Elective Course
Course Code: CS4103
Autumn 2025-26

Lecture #14

Artificial Intelligence for Data Science

Week-4: PROBLEM SOLVING BY SEARCH

Adversarial Search Problem--- Games [Part-II]

Course Instructor:

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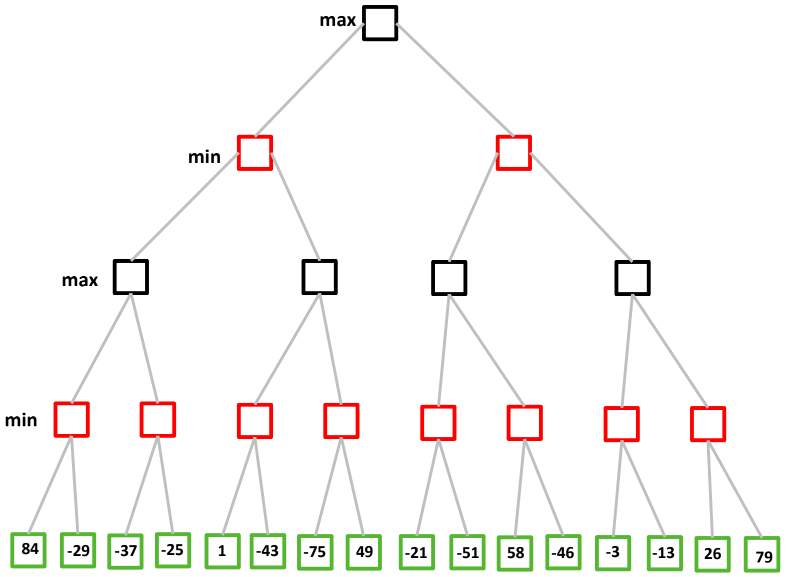
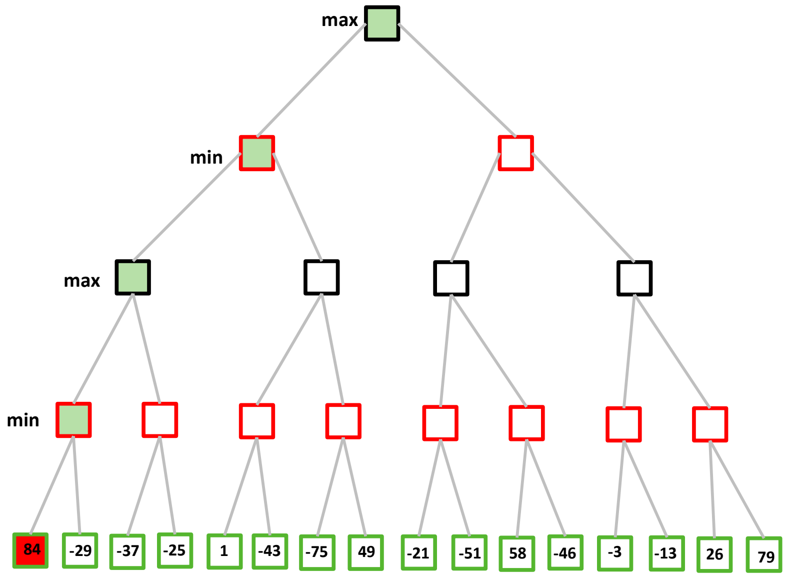
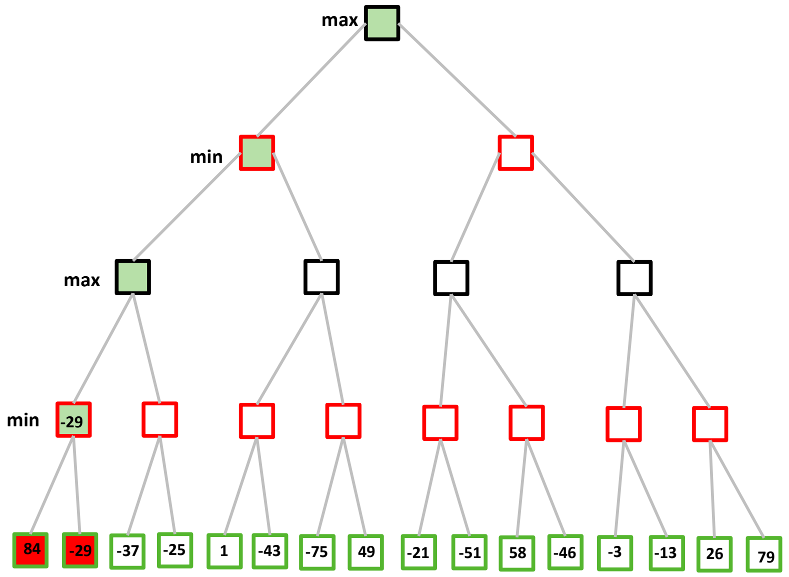
Minimax Example (revisited)



Minimax Example (revisited)

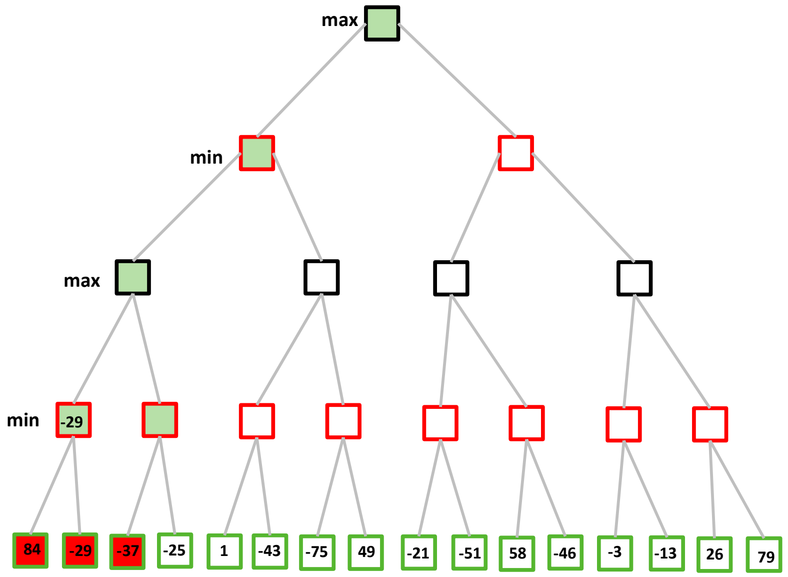


Minimax Example (revisited)



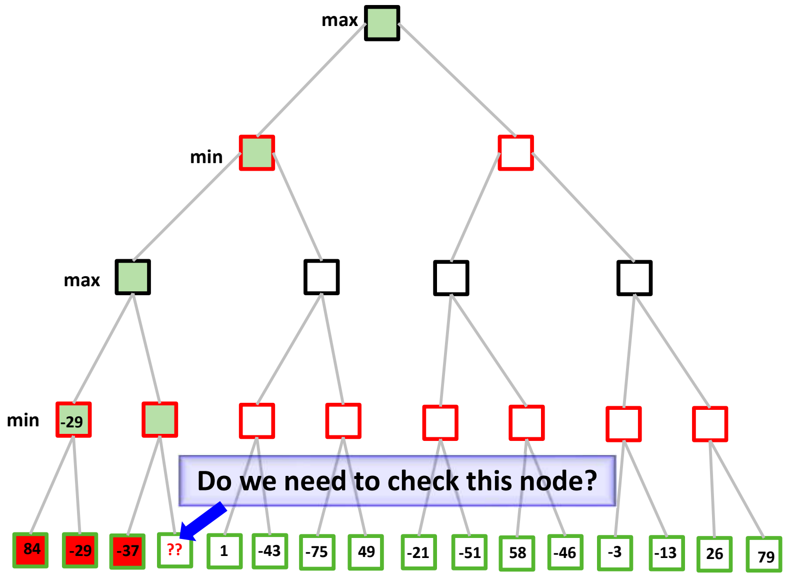
Minimax Example (revisited)

5



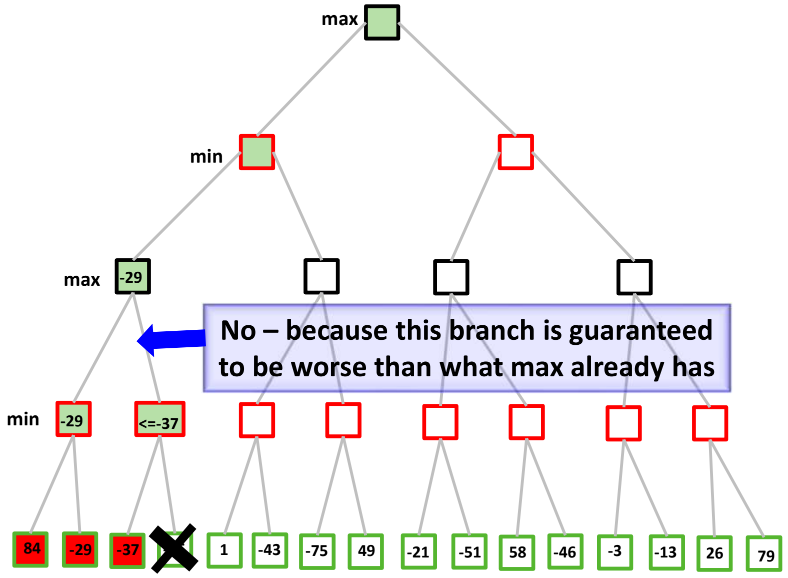
Minimax Example (revisited)

6



Do we need to check this node?

Minimax Example (revisited)



No – because this branch is guaranteed to be worse than what max already has

Alpha-Beta Procedure

- The alpha-beta procedure can speed up a depth-first minimax search.
- **Alpha:** a lower bound on the value that a max node may ultimately be assigned

$$v \geq \alpha$$

- **Beta:** an upper bound on the value that a min node may ultimately be assigned

$$v \leq \beta$$

Alpha-Beta Pruning

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- Traverse the search tree in depth-first order
- At each MAX node n, $alpha(n)$ = maximum value found so far
- At each MIN node n, $beta(n)$ = minimum value found so far
  - **Note:** The alpha values start at -infinity and only increase, while beta values start at +infinity and only decrease.
- **Beta cutoff:** Given a MAX node n, cut off the search below n (i.e., don't generate or examine any more of n's children) if $alpha(n) >= beta(i)$ for some MIN node ancestor i of n.
- **Alpha cutoff:** stop searching below MIN node n if $beta(n) <= alpha(i)$ for some MAX node ancestor i of n.

Alpha Beta Example

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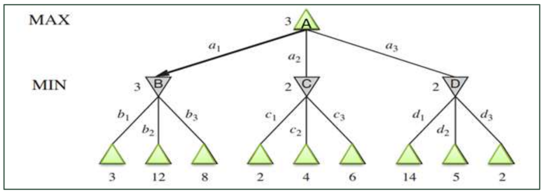
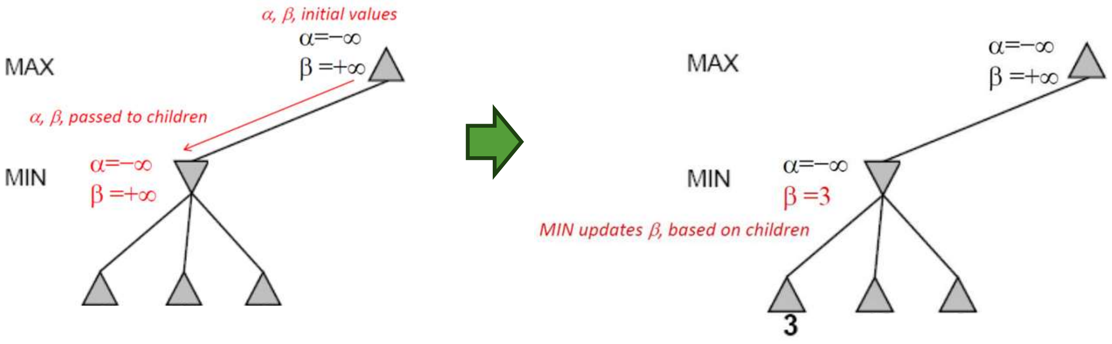
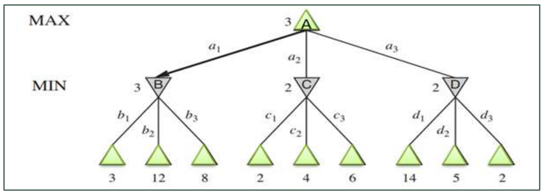
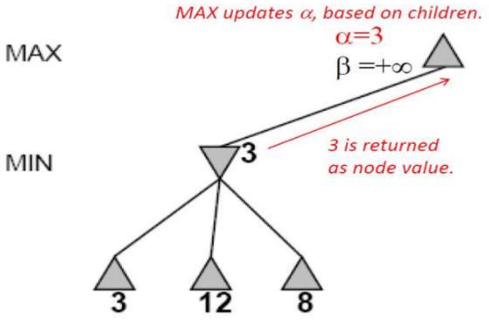
- Do depth-first search until first leaf





Alpha Beta Example



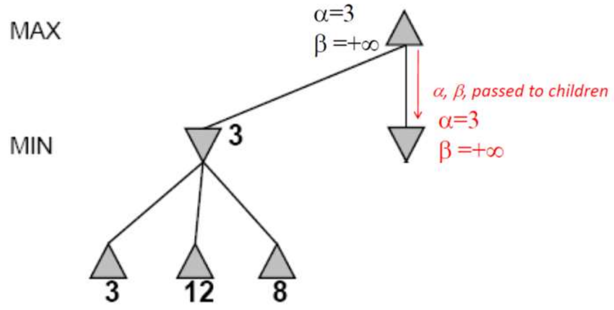


MAX updates α, based on children.

α=3

β =+∞

3 is returned as node value.



α=3

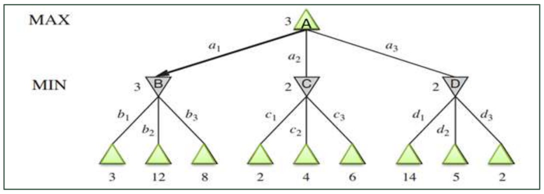
β =+∞

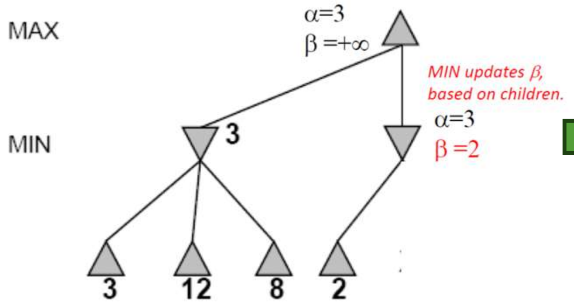
α, β, passed to children

α=3

β =+∞

Alpha Beta Example





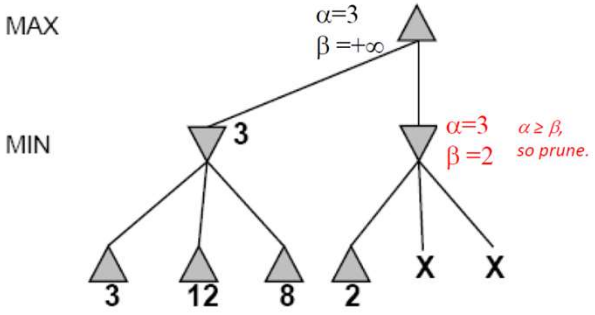
α=3

β =+∞

MIN updates β, based on children.

α=3

β =2



α=3

β =+∞

α=3

β =2

$\alpha \geq \beta$, so prune.

Alpha Beta Example

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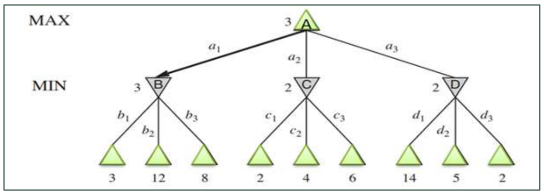

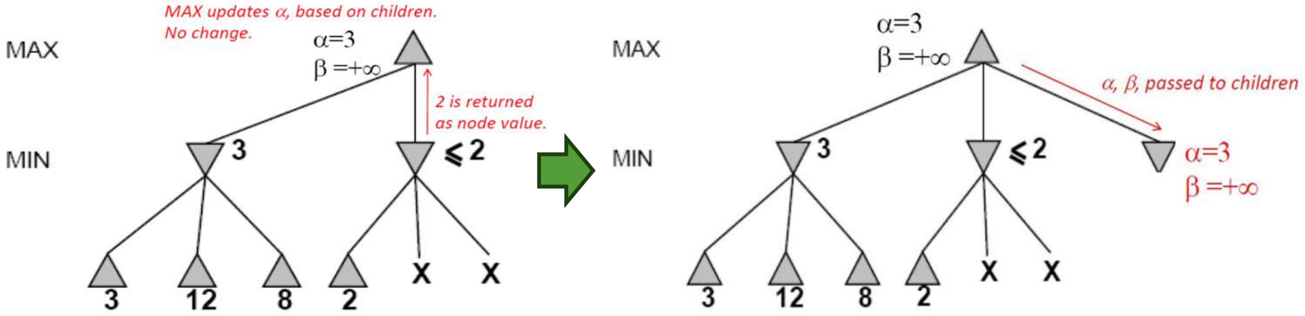

MAX updates α, based on children. No change.

2 is returned as node value.

α, β, passed to children

Alpha Beta Example

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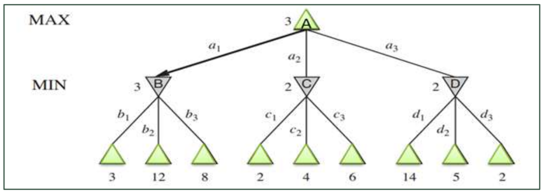

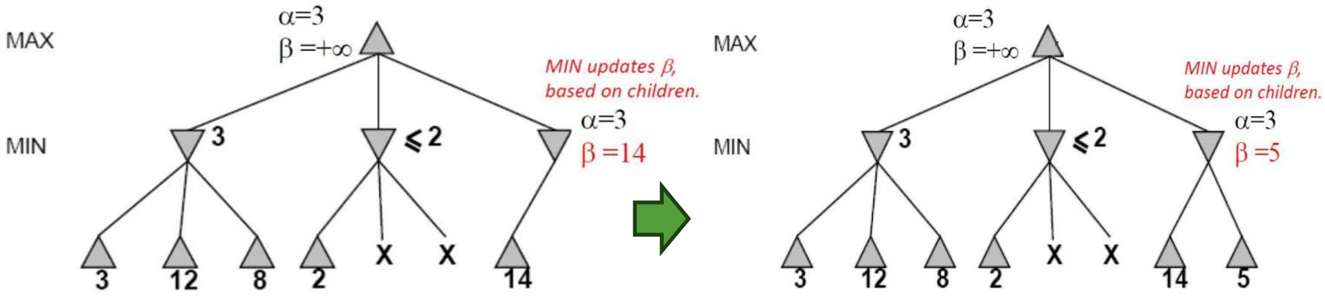

MIN updates β, based on children.

MIN updates β, based on children.

Alpha Beta Example

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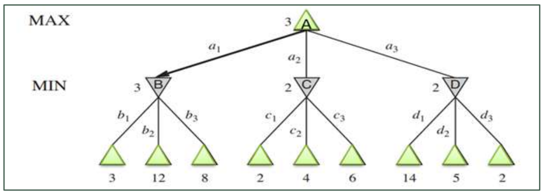
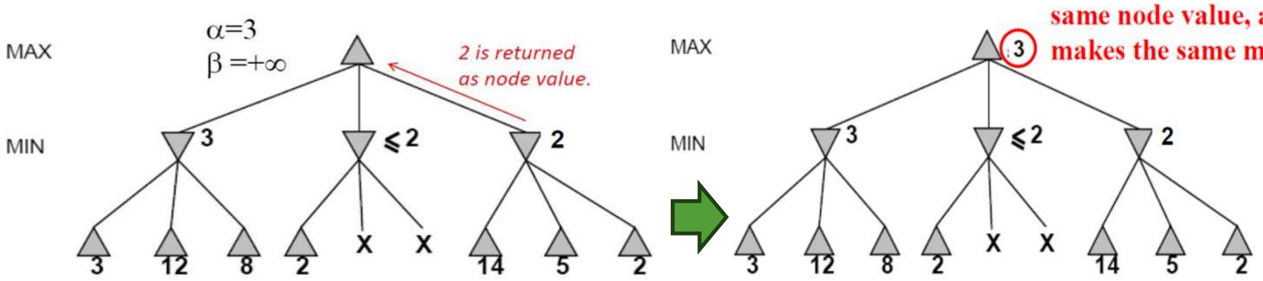


Max calculates the same node value, and makes the same move!



Alpha–Beta Search Algorithm

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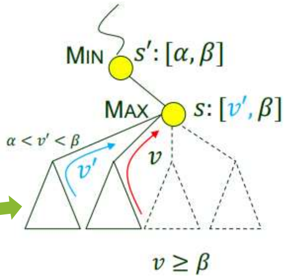
```
function ALPHA-BETA-SEARCH(state) returns an action
   v ← MAX-VALUE(state, −∞, +∞)
   return the action in ACTIONS(state) with value v

function MAX-VALUE(state, α, β) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for each a in ACTIONS(state) do
      v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
      if v ≥ β then return v
      α ← MAX(α, v)
   return v

function MIN-VALUE(state, α, β) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← +∞
   for each a in ACTIONS(state) do
      v ← MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
      if v ≤ α then return v
      β ← MIN(β, v)
   return v
```

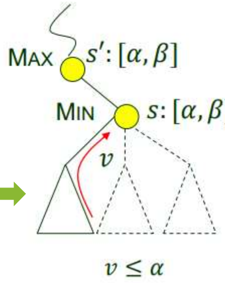
// no change of β value within MAX-VALUE()

$\alpha \leftarrow MAX(\alpha, v)$
$if\, \alpha \geq \beta\ then\ return\ v$

// no change of α value within MIN-VALUE()

$\beta \leftarrow MIN(\beta, v)$
$if\, \beta \leq \alpha\ then\ return\ v$



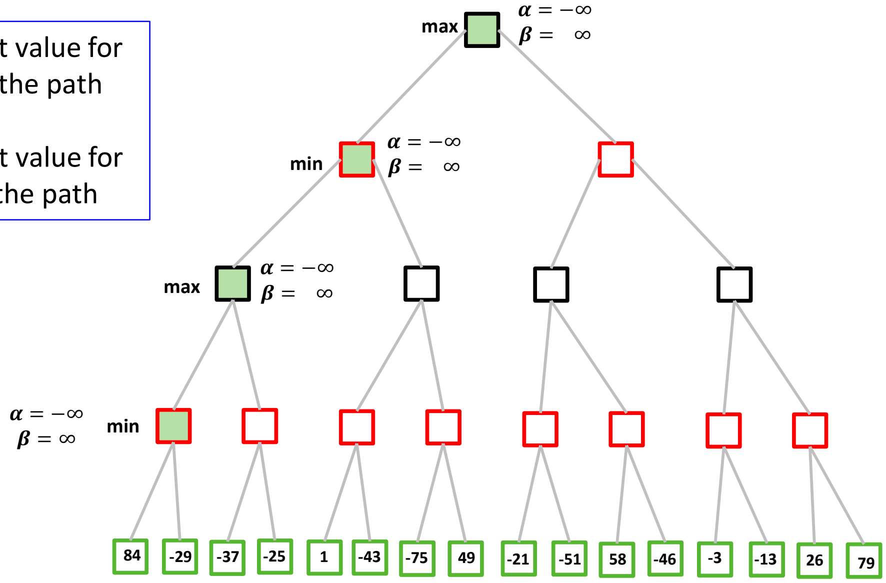


Example

17

α - the best value for max along the path

β - the best value for min along the path

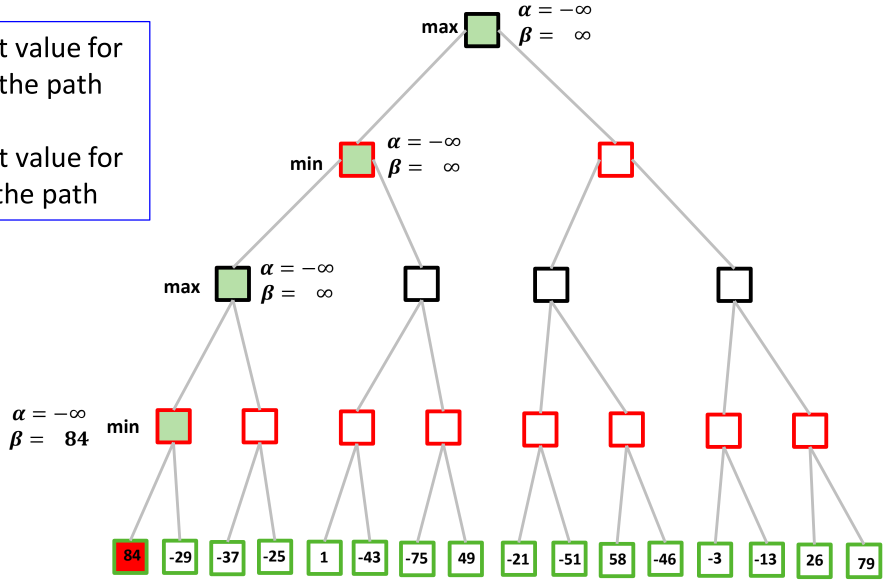


Example

18

α - the best value for max along the path

β - the best value for min along the path

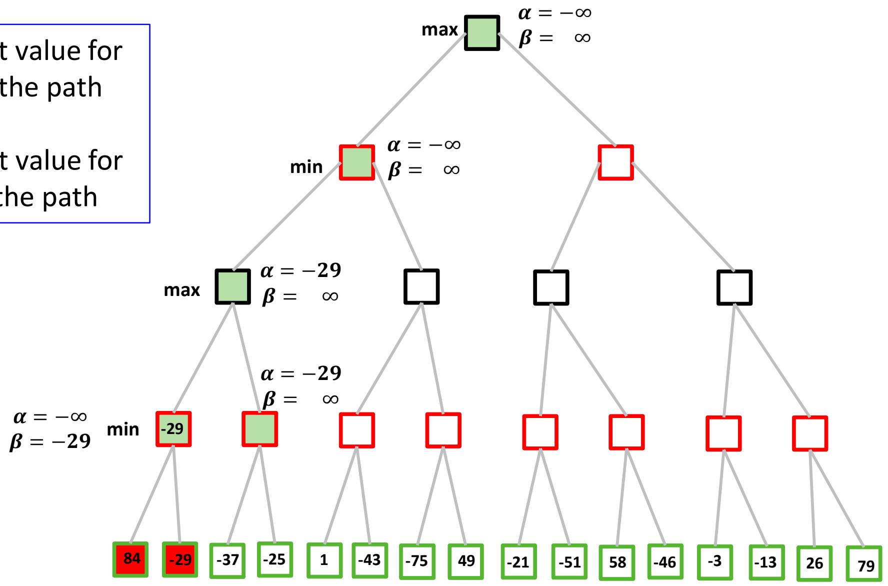


Example

19

α - the best value for max along the path

β - the best value for min along the path

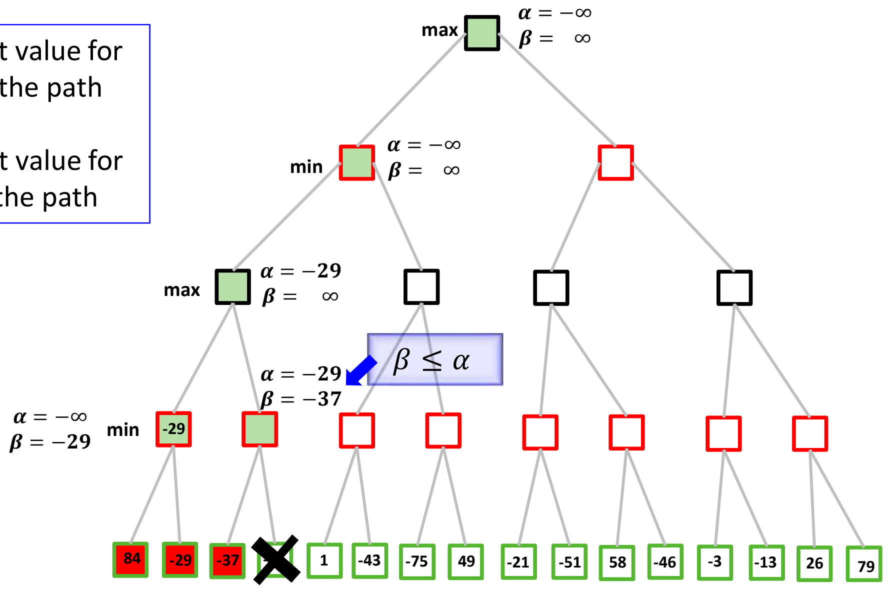


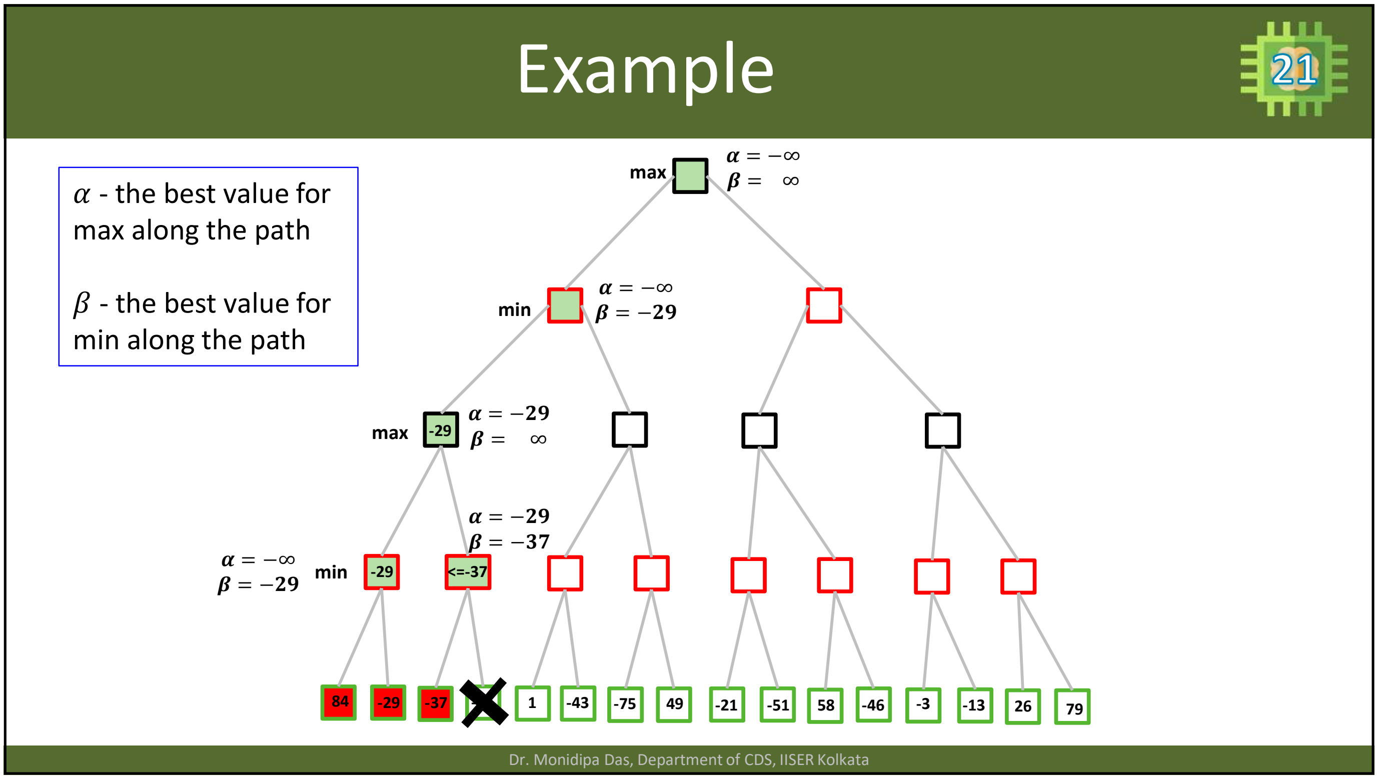
Example

20

α - the best value for max along the path

β - the best value for min along the path





Example

α - the best value for max along the path

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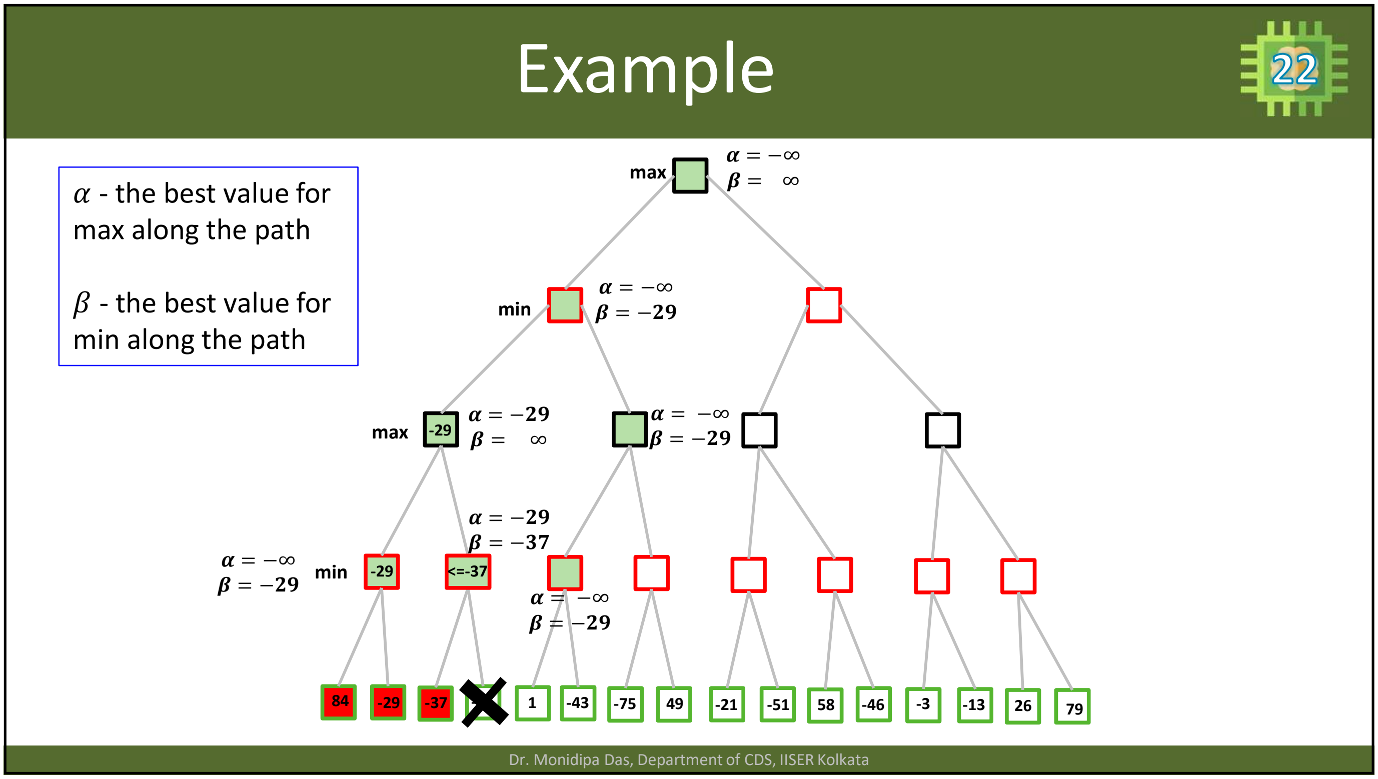


Example

α - the best value for max along the path

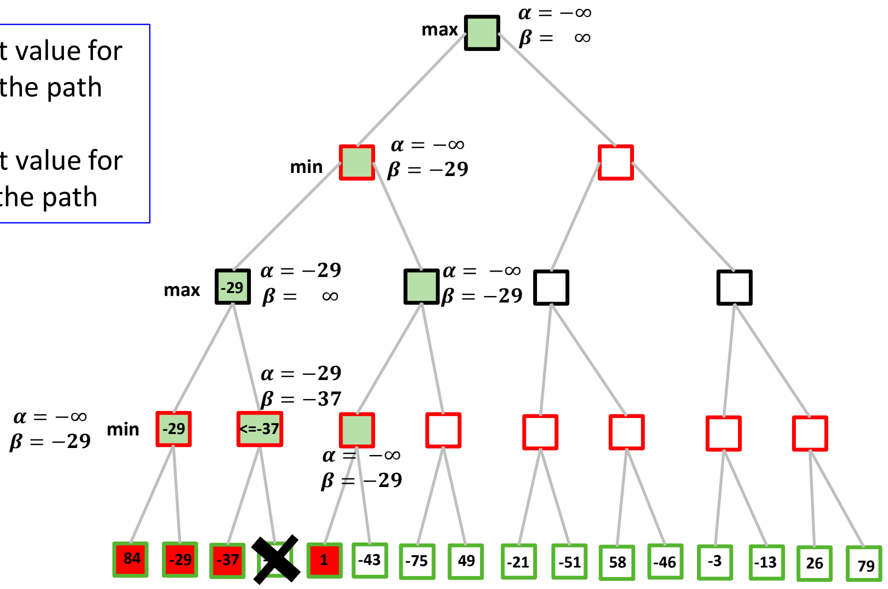
β - the best value for min along the path

Example

23

α - the best value for max along the path

β - the best value for min along the path



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Example

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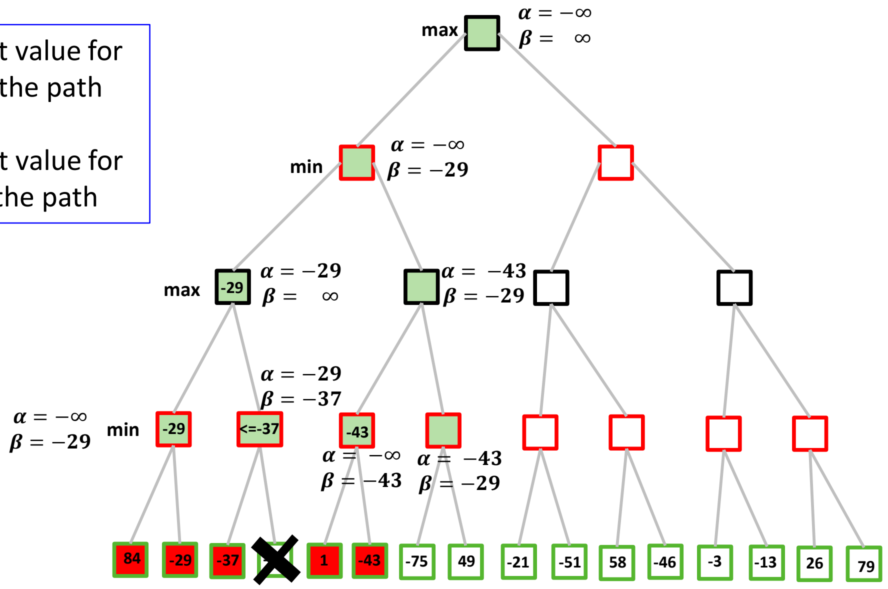
α - the best value for max along the path

β - the best value for min along the path



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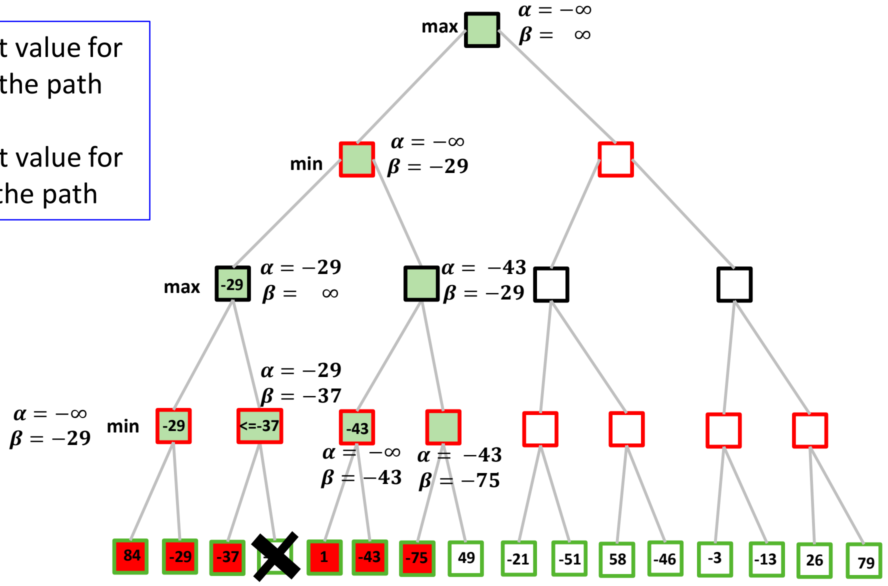
Example

25

α - the best value for max along the path

β - the best value for min along the path

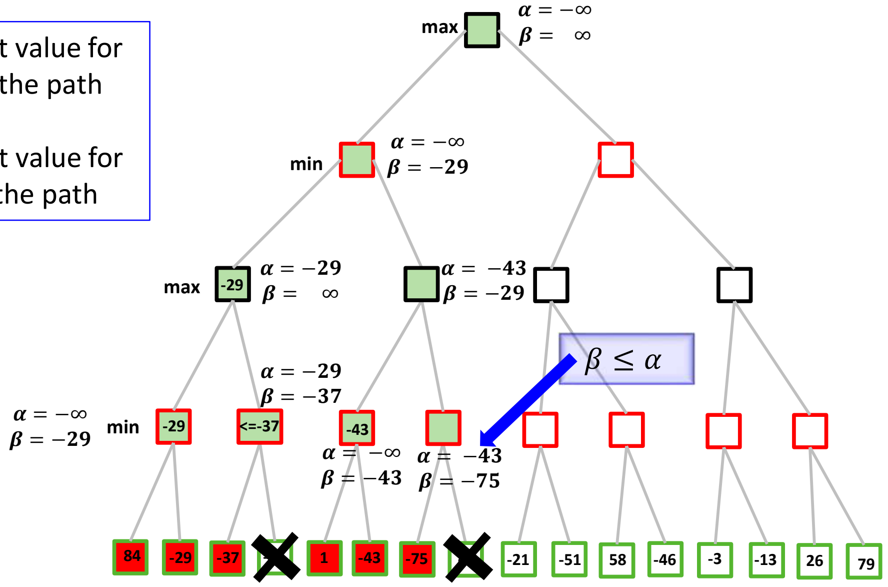


Example

26

α - the best value for max along the path

β - the best value for min along the path

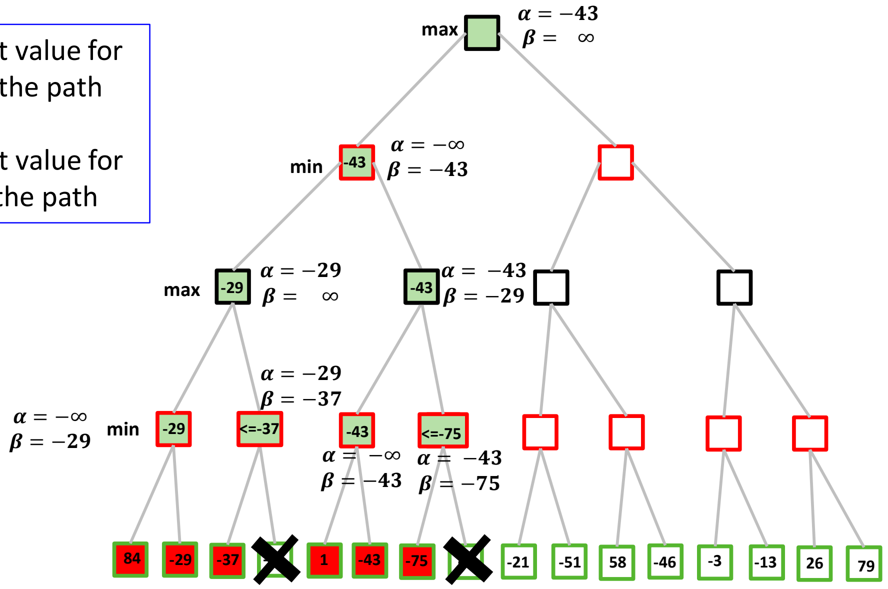


Example

27

α - the best value for max along the path

β - the best value for min along the path

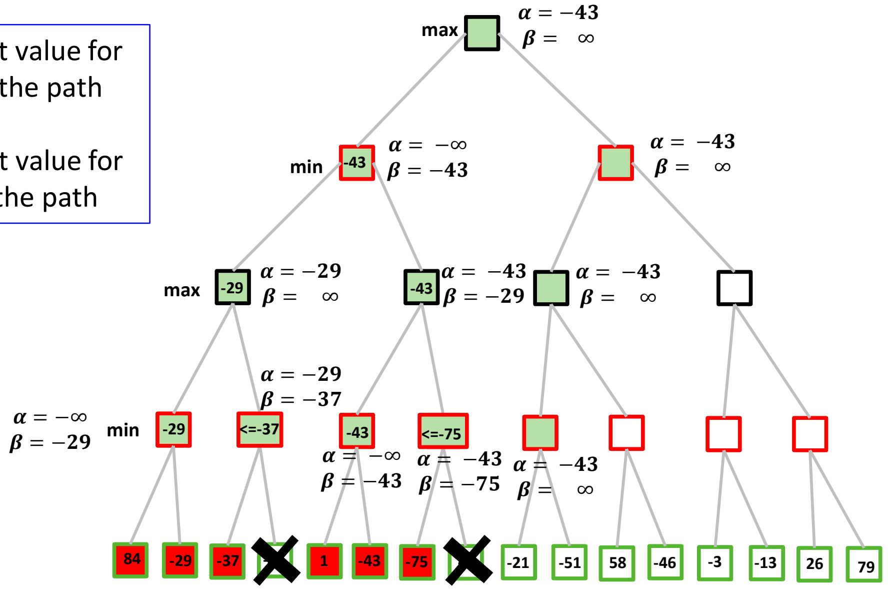


Example

28

α - the best value for max along the path

β - the best value for min along the path



Example

α - the best value for max along the path

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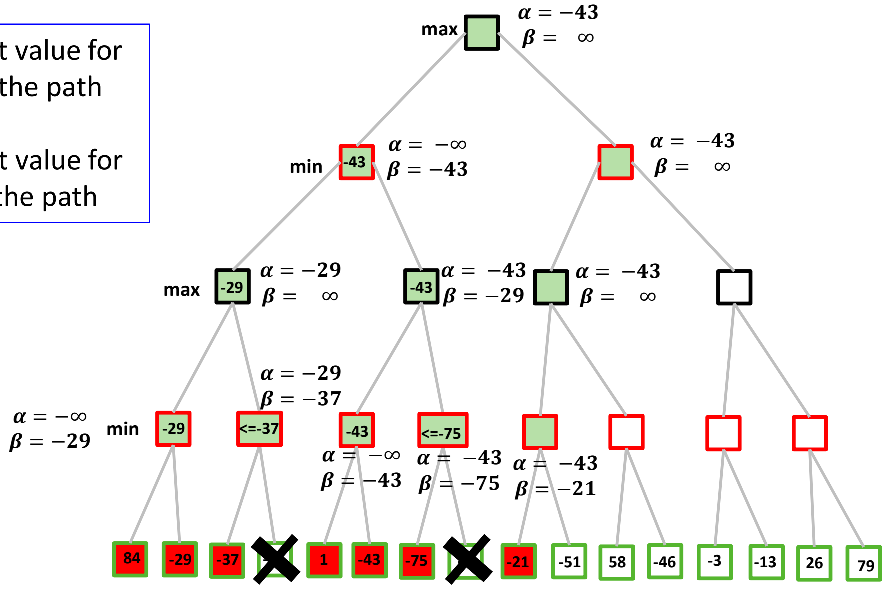

Example

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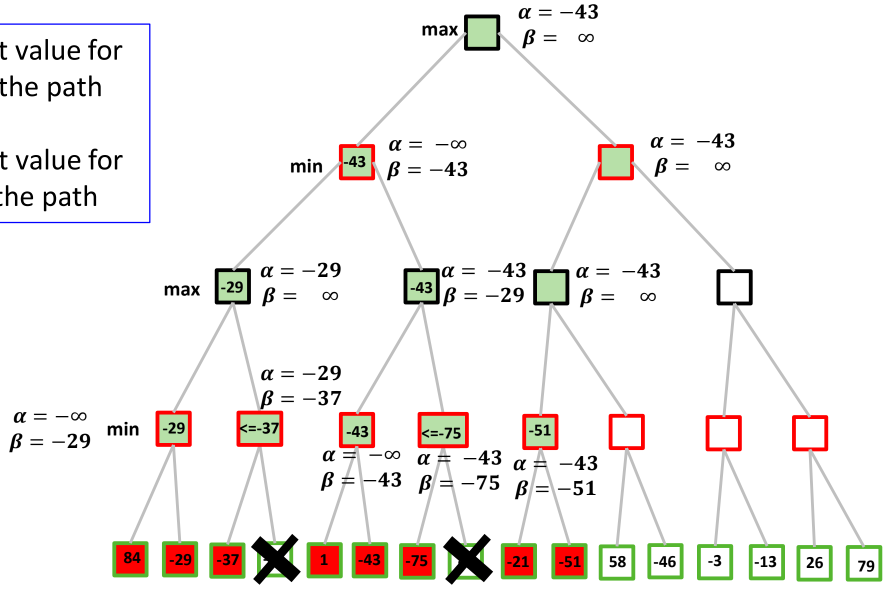

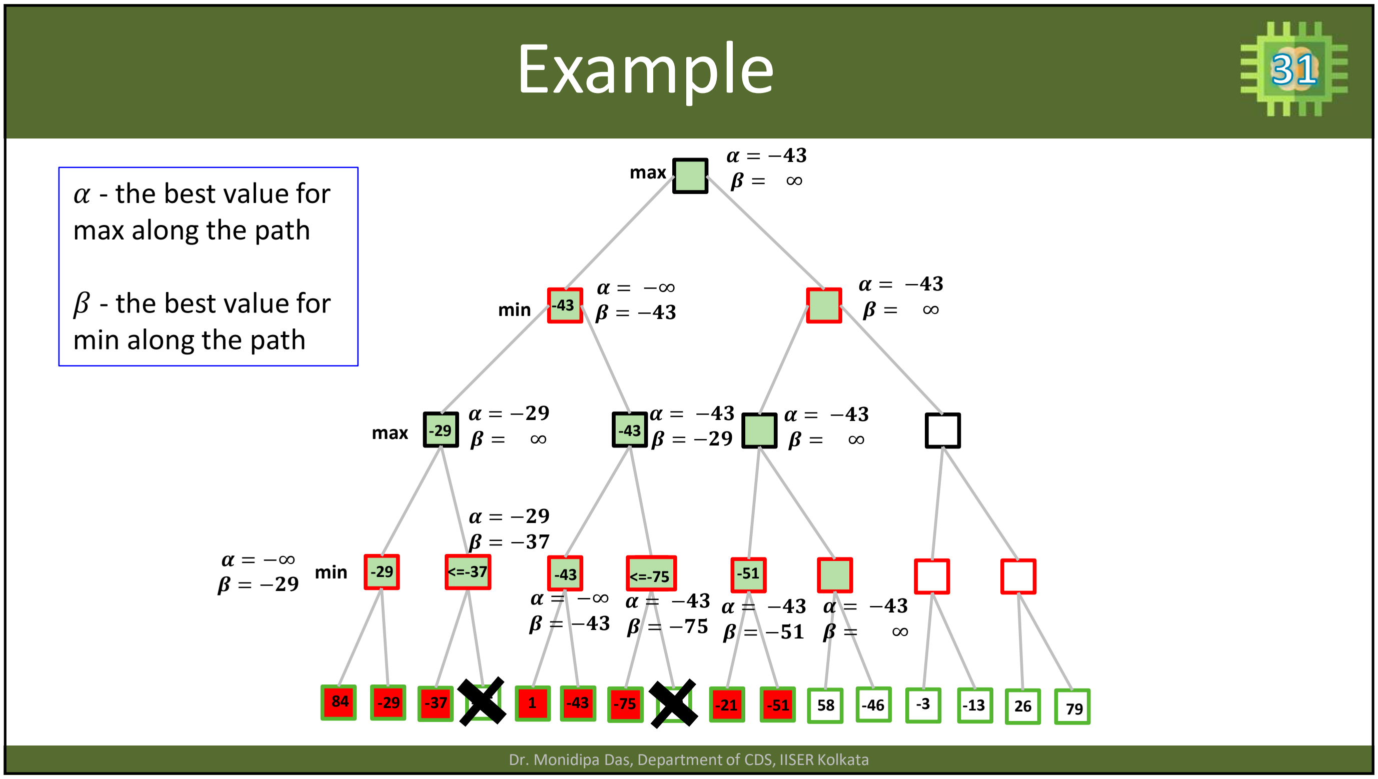

Example

31

α - the best value for max along the path

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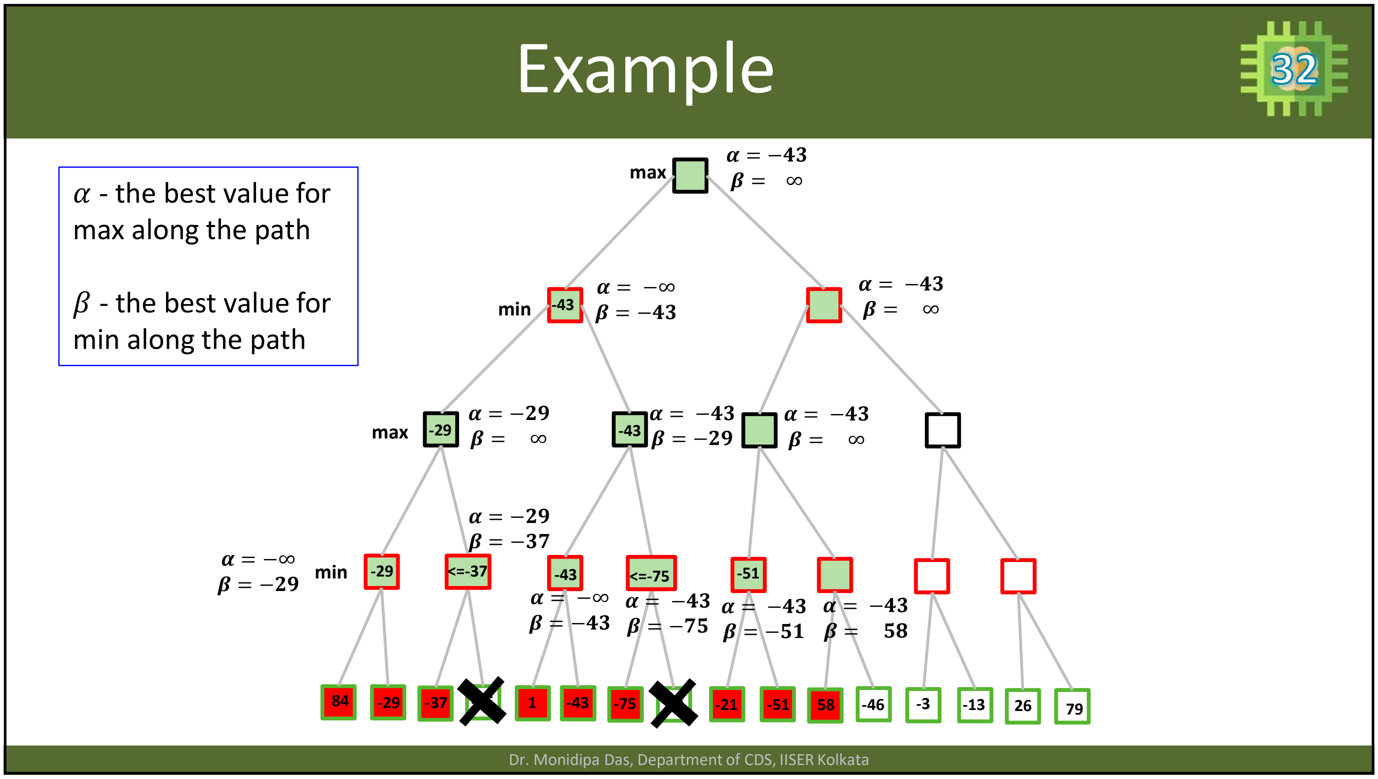

Example

32

α - the best value for max along the path

β - the best value for min along the path

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Example

33

α - the best value for max along the path

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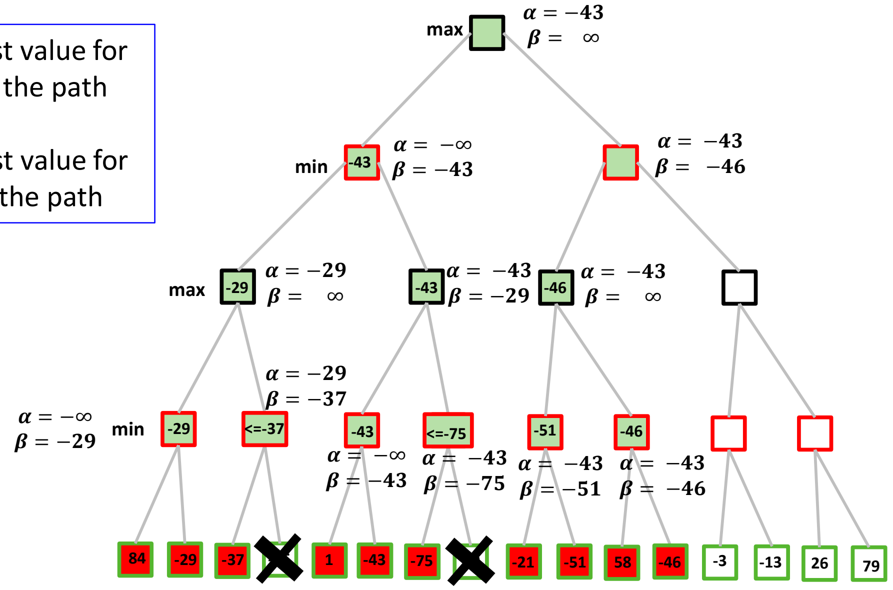

Example

34

α - the best value for max along the path

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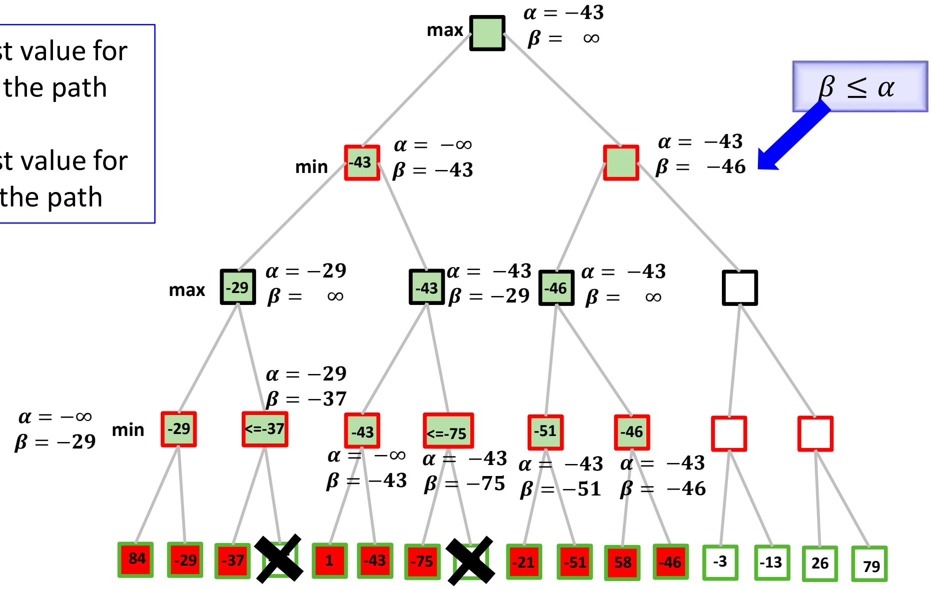
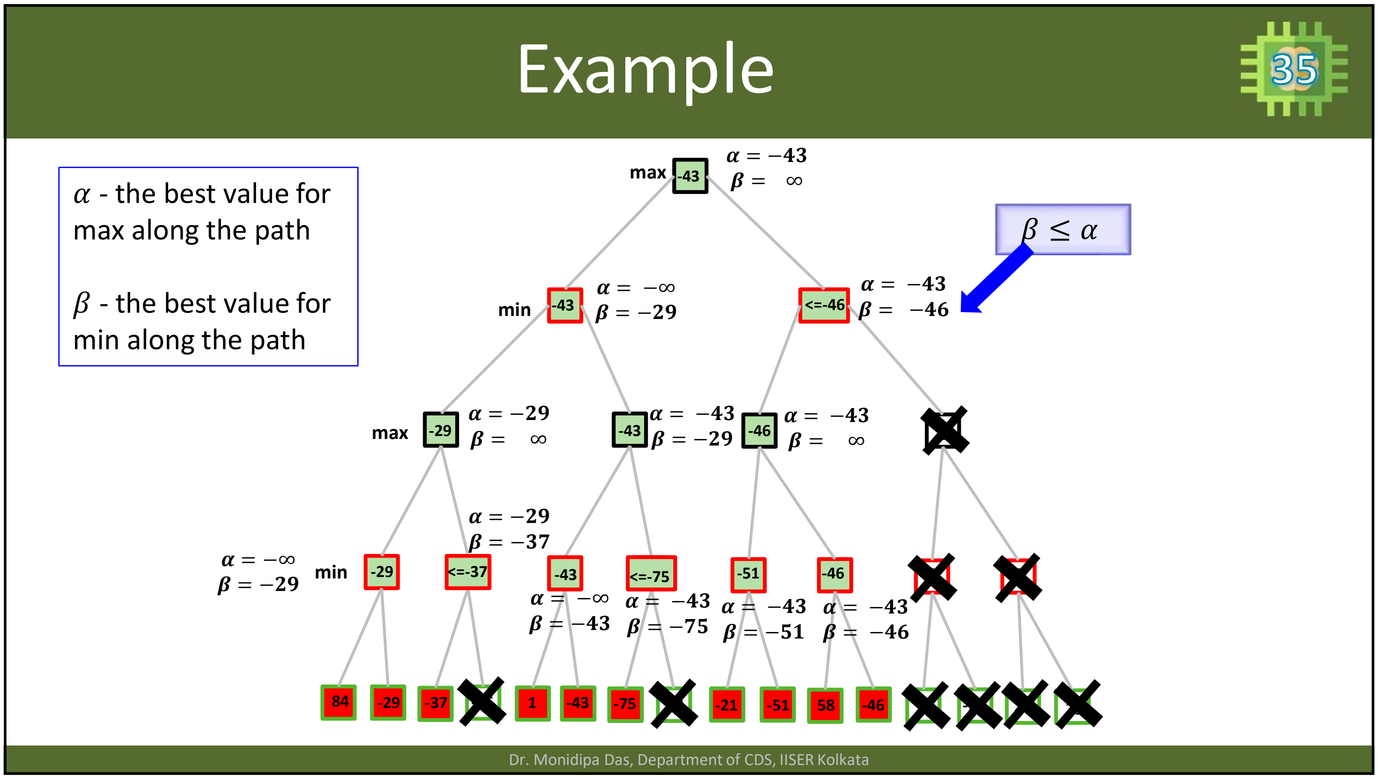

$\beta \leq \alpha$



Example

35

α - the best value for max along the path

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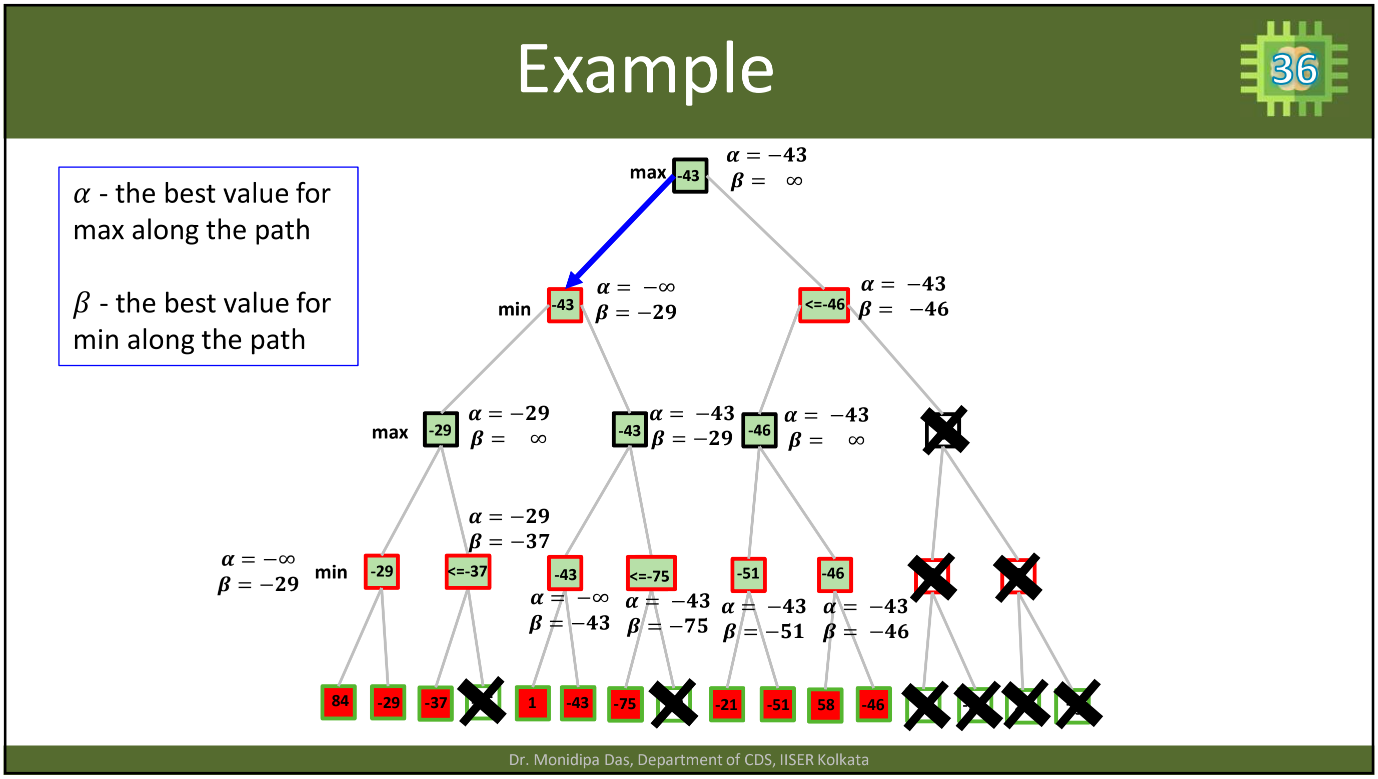

Example

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α - the best value for max along the path

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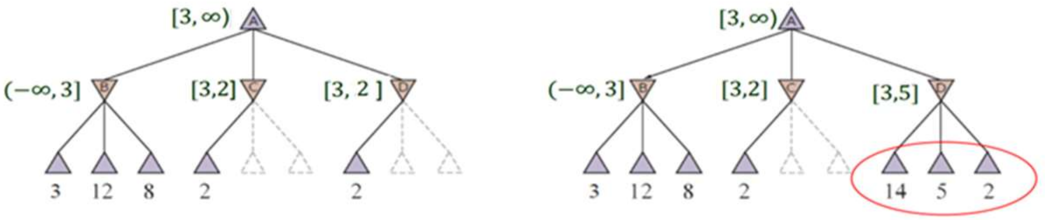
Effectiveness of alpha-beta

- Alpha-beta is guaranteed to compute the same value for the root node as computed by minimax, with less or equal computation
- **Worst case:** no pruning, examining b^m leaf nodes, where each node has b children and a m-ply search is performed
- **Best case:** examines only $O(b^{m/2})$ nodes.
  - Result is you can search twice as deep as minimax!
- **Best case** is when each player's best move is the first alternative generated

Move Ordering



Successors 14 and 5 would've been pruned had 2 been generated first.

- Effectiveness of pruning is highly dependent on the order in which successors are generated.
- "Perfect ordering" has effective branching factor $\sqrt{b}$, which limits examination to only $O(b^{m/2})$ nodes compared to $O(b^m)$ for minimax.
- $O(b^{3m/4}$) nodes for random move ordering.

Good Enough?

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- Chess (Minimax):
  - branching factor b≈35
  - game length m≈100
  - search space b m ≈ 35^{100}≈10^{154}
  - Exact solution completely infeasible
- Chess (Alpha Beta Pruning):
  - branching factor b≈35
  - game length m≈100
  - search space $b^{m/2} \approx 35^{50} \approx 10^{77}$

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Questions?