

1. **Axiom.** If A is a set then A is also an object. In particular, given two sets A and B , it is meaningful to ask whether A is also an element of B .
2. **Axiom (Empty set).** There exists a set ϕ , known as the empty set, which contains no elements, i.e., for every object x we have $x \notin \phi$.
3. **Axiom (Replacement).** Let A be a set. For any object $x \in A$, and any object y , suppose we have a statement $P(x, y)$ pertaining to x and y , such that for each $x \in A$ there is at most one y for which $P(x, y)$ is true. Then there exists a set $\{y : P(x, y) \text{ is true for some } x \in A\}$, such that for any object z ,

$$z \in \{y : P(x, y) \text{ is true for some } x \in A\} \\ \Leftrightarrow P \text{ is true for some } x \in A.$$

(De Morgan laws) We have $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$ and $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$.

And we use *cardinality* to describe the counts of the elements in a set.

Equality of functions. Two functions f, g with the same domain and range are said to be equal, if and only if for every $x \in X$, $f(x) = g(x)$.

Let's look at the function $f : \phi \rightarrow X$, from the empty set to an arbitrary set X . We can say that for every X , there's only one empty function.

Lemma (Composition is associative). Let $f : Z \rightarrow W$, $g : Y \rightarrow Z$, $h : X \rightarrow Y$ be functions. Then $f \circ (g \circ h) = (f \circ g) \circ h$.

A function is one-to-one (or injective) if

$$f(x) = f(x') \Rightarrow x = x'$$

A function is onto (or surjective) if

$$\forall y \in Y, \exists x \in X \text{ such that } f(x) = y$$

If f is both onto and one-to-one, then it's bijective.

If f is bijective, then for every $y \in Y$, there is exactly one x such that $f(x) = y$. This value of x is denoted as $f^{-1}(y)$. Thus f^{-1} is a function from Y to X , the reverse of f .