500$$

$$530 + 336 + 734 = 1600$$

$$SST = 1600 - 1500$$

$$SST = 100$$

N

SSC
between samples

$$\frac{(50)^2}{5} + \frac{(40)^2}{5} + \frac{(60)^2}{5} - \frac{(T)^2}{N}$$

$$\frac{2500}{5} + \frac{1600}{5} + \frac{3600}{5} - 1500$$

$$500 + 320 + 720 - 1500$$

$$1540 - 1500$$

$$= 40$$

SSC

$$SSE = 100 - 40$$

$$= 60$$

Sources of variation	Sum of Squares	Degree of freedom (V)	mean Square	Variance Ratio F
Between sample	40	$v_1 = 3 - 1 = 2$	$MSB = SSC = 40$	$F = MSB / MSE$
within samples	60	$v_2 = n - C = 15 - 3 = 12$	$MSE = 5$	$20 = 4$
Total	100			

Observations

	A	$(S_1)^2$	B	$(S_2)^2$	C	$(S_3)^2$
1	25	625	31	961	24	576
2	30	900	39	1521	30	900
3	36	1296	38	1444	28	784
4	38	1444	42	1764	25	625
5	31	961	35	1225	28	784

$$\sum = 160 \quad 5226 \quad 185 \quad 915 \quad 135 \quad 3669$$

$$160 + 185 + 135$$

$$\text{Correction Factor} = T^2 - \frac{(480)^2}{15} = 230400 - 15360 = 15360$$

$$SST = 961 + 1225 + 784$$

$$5226 + 915 + 3669$$

$$SST = 15810 - CF$$

$$15810 - 15360$$

$$SST = 450$$

~~x~~ C

between samples:

$$\frac{(31)^2}{5} + \frac{(35)^2}{5} + \frac{(28)^2}{5} - \frac{(T)^2}{N}$$

$$\frac{961}{5} + \frac{1225}{5} + \frac{784}{5} - \frac{15360}{15}$$

$$SSC = \frac{(160)^2}{5} + \frac{(185)^2}{5} + \frac{(135)^2}{5} - 15360$$

$$SSC = 250$$

$$SSE = SST - SSC$$

$$450 - 250$$

$$200$$

Sources of variation	Sum of squares	Degree of freedom	mean square	Variance Ratio F
Between samples	250	$v_1 = C - 1$	$MSC = \frac{SSC}{C-1}$	$F = MSC$
Within samples	200	$v_2 = n \cdot C - 1 = 15 - 3 = 12$	$\frac{200}{12} = 16.67$	125
Total	450			7.498

$$\text{table value} = 3.88$$

Calculated value > table value

Sample mean is significant

Two way ANOVA Table

Source of Variation	Sum of Square	Degrees of Freedom (v)	Mean Squares	Variance Ratio F
Between sample	SSC	$(C-1)$	$MSC = \frac{SSC}{(C-1)}$	$\frac{MSC}{MSE}$
Between Rows	SSR	$(\tau-1)$	$MSR = \frac{SSR}{(\tau-1)}$	$\frac{MSR}{MSE}$
Residual or Error	SSF	$(C-1)(\tau-1)$	$MSE = \frac{SSF}{(C-1)(\tau-1)}$	
Total	SST	$n-1$		

SSC = Sum of Squares between columns

SSR = Sum of squares between rows

SSF = Sum of squares due to error

SST = Total sum of squares

A tea company appoints 4 salesmen A, B, C, D and observes their sales in 3 seasons Summer, Winter, monsoon. The figures (in lakhs) are given in the following table

Salesmen	A	B	C	D	Season Total
Summer	36	36	21	36	129
winter	28	29	31	31	119
monsoon	26	28	29	29	112
Σ	90	93	81	96	360

- 1) Do the salesmen significantly differ in the performance
- 2) Is there significance between the seasons

	$A^2/(S_1)^2$	$B^2/(S_2)^2$	$C^2/(S_3)^2$	$D^2/(S_4)^2$	Season Total				
Summer	6 36 6 36 -9 81 6 36 9								
winter	-2 4 -1 1 1 1 1 1 -1								
monsoon	-4 16 -2 4 -1 1 -1 1 -8								
Σ	0	56	3	41	-9	83	6	38	0

$$SST = 56 + 41 + 83 + 38$$

$$SST = 218$$

SSC

Sum of Squares between samples

A B C

$$\frac{(0)^2}{3} + \frac{(3)^2}{3} + \frac{(-7)^2}{3} + \frac{(5)^2}{3} = 0$$

$$0 + 3 + \frac{81}{3} + \frac{25}{3}$$

$$3 + 27 + 12$$

$$SSC = 42$$

$$SSE = 218 - 42$$

Sum of Squares = $(9)^2 + (-1)^2 + (-8)^2$

$$b/w \text{ seasons } \frac{81}{4} + \frac{1}{4} + \frac{64}{4}$$

$$81 + 1 + 64$$

$$\frac{146}{4}$$

$$SSR = 36.5$$

$$SST = 218 \quad SST = (SSR + SSC)$$

$$218 - (36.5 + 42)$$

$$SSE = 139.5$$

Source of Variation	Sum of Squares	Degree of Freedom(v)	Mean squares	Variance Ratio F
Between Sample	42	4-1 = 3	$MSB = \frac{SSB}{(c-1)}$ $\frac{42}{3} = 14$	$MSB = \frac{14}{MSW}$ 23.25
Within Sample	36.5	3-1 = 2	$MSW = \frac{SSW}{(n-1)}$ $\frac{36.5}{2} = 18.25$	$MSB = \frac{14}{MSW}$ 23.25
Residual or Error	139.5	(3)(2)	$MSE = \frac{SSE}{(n-1)(c-1)}$ $\frac{139.5}{6} = 23.25$	
Total	218	12-1 = 11		

Conclusion 1:

For the table value of F for $v_1 = 3$ and $v_2 = 6$ at 5% LOS

Calculated value < Table value

and we conclude that Sales of different Salesman do not differ significantly.

Conclusion 2

$$0.60 < 4.76$$

The table value of F for $v_1 = 2$ and $v_2 = 6$ at 5% LOS is 5.14

Calculated value < Table value

Hence there is no significant difference

as far as the sales are concerned.

$$0.78 < 5.14$$

F test:

F test \rightarrow two random samples were drawn from two normal population and their values close

A	$(x - \bar{x})$	$(x - \bar{x})^2$	B	$(x - \bar{x}_1)$	$(x - \bar{x}_2)^2$
66	-14	196	64	-19	361
67	-13	169	66	-17	289
75	-5	25	74	-9	81
76	-4	16	78	-5	25
82	2	4	62	-1	1
84	4	16	85	2	4
88	8	64	87	4	16
90	10	100	92	9	81
92	12	144	93	10	100
			95	12	144
			97	14	196

$$\sum x = 720 \quad \sum x_1 = 734 \quad \sum x_2 = 913 \\ \text{Avg} = 80 \quad \text{Avg} = 82 \quad \text{Avg} = 82.5 \quad \sum x = 1298$$

Test whether the two populations have the same variance at 5% LOS

$$F = 3.36 \text{ at } 5\% \quad F_{0.05} = v_1 = 10 \quad v_2 = 8$$

$$F = \frac{s_1^2}{s_2^2}$$

< Accept H₀
> Reject Hypothesis

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$$S_1^2 = 734$$

$\frac{1}{n-1}$

$$\frac{734}{9-1} = \frac{734}{8} = 91.75$$

$$S_2^2 = \frac{1298}{10} = 129.8$$

$$F = \frac{S_1^2}{S_2^2} = \frac{91.75}{129.8} = 0.7067$$

$$F_{0.05} v_1=10, v_2=8$$

$$F_{0.05} = 3.36$$

The calculated value of F is less than the table value the hypothesis is accepted.

Hence it may be calculated that the two populations have same variance.

F test

Two random samples were drawn from 2 normal populations and their values are given below. Test whether the two population have the same variance at 5% LOS

$$A (x - \bar{x}) \quad B (x - \bar{x})^2 \quad (y - \bar{y}) \quad (y - \bar{y})^2$$

$$16 -14 14 196 -19 361$$

$$17 -13 16 169 -17 289$$

$$25 -5 24 25 -9 81$$

$$26 -4 28 16 -5 25$$

$$32 -2 32 4 -1 1$$

$$34 4 35 16 2 4$$

$$38 8 37 64 4 16$$

$$40 10 42 100 9 81$$

$$42 12 43 144 10 100$$

$$45 12 144 12 144$$

$$47 14 196$$

$$\sum A = 270 \quad \sum B = 3363 \quad \sum S_1^2 = 734 \quad \sum S_2^2 = 1298$$

$$\text{Avg } A = 30 \quad \text{Avg } B = 33$$

Let us take the

Hypothesis that the 2 populations have the same variance. Applying F test, $F = \frac{S_1^2}{S_2^2}$

$$\text{Since } F = \frac{S_1^2}{S_2^2} \Rightarrow \frac{734}{1298}$$

$$S_1^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

$$S_1^2 = \frac{734}{9-1} = 734$$

$$S_1^2 = 91.75$$

$$F = \frac{91.75}{129.8}$$

$$F = 0.706856$$

calculated

$$v_1 = 10 \quad v_2 = 8 \quad F_{0.05} = 3.3$$

v_1 = degrees of freedom for sample having larger variance $v_1 = n-1 = 11-1 = 10$

v_2 = degrees of freedom for sample having smaller variance $v_2 = n-1 = 9-1 = 8$.

$$0.7068 < 3.3$$

$$m = v_1$$

$$n = v_2$$

Conclusion:

The calculated value of F is less than the table value therefore the hypothesis is accepted. Hence it may be concluded that the 2 populations have the same variance.

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F - Test

Suppose that you are working in a research company and want to know the level of carbon dioxide emission happening from two different brands of cigarettes and whether they are significantly different or not. In your analysis, you have collected the following information:

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xyz ABC

sample size

11 10

mean 15.4 15.6

Standard deviation 1.2 1.1

Calculate the variance for xyz and ABC

$$S_1^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

$$S_1^2 = (1.2)^2$$

$$S_1^2 = 1.44$$

$$S_2^2 = (1.1)^2$$

$$S_2^2 = 1.21$$

$$F = \frac{S_1^2}{S_2^2} = \frac{1.44}{1.21} = 1.190$$

$$1.190 < 3.1373$$

$$S = \sqrt{\text{variance}}$$

Hypothesis is accepted.

$$\log 5\% \quad v_1 = 11-1 = 10$$

$$v_2 = 10-1 = 9$$

Both cigarettes companies are producing same carbon emission

$$3.1373$$

s_1^2 depend. for dof: v_1, v_2
 s_2^2

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Kruskal-Wallis Test; H test:
(200 more samples)

If several independent samples are involved, analysis of variance is the usual procedure. Failure to meet the assumptions needed for the analysis of variance makes its value doubtful. An alternative technique was developed called the Kruskal-Wallis test (one way Analysis of Variance) or H test. This test helps in testing the null hypothesis that k independent random samples come from identical population against the alternative hypothesis that the mean of the samples are not all equal.

$$H = \frac{12}{N(N+1)} \left[\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} + \dots + \frac{R_k^2}{n_k} \right] - 3(N+1)$$

H - statistic

t - statistic

F statistic

N = Total of all the values

Company's trainees are randomly assigned to groups which are taught a certain industrial inspection procedures by 3 different methods, at the end of instructing period they are tested for inspection performance quality. The following are these scores.

method A : 80, 83, 79, 85, 90, 68

method B : 82, 84, 60, 72, 86, 67, 91

method C : 93, 65, 77, 78, 88

use the H test to determine at the 5% LOS whether the 3 methods are equally effective.

Vf values	Rank S	Rank of Sample A	Rank of Sample B	Rank of Sample C
GO	1	9	10	18
G5	2	11	12	2
G7	3	8	1	6
G8	4	13	5	7
G2	5	16	14	15
G7	6	4	3	11
G8	7		17	
G9	8	61	62	48
G0	9			
G2	10			
G3	11			
G4	12			
G5	13			
G6	14			
G8	15			
G9	16			
G1	17			
G3	18			

$$H = \frac{12}{18(18+1)} \left[\frac{(G_1)^2}{6} + \frac{(G_2)^2}{7} + \frac{(G_3)^2}{5} \right] - 3(18+1)$$

$$\frac{12}{18(19)} \left[\frac{3721}{6} + \frac{3844}{7} + \frac{282304}{5} \right] - 57$$

$$\frac{12}{342} [1630.1095] - 57$$

~~$$0.0350 (1630.1095) - 57$$~~

~~$$\frac{57.0538}{57} - 57$$~~

D-0197

$$dof = n-1 = 3-1=2$$

$$\chi^2_{0.05} \quad V=2$$

$$\text{table } 5.991$$

$$\chi^2_{\text{cal}} < \chi^2_{\text{table}}$$

Conclusion:

The calculated value is less than the table value
the Null hypothesis is accepted and we conclude
that the 3 methods are equally effective

$$M_A = M_B = M_C$$

The manufacturing unit 4 teams of operators
were randomly selected and sent to 4 different
facilities for machining techniques training.
After the training the supervisor conducted
the exam and recorded the test scores
At 95% confidence level does the scores are same
in all 4 facilities.

Facility 1	Facility 2	Facility 3	Facility 4
88	77	71	52
92	76	56	65
86	84	64	68
87	59	51	81

Values	Ranks	Rank of Facility 1	Rank of Facility 2	Rank of Facility 3	Rank of Facility 4
51	1	16	10	8	2
52	2	12	9	3	6
56	3	14	13	5	7
59	4	15	4	1	11
64	5	1	17	11	10
65	6	57	36	17	25
68	7				
71	8				
76	9				
77	10				
81	11				
82	12				
84	13				
86	14				
87	15				
88	16				

$$H = \frac{12}{1C(17)} \left[\frac{(52)^2}{4} + \frac{(36)^2}{4} + \frac{(17)^2}{4} + \frac{(25)^2}{4} \right] - 3(17)$$

$$H_{cal} = 9.7720$$

$$df = n-1 \\ 4-1 = 3$$

$$\chi^2_{tab} = 7.815$$

$$\chi^2_{cal} > \chi^2_{table}$$

Reject the null hypothesis

Friedman's Test:

Department of public health and safety monitors the measures taken to clean up drinking water were effective for Trichloromethane (THM) at 12 counties. Drinking water was compared before clean up one week later and 2 weeks after clean up.

County	Before Clean up	1 week later	2 weeks later
1	21.1 (3)	19.2 (2)	18.4 (1)
2	24.1 (3)	22.3 (2)	21.2 (1)
3	14.1 (3)	12.9 (1.5)	12.9 (1.5)
4	18.1 (3)	17.8 (2)	17.3 (1)
5	15.4 (3)	15.1 (2)	14.9 (1)
6	16.2 (3)	15.1 (1.5)	15.1 (1.5)
7	2.4 (3)	7.2 (2)	6.8 (1)
8	7.5 (3)	6.7 (2)	6.1 (1)
9	14.2 (3)	13.6 (2)	13.1 (1)
10	21.3 (3)	20.9 (2)	20.4 (1)
11	9.5 (2)	9.8 (3)	9.2 (1)
12	11.9 (3)	10.5 (2)	10.1 (1)
	35	24	13

Let the null hypothesis be H_0 : The clean up system had no effect on the THMs

Alternate hypothesis: the clean up had effected the THMs

$$Q = \frac{12}{N(N+1)} \left[\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \dots + \frac{R_k^2}{n_k} \right] - 3(N+1)$$

Kruskal wallis test:

$$Q = \frac{12}{nk(k+1)} \left[R_1^2 + R_2^2 + \dots + R_k^2 \right] - 3n(k+1)$$

Friedman's Test:

$n = \text{no. of countries}$

$$\Phi = \frac{12}{nk(k+1)} \sum_{j=1}^k R_j^2 - 3n(k+1)$$

$$\begin{aligned}\Phi &= 12 \left(35^2 + 24^2 + 13^2 \right) - 3 \times 12(4) \\ &= 12 \times 3(3+1) \\ \Phi &= 20.16\end{aligned}$$

Friedman table

G.500

20.16 > G.5

critical > critical

calculated Φ value is greater than critical value of Φ for 5% LOS

Hence Reject the null hypothesis.

So it is concluded the clean up system effected the TAC's of drinking water.

Q18. 7 random people were given 3 different drugs and for each person, the reaction time corresponding to the drugs were noted. ie st. the claim at the 5% significance level that all the 3 drugs have the same probability distribution.

Drug A Drug B Drug C

~~11~~
~~12~~
~~13~~

Drug A Drug B Drug C

1	1.24 (1)	1.50 (2)	1.62 (3)
2	1.71 (1)	1.85 (2)	2.05 (3)
3	1.37 (1)	2.12 (2)	1.68 (3)
4	2.53 (2)	1.87 (1)	2.62 (3)
5	1.23 (1)	1.34 (2)	1.51 (3)
6	1.94 (1)	2.33 (2)	2.86 (3)
7	1.72 (2)	1.43 (1)	2.86 (3)

$$\sum \varepsilon = 9 \quad \sum \varepsilon = 13 \quad \sum \varepsilon = 20$$

$$\Phi = \frac{12}{nk(k+1)} \sum_{j=1}^k R_j^2 - 3n(k+1)$$

$$\Phi = 12 \left(9^2 + 13^2 + 20^2 \right) - 3 \times 7(3+1)$$

$$7 \times (3+1)$$

$$= \cancel{12} \times \cancel{3} \left(81 + 169 + 400 \right) - \cancel{21}(4)$$

$$= \cancel{12} \times \cancel{3}$$

$$= \cancel{3} (450) - 84$$

$$= \cancel{1350} - \cancel{84}$$

$$\text{cal} = 84.857$$

$$\text{tab} = 7.143$$

Conclusion: calculated value of Φ is greater than table critical value of Φ . Hence Reject the hypothesis. They do not have the same probability distribution.

am

Nbr Non parametric tests

(contd.)

Statisticians have devised alternative procedures which can be used to test hypotheses about data which are non normal or for which meaningful sample statistic cannot be calculated since these tests do not depend on the shape of the distribution they are called distribution free tests. These tests do not depend upon the population parameters such as mean and the variance and thus they are called as non parametric test.

Some of the non-parametric tests are:

- 1) Sign Test
- 2) Kruskal Wallis Test / H test
- 3) Spearman Rank Correlation Test
- 4) Friedman's Test

Advantages of non parametric tests:

- ① Non parametric test are distribution free i.e. they do not require any assumptions to be made about population following normal or any other distribution.
- ② They are simple to understand and easy to apply when the sample size is small.
- ③ They do not require lengthy calculations. They are applicable.
- 4 They are applicable to all types of data Qualitative data in the rank form as well as in the interval or ratio form.

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Limitations of non parametric test:

①

It should be noted that the statistical method which require no assumptions about the populations from which we are sampling are usually less efficient than the corresponding standard techniques. It is generally true that the more we assume the more we can infer from a sample however at the same time the more we assume the more we have to be cautious about the applicability of the methods and the inferences based on them. It is disadvantageous to use non parametric tests when all the assumptions of the classical procedures (t test, z test) can be met and the data are measured either on interval or ratio scale.

Stern and leaf

weights

· 7 7	4	6
· 6 2	5	5 6 8
· 4 6	6	2 7 9
· 5 5	7	6 7
· 6 7	8	9
· 5 6		
· 8 9		
· 6 9		
5 8		
7 6		