Time Series Analysis

Fall 2017

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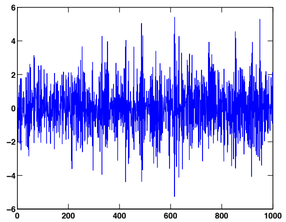
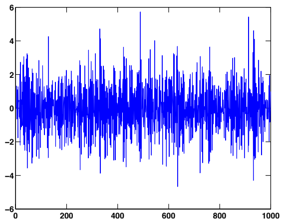
Multivariate systems





We consider a VAR(2), with

$$\mathbf{y}_t + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} = \mathbf{e}_t$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 0.5 & 0.4 \\ 0.1 & 0.8 \end{bmatrix} \qquad \mathbf{A}_2 = \begin{bmatrix} -0.2 & -0.1 \\ 0.3 & 0.6 \end{bmatrix}$$

Multivariate systems

	$k=1$	$k=2$	$k=3$	$k=4$
$\boldsymbol{\rho}_{\mathbf{y}}(k)$	$\begin{bmatrix} -0.67 & -0.36 \\ 0.31 & -0.57 \end{bmatrix}$	$\begin{bmatrix} 0.41 & 0.50 \\ -0.43 & -0.08 \end{bmatrix}$	$\begin{bmatrix} -0.14 & -0.37 \\ 0.33 & 0.43 \end{bmatrix}$	$\begin{bmatrix} 0.00 & 0.13 \\ -0.13 & -0.38 \end{bmatrix}$
	$\begin{bmatrix} - & - \\ + & - \end{bmatrix}$	$\begin{bmatrix} + & + \\ - & - \end{bmatrix}$	$\begin{bmatrix} - & - \\ - & + \end{bmatrix}$	$\begin{bmatrix} \cdot & + \\ - & - \end{bmatrix}$
$\boldsymbol{\phi}_{k,k}$	$\begin{bmatrix} -0.67 & 0.31 \\ -0.36 & -0.57 \end{bmatrix}$	$\begin{bmatrix} 0.16 & 0.10 \\ -0.11 & -0.52 \end{bmatrix}$	$\begin{bmatrix} -0.01 & -0.04 \\ 0.05 & -0.04 \end{bmatrix}$	$\begin{bmatrix} 0.07 & -0.01 \\ -0.03 & 0.01 \end{bmatrix}$
	$\begin{bmatrix} - & + \\ - & - \end{bmatrix}$	$\begin{bmatrix} + & + \\ - & - \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} + & \cdot \\ \cdot & \cdot \end{bmatrix}$

Our identification indicates that:

- The Box-Cox plot indicates that no transformation is required.
- No detrending seems to be required.
- We estimate ACF and PACF to determine model structure; these suggest a VAR structure of order 2 (here, $2/\sqrt{N} \approx 0.063$).
- The multivariate Jarque-Bera test indicates that the ACF and PACF are normal distributed (use `mjbtest`).

Multivariate systems

Model order selection:

$$Q^* = N^2 \sum_{\ell=1}^{K} (N-\ell)^{-1} tr \left\{ \hat{\mathbf{R}}_e^T(\ell)\hat{\mathbf{R}}_e^{-1}(0)\hat{\mathbf{R}}_e^T(\ell)\hat{\mathbf{R}}_e^{-1}(0) \right\}$$

$$M_p = -\left(N - p - mp - \frac{1}{2}\right) \ln \left[\frac{|\hat{\mathbf{\Sigma}}_p|}{|\hat{\mathbf{\Sigma}}_{p-1}|}\right]$$

$$AIC(p) = N \ln \left[|\hat{\mathbf{\Sigma}}_p|\right] + 2pm^2$$

$$BIC(p) = N \ln \left[|\hat{\mathbf{\Sigma}}_p|\right] + pm^2 \ln N$$

$$FPE(p) = \left[\frac{N + mp + 1}{N - mp - 1}\right]^m |\hat{\mathbf{\Sigma}}_p|$$

where $Q^* \in \chi^2_{1-\alpha}\left\{m^2(K-p-q)\right\}$ (use `lbpTest`) and $M_p \in \chi^2_{1-\alpha}(m^2)$

Here, $\hat{\mathbf{\Sigma}}_p$ denotes the covariance matrix of the residual when using a model of order p, i.e.,

$$\hat{\mathbf{\Sigma}}_p = \frac{1}{N-p}\left(\mathbf{Y} - \mathbf{X}_p^*\hat{\boldsymbol{\theta}}_p\right)^*\left(\mathbf{Y} - \mathbf{X}_p^*\hat{\boldsymbol{\theta}}_p\right) = \frac{\mathbf{Y}^*\mathbf{\Pi}^{\perp}_{\mathbf{X}_p}\mathbf{Y}}{N-p}$$

Multivariate systems

p	0	1	2	3	4	5
Q^*	393	315	15	18	16	12
M_p		1404	412	3.7	6.4	4.3
AIC	1602	352	21	27	30	34
BIC	1622	391	80	106	128	152
FPE	8.614	1.626	1.021	1.029	1.035	1.037

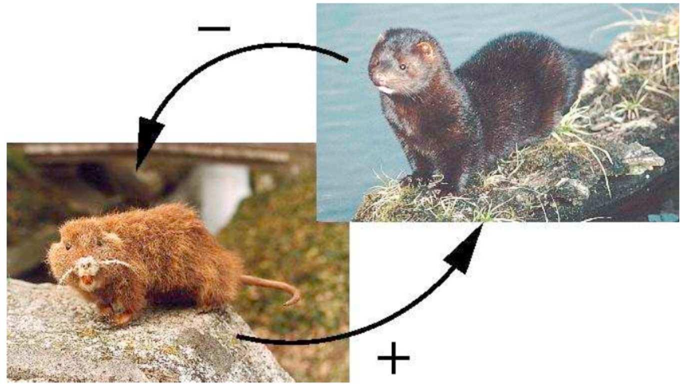
Here, the 95% quantile for Q^* is approximately 102, and for M_p about 9.5.

Note that the AIC, BIC, and FPE all have minimum for order 2.

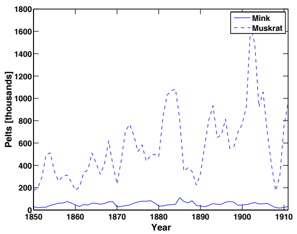
Using LS (use `lsVAR`), we estimate the unknown parameters

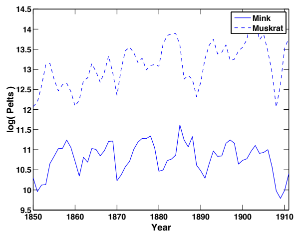
$$\mathbf{A}_1 = \begin{bmatrix} 0.52 & 0.36 \\ 0.07 & 0.28 \end{bmatrix} \qquad \mathbf{A}_2 = \begin{bmatrix} -0.17 & -0.12 \\ 0.24 & 0.58 \end{bmatrix}$$

Multivariate systems



Multivariate systems





Denote the muskrat and mink time series $y_{t,1}$ and $y_{t,2}$, respectively, and form

$$\mathbf{z}_t = \begin{bmatrix} \log y_{t,1} - \hat{m}_{\tilde{y}_1} \\ \log y_{t,2} - \hat{m}_{\tilde{y}_2} \end{bmatrix}$$

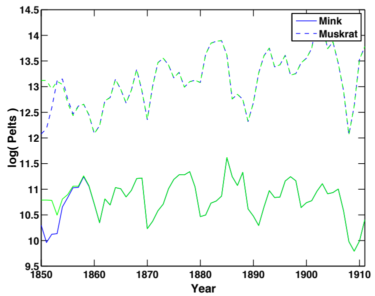
where $\hat{m}_{\tilde{y}_1} = 10,79$ and $\hat{m}_{\tilde{y}_2} = 13,12$ denote the estimated mean of the transformed muskrat and mink data sets, respectively.

Multivariate systems

	$k=1$	$k=2$	$k=3$	$k=4$
$\rho_{\mathbf{y}}(k)$	$\begin{bmatrix} 0.68 & 0.47 \\ -0.13 & 0.73 \end{bmatrix}$	$\begin{bmatrix} 0.29 & 0.38 \\ -0.27 & 0.37 \end{bmatrix}$	$\begin{bmatrix} -0.02 & 0.25 \\ -0.16 & 0.20 \end{bmatrix}$	$\begin{bmatrix} -0.25 & 0.08 \\ 0.03 & 0.13 \end{bmatrix}$
	$\begin{bmatrix} + & + \\ \cdot & + \end{bmatrix}$	$\begin{bmatrix} + & + \\ - & + \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$
$\phi_{k,k}$	$\begin{bmatrix} 0.68 & -0.13 \\ 0.47 & 0.73 \end{bmatrix}$	$\begin{bmatrix} -0.17 & -0.27 \\ 0.11 & -0.18 \end{bmatrix}$	$\begin{bmatrix} -0.24 & 0.10 \\ 0.08 & 0.15 \end{bmatrix}$	$\begin{bmatrix} -0.27 & -0.19 \\ 0.07 & -0.07 \end{bmatrix}$
	$\begin{bmatrix} + & \cdot \\ + & + \end{bmatrix}$	$\begin{bmatrix} \cdot & - \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} - & \cdot \\ \cdot & \cdot \end{bmatrix}$

	$k=5$	$k=6$	$k=7$	$k=8$
$\rho_{\mathbf{y}}(k)$	$\begin{bmatrix} -0.34 & -0.11 \\ 0.23 & 0.08 \end{bmatrix}$	$\begin{bmatrix} -0.30 & -0.33 \\ 0.35 & 0.05 \end{bmatrix}$	$\begin{bmatrix} -0.12 & -0.45 \\ 0.41 & 0.11 \end{bmatrix}$	$\begin{bmatrix} 0.10 & -0.47 \\ 0.40 & 0.25 \end{bmatrix}$
	$\begin{bmatrix} + & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} - & - \\ + & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & - \\ + & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & - \\ + & \cdot \end{bmatrix}$
$\phi_{k,k}$	$\begin{bmatrix} -0.21 & -0.12 \\ 0.00 & -0.22 \end{bmatrix}$	$\begin{bmatrix} -0.18 & -0.23 \\ -0.20 & -0.11 \end{bmatrix}$	$\begin{bmatrix} 0.02 & -0.15 \\ -0.03 & -0.09 \end{bmatrix}$	$\begin{bmatrix} -0.02 & -0.22 \\ 0.04 & 0.11 \end{bmatrix}$
	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$

Multivariate systems



p	0	1	2	3	4	5
Q^*	41	24	29	20	18	16
M_p		122.0	12.6	8.5	8.7	5.3
AIC	-189	-317	-324	-327	-330	-330
BIC	-181	-300	-299	-293	-288	-279
FPE	0.0506	0.0065	0.0057	0.0055	0.0052	0.0053

Here, the 95% quantile for Q^* is approximately 55.8, and for M_p about 9.5.