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A.Y. 2022-23 Subject - EM-III

ASSIGNMENT NO. 1

1. $\int_0^\infty e^{-t} \int_0^t e^{-2u} \cdot \frac{\sin u}{u} \, du \, dt$

$$L(f(t)) = \int_0^\infty e^{-t} \int_0^t e^{-2u} \cdot \frac{\sin u}{u} \, du . dt$$

$$L\left[\int_0^t e^{-2u} \cdot \frac{\sin u}{u} . du\right] = \frac{1}{s} F(s)$$

$$F_1(s) = L\left[\frac{e^{-2u} \sin u}{u}\right]$$

$$= F(s+2)$$

$$F_2(s) = L\left[\frac{\sin u}{u}\right]$$

$$= \int_s^\infty L[\sin u] . ds$$

$$= \int_s^\infty \frac{1}{s^2+1} . ds$$

$$= \left[\tan^{-1}(s)\right]_s^\infty$$

$$= \frac{\pi}{2} - \tan^{-1}(s) = \cot^{-1}(s)$$

$$F(s+2) = \cot^{-1}(s+2)$$

On comparing $\int_0^\infty e^{-t} \int_0^t e^{-2u} \frac{\sin u}{u} \, du . dt$ with $\int_0^\infty e^{-st} f(t) . dt$

$s = 1$

$$\int_0^\infty e^{-t} \cdot \int_0^t e^{-2u} \cdot \frac{\sin u}{u}\, du\, dt = \cot^{-1}(s+2)$$

$$= \cot^{-1}(1+2)$$

$$= \cot^{-1}(3)$$

$$\int_0^\infty \frac{e^{-t} \cdot \sin^2 t}{t}\, dt = \frac{1}{4} \log 5$$

$$L\left[\frac{\sin^2 t}{t}\right] = \int_s^\infty L[\sin^2 t]\, ds.$$

$$= \int_s^\infty L\left[\frac{1-\cos 2t}{2}\right] ds.$$

$$= \frac{1}{2}\int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+4}\right)$$

$$= \frac{1}{2}\left[\log s - \frac{1}{2}\log|s^2+4|\right]_s^\infty$$

$$= \frac{1}{2}\left[\log s - \log\sqrt{s^2+4}\right]_s^\infty$$

$$= \frac{1}{2}\left[\log \frac{s}{\sqrt{s^2+4}}\right]_s^\infty$$

$$= \frac{1}{2}\left[\log \frac{\cancel{s}\, 1}{\cancel{s}\sqrt{1+\frac{4}{s^2}}}\right]_s^\infty$$

$$= \frac{1}{2}\left[\log 1 - \log \frac{s}{\sqrt{s^2+4}}\right]$$

$$= -\frac{1}{2} \log \frac{s}{\sqrt{s^2+4}}$$

$$\frac{1}{4}\log 5 = \frac{-1}{2}\log\frac{s}{\sqrt{s^2+4}}$$

$$\log(5)^{1/4} = \log\left(\frac{s}{\sqrt{s^2+4}}\right)^{-1/2}$$

$$(5)^{1/4} = \left(\frac{s}{\sqrt{s^2+4}}\right)^{-1/2}$$

$$(5)^{1/4} = \left(\frac{\sqrt{s^2+4}}{s}\right)^{1/2}$$

Taking fourth root on both side

$$5 = \frac{(\sqrt{s^2+4})^2}{s^2}$$

$$5s^2 = s^2+4$$

$$4s^2 = 4$$

$$s^2 = 1$$

$$s = \pm 1$$

3. $L\left[t^4 \sinh 2t \cosh 2t\right]$

$$L\left[t^4\left(\frac{e^{2t}-e^{-2t}}{2}\right)\left(\frac{e^{2t}+e^{-2t}}{2}\right)\right]$$

$$\frac{1}{4}L\left[t^4\left((e^{2t})^2-(e^{-2t})^2\right)\right]$$

$$\frac{1}{4}L\left[t^4\left(e^{4t}-e^{-4t}\right)\right]$$

$$\frac{1}{4}\left[L(e^{4t}.t^4) - L(e^{-4t}.t^4)\right]$$

$$\frac{1}{4}\left[F(s-4) - F(s+4)\right]$$

$$F(s) = L(f(t)) = L(t^4)$$

$$L(f(t)) = L(t^4)$$
$$= \frac{4!}{s^5}$$

$$= \frac{1}{4}\left[\frac{4!}{(s-4)^5} - \frac{4!}{(s+4)^5}\right]$$

4.] $L\left[\int_0^t \frac{e^t \sin t}{t}\, du\right]$

$$= \frac{1}{s} F(s)$$

$$F(s) = L\left[\frac{e^t \sin t}{t}\right]$$

$$= \cancel{F_1(s-1)}\ F(s-1)$$

$$F_1(s) = L\left[\frac{\sin t}{t}\right]$$

$$= \int_s^\infty L[\sin t]\, .ds$$

$$= \int_s^\infty \frac{1}{s^2+1}\, .ds$$

$$= \left[\tan^{-1}(s)\right]_s^\infty$$

$$= \frac{\pi}{2} - \tan^{-1}(s)$$

$$= \cot^{-1}(s)$$

$$F(s) = F(s-1) = \cot^{-1}(s-1)$$

$$\therefore L\left[\int_0^t \frac{e^t \sin t}{t}.du\right] = \frac{1}{s}\cot^{-1}(s-1)$$

5.] $L\left[\frac{e^{-2t}\sin 2t \cosh t}{t}\right]$

$$= L\left[e^{-2t}\frac{\sin 2t}{t}\left(\frac{e^{-t}+e^{-3t}}{2}\right)\right]$$

$$= \frac{1}{2}L\left[e^{-t}\frac{\sin 2t}{t} + e^{-3t}.\frac{\sin 2t}{t}\right]$$

$$= \frac{1}{2}\left[F(s+1) + F(s+3)\right]$$

$$L\left[\frac{\sin 2t}{t}\right] = \int_s^\infty F(s).ds$$

$$= \int_s^\infty L(\sin 2t).ds$$

$$= \int_s^\infty \frac{2}{s^2+4}.ds$$

$$= 2\times\left[\frac{1}{2}\tan^{-1}\left(\frac{s}{2}\right)\right]_s^\infty$$

$$= 1.\left[\tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{2}\right)\right]$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{2}\right)$$

$$= \cot^{-1}\left(\frac{s}{2}\right)$$

$$= \frac{1}{2}\left[\cot^{-1}\left(\frac{s+1}{2}\right) + \cot^{-1}\left(\frac{s+3}{2}\right)\right]$$

6.) $f(t) = \begin{cases} t+1 & 0 \le t \le 2 \\ 3 & t > 2 \end{cases}$

$L[f'(t)] = s\,L(f(t)) - f(0)$

$f(0) = 1.$

$L(f(t)) = \int_0^\infty e^{-st} f(t).dt$

$= \int_0^2 e^{-st}(t+1).dt + \int_2^\infty e^{-st}.(3).dt$

$= \left[(t+1).\frac{e^{-st}}{(-s)} - 1.\frac{e^{-st}}{s^2} \right]_0^2 + 3\left[\frac{e^{-st}}{-s} \right]_2^\infty$

$= \left[3.\frac{e^{-2s}}{(-s)} - \frac{1}{s^2} e^{-2s} - \frac{1}{s} e^0 + \frac{1}{s^2} e^0 \right] + 3\left[0 - \frac{e^{-2s}}{(-s)} \right]$

$= -3.\frac{e^{-2s}}{s} - \frac{1}{s^2}.e^{-2s} - \frac{1}{s} + \frac{1}{s^2} + 3.\frac{e^{-2s}}{s}.$

$= \frac{-1}{s^2} e^{-2s} - \frac{1}{s} + \frac{1}{s^2}$

$L[f'(t)] = s\,L(f(t)) - f(0)$

$= s\left[\frac{-1}{s^2}.e^{-2s} - \frac{1}{s} + \frac{1}{s^2} \right] - 1$

$= \frac{-1}{s} e^{-2s} - 1 + \frac{1}{s} - 1$

$= -\frac{e^{-2s}}{s} + \frac{1}{s} - 2$

7.) $L[\sin^5 t]$

$$L[(\sin t)^5] = L\left[\left(\frac{e^{it}-e^{-it}}{2i}\right)^5\right]$$

$$= \frac{1}{(2i)^5} L[(e^{it}-e^{-it})^5]$$

$$= \frac{1}{32i} L[(e^{it}-e^{-it})^5]$$

$$(e^{it}-e^{-it})^5 = (e^{it})^5 - 5(e^{it})^4 . e^{-it} + 10.(e^{it})^3(e^{-it})^2 - 10.(e^{it})^2(e^{-it})^3 + 5.(e^{it})(e^{-it})^4 - (e^{-it})^5$$

$$= e^{5it} - 5.e^{4it-it} + 10.e^{3it-2it} - 10.e^{2it-3it} + 5.e^{it-4it} - \cancel{8}e^{-5it}$$

$$= e^{5it} - 5e^{3it} + 10.e^{it} - 10.e^{-it} + 5.e^{-3it} - e^{-5it}$$

$$L[(e^{it}-e^{-it})^5] = \frac{1}{S-5i} - \frac{5}{S-3i} + \frac{10}{S-i} - \frac{10}{S+i} + \frac{5}{S+3i} - \frac{1}{S+5i}$$

8.] $L^{-1}\left[\frac{1}{(s^2+9)(s^2+1)}\right]$

$$= L^{-1}\left[\frac{1}{(s^2+9)} \times \frac{1}{(s^2+1)}\right]$$

$$L^{-1}\left[\frac{1}{s^2+9}\right] = \frac{1}{3}\sin 3t = f_1(t)$$

$$L^{-1}\left[\frac{1}{s^2+1}\right] = \sin t = f_2(t)$$

By method of convolution

$$L^{-1}(F(s)) = \int_0^t f_1(u)\, f_2(t-u).du$$

$$= \int_0^t \frac{1}{3}\sin 3u .\ \sin(t-u)\, du$$

$$= \frac{1}{3}\int_0^t \sin 3u\ \sin(t-u) .du$$

$$= \frac{1}{3}\int_0^t \frac{1}{2}[\cos(3u-t+u) - \cos(3u+t-u)]\, du$$

$$= \frac{1}{6}\int_0^t [\cos(4u-t) - \cos(2u+t)]du$$

$$= \frac{1}{6}\left[\frac{\sin(4u-t)}{4} - \frac{\sin(2u+t)}{2}\right]_0^t$$

$$= \frac{1}{6}\left[\frac{\sin(4t-t)}{4} - \frac{\sin(2t+t)}{2} - \frac{\sin(-t)}{4} + \frac{\sin(+t)}{2}\right]$$

$$= \frac{1}{6}\left[\frac{\sin 3t}{4} - \frac{\sin 3t}{2} + \frac{\sin t}{4} + \frac{\sin t}{2}\right]$$

$$= \frac{1}{6}\left[\frac{\sin 3t}{4} - \frac{2\sin 3t}{4} + \frac{\sin t}{4} + \frac{2\sin t}{4}\right]$$

$$= \frac{1}{24}\left[-\sin 3t + 3\sin t\right]$$

9.) $L^{-1}\left[\frac{1}{(s^2+1)(s+9)}\right]$

$L^{-1}\left[\frac{1}{(s+9)} \times \frac{1}{(s^2+1)}\right]$

$L^{-1}\left[\frac{1}{s^2+1}\right] = \sin t = f_1(t)$ $\qquad$ $L^{-1}\left(\frac{1}{s+9}\right) = e^{-9t} = f_2(t)$

By method of convolution

$$L^{-1}(F(s)) = \int_0^t f_1(u)\, f_2(t-u)\,du$$

$$= \int_0^t \sin u \cdot e^{-9(t-u)}\,du$$

$$= \int_0^t e^{-9t+9u} \cdot \sin u\,du$$

$$= e^{-9t} \int_0^t e^{9u} \cdot \sin u\,du$$

$$= e^{-9t} \left[\frac{e^{9u}}{9^2+1^2} (9\sin u - \cos u) \right]_0^t$$

$$= e^{-9t} \left[\frac{e^{9t}}{81+1} (9\sin t - \cos t) - \frac{e^0}{81+1} (9\sin 0 - \cos 0) \right]$$

$$= e^{-9t} \left[\frac{e^{9t}}{82} (9\sin t - \cos t) - \frac{1}{82}(-1) \right]$$

$$= \frac{e^{-9t}}{82} \left[\frac{e^{9t}(9\sin t - \cos t)}{1} + 1 \right]$$

10.) $L^{-1}\left(\dfrac{1}{(s-4)^4 (s+3)} \right)$

$$F_1(s) = \frac{1}{(s-4)^4} \qquad F_2(s) = \frac{1}{s+3}$$

$$L^{-1}\left(\frac{1}{(s-4)^4}\right) = e^{4t} L^{-1}\left(\frac{1}{s^4}\right)$$

$$= e^{4t} \cdot \frac{t^3}{\lceil 4} = e^{4t} \cdot \frac{t^3}{6} = f_1(t)$$

$$L^{-1}\left(\frac{1}{(s+3)}\right) = e^{-3t}$$

$$= f_2(t)$$

By method of convolution

$$L^{-1}(F(s)) = \int_0^t f_1(u)\, f_2(t-u).\, du$$

$$= \int_0^t \frac{e^{4u}.\,u^3}{6}\, e^{-3(t-u)}.du$$

$$= \int_0^t \frac{e^{4u}.\,u^3}{6}.\, e^{-3t+3u}.du$$

$$= \frac{e^{-3t}}{6} \int_0^t e^{7u}.\,u^3\, du$$

$$= \frac{e^{-3t}}{6}\left[\frac{u^3.\,e^{7u}}{7} - \frac{3u^2 e^{7u}}{49} + \frac{6u e^{7u}}{343} - \frac{6e^{7u}}{2401}\right]_0^t$$

$$= \frac{e^{-3t}}{6}\left[\frac{t^3.\,e^{7t}}{7} - \frac{3t^2 e^{7t}}{49} + \frac{6t.\,e^{7t}}{343} - \frac{6e^{7t}}{2401} + \frac{6.\,e^0}{2401}\right]$$

$$= \frac{e^{-3t}}{6}\left[\frac{t^3.\,e^{7t}}{7} - \frac{3t^2 e^{7t}}{49} + \frac{6t.\,e^{7t}}{343} - \frac{6e^{7t}}{2401} + \frac{6}{2401}\right]$$

$$= \frac{1}{6}\left[\frac{t^3.\,e^{4t}}{7} - \frac{3t^2.\,e^{4t}}{49} + \frac{6t.\,e^{4t}}{343} - \frac{6e^{4t}}{2401} + \frac{6e^{-3t}}{2401}\right]$$

$$L^{-1}\left(\frac{1}{(s^2+4s+13)^2}\right)$$

$$F_1(s) = \frac{1}{(s^2+4s+13)}$$

$$F_2(s) = \frac{1}{(s^2+4s+13)}$$

$$L^{-1}\left(\frac{1}{s^2+4s+13}\right) = L^{-1}(F_1(s)) = L^{-1}(F_2(s))$$

$$= L^{-1}\left(\frac{1}{s^2+4s+4-4+13}\right)$$

$$= L^{-1}\left(\frac{1}{(s+2)^2+3^2}\right)$$

$$= e^{-2t}\, L^{-1}\left(\frac{1}{s^2+3^2}\right)$$

$$= e^{-2t}\,\frac{1}{3}\sin 3t = f_1(t) = f_2(t)$$

By method of convolution

$$L^{-1}(F(s)) = \int_0^t f_1(t)\, f_2(t-u)\,.du$$

$$= \int_0^t \frac{e^{-2u}\sin u}{3}\,.\,\frac{e^{-2(t-u)}}{3}\sin(t-u)\,.du$$

$$= \frac{e^{-2t}}{9} \int_0^t e^{-2u} \sin 3u . e^{2u} \sin(t-u) . du$$

$$= \frac{e^{-2t}}{9} \int_0^t \sin 3u . \sin(t-u) . du$$

$$= \frac{e^{-2t}}{9} \int_0^t \frac{1}{2} [\cos(3u - t + u) - \cos(3u + t - u)] . du$$

$$= \frac{e^{-2t}}{18} \int_0^t [\cos(4u - t) - \cos(2u + t)] . du$$

$$= \frac{e^{-2t}}{18} \left[\frac{\sin(4u-t)}{4} - \frac{\sin(2u+t)}{2} \right]_0^t$$

$$= \frac{e^{-2t}}{18} \left[\frac{\sin 3t}{4} - \frac{\sin 3t}{2} - \frac{\sin(-t)}{4} + \frac{\sin t}{2} \right]$$

$$= \frac{e^{-2t}}{18} \left[-\frac{\sin 3t}{4} + \frac{3 \sin t}{4} \right]$$

$$= \frac{e^{-2t}}{72} \left[-\sin 3t + 3 \sin t \right]$$

12.) $L^{-1}\left[\dfrac{5s^2+8s-1}{(s+3)(s^2+1)}\right]$

$$\frac{5s^2+8s-1}{(s+3)(s^2+1)} = \frac{A}{(s+3)} + \frac{Bs+C}{s^2+1}$$

$$5s^2+8s-1 = A(s^2+1)+(Bs+C)(s+3)$$
$$= As^2+A+Bs^2+3Bs+Cs+3C$$
$$= (A+B)s^2+(3B+C)s+A+3C$$

On comparing

$A+B=5$ —(1) $\quad 3B+C=8$ —(2) $\quad A+3C=-1$ —(3)

Solving (1), (2) & (3)

$A=2$

$B=3$

$C=-1$

$$L^{-1}\left[\frac{5s^2+8s-1}{(s+3)(s^2+1)}\right] = L^{-1}\left[\frac{2}{s+3}+\frac{3s-1}{s^2+1}\right]$$
$$= L^{-1}\left[\frac{2}{s+3}+\frac{3s}{s^2+1}-\frac{1}{s^2+1}\right]$$
$$= 2e^{-3t}+3\cos t-\sin t$$

13.) $L^{-1}\left[\dfrac{6s-4}{s^2-4s+20}\right]$

$$L^{-1}\left[\frac{6s-4}{s^2-4s+4-4+20}\right]$$

$$L^{-1}\left[\frac{6s-4}{(s-2)^2+16}\right]$$

$$L^{-1}\left[\frac{6(s-2)+12-4}{(s-2)^2+16}\right]$$

$$e^{2t}\, L^{-1}\left[\frac{6s+8}{s^2+16}\right]$$

$$e^{2t}\, L^{-1}\left[\frac{6s}{s^2+16}+\frac{8}{s^2+16}\right]$$

$$e^{2t}\cdot\left[6\cos 4t+\cancel{2}\,\frac{8}{4}\sin 4t\right]$$

$$e^{2t}\left[6\cos 4t+2\sin 4t\right]$$

14.) $L^{-1}\left[\dfrac{3s-7}{s^2-6s+8}\right]$

$$\frac{3s-7}{s^2-6s+8}=\frac{3s-7}{(s-4)(s-2)}=\frac{A}{s-4}+\frac{B}{s-2}$$

$$(3s-7)=A(s-2)+B(s-4)$$

Put $s=2 \Rightarrow 6-7=B(2-4) \Rightarrow -1=-2B \Rightarrow B=\frac{1}{2}$.

Put $s=4 \Rightarrow 12-7=2A \Rightarrow 5=2A \Rightarrow A=\frac{5}{2}$

$$L^{-1}\left[\frac{3s-7}{s^2-6s+8}\right]=\frac{1}{2}L^{-1}\left[\frac{1}{s-4}\right]+\frac{5}{2}L^{-1}\left[\frac{1}{s-2}\right]$$

$$=\frac{1}{2}e^{4t}+\frac{5}{2}e^{2t}$$

15.) $L^{-1}\left[\dfrac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right]$

$$L^{-1}\left[\frac{s^2+2s+1+2}{(s^2+2s+1+1)(s^2+2s+1+4)}\right]$$

$$L^{-1}\left[\frac{(s+1)^2+2}{((s+1)^2+1)((s+1)^2+4)}\right]$$

$$e^{-t}\,L^{-1}\left[\frac{s^2+2}{(s^2+1)(s^2+4)}\right]$$

Put $s^2 = x$

$$\frac{s^2+2}{(s^2+1)(s^2+4)} = \frac{x+2}{(x+1)(x+4)} = \frac{A}{x+1} + \frac{B}{x+4}$$

$$(x+2) = A(x+4) + B(x+1)$$

Put $x=-4 \Rightarrow -4+2 \Rightarrow -3B \Rightarrow B = 2/3$

Put $x = -1 \Rightarrow 1 = 3A \Rightarrow A = 1/3$

$$\frac{x+2}{(x+1)(x+4)} = \frac{1}{3(x+1)} + \frac{2}{3(x+4)}$$

Resubstitute $x = s^2$

$$= \frac{1}{3(s^2+1)} + \frac{2}{3(s^2+4)}$$

$$e^{-t}L^{-1}\left[\frac{1}{3(s^2+1)} + \frac{2}{3}\times\frac{1}{(s^2+4)}\right] = e^{-t}\left[\frac{1}{3}\sin t + \frac{1}{3}\sin 2t\right]$$

[illegible]

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17.) $R = 0.8$

$n = 10$

$$R = 1 - \frac{6\sum di^2}{n(n^2-1)}$$

$$0.8 = 1 - \frac{6\sum di^2}{10(100-1)}$$

$$\frac{6\sum di^2}{10\times99} = 0.2$$

$$\sum di^2 = \frac{0.2\times99\times10}{6}$$

$$= \frac{2\times99}{6}$$

$$\sum di^2 = 33$$

Corrected $\sum di^2 = 33 - (6)^2 + (8)^2$

$= 61.$

Corrected rank

$$R = 1 - \frac{6\sum di^2}{n(n^2-1)}$$

$$= 1 - \frac{6\times61}{990}$$

$$= 0.63$$

18.)

X	Y	R_1	R_2	$(R_1-R_2)^2$
10	12	6	6	0
12	18	5	5	0
18	25	2.5	3	0.25
18	25	2.5	3	0.25
15	50	4	1	9
40	25	1	3	4
				13.5

$$R = 1 - \frac{6\left(\Sigma d_i^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2)\right)}{n(n^2-1)}$$

$$= 1 - \frac{6\left(13.5 + \frac{1}{12}(8-2) + \frac{1}{12}(27-3)\right)}{6(36-1)}$$

$$= 1 - \frac{\cancel{6}\left(13.5 + \frac{1}{2} + 2\right)}{\cancel{6}(35)}$$

$$= 0.543$$

19.)

X	Y	X^2	XY	Y^2
2	5	4	10	25
4	7	16	21	49
6	8.5	36	51	72.25
8	11	64	88	121
20	31.5	120	170	267.25

line of regression of Y on X

$y = a + bx$

$\Sigma y = aN + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$31.5 = a(4) + b(20)$

$4a + 20b = 31.5$ —(1)

$170 = a(20) + b(120)$

$20a + 120b = 170$ —(2)

Solving (1) & (2)

$a = 4.75$, $b = 0.625$

$$\boxed{y = 4.75 + 0.625x}$$

For $x = 9 \Rightarrow y = 4.75 + 0.625 \times 9$

$= 10.375$

line of regression of x on y

$x = a + by$

$\Sigma x = aN + b\Sigma y$

$\Sigma xy = a\Sigma x + b\Sigma y^2$

$20 = a(4) + 31.5(b)$ —(3)

$170 = a(31.5) + b(267.25)$ —(4)

Solving (3) & (4)

$a = -0.13$, $b = 0.651$

$$\boxed{x = -0.13 + 0.651y}$$

For $x = 9$

$9 = -0.13 + 0.651y$

$y = 14.02$

20.) $3x + 2y = 26$

$6x + y = 31$

Let $3x + 2y = 26$ be the line of regression of Y on X

$$y = a + bx$$

$$2y = 26 - 3x$$

$$y = 13 - \frac{3}{2}x$$

$$b_{yx} = -\frac{3}{2}$$

Let $6x + y = 31$ be the line of regression of X on Y

$$x = a + by$$

$$6x = 31 - y$$

$$x = \frac{31}{6} - \frac{1}{6}y$$

$$b_{xy} = -\frac{1}{6}$$

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$= \sqrt{-\frac{\cancel{3}}{2} \times -\frac{1}{\cancel{6}\,2}}$$

$$= \sqrt{\frac{1}{4}}$$

$$= \pm \frac{1}{2} = \pm 0.5 = -0.5$$

21.)

X	Y	X^2	XY	X^3	X^2Y	X^4
1	2	1	2	1	2	1
2	6	4	12	8	24	16
3	7	9	21	27	63	81
4	8	16	32	64	128	256
5	10	25	50	125	250	625
6	11	36	66	216	396	1296
7	11	49	77	343	539	2401
8	10	64	80	512	640	4096
9	9	81	81	729	729	6561
45	74	285	421	2025	2771	15333

$$\Sigma Y = a\Sigma X^2 + b\Sigma X + nc$$
$$\Sigma XY = a\Sigma X^3 + b\Sigma X^2 + c\Sigma X$$
$$\Sigma X^2Y = a\Sigma X^4 + b\Sigma X^3 + c\Sigma X^2$$

$$74 = a(285) + b(45) + 9c \quad \text{---(1)}$$
$$421 = a(2025) + b(285) + c(45) \quad \text{---(2)}$$
$$2771 = a(15333) + b(2025) + c(285) \quad \text{---(3)}$$

Solving (1), (2) & (3)

$a = -0.27$

$b = 3.52$

$c = -0.93$

$$\boxed{y = -0.27x^2 + 3.52x - 0.93}$$

22.)

X	Y	XY	X^2
5	11	55	25
6	14	84	36
7	14	98	49
8	15	120	64
9	12	108	81
10	17	170	100
11	16	176	121
56	99	811	476

Regression of Y on X

$\Sigma Y = aN + b\Sigma X$

$\Sigma XY = a\Sigma X + b\Sigma X^2$

$99 = a(7) + b(56)$ — (1)

$811 = a(56) + b(476)$ — (2)

Solving (1) & (2)

$a = 8.71$; $b = 0.678$.

$y = a + bx$

$y = 8.71 + 0.678x$