Here's a **mathematical example** of backtesting a **Simple Moving Average (SMA) Crossover strategy** with a given stock price over a period of time:

Problem Setup:

You have the following stock prices over 10 days:

Day	Stock Price ($)
1	100
2	102
3	104
4	106
5	108
6	110
7	112
8	115
9	117
10	119

Now, we backtest the **SMA Crossover Strategy** with the following parameters:

- **Short-term SMA**: 3-day SMA (for simplicity).
- **Long-term SMA**: 5-day SMA.
- **Buy Signal**: When the short-term SMA crosses above the long-term SMA.
- **Sell Signal**: When the short-term SMA crosses below the long-term SMA.

Step 1: Calculate SMAs

3-Day SMA:

- The 3-day SMA is calculated by averaging the last 3 days' closing prices.

Day	Stock Price ($)	3-Day SMA
1	100	-
2	102	-
3	104	(100+102+104)/3 = 102
4	106	(102+104+106)/3 = 104
5	108	(104+106+108)/3 = 106
6	110	(106+108+110)/3 = 108
7	112	(108+110+112)/3 = 110
8	115	(110+112+115)/3 = 112.33
9	117	(112+115+117)/3 = 114.67
10	119	(1 ↓ 17+119)/3 = 117

5-Day SMA:

- The 5-day SMA is calculated by averaging the last 5 days' closing prices.

Day	Stock Price ($)	5-Day SMA
1	100	-
2	102	-
3	104	-
4	106	-
5	108	(100+102+104+106+108)/5 = 104
6	110	(102+104+106+108+110)/5 = 106
7	112	(104+106+108+110+112)/5 = 108
8	115	(106+108+110+112+115)/5 = 110.2
9	117	(108+110+112+115+117)/5 = 112.4
10	119	(110+112+115+117+119)/5 = 114.6

Step 2: Generate Buy/Sell Signals

- **Buy Signal**: When the 3-day SMA crosses above the 5-day SMA.
- **Sell Signal**: When the 3-day SMA crosses below the 5-day SMA.

Looking at the values of the **3-day SMA** and **5-day SMA**:

- On **Day 3**, the 3-day SMA (102) crosses above the 5-day SMA (104) → **Buy Signal**.
- On **Day 8**, the 3-day SMA (112.33) crosses above the 5-day SMA (110.2) → **Buy Signal** again.
- On **Day 9**, the 3-day SMA (114.67) is higher than the 5-day SMA (112.4), but we already have an open position.
- On **Day 10**, the 3-day SMA (117) continues to stay above the 5-day SMA (114.6), so no action is needed.

Step 3: Calculate Portfolio Value

Let's assume an initial investment of $10,000 and that you are buying the stock with your entire capital when the buy signal occurs.

At Buy Signal on Day 3:

- Stock Price on Day 3 = $104.
- Number of shares bought = $10,000 / $104 ≈ 96 shares.

At Buy Signal on Day 8:

- Stock Price on Day 8 = $115.
- You will add to your position, so you buy additional shares.
- Additional shares bought = $10,000 / $115 ≈ 87 shares.

Final Portfolio Value (Day 10):

- On Day 10, the stock price is $119.
- Total shares owned = 96 + 87 = 183 shares.
- Portfolio Value = 183 * $119 = $21,807.

Step 4: Performance Metrics

1. **Total Return**:
   Total Return = (Final Portfolio Value - Initial Capital) / Initial Capital
   Total Return = ($21,807 - $10,000) / $10,000 = **118.07%**.
2. **Annualized Return (CAGR)**: To calculate CAGR, assume the backtest period is 10 days, and the initial capital is $10,000.
   CAGR = (Final Portfolio Value / Initial Capital)^(1 / Years) - 1
   For this 10-day period, CAGR ≈ (21,807 / 10,000) ^ (1 / (10/365)) - 1 ≈ **365.37%**.

Conclusion:

- **Buy Signal** occurred on **Days 3 and 8**.
- **Portfolio Value** grew from $10,000 to **$21,807** by Day 10.
- **Total Return** was **118.07%** over the 10-day period.
- The **CAGR** is **365.37%**, showing exceptional performance for the short-term period.

This example illustrates how a simple **SMA Crossover strategy** can be backtested mathematically to evaluate performance based on specific buy/sell signals and portfolio value tracking.