Evolutionary Game Theory :-

lets understand basics of evolutionary biology first.

Idea: Genes are the prime determinants of observable characterstics [called fitness] in a given environment.

→ More fit organism [on average] produce more offsprings(kids)

→ meaning that the genes which provide greater fitness get increased representation in the population.

Regular V/s Evolutionary game Theory :

Game Theory	Evolutionary Game Theory
• Rational players make decisions consciously & strategically.	• Game theoretic concepts continues to apply even if no player is using any reasoning or making concious decisions.
fundamental Question → How does strategic interaction among the players decide the outcome of a game?	fundamental Ques → which behaviours will persist in a population?

*→ Natural selection replaces the rational behaviour.

*→ Survival of the fittest.

Better fitness means → better reproductive success.

So, In Evolutionary Game theory,

→ In a population, the success of an organism depends on how it interacts with other organisms.

→ It does not make sense to measure fitness of an individual organism, so fitness must be evaluated in the context of full population in which it lives.

Evolutionary Game theory is analogeous to regular game theory as :-

Player → living organism, animals, plants
strategy → encoded behaviour or characteristics
payoff → fitness
Game Matrix → fitness evaluated w.r.t. to other animals.

Example : Beetle's world

→ Beetle's fitness depends on finding and processing food effectively.

→ lets say all the beetles are of the same size.

→ Now suppose a particular mutation is introduced into the population, causing beetles with the mutations to grow a significantly larger body size.

→ Thus we now have two distinct kinds of beetles in the population — small & large.

* It is difficult for the large beetles to maintain the metabolic requirements of their larger body size – so this has a negative effect on the fitness.

What should be consequences of this mutation in the beetle's world?

→ One conclusion can be → that the large body size mutation is decreasing fitness, so it will be wiped out from the population eventually.

→ If we use game theortic notion →
The beetles in the population compete with each other for food — when they come upon a food source, there's crowding among the beetle's as they each try to get as much as food as they can. And the beetle's with large body size are more effective at claiming the ~~food~~. above average share of food.

In case of competition for food between two beetles, lets assume:

→ If both beetle's of same size, they both get equal share.
→ If a beetle is large & other is small, the large beetle gets majority share.
→ A large beetle experience less fitness in comparison to a small beetle for the same quantity of food.

→ Thus the fitness that each beetle gets from a food related interaction can be considered as payoff in a two player beetle game.
→ Both the beetles can play two stratagies: Large & small.

Beetle 2

		Small	Large
Beetle 1	Small	5,5	1,8
	Large	8,1	3,3

6,6	1,10
10,1	4,4

Large, Large = 3,3 ⇒ Since a large beetle has to spend extra energy to compete to another large beetle.

In this game, the beetle's aren't asking themselves, "What do I want my body size to be in this interaction?". Rather each beetle is genetically hard-wired to play one of these two stratagies (small, large) for its whole lifetime.

This is fundamentaly different than regular game theory. (Eg: Prisoner's dillema → Prisoner thinks of whether to confess or not)

Given this difference, the idea of choosing the stratagies -(which was central to our formulation of game theory)- is missing from the biological side of analogy.

AS A RESULT, in place of idea of Nash Equillibrium (which was based on relative benefit of changing one's own personal strategy) we use Evolutionarily Stable Stratagies. - A genetically determined stratagy that tends to persist once it is prevalent (Stable) in a population.

*

		Beetle 2	
		Small	Large
Beetle 1	Small	5,5	1,8
	Large	8,1	3,3

Here we dont need to write the payoff of 2^{nd} player, It can be obtained by transposing the payoffs of 1^{st} player

$$\underset{\text{Player 1}}{\begin{matrix} 5 & 1 \\ 8 & 3 \end{matrix}} \xrightarrow{\text{transpose}} \underset{\text{Player 2.}}{\begin{matrix} 5 & 8 \\ 1 & 3 \end{matrix}}$$

* Suppose in our example, ~~that~~ each beetle is repeatedly paired off with other beetle in food competition.

* Beetle fitness = Average fitness from food interaction = reproductive success

* A given strategy is Evolutionary stable if when the whole population is using ~~that~~ one strategy, any small group of invaders using a different strategy will eventually die off over multiple generations.

lets say -
→ whole population is using strategy S.
→ small group of invaders using strategy T.

Definition

- Strategy T (large Beetle) invades a strategy S (small Beetle) at level x (for small x) if:
  - x fraction of population uses T
  - $1-x$ fraction of population uses S

- Strategy S is evolutionary stable if there is some number y such that:
  - when any other strategy T invades S at any level $x < y$, the fitness of an organism playing S is strictly greater than the fitness of an ~~organism~~ organism playing T.

Beetle 2

		small $(1-x)$	large (x)
Beetle 1	small	5,5	1,8
	large	8,1	3,3

* Is small an evolutionary stable strategy?

Suppose for small no. x,

$(1-x)$ fraction of population use SMALL

x ——— use LARGE

Means, small population of Large Beetles(x) invades the population of small Beetles. If small does better than large then we say that small is evolutionary stable strategy.

~~[illegible]~~ expected payoff of small beetle in a random interaction →

Small meets small with prob. $(1-x)$ ⇒ $5(1-x)$

Small ——— large ——— x ⇒ $1(x)$

∴ Expected payoff $= 5(1-x) + 1(x) = 5-4x$

expected payoff of large beetle in a random interaction →

$$8(1-x) + 3x = 8-5x$$

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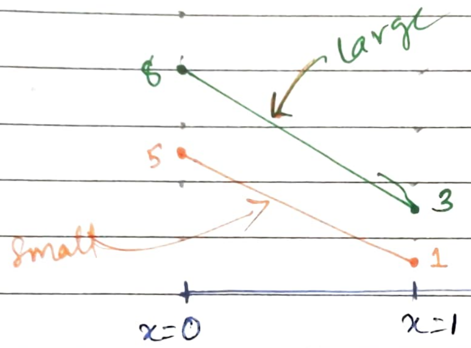
So, expected payoff of small Beetle = $5-4x$
large —— = $8-5x$



Here, a large beetle always does better than small beetle. So no matter how small proportion of large beetles are introduced, in the introduced, they would have better fitness on average, so over the time their population would increase & they would eventually wipe out the population of small beetle.

small	large
$5-4x$	$8-5x$
$x=0 \Rightarrow 5$	$x=0 \Rightarrow 8$
$x=1 \Rightarrow 1$	$x=1 \Rightarrow 3$

So, clearly small is not evolutionary stable.

* Is large an evolutionary stable strategy?

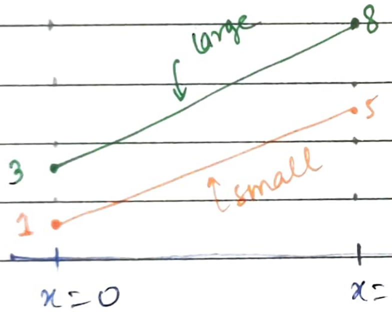
Suppose for small no. x,
$(1-x)$ fraction of population uses LARGE
x —— SMALL

Means, small population of small beetles, invades the population of large beetles. So if large does better than small, then we say that large is evolutionary stable strategy.

	(x) small	$(1-x)$ large
small	5,5	1,8
large	8,1	3,3

expected payoff of small beetles $= 5x + (1-x)1$
$= 4x + 1$

——— large ——— $= 8x + (1-x)\cdot 3$
$= 5x + 3$



Here, large beetle always has better fitness than small beetle, so a small beetle would not be able to invade the population of large beetle.

So, large is evolutionary stable.

large
$x = 0 \to 3$
$x = 1 \to 8$

small
$x = 0 \to 1$
$x = 1 \to 5$

Summary →

* If a few large beetles get introduced into a population of small beetles → large will do really well cause they rarely meet each other and they get most of the food in most competitions.
* conversely, if few small beetles are introduced in a population of large beetles → Small will do badly cause they will loose almost every food competition.