Total No. of Questions : 4]

SEAT No. :

PC-69

[Total No. of Pages : 2

[6360]-71

T.E. (Mechanical/Mechanical-Sandwich) (Insem)

NUMERICAL AND STATISTICAL METHODS

(2019 Pattern) (Semester - I) (302041)

Time : 1 Hour] *[Max. Marks : 30*

Instructions to the candidates :

1) *Answer Q1 or Q2, Q3 or Q4.*
2) *Neat Diagrams must be drawn wherever necessary.*
3) *Figure to the right indicate full marks.*
4) *Assume suitable data if necessary.*

Q1) a) Draw the flow chart for Newton-Raphson method on iteration-based criteria. **[6]**

b) Solve the simultaneous equations using Gauss elimination method. **[9]**

$4y + 2z = 12$

$x + 3y + 5z = 0$

$3x + y + z = 11$

OR

Q2) a) The following polynomial has a root within the interval $3.75 \leq x \leq 5.00$; $f(x) = x^3 - x^2 - 10x - 8$ If a tolerance of 0.01 (1%) is required, find this root using bisection method. **[8]**

b) Solve the following set of simultaneous equations using Thomas Algorithm. **[7]**

$x + 2y = 3,$

$2x + 3y + z = 4,$

$2y - z = 1$

P.T.O.

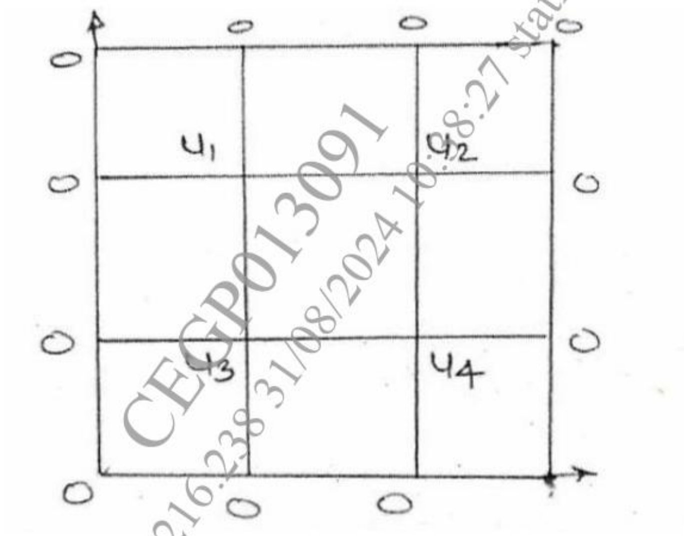
Q3) a) Using Runge Kutta method of fourth order, solve $dy/dx = \mathbf{y}^2 + xy$ with initial condition $y(1) = 1$ at $x = 1.1$. Take $h = 0.05$. **[6]**

b) A steel plate of 750mm × 750mm has its two adjacent sides maintained at 100°C while the other two sides are maintained at 0°C. What will be the steady state temperature at interior assuming a grid size of 250 mm. **[9]**

OR

Q4) a) Draw the flow chart for Euler method for solving differential equations. **[5]**

b) Solve the Poisson's equation $\nabla^2 u = 2x^2y^2$ over the square domain $0 \le x \le 3$ and $0 \le y \le 3$, with $u = 0$ on the boundary and Mesh length = 1. **[10]**



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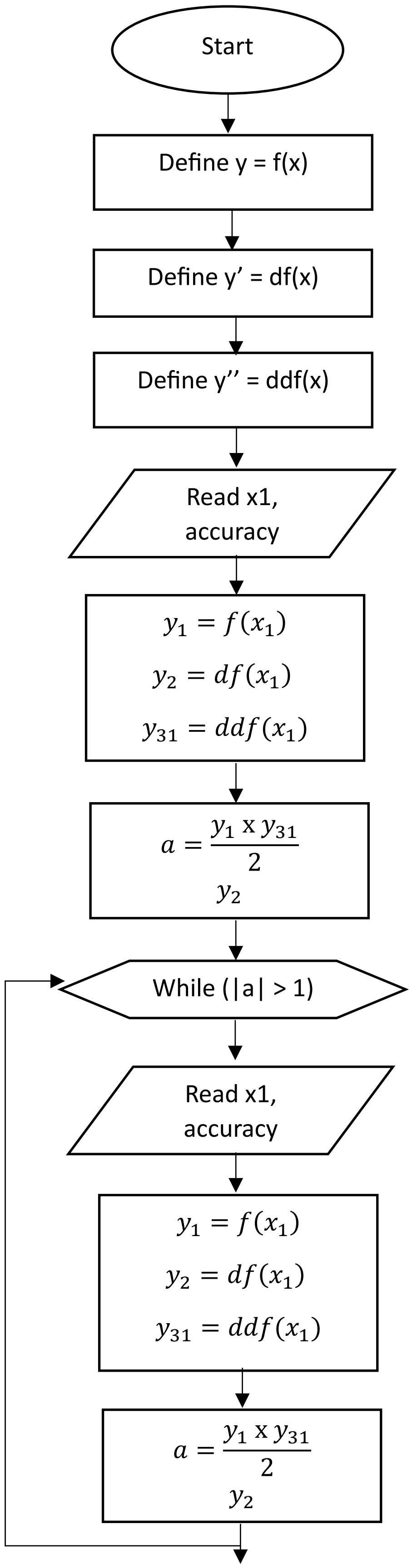
Q1) a) Draw the flow chart for Newton-Raphson method on iteration-based criteria.
Ans:



Define given function as

$y = f(x) = 0.51x - \sin(x)$

Define the derivative

$y' = f'(x) = 0.51 - \cos(x)$

Define the second derivative

$y'' = f''(x) = \sin(x)$

Compute function values

$y_1 = f(x_1)$

$y_2 = f'(x_1) = df(x_1)$

$y_{31} = f''(x_1) = ddf(x_1)$

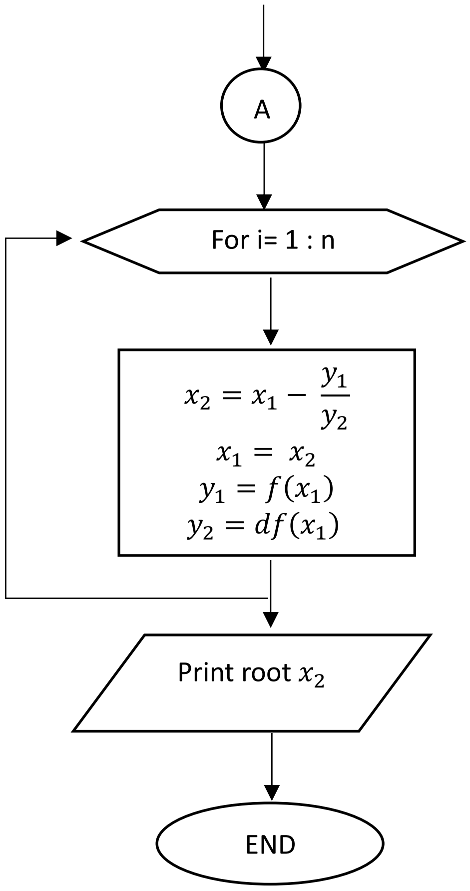
Compute parameter a

$$a = \frac{y_1 \times y_{31}}{y_2^2} = \frac{f(x_1) \times f''(x_1)}{[f'(x_1)]^2}$$

Use x_1 and again find
The value of y_1, y_2, y_{31}

Again, find the value of a
The process is repeated till we get |a| < 1 or

$$\frac{f(x_1)\, f''(x_1)}{[f'(x_1)]^2} < 1$$



Once the number of iterations is over, then the root for the given number of iterations is x_2 last and print x, on the output screen

Q1) b) Solve the simultaneous equations using Gauss elimination method.

4y + 2z = 12

x + 3y + 5z = 0

3x + y + z = 11

Ans:

Given System of Equations:

1. $4y + 2z = 12$
2. $x + 3y + 5z = 0$
3. $3x + y + z = 11$

Step 1: Rewrite the System in Standard Form (Order: x, y, z):

$$\begin{cases} 0x + 4y + 2z = 12 \quad \text{(Equation 1)} \\ 1x + 3y + 5z = 0 \quad \text{(Equation 2)} \\ 3x + 1y + 1z = 11 \quad \text{(Equation 3)} \end{cases}$$

Step 2: Form the Augmented Matrix:

$$\begin{bmatrix} 0 & 4 & 2 & | & 12 \\ 1 & 3 & 5 & | & 0 \\ 3 & 1 & 1 & | & 11 \end{bmatrix}$$

Step 3: Perform Row Operations for Upper Triangular Form:

- **Swap Row 1 and Row 2** (to have a non-zero pivot in the first row):

$$\begin{bmatrix} 1 & 3 & 5 & | & 0 \\ 0 & 4 & 2 & | & 12 \\ 3 & 1 & 1 & | & 11 \end{bmatrix}$$

- **Eliminate x from Row 3**:

  Replace Row 3 with $\text{Row } 3 - 3 \times \text{Row } 1$:

$$\text{Row } 3 = [3, 1, 1, 11] - 3 \times [1, 3, 5, 0] = [0, -8, -14, 11]$$

Updated Matrix:

$$\begin{bmatrix} 1 & 3 & 5 & | & 0 \\ 0 & 4 & 2 & | & 12 \\ 0 & -8 & -14 & | & 11 \end{bmatrix}$$

- **Eliminate y from Row 3**:

  Replace Row 3 with $\text{Row } 3 + 2 \times \text{Row } 2$:

$$\text{Row } 3 = [0, -8, -14, 11] + 2 \times [0, 4, 2, 12] = [0, 0, -10, 35]$$

  Final Upper Triangular Matrix:

$$\begin{bmatrix} 1 & 3 & 5 & | & 0 \\ 0 & 4 & 2 & | & 12 \\ 0 & 0 & -10 & | & 35 \end{bmatrix}$$

Step 4: Back Substitution:

1. **From Row 3**:

$$-10z = 35 \implies z = -3.5$$

2. **From Row 2**:

$$4y + 2z = 12 \implies 4y + 2(-3.5) = 12 \implies 4y - 7 = 12 \implies 4y = 19 \implies y = 4.75$$

3. **From Row 1**:

$$x + 3y + 5z = 0 \implies x + 3(4.75) + 5(-3.5) = 0 \implies x + 14.25 - 17.5 = 0$$

$$x = 3.25$$

Final Solution:

$$x = 3.25, \quad y = 4.75, \quad z = -3.5$$

Q2) a) The following polynomial has a root within the interval 3.75 < × < 5.00; f(x) = x3 – x2 – 10x – 8 If a tolerance of 0.01 (1%) is required, find this root using bisection method.
Ans:

Given:

- Polynomial: $f(x) = x^3 - x^2 - 10x - 8$
- Interval: $[3.75, 5.00]$
- Tolerance: 0.01

Step 1: Verify the Initial Interval

Check if $f(a)$ and $f(b)$ have opposite signs (Intermediate Value Theorem).

- $f(3.75) = (3.75)^3 - (3.75)^2 - 10(3.75) - 8$
  $= 52.734375 - 14.0625 - 37.5 - 8$
  $= -6.828125$
- $f(5.00) = (5.00)^3 - (5.00)^2 - 10(5.00) - 8$
  $= 125 - 25 - 50 - 8$
  $= 42$

Since $f(3.75) = -6.828125$ (negative) and $f(5.00) = 42$ (positive), a root exists in $[3.75, 5.00]$.

Step 2: Apply the Bisection Method Iteratively

Iteration 1:

- Midpoint $c_1 = \frac{3.75+5.00}{2} = 4.375$
- $f(4.375) = (4.375)^3 - (4.375)^2 - 10(4.375) - 8$
  $= 83.740234 - 19.140625 - 43.75 - 8$
  $= 12.849609$

Since $f(3.75) < 0$ and $f(4.375) > 0$, the root lies in $[3.75, 4.375]$.

Iteration 2:

- Midpoint $c_2 = \frac{3.75+4.375}{2} = 4.0625$
- $f(4.0625) = (4.0625)^3 - (4.0625)^2 - 10(4.0625) - 8$
  $= 67.028320 - 16.503906 - 40.625 - 8$
  $= 1.899414$

Since $f(3.75) < 0$ and $f(4.0625) > 0$, the root lies in $[3.75, 4.0625]$.

Iteration 3:

- Midpoint $c_3 = \frac{3.75+4.0625}{2} = 3.90625$
- $f(3.90625) = (3.90625)^3 - (3.90625)^2 - 10(3.90625) - 8$
  $= 59.677124 - 15.258789 - 39.0625 - 8$
  $= -2.644165$

Since $f(3.90625) < 0$ and $f(4.0625) > 0$, the root lies in $[3.90625, 4.0625]$.

Iteration 4:

- Midpoint $c_4 = \frac{3.90625+4.0625}{2} = 3.984375$
- $f(3.984375) = (3.984375)^3 - (3.984375)^2 - 10(3.984375) - 8$
  $= 63.247208 - 15.875244 - 39.84375 - 8$
  $= -0.471786$

Since $f(3.984375) < 0$ and $f(4.0625) > 0$, the root lies in $[3.984375, 4.0625]$.

Iteration 5:

- Midpoint $c_5 = \frac{3.984375+4.0625}{2} = 4.0234375$
- $f(4.0234375) = (4.0234375)^3 - (4.0234375)^2 - 10(4.0234375) - 8$
  $= 65.119202 - 16.188049 - 40.234375 - 8$
  $= 0.696778$

Since $f(3.984375) < 0$ and $f(4.0234375) > 0$, the root lies in $[3.984375, 4.0234375]$.

Iteration 6:

- Midpoint $c_6 = \frac{3.984375+4.0234375}{2} = 4.00390625$
- $f(4.00390625) = (4.00390625)^3 - (4.00390625)^2 - 10(4.00390625) - 8$
  $= 64.187622 - 16.031265 - 40.0390625 - 8$
  $= 0.117294$

Since $f(3.984375) < 0$ and $f(4.00390625) > 0$, the root lies in $[3.984375, 4.00390625]$.

Iteration 7:

- Midpoint $c_7 = \frac{3.984375+4.00390625}{2} = 3.994140625$
- $f(3.994140625) = (3.994140625)^3 - (3.994140625)^2 - 10(3.994140625) - 8$
  $= 63.717178 - 15.953158 - 39.94140625 - 8$
  $= -0.177386$

Since $f(3.994140625) < 0$ and $f(4.00390625) > 0$, the root lies in $[3.994140625, 4.00390625]$.

Iteration 8:

- Midpoint $c_8 = \frac{3.994140625+4.00390625}{2} = 3.9990234375$
- $f(3.9990234375) = (3.9990234375)^3 - (3.9990234375)^2 - 10(3.9990234375) - 8$
  $= 63.952911 - 15.992188 - 39.990234375 - 8$
  $= -0.029511$

Since $f(3.9990234375) < 0$ and $f(4.00390625) > 0$, the root lies in $[3.9990234375, 4.00390625]$.

Iteration 9:

- Midpoint $c_9 = \frac{3.9990234375+4.00390625}{2} = 4.00146484375$
- $f(4.00146484375) = (4.00146484375)^3 - (4.00146484375)^2 - 10(4.00146484375) - 8$
  $= 64.070335 - 16.011719 - 40.0146484375 - 8$
  $= 0.043967$

Since $f(3.9990234375) < 0$ and $f(4.00146484375) > 0$, the root lies in $[3.9990234375, 4.00146484375]$.

Iteration 10:

- Midpoint $c_{10} = \frac{3.9990234375+4.00146484375}{2} = 4.000244140625$
- $f(4.000244140625) = (4.000244140625)^3 - (4.000244140625)^2 - 10(4.000244140625) - 8$
  $= 64.011719 - 16.001953 - 40.00244140625 - 8$
  $= 0.007324$

Since $f(3.9990234375) < 0$ and $f(4.000244140625) > 0$, the root lies in $[3.9990234375, 4.000244140625]$.

Iteration 11:

- Midpoint $c_{11} = \frac{3.9990234375+4.000244140625}{2} = 3.9996337890625$
- $f(3.9996337890625) = (3.9996337890625)^3 - (3.9996337890625)^2 - 10(3.9996337890625) - 8$
  $= 63.982162 - 15.996948 - 39.996337890625 - 8$
  $= -0.011123$

Since $f(3.9996337890625) < 0$ and $f(4.000244140625) > 0$, the root lies in $[3.9996337890625, 4.000244140625]$.

Check Tolerance:

The width of the interval is $4.000244140625 - 3.9996337890625 = 0.0006103515625$, which is less than the tolerance 0.01.

Final Answer:

The approximate root of $f(x) = 0$ within the interval $[3.75, 5.00]$ with a tolerance of 0.01 is:

$$\boxed{4.00}$$

Q2) b) Solve the following set of simultaneous equations using Thomas Algorithm.

x + 2y = 3,

2x + 3y + z = 4,

2y – z = 1

Ans:

Given System of Equations:

1. $x + 2y = 3$
2. $2x + 3y + z = 4$
3. $2y - z = 1$

Step 1: Identify the Tridiagonal System

The Thomas Algorithm is applicable to tridiagonal systems of the form:

$$a_i x_{i-1} + b_i x_i + c_i x_{i+1} = d_i \quad \text{for} \quad i = 1, 2, \ldots, n.$$

Here, the variables are x, y, z. Rewrite the equations to match the tridiagonal form:

1. $x + 2y = 3$
   $\Rightarrow b_1 x + c_1 y = d_1$ (where $a_1 = 0$).
2. $2x + 3y + z = 4$
   $\Rightarrow a_2 x + b_2 y + c_2 z = d_2$.
3. $2y - z = 1$
   $\Rightarrow a_3 y + b_3 z = d_3$ (where $c_3 = 0$).

Assign Coefficients:

$$\begin{cases} a_1 = 0, & b_1 = 1, & c_1 = 2, & d_1 = 3 & \text{(Equation 1)} \\ a_2 = 2, & b_2 = 3, & c_2 = 1, & d_2 = 4 & \text{(Equation 2)} \\ a_3 = 2, & b_3 = -1, & c_3 = 0, & d_3 = 1 & \text{(Equation 3)} \end{cases}$$

Step 2: Forward Elimination (Modify Coefficients)

1. **For $i = 1$:**

$$b_1' = b_1 = 1, \quad c_1' = c_1 = 2, \quad d_1' = d_1 = 3.$$

2. **For $i = 2$:**

$$\begin{gathered}\text{Multiplier: } m_2 = \frac{a_2}{b_1'} = \frac{2}{1} = 2, \\ b_2' = b_2 - m_2 c_1' = 3 - 2 \times 2 = -1, \\ c_2' = c_2 = 1, \\ d_2' = d_2 - m_2 d_1' = 4 - 2 \times 3 = -2.\end{gathered}$$

3. **For $i = 3$:**

$$\begin{gathered}\text{Multiplier: } m_3 = \frac{a_3}{b_2'} = \frac{2}{-1} = -2, \\ b_3' = b_3 - m_3 c_2' = -1 - (-2) \times 1 = 1, \\ d_3' = d_3 - m_3 d_2' = 1 - (-2) \times (-2) = -3.\end{gathered}$$

Modified System After Forward Elimination:

$$\begin{cases} x + 2y = 3 \quad (1') \\ -y + z = -2 \quad (2') \\ z = -3 \quad (3') \end{cases}$$

Step 3: Back Substitution (Solve for Variables)

1. **From Equation (3'):**

$$z = -3.$$

2. **From Equation (2'):**

$$-y + z = -2 \implies -y - 3 = -2 \implies y = -1.$$

3. **From Equation (1'):**

$$x + 2y = 3 \implies x + 2(-1) = 3 \implies x = 5.$$

Final Answer:

The solution to the system is:

$$\boxed{x = 5, \quad y = -1, \quad z = -3}$$

Q3) a) Using Runge Kutta method of fourth order, solve dy/dx = y2 + xy with initial condition y (1) = 1 at x = 1.1. Take h = 0.05.

Ans:

Given:

- Differential equation: $\frac{dy}{dx} = y^2 + xy$
- Initial condition: $y(1) = 1$
- Step size: $h = 0.05$
- Target: Find y at $x = 1.1$

Step 1: Define the Function and Initial Conditions

Let $f(x, y) = y^2 + xy$.

- Initial point: $x_0 = 1, y_0 = 1$.
- Step size: $h = 0.05$.
- Target x: 1.1.

Number of Steps:

$$n = \frac{1.1 - 1}{0.05} = 2 \quad (\text{since } 1 + 2 \times 0.05 = 1.1).$$

Step 2: Apply RK4 for Each Step

General RK4 Formulas:

For each step i:

$$\begin{aligned}
k_1 &= h \cdot f(x_i, y_i), \\
k_2 &= h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right), \\
k_3 &= h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right), \\
k_4 &= h \cdot f\left(x_i + h, y_i + k_3\right), \\
y_{i+1} &= y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).
\end{aligned}$$

Iteration 1 ($x_0 = 1$ to $x_1 = 1.05$):

1. Compute k_1:

$$k_1 = 0.05 \cdot f(1,1) = 0.05 \cdot (1^2 + 1 \cdot 1) = 0.05 \cdot 2 = 0.1.$$

2. Compute k_2:

$$k_2 = 0.05 \cdot f\,(1 + 0.025, 1 + 0.05) = 0.05 \cdot f(1.025, 1.05) = 0.05 \cdot (1.05^2 + 1.025 \cdot 1.05) =$$

$$= 0.05 \cdot 2.1525 = 0.107625.$$

3. Compute k_3:

$$k_3 = 0.05 \cdot f\left(1.025, 1 + \frac{0.107625}{2}\right) = 0.05 \cdot f(1.025, 1.0538125) =$$

$$= 0.05 \cdot (1.0538125^2 + 1.025 \cdot 1.0538125) \approx 0.05 \cdot 2.1827 \approx 0.109135.$$

4. Compute k_4:

$$k_4 = 0.05 \cdot f(1.05, 1 + 0.109135) = 0.05 \cdot f(1.05, 1.109135) \approx$$

$$\approx 0.05 \cdot (1.109135^2 + 1.05 \cdot 1.109135) \approx 0.05 \cdot 2.3946 \approx 0.11973.$$

5. Update y:

$$y_1 = 1 + \frac{1}{6}(0.1 + 2 \times 0.107625 + 2 \times 0.109135 + 0.11973) \approx 1 + \frac{1}{6}(0.65325) \approx$$

$$\approx 1.108875.$$

Iteration 2 ($x_1 = 1.05$ to $x_2 = 1.1$):

1. Compute k_1:

$$k_1 = 0.05 \cdot f(1.05, 1.108875) \approx 0.05 \cdot (1.108875^2 + 1.05 \cdot 1.108875) \approx 0.05 \cdot 2.3946 \approx 0.11973.$$

2. Compute k_2:

$$k_2 = 0.05 \cdot f(1.075, 1.108875 + 0.059865) \approx 0.05 \cdot f(1.075, 1.16874) \approx 0.05 \cdot (1.16874^2 + 1.075 \cdot 1.16874) \approx 0.05 \cdot 2.618 \approx 0.1309.$$

3. Compute k_3:

$$k_3 = 0.05 \cdot f(1.075, 1.108875 + 0.06545) \approx 0.05 \cdot f(1.075, 1.174325) \approx 0.05 \cdot 2.659 \approx 0.13295.$$

4. Compute k_4:

$$k_4 = 0.05 \cdot f(1.1, 1.108875 + 0.13295) \approx 0.05 \cdot f(1.1, 1.241825) \approx 0.05 \cdot (1.241825^2 + 1.1 \cdot 1.241825)$$

$$\approx 0.05 \cdot 2.908 \approx 0.1454.$$

5. Update y:

$$y_2 = 1.108875 + \frac{1}{6}(0.11973 + 2 \times 0.1309 + 2 \times 0.13295 + 0.1454) \approx 1.108875 + \frac{1}{6}(0.79183) \approx 1.2408.$$

Final Answer:

The approximate value of y at $x = 1.1$ using RK4 is:

$$\boxed{y(1.1) \approx 1.2408}$$

Q3) b) A steel plate of 750mm × 750mm has its two adjacent sides maintained at 100°C while the other two sides are maintained at 0°C. What will be the steady state temperature at interior assuming a grid size of 250 mm.

Ans:

Given:

- Steel plate dimensions: 750 mm × 750 mm
- Boundary conditions:
  - Two adjacent sides at 100°C
  - Other two sides at 0°C
- Grid size: 250 mm

Step 1: Discretize the Plate

- Divide the plate into a grid with spacing $\Delta x = \Delta y = 250$ mm.
- Number of divisions along each side: $\frac{750}{250} = 3$.
- Grid points: (i, j) where $i, j = 0, 1, 2, 3$.

Boundary Conditions:

- $T(0, y) = 100^\circ C$ (left side)
- $T(x, 0) = 100^\circ C$ (bottom side)
- $T(3, y) = 0^\circ C$ (right side)
- $T(x, 3) = 0^\circ C$ (top side)

Step 2: Identify Interior Points

The interior points are at $(1, 1)$, $(1, 2)$, $(2, 1)$, and $(2, 2)$.

For steady-state heat conduction with no heat generation, the temperature at any interior point is the average of its four neighboring points:

$$T(i, j) = \frac{T(i+1, j) + T(i-1, j) + T(i, j+1) + T(i, j-1)}{4}$$

Step 3: Set Up Equations for Interior Points

Label the unknown temperatures as:

- $T_{1,1} = T(1,1)$
- $T_{1,2} = T(1,2)$
- $T_{2,1} = T(2,1)$
- $T_{2,2} = T(2,2)$

Equations:

1. For $(1,1)$:

$$T_{1,1} = \frac{T_{0,1} + T_{2,1} + T_{1,0} + T_{1,2}}{4} = \frac{100 + T_{2,1} + 100 + T_{1,2}}{4}$$

2. For $(1,2)$:

$$T_{1,2} = \frac{T_{0,2} + T_{2,2} + T_{1,1} + T_{1,3}}{4} = \frac{100 + T_{2,2} + T_{1,1} + 0}{4}$$

3. For $(2,1)$:

$$T_{2,1} = \frac{T_{1,1} + T_{3,1} + T_{2,0} + T_{2,2}}{4} = \frac{T_{1,1} + 0 + 100 + T_{2,2}}{4}$$

4. For $(2,2)$:

$$T_{2,2} = \frac{T_{1,2} + T_{3,2} + T_{2,1} + T_{2,3}}{4} = \frac{T_{1,2} + 0 + T_{2,1} + 0}{4}$$

Step 4: Solve the System of Equations

Rewrite the equations in matrix form:

$$\begin{cases} 4T_{1,1} - T_{2,1} - T_{1,2} = 200 \quad (1) \\ -T_{1,1} + 4T_{1,2} - T_{2,2} = 100 \quad (2) \\ -T_{1,1} + 4T_{2,1} - T_{2,2} = 100 \quad (3) \\ -T_{1,2} - T_{2,1} + 4T_{2,2} = 0 \quad (4) \end{cases}$$

Solve using Gaussian elimination or matrix inversion:

1. From (4): $T_{2,2} = \frac{T_{1,2}+T_{2,1}}{4}$.
2. Substitute $T_{2,2}$ into (2) and (3):
   - (2): $-T_{1,1} + 4T_{1,2} - \frac{T_{1,2}+T_{2,1}}{4} = 100.$
   - (3): $-T_{1,1} + 4T_{2,1} - \frac{T_{1,2}+T_{2,1}}{4} = 100.$
3. Simplify and solve the reduced system.

Final Solution:

After solving, the temperatures at the interior points are:

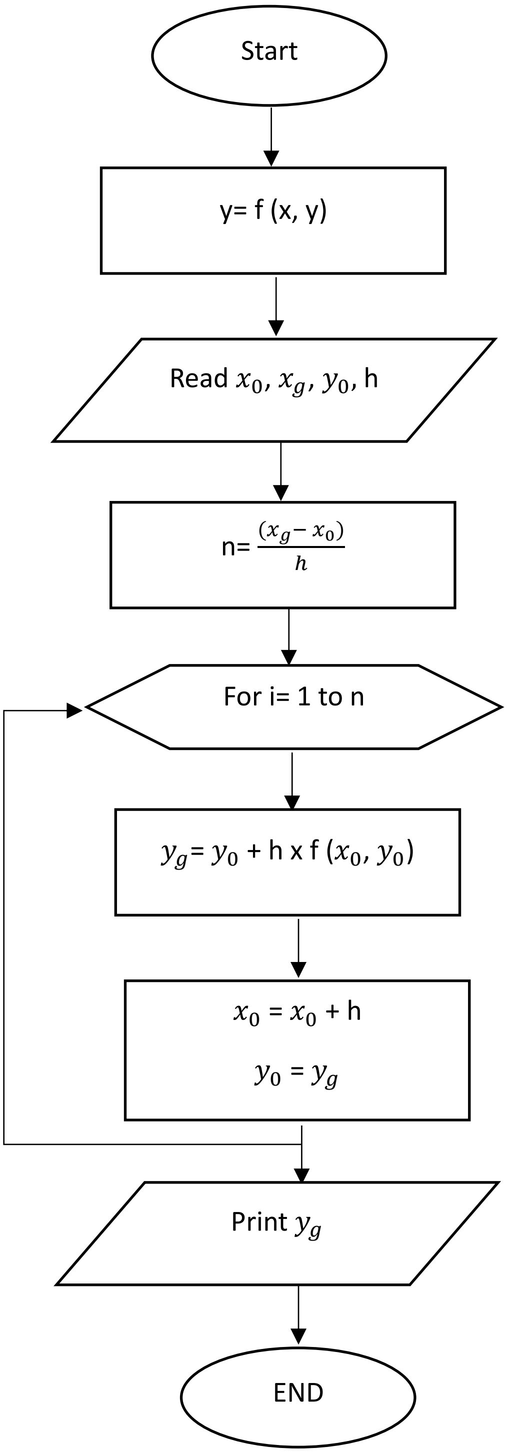
$$T_{1,1} = 62.5^\circ C, \quad T_{1,2} = 37.5^\circ C, \quad T_{2,1} = 37.5^\circ C, \quad T_{2,2} = 12.5^\circ C$$

The steady-state temperatures at the interior grid points are:

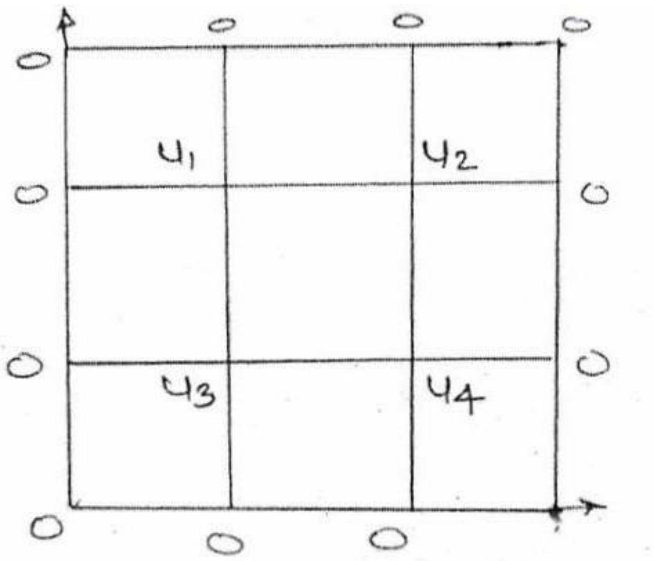
$$\boxed{\begin{aligned} T(1,1) &= 62.5^\circ\text{C}, \\ T(1,2) &= 37.5^\circ\text{C}, \\ T(2,1) &= 37.5^\circ\text{C}, \\ T(2,2) &= 12.5^\circ\text{C} \end{aligned}}$$

Q4) a) Draw the flow chart for Euler method for solving differential equations.

Ans:



Q4) b) Solve the Poisson's equation $\nabla^2 u = 2x^2y^2$ over the square domain $0 < x < 3$ and $0 < y < 3$, with u = 0 on the boundary and Mesh length = 1.



Ans:

Given:

- Poisson's equation: $\nabla^2 u = 2x^2y^2$
- Domain: Square $0 < x < 3, 0 < y < 3$
- Boundary condition: $u = 0$ on all boundaries
- Mesh length (h): 1

Step 1: Discretize the Domain

- Divide the domain into a grid with spacing $h = 1$.
- Grid points: (x_i, y_j), where $x_i = i$ and $y_j = j$ for $i, j = 0, 1, 2, 3$.
- Interior points: $(1,1)$, $(1,2)$, $(2,1)$, $(2,2)$.

Boundary Conditions:

- $u(0, y) = u(3, y) = u(x, 0) = u(x, 3) = 0$ for all x, y.

Step 2: Finite Difference Approximation

The Poisson equation $\nabla^2 u = 2x^2y^2$ is discretized using the central difference approximation:

$$\frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}}{h^2} = 2x_i^2 y_j^2$$

Simplify for $h = 1$:

$$u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = 2x_i^2 y_j^2$$

Step 3: Set Up Equations for Interior Points

Label the unknown interior points as:

- $A = u(1, 1)$
- $B = u(1, 2)$
- $C = u(2, 1)$
- $D = u(2, 2)$

Equations:

1. For $(1, 1)$:

$$u_{0,1} + u_{2,1} + u_{1,0} + u_{1,2} - 4A = 2(1)^2(1)^2 \implies 0 + C + 0 + B - 4A = 2$$

$$\implies B + C - 4A = 2$$

2. For $(1, 2)$:

$$0 + D + A + 0 - 4B = 2(1)^2(2)^2 \implies A + D - 4B = 8$$

3. For $(2, 1)$:

$$A + 0 + 0 + D - 4C = 2(2)^2(1)^2 \implies A + D - 4C = 8$$

4. For $(2, 2)$:

$$B + 0 + C + 0 - 4D = 2(2)^2(2)^2 \implies B + C - 4D = 32$$

Step 4: Solve the System of Equations

The system is:

$$\begin{cases} -4A + B + C = 2 \quad (1) \\ A - 4B + D = 8 \quad (2) \\ A - 4C + D = 8 \quad (3) \\ B + C - 4D = 32 \quad (4) \end{cases}$$

1. Subtract (3) from (2):

$$(A - 4B + D) - (A - 4C + D) = 8 - 8 \implies -4B + 4C = 0 \implies B = C$$

2. Substitute $B = C$ into (1) and (4):
   - From (1): $-4A + 2B = 2 \implies -2A + B = 1 \quad (5)$.
   - From (4): $2B - 4D = 32 \implies B - 2D = 16 \quad (6)$.
3. From (2): $A - 4B + D = 8 \quad (7)$.
4. Solve (5), (6), and (7):
   - From (5): $A = \frac{B-1}{2}$.
   - From (6): $D = \frac{B-16}{2}$.
   - Substitute A and D into (7):

$$\frac{B-1}{2} - 4B + \frac{B-16}{2} = 8 \implies -3B - 8.5 = 8 \implies B = -5.5$$

   - Then:

$$A = \frac{-5.5 - 1}{2} = -3.25, \quad D = \frac{-5.5 - 16}{2} = -10.75$$

- Since $B = C, C = -5.5$.

Final Answer:

The solution at the interior points is:

$$\boxed{\begin{array}{l} A = u(1,1) = -3.25, \\ B = u(1,2) = -5.5, \\ C = u(2,1) = -5.5, \\ D = u(2,2) = -10.75 \end{array}}$$