Time Series Analysis

Fall 2017

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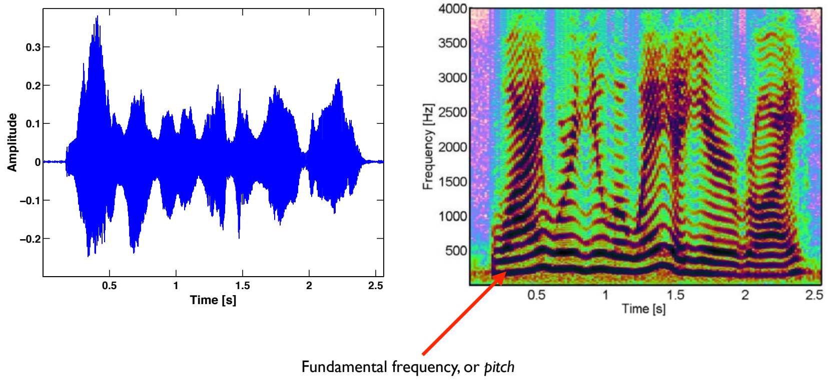
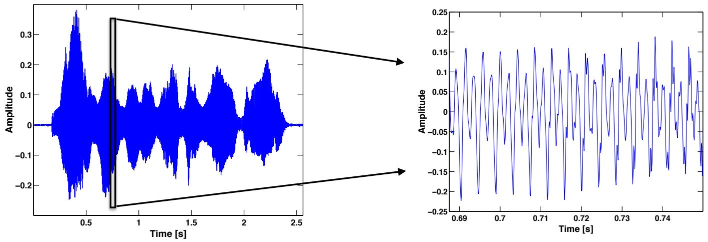
”Why were you away a year, Roy?”



”Why were you away a year, Roy?”



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- Such segments can be modelled as *voiced* or *unvoiced*.

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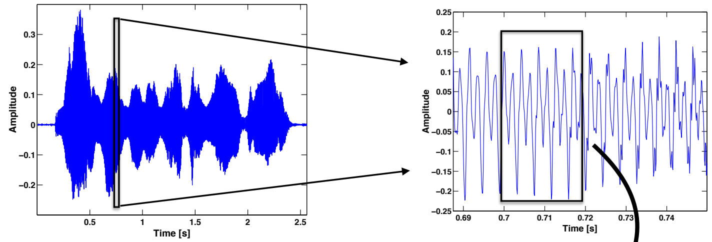

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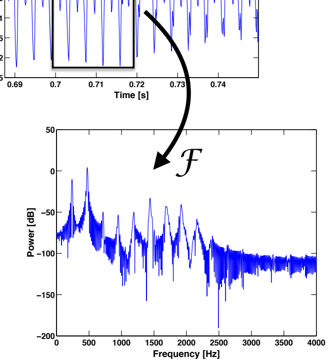
- Such segments can be modelled as *voiced* or *unvoiced*.
- Voiced speech can be well modelled as a sum of sinusoids.



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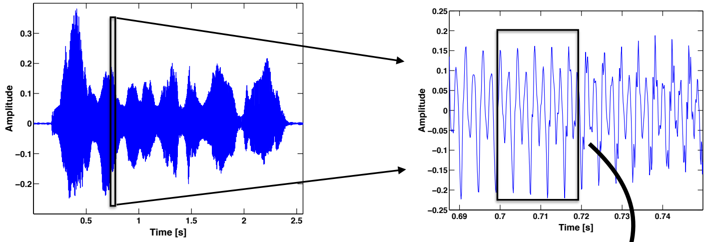
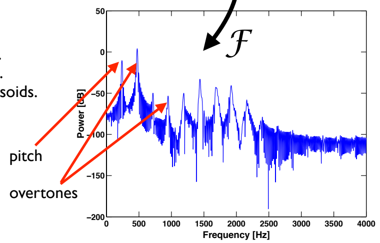




- Speech is reasonably stationary over 20 ms intervals.
- Such segments can be modelled as *voiced* or *unvoiced*.
- Voiced speech can be well modelled as a sum of sinusoids.

We need to determine the pitch frequency as well as the phases and amplitudes of the sinusoids.

Estimating the unknown parameters

Using a complex-valued notation, the signal may be expressed as

$$
\begin{aligned}
y_t &= \sum_{k=1}^{d} \alpha_k e^{i\omega_0 kt} + e_t \\
&= \begin{bmatrix} e^{i\omega_0 t} & \dots & e^{i\omega_0 dt} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_d \end{bmatrix} + e_t \\
&= \mathbf{x}_t^T \boldsymbol{\theta} + e_t
\end{aligned}
$$

Gathering N samples in a vector

$$
\begin{aligned}
\mathbf{y} &= \begin{bmatrix} y_1 & \dots & y_N \end{bmatrix}^T \\
&= \begin{bmatrix} e^{i\omega_0} & \dots & e^{i\omega_0 d} \\ \vdots & \ddots & \vdots \\ e^{i\omega_0 N} & \dots & e^{i\omega_0 dN} \end{bmatrix} \boldsymbol{\theta} + \mathbf{e} \\
&= \mathbf{X}\boldsymbol{\theta} + \mathbf{e}
\end{aligned}
$$

Estimating the unknown parameters

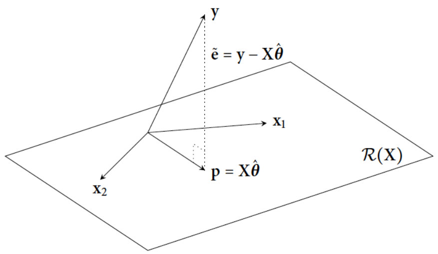
The signal can thus be expressed as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \mathbf{e}$$

where the regressor $\mathbf{X}$ depends on the unknown pitch ω_0. Assuming for the moment that we know ω_0, the (unknown) complex-valued amplitudes $\boldsymbol{\theta}$ may be estimated as

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}$$

where $\mathbf{p} = \mathbf{X}\hat{\boldsymbol{\theta}} = \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}$ is the projection of $\mathbf{y}$ onto $\mathbf{X}$.



Estimating the unknown parameters

However, we don't know ω_0. To solve this, find the ω_0 that minimize the difference between $\mathbf{y}$ and its projection, i.e.,

$$
\begin{aligned}
\hat{\omega}_0 &= \arg\min_{\omega_0} \left\| \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\theta}} \right\|_2^2 \\
&= \arg\min_{\omega_0} \mathbf{y}^T \left(\mathbf{I} - \boldsymbol{\Pi}_{\mathbf{X}}\right)\mathbf{y} \\
&= \arg\max_{\omega_0} \mathbf{y}^T \boldsymbol{\Pi}_{\mathbf{X}} \mathbf{y}
\end{aligned}
$$

where $\boldsymbol{\Pi}_{\mathbf{X}} = \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T$ is the projection matrix onto $\mathbf{X}$.