

Numerical Simulation of Pendulum Using RK4 Method

Introduction

In classical mechanics, the time period T of a simple pendulum undergoing small oscillations is given by the linear approximation:

$$T = 2\pi\sqrt{L/g}$$

where:

- L is the length of the pendulum
- g is the acceleration due to gravity

However, this formula is only accurate for small angles (typically less than 15 degrees). For larger amplitudes, the motion becomes nonlinear, and a more accurate solution requires solving the nonlinear differential equation numerically.

Nonlinear Pendulum Equation

The true equation of motion for a pendulum without the small-angle approximation is:

$$d^2\theta/dt^2 + (g/L) * \sin(\theta) = 0$$

This is a second-order nonlinear ODE. To solve it using numerical methods, we convert it into two coupled first-order equations:

Let $u = \theta$ (angle), $v = d\theta/dt$ (angular velocity)

Then:

$$du/dt = v$$

$$dv/dt = -(g/L) * \sin(u)$$

Using RK4 Method

The fourth-order Runge-Kutta (RK4) method is a powerful numerical method to solve differential equations. We use RK4 to solve the two coupled equations above. At each time step, we compute intermediate values (K_1, K_2, K_3, K_4 and L_1, L_2, L_3, L_4) and update u and v accordingly.

Estimating Time Period from RK4 Simulation

To estimate the time period:

1. Start with an initial angle (e.g., $\theta_0 = 1$ radian) and zero initial velocity.
2. Integrate the system using RK4.
3. Monitor the solution to detect when the angle θ returns close to its initial value with positive angular velocity. This indicates the pendulum has completed one full oscillation.

4. Record the time t at this point as the numerical time period.

This approach works for both small and large angles since it solves the full nonlinear equation.

Conclusion

The RK4 method allows accurate simulation of pendulum motion beyond the limits of the small-angle approximation. It numerically integrates the nonlinear ODE and enables us to estimate the time period for large amplitude oscillations, where the linear formula fails.