LUND UNIVERSITY

Time Series Analysis

Fall 2017

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Prediction

The optimal k-step predictor of the ARMA process $A(z)y_t = C(z)e_t$ is obtained as

$$\hat{y}_{t+k} = \frac{G(z)}{C(z)} y_t$$

where $C(z) = A(z)F(z) + z^{-k}G(z)$. Here,

$$ord(G) = \max(p-1, q-k)$$
$$ord(F) = k-1$$

The prediction error is given as

$$\epsilon_{t+k|t} = y_{t+k} - \hat{y}_{t+k|t} = F(z)e_{t+k}$$

Note that this is an MA(k-1) process!

Prediction

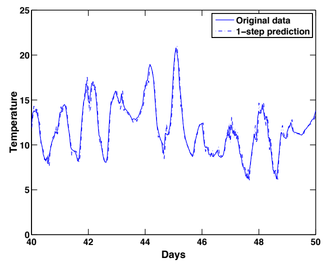
Matlab example of 5-step prediction of

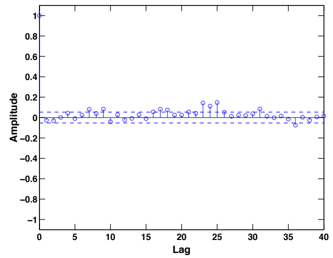
$$(1-0.2z^{-1})\nabla_{12} y_t = (1-0.3z^{-12})e_t$$

Code:

```
s = 12;
k = 5;
C = [ 1 zeros(1,11) -0.3 ];
A = conv( [1 -0.2 ], [ 1 zeros(1,s-1) -1 ] );
[A,C] = equalLength( A, C );
[ F, G ]= deconv( conv( [ 1 zeros(1,k-1) ], C ), A );
```


Prediction



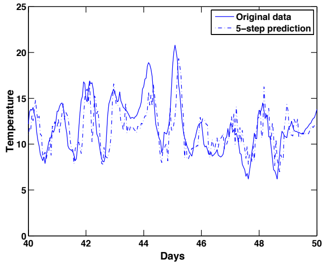


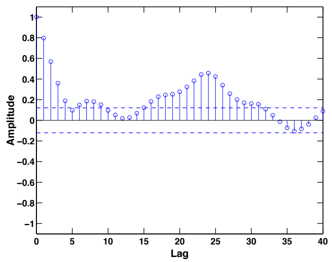
Modeling the temperature in Svedala - first model:

$$A(z) = 1 - 1.79z^{-1} + 0.84z^{-2}$$
$$C(z) = 1 - 0.18z^{-1} - 0.11z^{-2}$$

One-step prediction... Is the prediction residual white? It is deemed normal distributed.

Prediction





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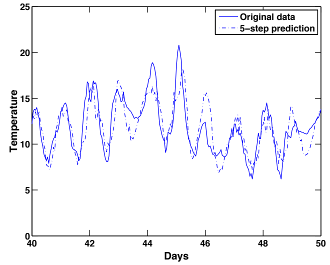
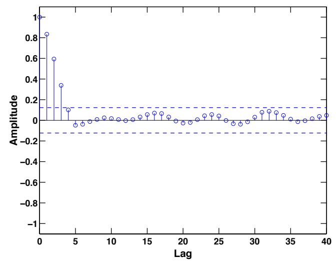
Five-step prediction... Is the prediction residual an MA(4)?

Prediction





Modeling the temperature in Svedala - second model:

$$\begin{aligned} A(z) =& 1 - 3.7511z^{-1} + 5.3368z^{-2} - 3.4079z^{-3} + 0.8224z^{-4} \\ C(z) =& 1 - 2.3056z^{-1} + 1.4174z^{-2} + 0.0773z^{-3} - \\ & 0.0633z^{-4} - 0.1176z^{-5} \end{aligned}$$

Five-step prediction... Is the prediction residual an MA(4)?

Prediction

Similarly, an ARMAX process $A(z)y_t = B(z)x_t + C(z)e_t$ can be predicted as

$$\hat{y}_{t+k} = \hat{F}(z)E\{x_{t+k}|\mathbf{Y}_t, \mathbf{X}_t\} + \frac{\hat{G}(z)}{C(z)}x_t + \frac{G(z)}{C(z)}y_t$$

where

$$F(z)B(z) = C(z)\hat{F}(z) + z^{-k}\hat{G}(z)$$

with

$$ord(\hat{G}) = \max\Big(ord(C) - 1, ord(FB) - k\Big)$$

Note that this is the same order rules as before, but for the new polynomials.

Prediction

Matlab example:

$$(1 - 0.2z^{-1})\nabla_{12}y_t = (1 - 0.3z^{-12})e_t + (1 + 0.3z^{-1} + 0.4z^{-3})x_{t-4}$$

Then,

$$B(z) = z^{-4} + 0.3z^{-5} + 0.4z^{-7}$$

We obtain F(z) and G(z) as before, and

```
B = [ 0 0 0 0 1 0.3 0 0.4 ];
BF = conv( B, F );
[C,BF] = equalLength( C, BF );
[Fhat,Ghat] = deconv( conv( [1 zeros(1,k-1) ], BF ), C );
```