

Complex Integration

Evaluation of Line Integral

- ① Evaluate $\int_C |z| dz$, where C is the left half of unit circle $|z|=1$ from $z=-i$ to $z=i$.
- ② Evaluate $\int_C (z + \frac{2}{z}) dz$ along the circle $x^2 + y^2 = 1$
- ③ Evaluate $\int_C \frac{z^2 + 3}{z} dz$, where C is the
- (i) upper half of the circle $|z|=2$
 - (ii) lower half of the circle $|z|=2$
 - (iii) the whole circle in anti-clock-wise direction.
- ④ Evaluate $\int_C (z - z^2) dz$, where C is the upper half of the circle $|z|=1$. What is the value of the integral for the lower half of the same circle?
- ⑤ Show that $\int_C \log z dz = 2\pi i$, where C is the

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unit circle in the z -plane.

⑥ Evaluate $\int_C z^2 dz$, where C is the arc of the circle $x=r\cos\theta, y=r\sin\theta$ from $\theta=0$ to $\theta=\pi/3$.

⑦ Evaluate $\int_C (z^2 - 2z + 1) dz$, where C is the circle $x^2 + y^2 = \rho^2$.

⑧ Evaluate $\int_0^{+i} (x-y+iy^2) dz$

i) along the line from $z=0$ to $z=1+i$

ii) along the real axis from $z=0$ to $z=1$ and then along the line parallel to the imaginary axis from $z=1$ to $z=1+i$.

iii) along the imaginary axis from $z=0$ to $z=i$,

and then along the line parallel to the real axis from $z=i$ to $z=i+1$.

iv) along the parabola $y^2 = x$.

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(B)

- ⑨ Evaluate $\int_{1-i}^{2+i} (2x+iy+1) dz$, along
- the straight line joining $(1-i)$ to $(2+i)$
 - $x=t+1$, $y=2t^2-1$ a parabola.

- ⑩ Integrate the function $f(z) = z^2 + \ln y$ from $A(1,1)$ to $B(2,4)$ along the curve $x=t$, $y=t^2$.

- ⑪ If O is the origin, L is the point $z=3$, M is the point $z=3+i$, evaluate $\int_C z^2 dz$,
- along i) the path OM ii) the path OLM
- iii) the path $DLMO$.

- ⑫ Evaluate $\int_0^{3+i} z^2 dz$
- i) along the real axis from 0 to 3 & then vertically to $3+i$.
- ii) along the imaginary axis from 0 to i & then horizontally to $3+i$

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iii) along the parabola $x=3y^2$.

(13) Evaluate $\int_C |z|^2 dz$ where C is the boundary of the square C with vertices $(0,0), (1,0), (1,1), (0,1)$

(14) Evaluate $\int_0^{1+2i} z^2 dz$ along the curve $2y^2 = x$.

(15) Evaluate $\int_{1-i}^{1+i} (ix + iy + i) dx$ along straight line joining $(1-i)$ to $(1+i)$.

Cauchy's Theorem

(1) Evaluate $\int_C \frac{\sin^b z}{(z - \pi/2)^3} dz$ where C is the circle $|z|=2$.

(2) Evaluate $\int_C \frac{\tan z/2}{(z-a)^2} dz, (-2 < a < 2)$

where C is the boundary of the square with centre at the origin & sides of length a .

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③ Evaluate $\int\limits_C \frac{e^{2z}}{(z-1)(z-2)} dz$, where C is the circle $|z|=3$.

④ Evaluate $\int\limits_C \frac{z^2 + 2z + 1}{z^2 - z + 2} dz$, where C is the ellipse $25x^2 + 16y^2 = 1$.

⑤ Evaluate $\int\limits_C \frac{z+3}{z^2 + 2z + 5} dz$, where C is the circle i) $|z|=1$, ii) $|z+1-i|=2$.

⑥ Evaluate $\int\limits_C \frac{4z-1}{z^2-3z-4} dz$, where C is the ellipse $x^2 + 4y^2 = 4$.

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