Here are **conceptual + computational questions**:

**1. Central Limit Theorem in Random Walks**

(a) Generate N random velocities sampled from a uniform distribution between [−1,1].
(b) Compute their average over many trials (say 10,000).
(c) Plot the histogram of these averages.
(c) Explain how the distribution changes as N increases, and relate this to the Central Limit Theorem.

**2. Compute the speed:** $v = \sqrt{v_x^2 + v_y^2 + v_z^2}.$

(a) Plot the histogram of $v$.
(b) Compare the histogram to the Maxwell-Boltzmann theoretical curve.
(c) Explain why the distribution of speed is not Gaussian, even though each component $vx$,$vy$,$vz$ is Gaussian.

Model Questions (Midsem: Mark-40)

Here are **conceptual + computational questions**:

1. Derive the Verlet Integration Method
2. Using the above Integration method solve equation of motion of multi-particle treating them as harmonic oscillators in 3D.
3. Observation based Questions from the code:

**(i) Energy conservation:**

Does total energy stay constant?

How does changing $\Delta t$ affect energy conservation?

**(ii) Stability of Verlet:**

What happens if you increase $\Delta t$ too much?

Why does the trajectory blow up?

**(iii) Phase space:**

Plot velocity vs position for one particle. What shape do you see?

Why is it an ellipse for a harmonic oscillator?

**4. Comparison with Euler:**

(i) Implement simple Euler instead of Verlet.

(ii) Compare total energy evolution between Euler and Verlet.

(Indicative Answer: Euler's method shows instability because it does not respect the circular nature of harmonic oscillator motion in phase space. Each update stretches/shrinks trajectories, causing the total energy to drift over time. Verlet, being symplectic, keeps energy bounded with small oscillations instead of long-term drift.)

**Understanding Symplectic Nature of an Algorithm:**

- In a harmonic oscillator, motion is a **rotation in phase space** (the X-V plane).

Exact solution: trajectory is a circle (constant energy).

- Euler's method does not preserve this rotation. Instead of rotating, it spirals either outward (energy grows) or inward (energy shrinks).
- Mathematically, it can be shown that the update matrix of Euler has eigenvalues with magnitude not equal to 1. That means instead of preserving phase-space length (constant energy), the method stretches or contracts each step.

Thus, energy drifts linearly or exponentially with time.

A few more **conceptual + computational questions**:

**5. How do you understand Symplectic behaviour** of an algorithm?

Symplectic means: they preserve the area/geometry of phase space. That guarantees bounded energy oscillations instead of drift. So energy "wiggles" around the true constant value, but never runs away.

6. **draw the phase space diagram (X vs V)** and compare Euler vs Verlet ?
It's the clearest way to show why Euler spirals while Verlet circles.
7. Write a Python script that uses your Verlet integrator to plot the velocity histogram for a single oscillator vs. many oscillators. What do you observe.