1. **Axiom.** *If A is a set then A is also an object. In particular, given two sets A and B, it is meaningful to ask whether A is also an element of B.*

2. **Axiom (Empty set).** *There exists a set ϕ, known as the empty set,which contains no elements, i.e., for every object x we have $x \notin \phi$.*

3. **Axiom (Replacement).** *Let A be a set. For any object $x \in A$, and any object y, suppose we have a statement $P(x, y)$ pertaining to x and y, such that for each $x \in A$ there is at most one y for which $P(x, y)$ is true. Then there exsists a set $\{y : P(x, y)\ is\ true\ for\ some\ x \in A\}$, such that for any object z,*

$$z \in \{y : P(x, y)\ is\ true\ for\ some\ x\ \in A\}$$
$$\Leftrightarrow P\ is\ true\ for\ some\ x \in A.$$

(De Morgan laws) We have $X \backslash (A \cup B) = (X \backslash A) \cap (X \backslash B)$ and $X \backslash (A \cap B) = (X \backslash A) \cup (X \backslash B)$.

And we use *cardinality*to describe the counts of the elements in a set.

Equality of functions. *Two functions f, g with the same domain and rage are said to be equal, if and only if for every $x \in X$, $f(x) = g(x)$.*

Let's look at the function $f : \phi \to X$, from the empty set to an arbitrary set X. We can say that for every X, there's only one empty function.

Lemma (Composition is associative). *Let $f : Z \to W$, $g : Y \to Z$, $h : X \to Y$ be funcitons. Then $f \circ (g \circ h) = (f \circ g) \circ h$.*

A function is *one-to-one*(or *injective*) if

$$f(x) = f(x') \Rightarrow x = x'$$

A function is *onto*(or *surjective*) if

$$\forall y \in Y, \exists x \in X such that f(x) = y$$

If f is both onto and one-to-one, then it's *bijective*.

If f is bijective, then for every $y \in Y$, there is exactly one x such that $f(x) = y$. This value of x is denoted as $f^{-1}(y)$. Thus f^{-1} is a function from Y to X, the reverse of f.