Lotka - Volterra Equations

The Lotka-Volterra are a pair of first order ordinary differential equations.These equations come from the simple population model proposed Thomas Malthus, where the constant *k* is determined by the birth and death rate. Thus each parameter influences the birth and death rate for the prey and predators.

In exploring the values for alpha and beta, the initial assumption the population dynamic change was wrong. Even though alpha emulates the birth rate of the prey, the prey's population dynamic decreases in relation to the predators. This is because increasing the prey numbers also increases the predator numbers because the number of prey (the variable x) directly calculates the birth rate for the predators. Since the predator population determines the prey death rate, the prey population dynamic ultimately decreases. Similarly increasing beta (the predation rate) increases the prey dynamic in relation to the predators. This is because increasing the amount of prey the predators consume decreases the prey population, and then as a result decreases the food supply which then makes the predator dynamics decrease. When exploring the values for delta and gamma, the initial assumptions of how the population dynamics would change were accurate. Increasing delta, the reproduction rate of the predators per prey eaten, the predator dynamic increased in relation to the prey dynamics. Similarly when increasing gamma, the predator mortality rate, the prey dynamic increased relative to the predator dynamics.

When graphing these results, two very different graphs can be produced. One graph, plotting population over time, produces a series of cycles through time. Changing each of the parameters also influenced the number of cycles per time. Depending on the nature of the ecosystems, increasing or decreasing the number of cycles can either help or hinder the ecosystem, as some may be able to adapt to rapid dynamic changes and others are not able to accomodate for these faster cycles. Alpha and gamma are the parameters that influence this cyclical nature, whereas beta and delta do not change the cyclical nature. Plotting predators over prey resulted in a graph that was very similar to the shape of ellipses, where each ring changed as the integration constant changed.

Looking at the graphs that were produced by numerically approximating the solution, it appeared that there was an overall downward sloping trend. It was initially assumed that this trend indicated that this model would naturally approach equilibrium. However, this assumption was wrong. This version of the Lotka - Volterra model depicts the dynamics of predators and prey at a point that is a certain distance from equilibrium. The behavior at the equilibrium point depends on the values of the parameters, and this point can behave in three different ways, it can either spiral outwards, spiral inwards, or stabilize. In this application, the cycles stabilized at a fixed ellipse. In order to understand these graphs, the nonlinear ordinary differential equations had to be linearized. Linearizing these functions enables interpretations of the behavior of the graph at both stability points. By finding eigenvalues and eigenvectors and using the chain rule, it was understood that for each of the graphs created, the peaking point should be the same. The graphs that were initially plotted were incorrect, in that the error created a downward trend of the peaking points during each cycle in time. This indicates the importance of taking in account the effect that error has when doing a numerical approximation.