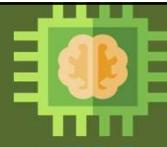


Elective Course

Course Code: CS4103

Autumn 2025-26

**Lecture #12**

Artificial Intelligence for Data Science

Week-3: PROBLEM SOLVING BY SEARCH

Introduction to Informed Search [Part-IV] &
Adversarial Search--- Games [Part-I]

Course Instructor:**Dr. Monidipa Das**

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Simulated Annealing Algorithm



```

function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
           schedule, a mapping from time to “temperature”

  current  $\leftarrow$  MAKE-NODE(problem.INITIAL-STATE)
  for t = 1 to  $\infty$  do
    T  $\leftarrow$  schedule(t)
    if T = 0 then return current
    next  $\leftarrow$  a randomly selected successor of current
     $\Delta E \leftarrow$  next.VALUE - current.VALUE
    if  $\Delta E > 0$  then current  $\leftarrow$  next
    else current  $\leftarrow$  next only with probability  $e^{\Delta E/T}$ 

```

Simulated Annealing Search



- Probability of a move decreases with the amount ΔE by which the evaluation is worsened
- A second parameter T is also used to determine the probability: high T allows more worse moves, T close to zero results in few or no bad moves
- *Schedule* input determines the value of T as a function of the completed cycles

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Simulated Annealing Search



- **Idea:** improve hill-climbing by allowing occasional down-hill steps, to minimize the probability of getting stuck in a local maximum
- Down-hill steps taken randomly but with probability that decreases with time
- Probability controlled by a given annealing schedule for a temperature parameter T
- If schedule lowers T slowly enough, search is guaranteed to end in a global maximum
- **Catch:** it may take several tries with test problems to devise a good annealing schedule
- Simulated Annealing vs. Hill Climbing

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Adversarial Search--- Games

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Search vs. Games

- **Search- No adversary**
 - Solution is a path from start to goal, or a series of actions from start to goal
 - Heuristics and search techniques can find optimal solution
 - Evaluation function: estimate of cost from start to goal through given node
 - Actions have cost
 - Example: Path planning, Scheduling activities

- **Games- Adversary**
 - Solution is strategy
 - Strategy specifies move for every possible opponent reply
 - Time limits force an approximate solution
 - Evaluation function: evaluate “goodness” of game position
 - Board configurations have utility
 - Examples: Chess, Checkers, Othello, Backgammon

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Why Study Games in AI?



- Problems are formalized
- Real world knowledge (common sense knowledge) is not too important
- Rules are fixed
- Adversary modeling is of general importance
 - opponent introduces uncertainty
 - programs must deal with the contingency problem
- Complexity of games?
 - number of nodes in a search tree (e.g., 10^{40} legal positions in chess)
 - specification is simple (no missing information, well-defined problem)

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Types of games



	Deterministic	Chance
Perfect Information	Chess, Checkers, Othello, Go	Backgammon, Monopoly
Imperfect Information		Bridge, Poker

Not considered: Physical games like tennis, croquet, ice hockey, etc.

**We restrict our analysis to a very specific set of games:
2-player zero-sum discrete finite deterministic games of perfect information**

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2-player zero-sum discrete finite deterministic games of perfect information



- **What does it mean?**
 - **2-player:** Two players involved (multi-agent environment)
 - **Zero-sum:** In any outcome of any game, Player A's gains equal player B's losses.
 - **Discrete:** All game states and decisions are discrete values.
 - **Finite:** Only a finite number of states and decisions.
 - **Deterministic:** No chance (no die rolls).
 - **Perfect information:** Both players can see the state, and each decision is made sequentially (no simultaneous moves).

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Games as Adversarial Search



- **States:**
 - board configurations
- **Initial state:**
 - the board position and which player will move
- **Successor function:**
 - returns list of (action, state) pairs, each indicating a legal move and the resulting state
- **Terminal test:**
 - determines when the game is over
- **Utility function:**
 - gives a numeric value in terminal states
(e.g., -1, 0, +1 for loss, tie, win)

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Evaluation function



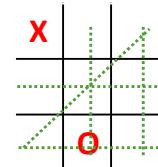
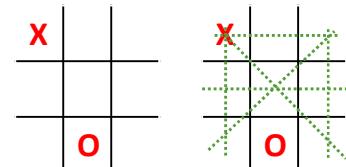
- **Evaluation function** or **static evaluator** is used to evaluate the “goodness” of a game position.
 - Contrast with heuristic search where the evaluation function was a non-negative estimate of the cost from the start node to a goal and passing through the given node
- The **zero-sum assumption** allows us to use a single evaluation function to describe the goodness of a board with respect to both players.
 - $f(n) >> 0$: position n good for player A and bad for player B
 - $f(n) << 0$: position n bad for player A and good for player B
 - $f(n)$ near 0: position n is a neutral position
 - $f(n) = +1$: win for player A
 - $f(n) = -1$: win for player B

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Evaluation function examples

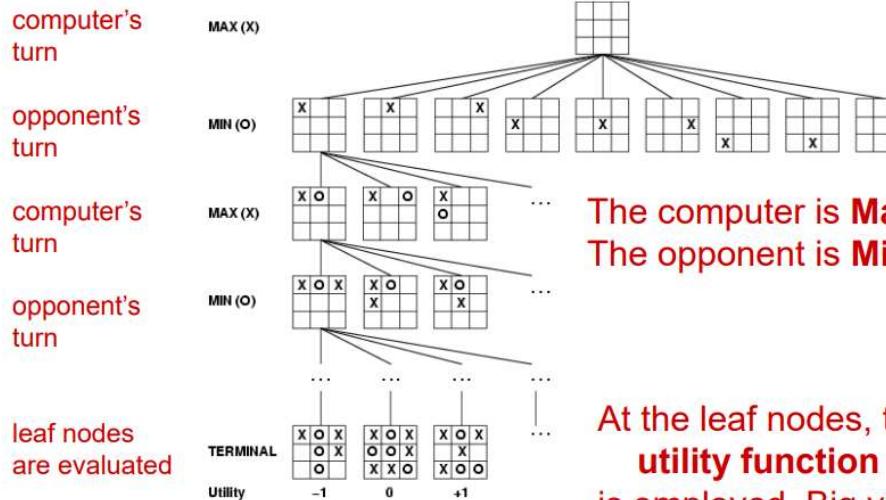


- Example of an evaluation function for **Tic-Tac-Toe**:
 - $f(n) = [\# \text{of win path for player A}] - [\# \text{of win path for player B}]$
where a win path is a complete row, column, or diagonal
[E.g. $f(n) = 6 - 5 = 1$ for the state given in this slide.
Here, player A gives cross and player B naught]
- Alan Turing’s function for **chess**:
 - $f(n) = w(n)/b(n)$
where $w(n)$ = sum of the point value of white’s pieces and $b(n)$ = sum of black’s
- Most evaluation functions are specified as a weighted sum of position features:
 - $f(n) = w_1 * \text{feature1}(n) + w_2 * \text{feature2}(n) + \dots + w_k * \text{featurek}(n)$
Example features for chess are piece count, piece placement, squares controlled, etc.
- Deep Blue had over 8000 features in its evaluation function



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Game Tree (2-player, Deterministic, Turns)



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MiniMax Terminology



- **Move:** a move considering both players
- **Ply:** a half-move
- **Utility function:** the function applied to leaf nodes
- **Backed-up value**
 - of a max-position: the value of its largest successor
 - of a min-position: the value of its smallest successor
- **Minimax procedure:** search down several levels; at the bottom level apply the utility function, back-up values all the way up to the root node, and that node selects the move.

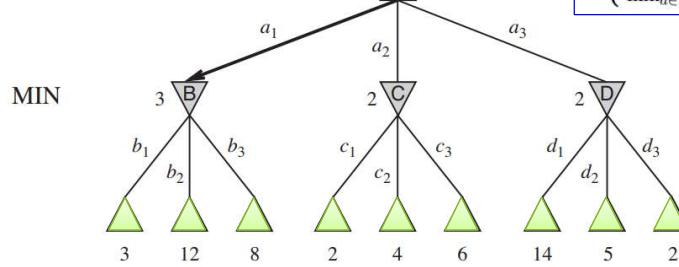
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Minimax



- Perfect play for deterministic games
- **Idea:** choose move to position with highest minimax value
= best achievable payoff against best play
- E.g., 2-ply game:

MAX



```

MINIMAX(s) =
  { UTILITY(s)
    maxa ∈ Actions(s) MINIMAX(RESULT(s, a))
    mina ∈ Actions(s) MINIMAX(RESULT(s, a))
  }
  if TERMINAL-TEST(s)
  if PLAYER(s) = MAX
  if PLAYER(s) = MIN

```

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Minimax algorithm Adversarial analogue of DFS



```

function MINIMAX-DECISION(state) returns an action
  return arg maxa ∈ ACTIONS(s) MIN-VALUE(RESULT(s, a))

function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s, a)))
  return v

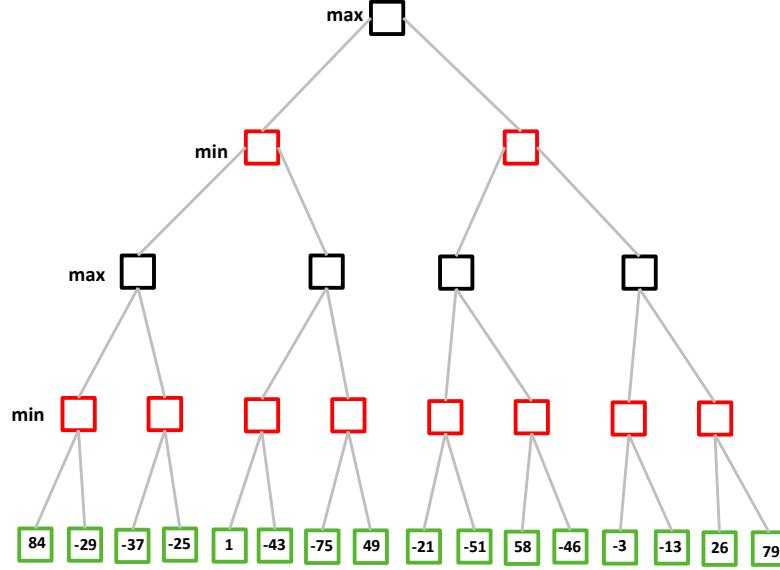
function MIN-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← ∞
  for each a in ACTIONS(state) do
    v ← MIN(v, MAX-VALUE(RESULT(s, a)))
  return v

```

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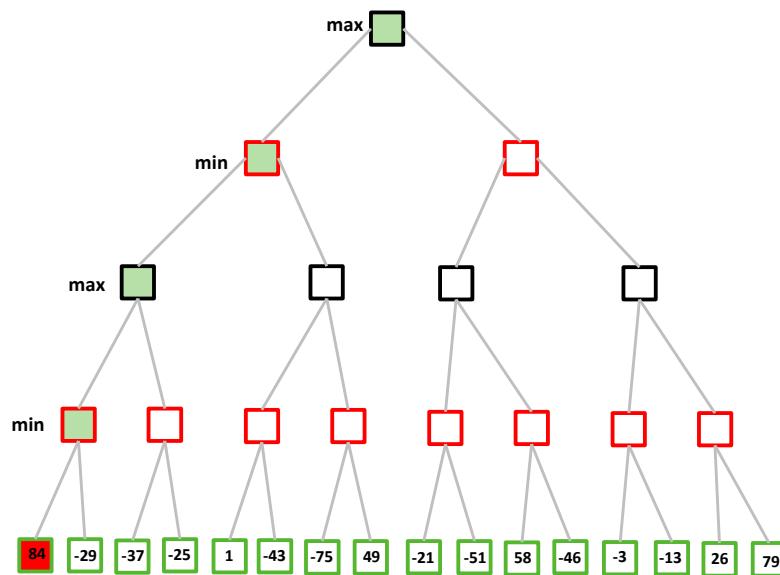
Example



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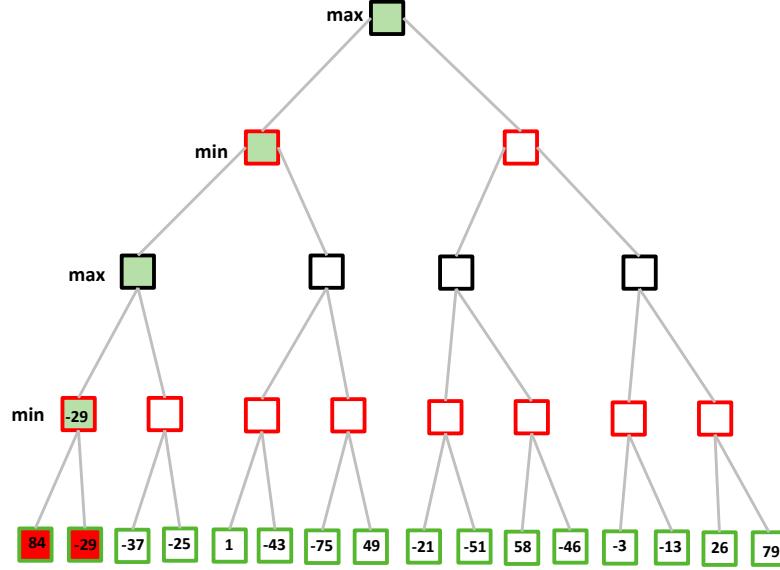
Example



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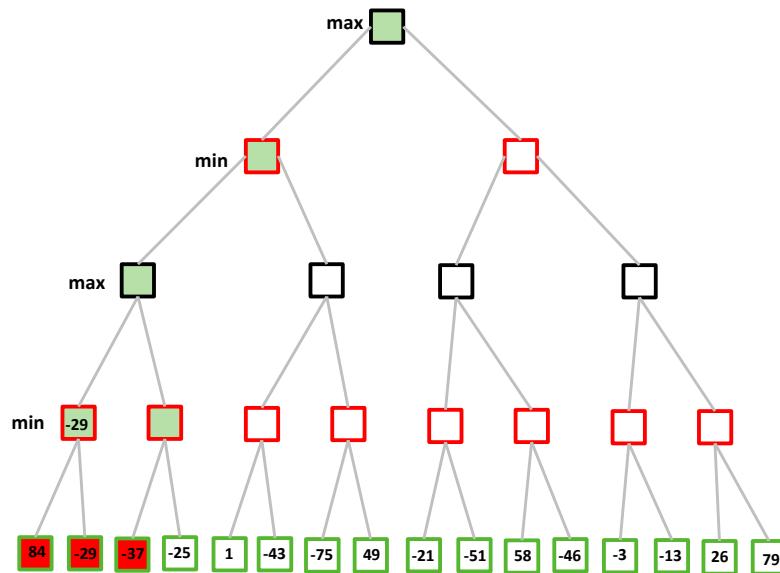
Example



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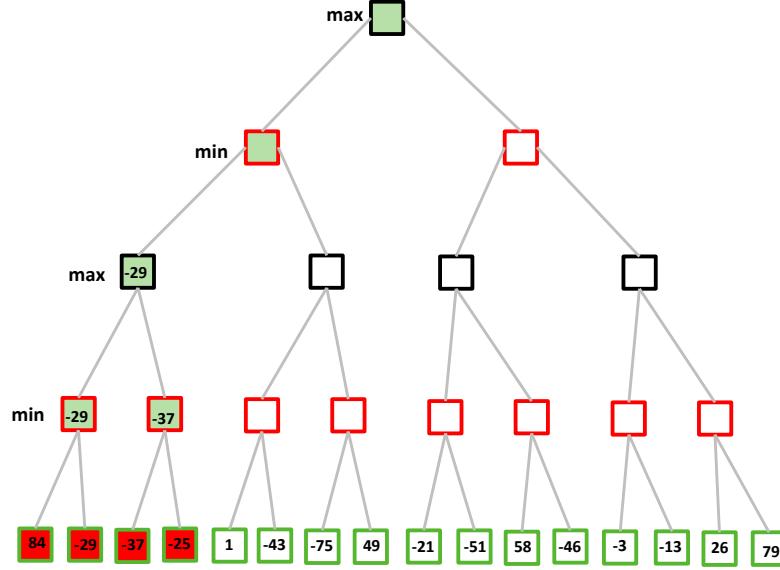
Example



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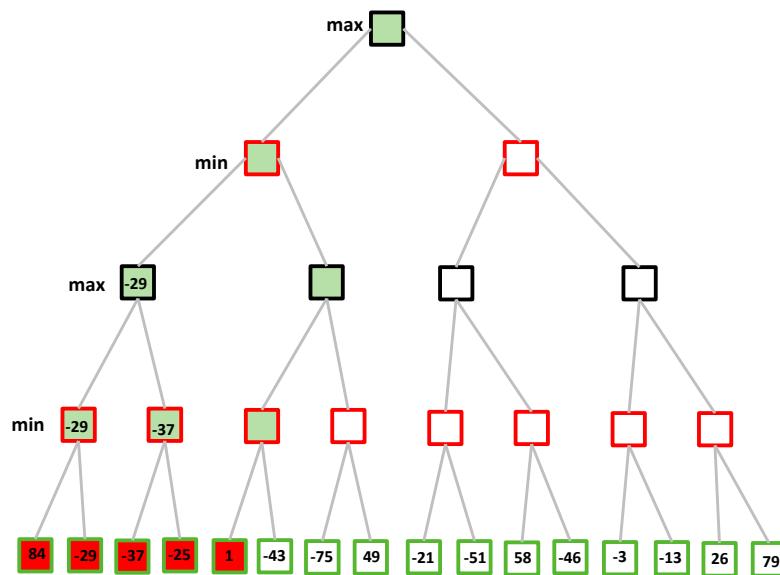
Example



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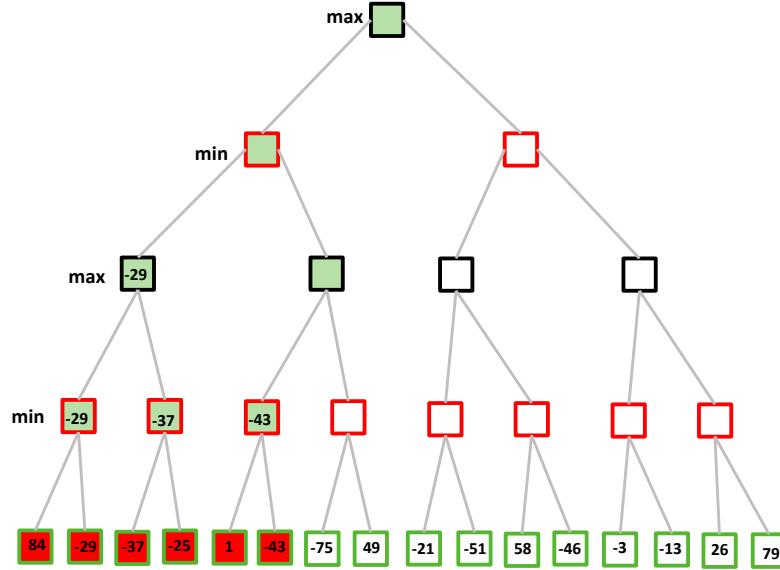


Example



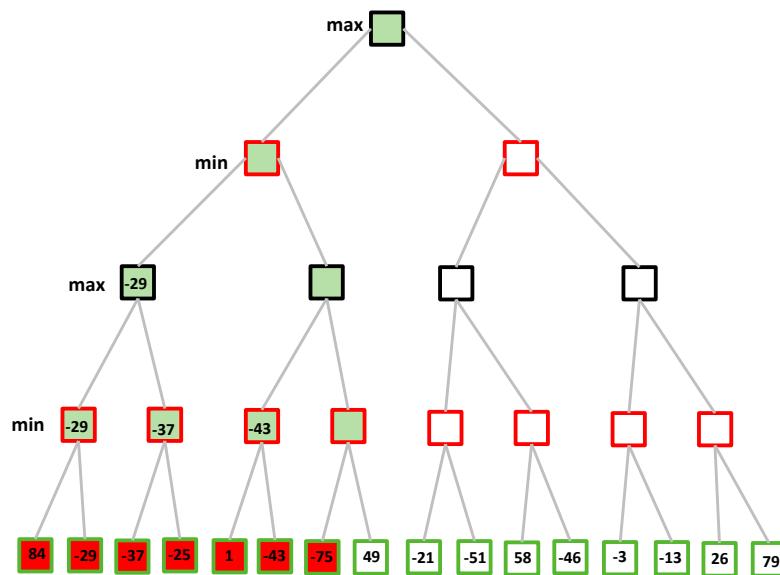
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Example



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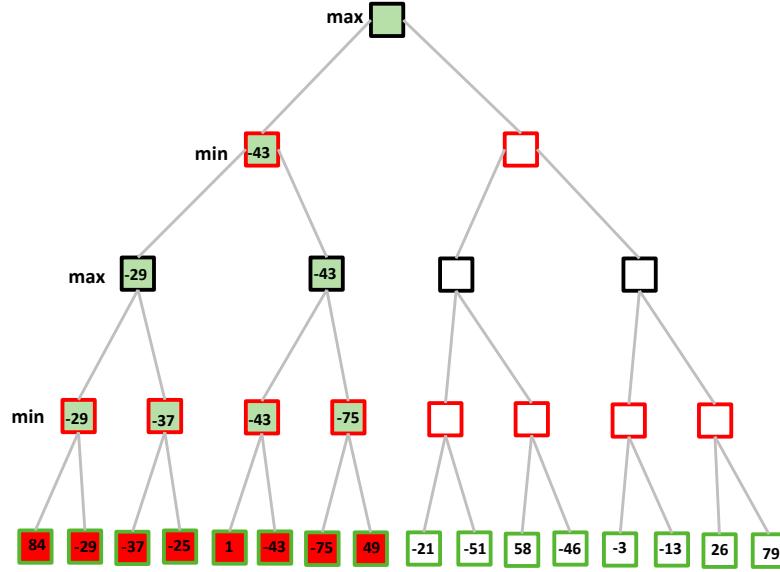
Example



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Example

25

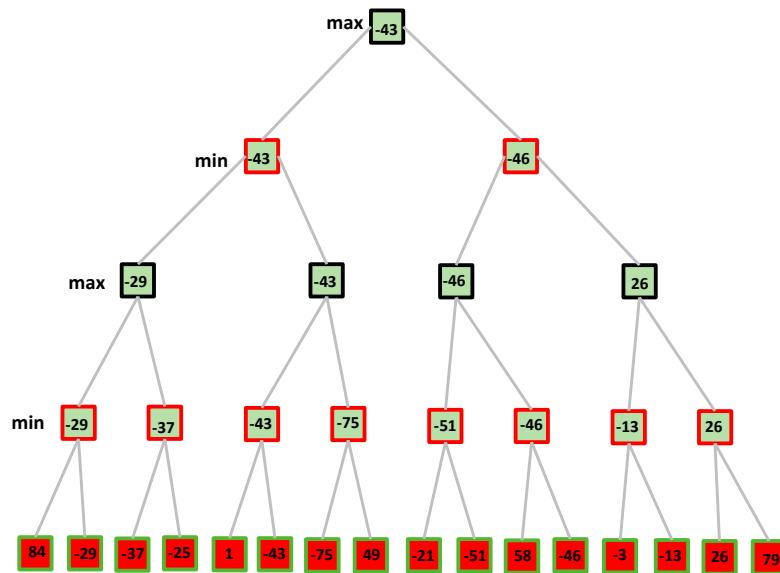


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Example

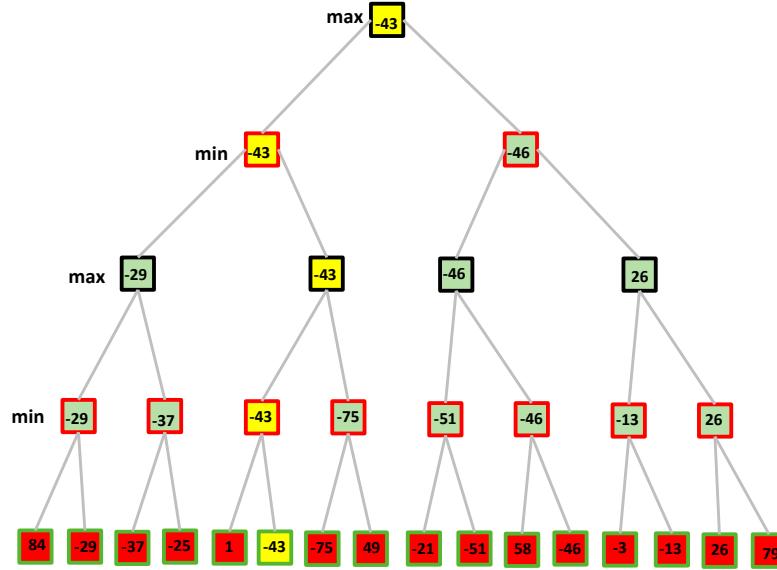
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...similarly



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Example



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Properties of Minimax



- **Complete?**
 - Yes (if tree is finite)
- **Optimal?**
 - Yes (against an optimal opponent)
 - Can it be beaten by an opponent playing sub-optimally?
 - No
- **Time Complexity?**
 - $O(b^m)$
- **Space Complexity?**
 - $O(bm)$ [depth-first search, generate all actions at once]
 - $O(m)$ [backtracking search, generate actions one at a time]

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Good Enough?



- **Tic-Tac-Toe:**
 - branching factor $b \approx 5$ (on average)
 - game length $m \approx 9$
 - search space $\approx 5^9 \approx 1,953,125$
 - Exact solution quite reasonable
- **Chess:**
 - branching factor $b \approx 35$
 - game length $m \approx 100$
 - search space $\approx 35^{100} \approx 10^{154}$
 - **Exact solution completely infeasible**

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Questions?

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