3-5 DATA RATE LIMITS

A very important consideration in data communications is how fast we can send data, in bits per second, over a channel. Data rate depends on three factors:

1. ***The bandwidth available***
2. ***The level of the signals we use***
3. ***The quality of the channel (the level of noise)***

Topics discussed in this section:

- **Noiseless Channel: Nyquist Bit Rate**
- **Noisy Channel: Shannon Capacity**
- **Using Both Limits**

Note

Increasing the levels of a signal increases the probability of an error occurring, in other words it reduces the reliability of the system. Why??

Capacity of a System

- The bit rate of a system increases with an increase in the number of signal levels we use to denote a symbol.
- A symbol can consist of a single bit or "n" bits.
- The number of signal levels = 2^n.
- As the number of levels goes up, the spacing between level decreases -> increasing the probability of an error occurring in the presence of transmission impairments.

Nyquist Theorem

- Nyquist gives the upper bound for the bit rate of a transmission system by calculating the bit rate directly from the number of bits in a symbol (or signal levels) and the bandwidth of the system (assuming 2 symbols/per cycle and first harmonic).
- Nyquist theorem states that for a noiseless channel:

$$C = 2\ B \log_2 2^n$$

C= capacity in bps

B = bandwidth in Hz

Example 3.33

Does the Nyquist theorem bit rate agree with the intuitive bit rate described in baseband transmission?

Solution

They match when we have only two levels. We said, in baseband transmission, the bit rate is 2 times the bandwidth if we use only the first harmonic in the worst case. However, the Nyquist formula is more general than what we derived intuitively; it can be applied to baseband transmission and modulation. Also, it can be applied when we have two or more levels of signals.

Example 3.34

Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. The maximum bit rate can be calculated as

$$\boxed{\text{BitRate} = 2 \times 3000 \times \log_2 2 = 6000 \text{ bps}}$$

Example 3.35

Consider the same noiseless channel transmitting a signal with four signal levels (for each level, we send 2 bits). The maximum bit rate can be calculated as

$$\text{BitRate} = 2 \times 3000 \times \log_2 4 = 12{,}000 \text{ bps}$$

Example 3.36

We need to send 265 kbps over a noiseless channel with a bandwidth of 20 kHz. How many signal levels do we need?

Solution

We can use the Nyquist formula as shown:

$$265{,}000 = 2 \times 20{,}000 \times \log_2 L$$

$$\log_2 L = 6.625 \qquad L = 2^{6.625} = 98.7 \text{ levels}$$

Since this result is not a power of 2, we need to either increase the number of levels or reduce the bit rate. If we have 128 levels, the bit rate is 280 kbps. If we have 64 levels, the bit rate is 240 kbps.

Shannon's Theorem

- Shannon's theorem gives the capacity of a system in the presence of noise.

$$C = B \log_2(1 + SNR)$$

Example 3.37

Consider an extremely noisy channel in which the value of the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. For this channel the capacity C is calculated as

$$C = B \log_2(1 + \mathrm{SNR}) = B \log_2(1 + 0) = B \log_2 1 = B \times 0 = 0$$

This means that the capacity of this channel is zero regardless of the bandwidth. In other words, we cannot receive any data through this channel.

Example 3.38

We can calculate the theoretical highest bit rate of a regular telephone line. A telephone line normally has a bandwidth of 3000. The signal-to-noise ratio is usually 3162. For this channel the capacity is calculated as

$$C = B \log_2 (1 + \mathrm{SNR}) = 3000 \log_2 (1 + 3162) = 3000 \log_2 3163$$
$$= 3000 \times 11.62 = 34{,}860 \text{ bps}$$

This means that the highest bit rate for a telephone line is 34.860 kbps. If we want to send data faster than this, we can either increase the bandwidth of the line or improve the signal-to-noise ratio.

Example 3.39

The signal-to-noise ratio is often given in decibels. Assume that SNR_{dB} = 36 and the channel bandwidth is 2 MHz. The theoretical channel capacity can be calculated as

$$SNR_{dB} = 10 \log_{10} SNR \rightarrow SNR = 10^{SNR_{dB}/10} \rightarrow SNR = 10^{3.6} = 3981$$

$$C = B \log_2 (1 + SNR) = 2 \times 10^6 \times \log_2 3982 = 24 \text{ Mbps}$$

Example 3.40

For practical purposes, when the SNR is very high, we can assume that SNR + 1 is almost the same as SNR. In these cases, the theoretical channel capacity can be simplified to

$$\boxed{C = B \times \frac{\mathrm{SNR_{dB}}}{3}}$$

For example, we can calculate the theoretical capacity of the previous example as

$$\boxed{C = 2\ \mathrm{MHz} \times \frac{36}{3} = 24\ \mathrm{Mbps}}$$

Example 3.41

We have a channel with a 1-MHz bandwidth. The SNR for this channel is 63. What are the appropriate bit rate and signal level?

Solution

First, we use the Shannon formula to find the upper limit.

$$C = B \log_2 (1 + \text{SNR}) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 64 = 6 \text{ Mbps}$$

Example 3.41 (continued)

The Shannon formula gives us 6 Mbps, the upper limit. For better performance we choose something lower, 4 Mbps, for example. Then we use the Nyquist formula to find the number of signal levels.

$$\boxed{4 \text{ Mbps} = 2 \times 1 \text{ MHz} \times \log_2 L \quad \Rightarrow \quad L = 4}$$

Note

The Shannon capacity gives us the upper limit; the Nyquist formula tells us how many signal levels we need.

3-6 PERFORMANCE

One important issue in networking is the performance of the network—how good is it? We discuss quality of service, an overall measurement of network performance, in greater detail in Chapter 24. In this section, we introduce terms that we need for future chapters.

Topics discussed in this section:

- **Bandwidth - capacity of the system**
- **Throughput - no. of bits that can be pushed through**
- **Latency (Delay) - delay incurred by a bit from start to finish**
- **Bandwidth-Delay Product**
- **Jitter**

Note

In networking, we use the term bandwidth in two contexts.

- **The first, bandwidth in hertz, refers to the range of frequencies in a composite signal or the range of frequencies that a channel can pass.**
- **The second, bandwidth in bits per second, refers to the speed of bit transmission in a channel or link. Often referred to as Capacity.**

Example 3.42

The bandwidth of a subscriber line is 4 kHz for voice or data. The bandwidth of this line for data transmission can be up to 56,000 bps using a sophisticated modem to change the digital signal to analog.

Example 3.43

If the telephone company improves the quality of the line and increases the bandwidth to 8 kHz, we can send 112,000 bps by using the same technology as mentioned in Example 3.42.

Example 3.44

A network with bandwidth of 10 Mbps can pass only an average of 12,000 frames per minute with each frame carrying an average of 10,000 bits. What is the throughput of this network?

Solution

We can calculate the throughput as

$$\text{Throughput} = \frac{12{,}000 \times 10{,}000}{60} = 2\ \text{Mbps}$$

The throughput is almost one-fifth of the bandwidth in this case.

Propagation & Transmission delay

- Propagation speed - speed at which a bit travels though the medium from source to destination.
- Transmission speed - the speed at which all the bits in a message arrive at the destination. (difference in arrival time of first and last bit)

Propagation and Transmission Delay

- Propagation Delay = Distance/Propagation speed
- Transmission Delay = Message size/bandwidth bps
- Latency = Propagation delay + Transmission delay + Queueing time + Processing time

Example 3.45

What is the propagation time if the distance between the two points is 12,000 km? Assume the propagation speed to be 2.4 × 108 m/s in cable.

Solution

We can calculate the propagation time as

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

The example shows that a bit can go over the Atlantic Ocean in only 50 ms if there is a direct cable between the source and the destination.

Example 3.46

What are the propagation time and the transmission time for a 2.5-kbyte message (an e-mail) if the bandwidth of the network is 1 Gbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at 2.4 × 108 m/s.

Solution

We can calculate the propagation and transmission time as shown on the next slide:

Example 3.46 (continued)

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{2500 \times 8}{10^9} = 0.020 \text{ ms}$$

Note that in this case, because the message is short and the bandwidth is high, the dominant factor is the propagation time, not the transmission time. The transmission time can be ignored.

Example 3.47

What are the propagation time and the transmission time for a 5-Mbyte message (an image) if the bandwidth of the network is 1 Mbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at 2.4×10^8 m/s.

Solution

We can calculate the propagation and transmission times as shown on the next slide.

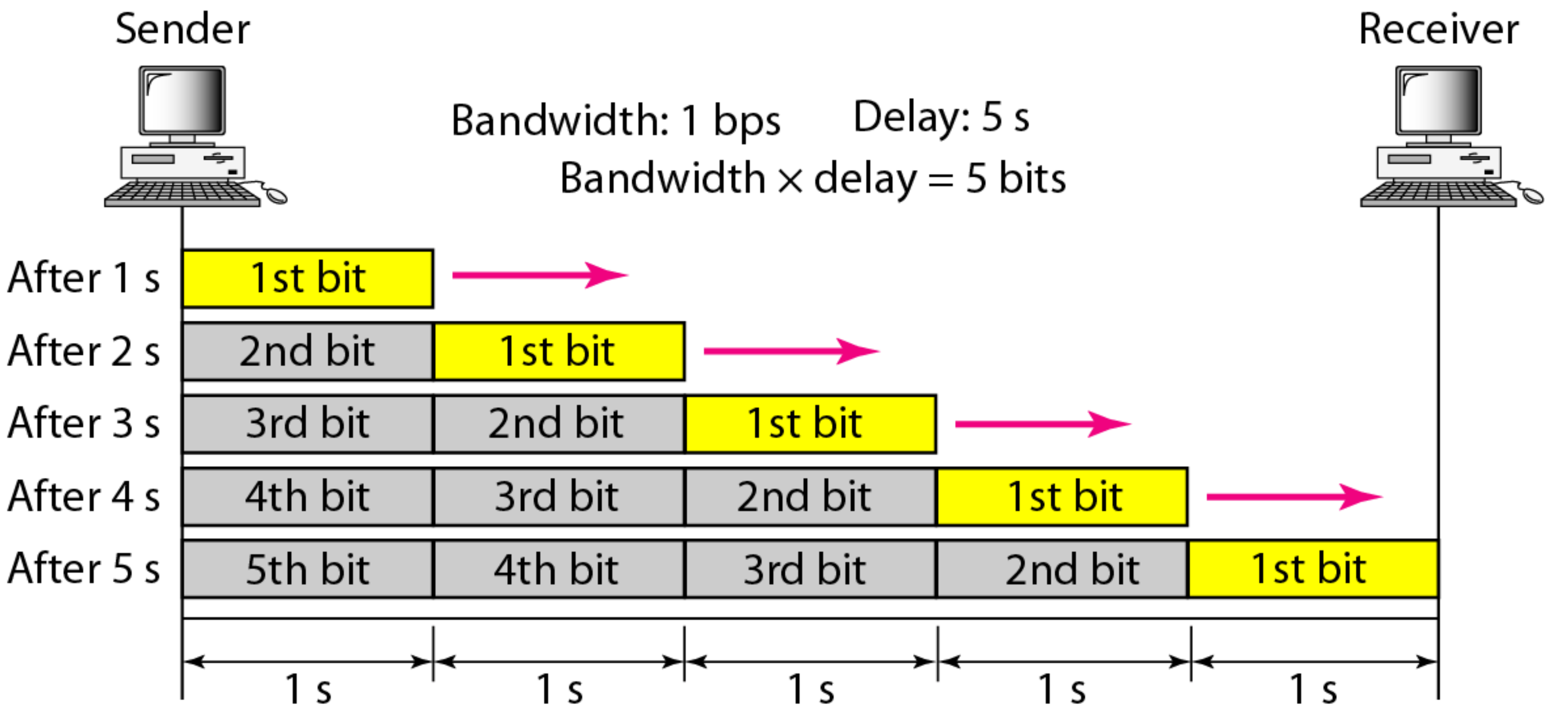
Example 3.47 (continued)

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{5{,}000{,}000 \times 8}{10^6} = 40 \text{ s}$$

Note that in this case, because the message is very long and the bandwidth is not very high, the dominant factor is the transmission time, not the propagation time. The propagation time can be ignored.

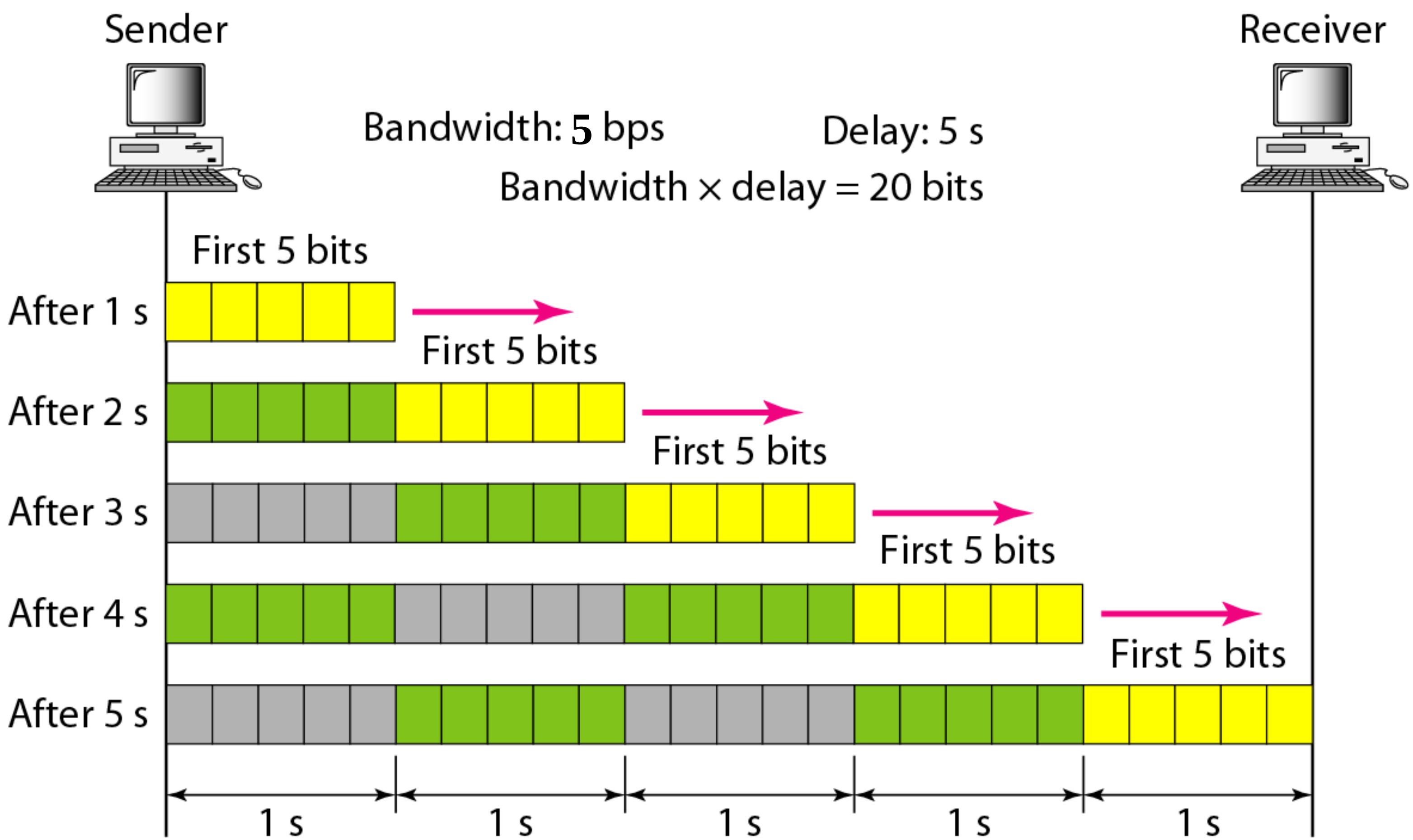
Figure 3.31 *Filling the link with bits for case 1*



Example 3.48

We can think about the link between two points as a pipe. The cross section of the pipe represents the bandwidth, and the length of the pipe represents the delay. We can say the volume of the pipe defines the bandwidth-delay product, as shown in Figure 3.33.

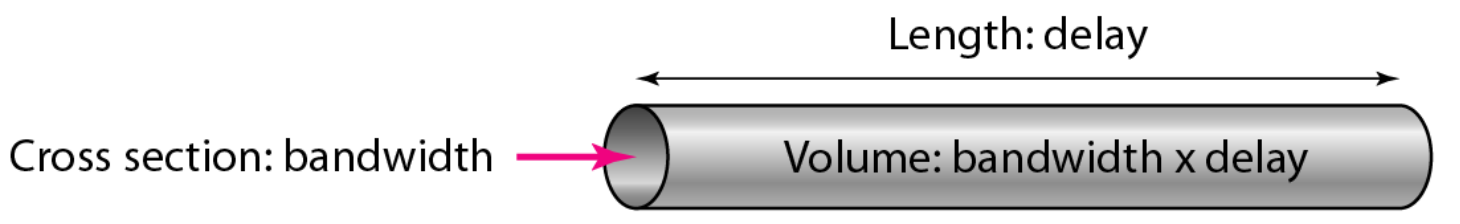
Figure 3.32 *Filling the link with bits in case 2*



Note

The bandwidth-delay product defines the number of bits that can fill the link.

Figure 3.33 *Concept of bandwidth-delay product*



Note

Jitter indicates uneven delay for different packets.