

1+3: Algorithm: An algorithm is a finite set of instruction that, accomplishes a particular task. In addition, all algorithms must satisfy the following criteria -

- ① Input
- ② Output
- ③ Definiteness
- ④ Finiteness
- ⑤ Effectiveness

Input: Zero or more quantities are externally supplied.

Output: At least one quantity is produced

Definiteness: Each instruction is clear and unambiguous.

Finiteness: For any input, the algorithm must terminate after a finite number of steps.

Effectiveness: Every instruction must be very basic so that it can be carried out in principle, by a person using only pencil and paper. It is not enough that each operation be definite as in criterion definiteness. It also must be feasible.

2: O-notation: Big O notation is used in computer science to describe the performance or complexity of an algorithm. Big O specifically describes the worst-case scenario and can be used to describe the execution time required or the space used by an algorithm.

Big O notation is something that can appear more confusing than it actually is. At its simplest, Big O notation is a way to represent the relative complexity of algorithm. also use for-

- ① Similar purpose
- ② More complex portion
- ③ Worst case scenario
- ④ Machine and Instance Independent
- ⑤ Not a speed comparison.

4. Best Case: Fastest time to complete, with optimal inputs chosen. For example the best case for a ~~strong~~ sorting algorithm would be data that's already sorted.

Worst Case: Slowest time to complete with pessimistic input chosen. For example the worst case for a sorting algorithm might be data that's sorted in reverse order.

Average Case: An estimate mean. Run the algorithm many times, using many different inputs of size  $n$  that come from some distribution that generates these inputs. Compute the total running time and divide by the number of trials.

Algorithm	Best Case	Average Case	Worst Case
Quick Sort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$
Merge Sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Heap Sort	$O(n)$	$O(n \log n)$	$O(n \log n)$
Bubble Sort	$O(n)$	$O(n^2)$	$O(n^2)$
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$

## 5.1 Asymptotic notation (O, -\Omega, \Theta)

Big-O: Big O commonly written as O, is an Asymptotic Notation for the worst case or ceiling of growth for a given function. It provides us with an asymptotic upper bound for the growth rate of runtime of an algorithm.

$f(n)$  is  $O(g(n))$  if for some real constants  $c(c > 0)$  and  $n_0$ ,  $f(n) \leq c * g(n)$  for every input size  $n (n > n_0)$

Big-Omega: Big Omega commonly written as  $\Omega$ , is an Asymptotic Notation for the best case, or a floor growth rate for a

Given function. It provides us with an asymptotic lower bound for the growth rate of runtime of a algorithm.

$f(n)$  is  $\Omega(g(n))$ , if for some real constants  $c(c > 0)$  and  $n_0(n_0 > 0)$ ,  $f(n) \geq c * g(n)$  for every input size  $n(n > n_0)$ .

Theta: Theta commonly written as  $\Theta$ , is a Asymptotic Notation to denote the asymptotically tight bound on the growth rate of runtime of an algorithm.

$f(n)$  is  $\Theta(g(n))$ , if for some real constants  $c_1, c_2$  and  $n_0$  ( $c_1 > 0, c_2 > 0, n_0 > 0$ )

$c_1^*(g(n)) \leq f(n) \leq c_2^*(g(n))$  for input size  $n(n > n_0)$

## Set - 2

### 1. Divide and Conquer Algorithm:

Like Greedy and Dynamic programming, Divide and conquer is an algorithmic paradigm. A typical Divide and Conquer algorithm solves a problem using following three steps-

- ① Divide: Divide the given problem into subproblems of some type.
- ② Conquer: Recursively solve these subproblem.
- ③ Combine: Appropriately combine the answers.

Following are some standard algorithms that are Divide and conquer algorithms.

- ① Binary Search
- ② Quicksort
- ③ Mergesort
- ④

## Compare between them

### Divide and Conquer

An algorithm that recursively breaks down a problem into two or more sub-problems of the same type until it becomes simple enough to be solved directly.

Sub-problem are independent of each other

### Recursive

More time consuming as it solves each sub-problem

Less efficient

Used by merge sort, quicksort and binary search

### Dynamic programming

A.D algorithm that helps to efficiently solve a class of problem that have overlapping subproblems and optimal

Sub-problems are inter-dependent

### Non - recursive

Less time-consuming as it solves problem

More efficient

Used by matrix chain multiplication, optimal binary search tree.

## ②: Heap Sort Algorithm:

Step 1: Start

Step 2: Store N integer values in array.

Step 3: Construct a binary tree with given list of elements.

Step 4: Transform the binary tree into min heap.

Step 5: Create a new node at the end of heap.

Step 6: Assign new value to the node

Step 7: Compare the value of the child node with its parent.

Step 8: If value of parent is less than child, then swap them.

Step 9: Repeat step 7 and step 4 until heap property holds.

Step 10: Put the deleted element into the sorted list.

Step 11: Display the sorted list.

Step 12: Stop.

### 3.1 Quick Sort Algorithm

Step 1 : Start

Step 2 : Store the input Array.

Step 3 : choose the highest index value has pivot.

Step 4 : Take two variables to point left

Step 5 : Left point to the high index

Step 6 : Right point to the less index

Step 7 : while value at left is less than pivot move right

Step 8 : while value at right is greater than pivot move left

Step 9 : If both step 7 and step 8 does not match swap left and right.

Step 10 : if left  $\geq$  right, the point where they met is new pivot.

Step 11 : display the sorted list.

Step 12 : stop.

## 41 Huffman Coding

example

a	b	c	d	e	f
5	9	12	13	16	45

Solution: There are mainly two major parts in Huffman Coding.

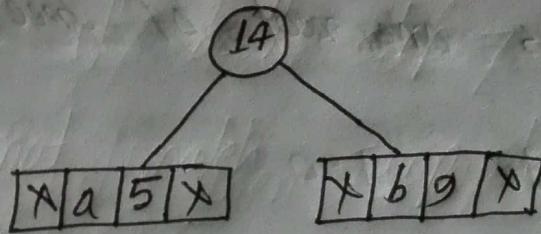
① Build a Huffman tree from input characters

② Traverse the Huffman tree and assign codes to characters.

① Build a Huffman tree:

Step 1: Build a min heap that contains 6 nodes where each node represents root of a tree with single node

Step 2: Extract two minimum frequency nodes from min heap and add a new internal node with frequency  $5+9=14$

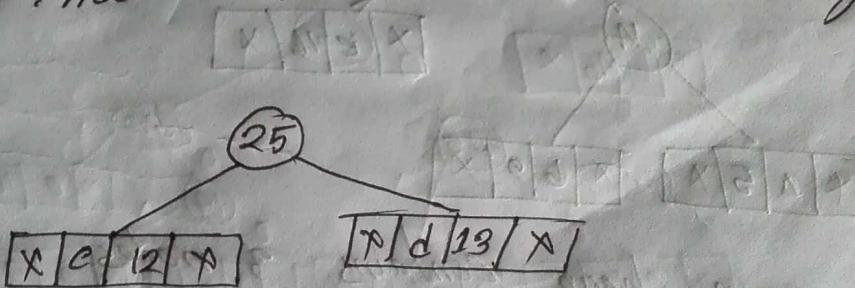


Now min heap contains 5 nodes where 4 nodes are roots of trees with single element each and one heap node is root of tree with 3 element.

c	d	IN	e	f
12	13	14	16	45

i.e. IN = Internal node.

Step 3: Extract two minimum frequency nodes from heap. Add a new node with frequency  $12+13=25$



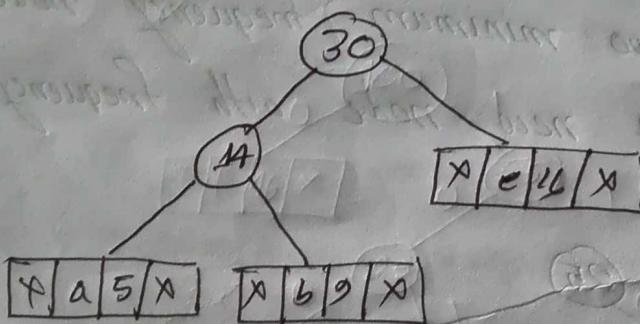
Now min heap contains 4 nodes where 2 nodes are roots of trees with single element each and

two heap nodes are root of tree with more than one nodes.



Internal Node value 14  
Internal Node value 16  
Internal Node value 25  
Internal Node value 45

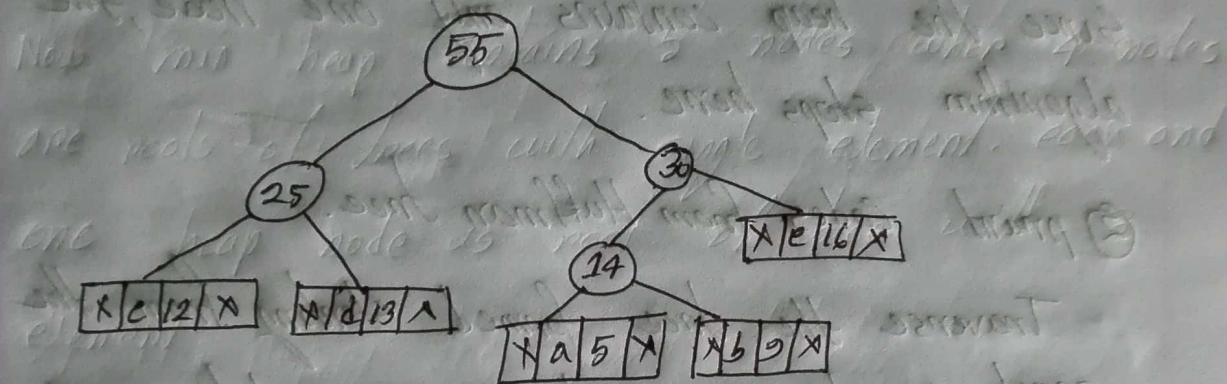
Step 4: Extract two minimum frequency nodes. Add new internal node with frequency  $14+16=30$



Now min heap contains 3 nodes

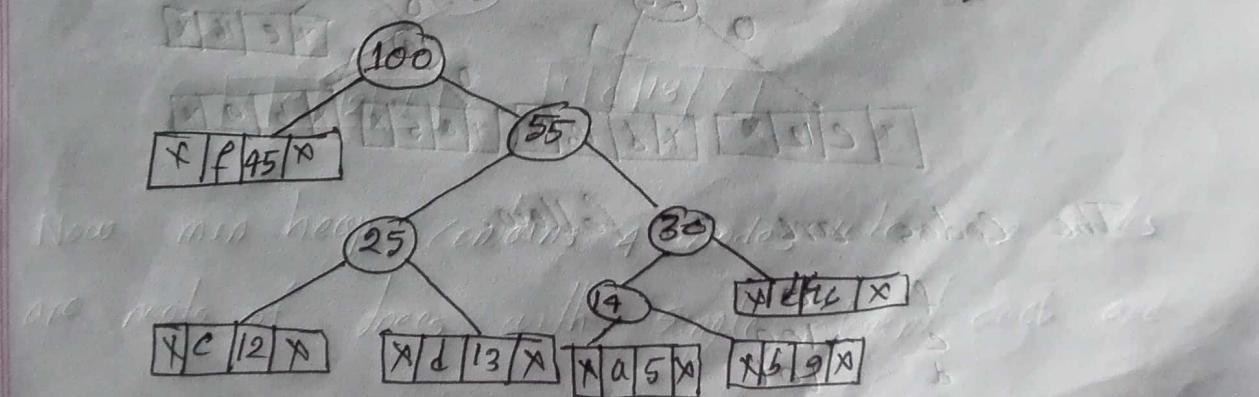
Internal Node  
Internal Node  
f  
45

Step 5: Extract two minimum frequency nodes. Add a new internal node with frequency  $25+30=55$



Now min heap contains 2 Nodes.

Step 6: Extract two minimum frequency nodes from min heap. Add a new internal node with frequency  $45+55=100$ .



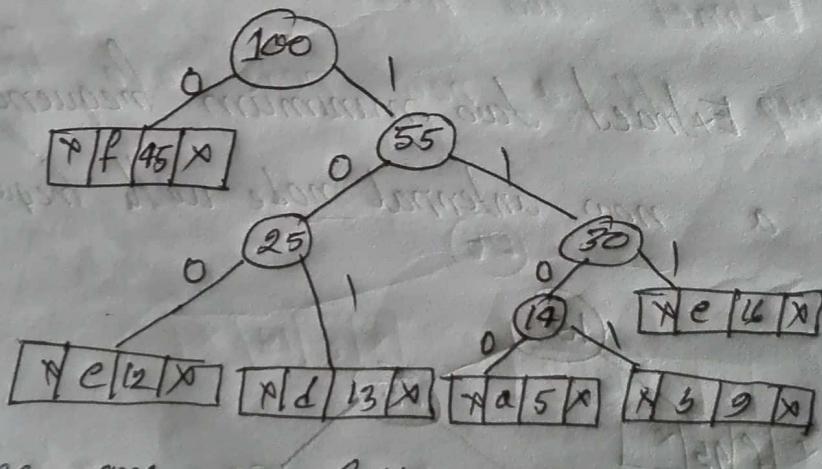
Now min heap contains only one node

Internal Node 100

Since the heap contains only one node, the algorithm stops here.

② print codes from Huffman tree.

Traverse the tree formed starting from the root. Maintain an auxiliary array. While moving to the left child, write 0 to the array. While moving to the right child, write 1 to the array.



The codes are as follow.

f	0
e	100
d	101
a	1100
b	1101
c	111

Algorithm: build Huffman tree from frequency table. Add a new node each time frequency  $25+50=55$

Huffman( $C$ )

1.  $n = |C|$

2.  $\emptyset = Q$

3. for  $i = 1$  to  $n-1$

4. allocate a new node  $z$

5.  $z.\text{left} = x = \text{Extract-Min}(Q)$

6.  $z.\text{right} = y = \text{Extract-Min}(Q)$

7.  $z.\text{freq} = x.\text{freq} + y.\text{freq}$

8. Insert  $(Q, z)$

9. return Extract-Min( $Q$ )

now internal node will frequency  $45+55=100$



11 Optimization: optimization is a program transformation technique, which tries to improve the code by making it consume less resource and deliver high speed.

In optimization, high-level general programming constructs are replaced by very efficient low-level programming codes. A code optimizing process must follow the three rules given below-

- ① The output code must not, in any way, change the meaning of the program.
- ② Optimization should increase the speed of the program and if possible, the program should demand less number of resources.
- ③ Optimization should itself be fast and should not delay the overall compilation process.

Optimization of Binary Search tree: A Binary Search tree is a tree where the key values are stored in the internal nodes. The external nodes are null nodes. The keys are ordered lexicographically. For each internal node all the keys in the left sub-tree are less than the keys in the node, and all the keys in the right sub-tree are greater.

When we know the frequency of searching each one of the keys, it is quite easy to compute the expected cost of accessing each node in the tree. An optimal binary search tree is a BST, which has minimal expected cost of locating each node.

Search time of an element in a BST is  $O(n)$ , whereas in a balanced-BST search time is  $O(\log n)$ . Again the search time can be improved in optimal cost binary search tree, placing the most frequently used data in the root and closer to the root element, while placing the least frequently used data near leaves and in leaves.

## 2.1 Term of optimum Solution:

An Optimal Solution is a feasible Solution where the objective function reaches its maximum (or minimum) value - for example, the most profit, or the least cost.

A globally optimal solution is one where there are no other feasible solutions with better objectives function values.

Two main properties of a problem suggest that the given problem can be solved using dynamic programming. These properties are overlapping sub-problems and optimal substructure.

Dynamic programming algorithm is designed using the following four steps - where the values are stored in the internal nodes. The optimal odds are

- ① characterize the structure of an optimal solution.
- ② Recursively define the value of an optimal solution.
- ③ Compute the value of an optimal solution, typically in a bottom-up fashion.
- ④ Construct an optimal solution from the computed information.

Search time of an element in a balanced binary tree is  $O(\log n)$ . If the tree is skewed, then the search time can be improved by optimal test. In binary search tree, we divide the data equally and data in the root and closer to the root elements are visited the least frequently used data near leaves and in leaves.

### 3.1 Matrix multiplication in dynamic programming:

We shall use the dynamic programming method to determine how to optimally parenthesize a matrix chain. In so doing, we shall follow the four-step sequence that we stated at the beginning of this chapter.

- ① Characterize the structure of an optimal solution.
- ② Recursively defines the value of an optimal solution.
- ③ Compute the value of an optimal solution.
- ④ Construct an optimal solution from computed information.

we shall go through these steps in order demonstrating clearly how we apply each step to the problem.

1:  $n := \text{length}[p] - 1$

2: for  $i := 1$  to  $n$

3:      $m[i, i] := 0$

4: for  $l := 2$  to  $n$

5:     for  $i := 1$  to  $\cancel{n-l+1}$

6:          $j := i+l-1$       $m[i, j] := \infty$

7:             for  $k := i$  to  $j-1$

8:                  $q := m[i, k] + m[k+1, j] + d_{i-1} \cdot d_k \cdot d_j$

9:                 if  $q < m[i, j]$

10:                      $m[i, j] := q$       $s[i, j] := k$

11: return  $m$  and  $s$

4.1

## Dynamic programming advantage:

Dynamic programming is used where we have problems, which can be divided into similar sub-problems, so that their results can be re-used. Mostly, these algorithms are used for optimization. Before solving the in-hand sub-problems, dynamic algorithm will try to examine the results of the previously solved sub-problems. The solutions of sub-problems are combined in order to achieve the best solution.

The following computer problems can be solved using dynamic programming approach

- ① Fibonacci number series
- ② Knapsack problem
- ③ Tower of Hanoi
- ④ All pair shortest path by Floyd Warshall
- ⑤ Shortest path by Dijkstra
- ⑥ Project scheduling

Dynamic programming can be used in both top-down and bottom-up manner. And of course, most of the times, referring to the previous solution output is cheaper than recomputing in terms of CPU cycles.

## 5.1 Longest Common Subsequence :-

We have two strings  $X = ABCD$  and  $Y = ABD$  to find the longest common subsequence.

String X	A	B	C	D
String Y	A	B	D	

Making completed LCS Table:

Step 1: This table is used to store the LCS sequence for each step of the calculation. The first column and first row have been filled in with 0, because when an empty sequence

is compared with a non-empty sequence, the LCS is always an empty sequence.

	A	B	C	D	
A	0	0	0	0	0
B	0				0
D	0				0

Step 2: A row completed

	A	B	C	D	
A	0	0	0	0	0
B	0	1	1	1	1
D	0				0

Step 3: B row completed

	A	B	C	D	
A	0	0	0	0	0
B	0	1	1	1	1
D	0				0

Step 4: Complete the D row ~~and~~

	A	B	C	D
A	0	0	0	0
B	0	1	2	2
D	0	1	2	2
				3

Making Trackback table:

	A	B	C	D
A	0	0	0	0
B	0	1	2	2
D	0	1	2	2
				3

ABD

Result: The longest string is 'ABD', whose length is 3.

# Longest Common Subsequence

## LCS-Length(x, y)

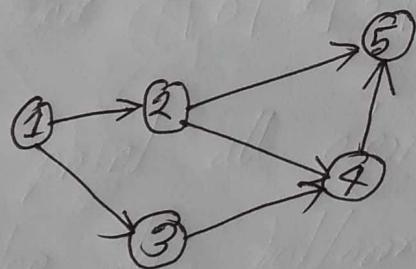
1.  $m = x.length$
2.  $n = y.length$
3. let  $b[1..m, 1..n]$  and  $e[0..m, 0..n]$  be new table
4. for  $i = 1$  to  $m$
5.      $e[i, 0] = 0$
6. for  $j = 0$  to  $n$
7.      $e[0, j] = 0$
8. for  $i = 1$  to  $m$
9.     for  $j = 1$  to  $n$
10.       if  $x_i == y_j$
11.            $e[i, j] = e[i-1, j-1] + 1$
12.        $b[i, j] = " \leftarrow "$
13.     else if  $e[i-1, j] \geq e[i, j-1]$
14.        $e[i, j] = e[i-1, j]$
15.        $b[i, j] = " \uparrow "$
16.     else
17.        $e[i, j] = e[i, j-1]$
18.        $b[i, j] = " \leftarrow "$
19. Return  $e$  and  $b$

Print\_LCS (b, x, i, j)

1. If  $i == 0$  or  $j == 0$
2. return
3. if  $b[i, j] == " \diagdown "$
4. print\_LCS (b, x, i-1, j-1)
5. print  $x_i$
6. elseif  $b[i, j] == " \uparrow "$
7. print\_LCS (b, x, i-1, j)
8. else print\_LCS (b, x, i, j-1).

## 1.1 Topological Sorting:

The topological sort algorithm takes a directed graph and returns an array of the nodes where each node appears before all the nodes it points to. The ordering of the nodes in the array is called a topological ordering.

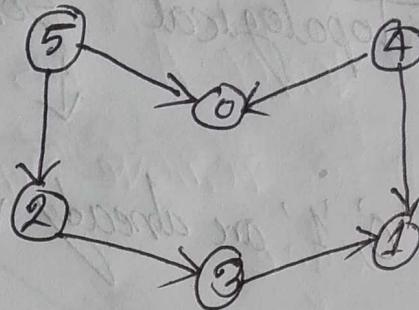


Since node 1 points to nodes 2 and 3, node 1 appears before them in the ordering. And, since nodes 2 and 3 both point to node 4, they appear before it in the ordering.

(2)

How it is working

Adjacent list (a)



$$0 \rightarrow$$

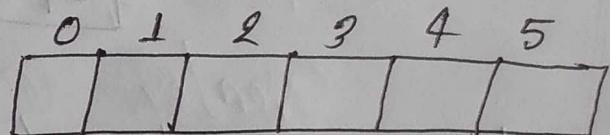
$$1 \rightarrow$$

$$2 \rightarrow 3$$

$$3 \rightarrow 1$$

$$4 \rightarrow 0, 1$$

$$5 \rightarrow 2, 0$$



Step 1: Topological sort (0), visited [0] = true

List is empty. No more recursion call

Stack

[0]

Step 2: Topological sort (1), visited [1] = true

List is empty. No more recursion call

Stack

[0 1]

Step 3: Topological sort (2), visited [2] = true

Topological sort (3), visited [3] = true

'4' is already visited. No more recursion call

Stack

[0 1 3 2]

Step 4: Topological sort(4), visited[4] = true

0, 1 are already visited, No more recursion call

Stack

0	1	3	2	4
---	---	---	---	---

Step 5: Topological Sort(5), visited[5] = true

0, 2 are already visited, No more recursion call

Stack

0	1	3	2	4	5
---	---	---	---	---	---

Step 6: print all elements of stack from top to bottom.

top to bottom.

1	0
---	---

done

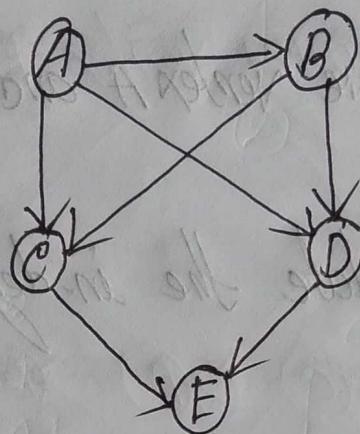
visited [0] bottom, (1) true

visited [1] bottom, (2) true

No more recursion calls. Stack is [1]

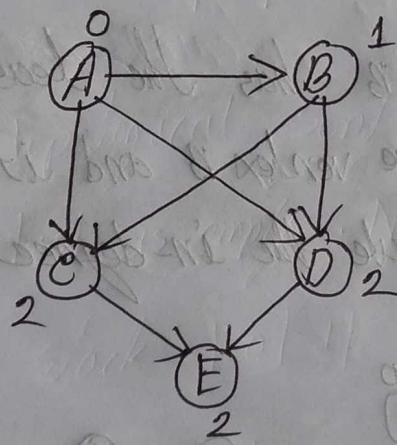
1	0	1
---	---	---

2] problem 4.

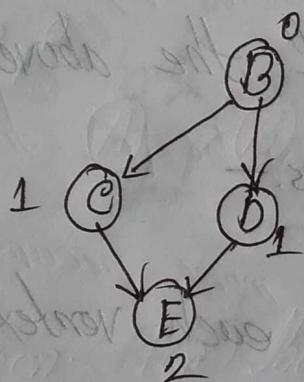


The topological ordering of the above graph are found in the following steps:-

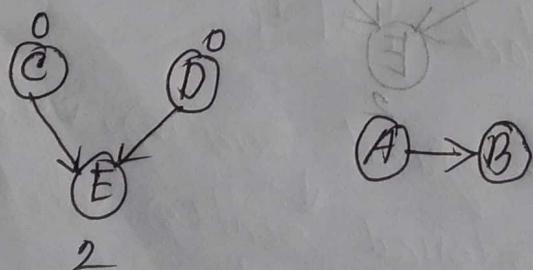
Step 1: Write in-degree of each vertex -



- ⑤
- Step 2:
- (i) Vertex A has the least in-degree
  - (ii) So, remove vertex A and its associated edge
  - (iii) Now update the in-degree of other vertex

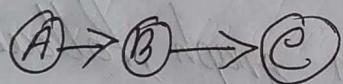
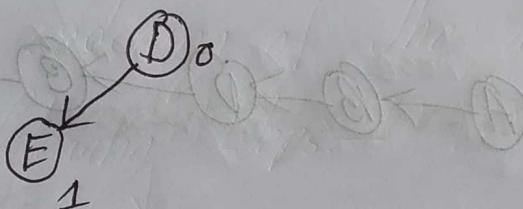


- Step 3:
- (i) Vertex B has the least in-degree
  - (ii) So, remove vertex B and its edge
  - (iii) Now, update the in-degree

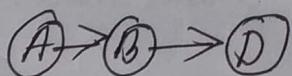
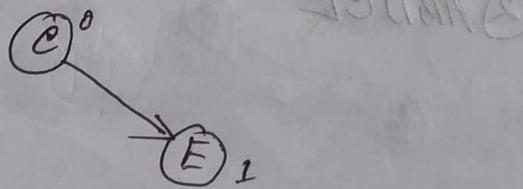


⑥ Step 4: There are two vertices with the least in-degree. So, following two case are possible.

in-Case 1: Remove vertex e and its associated edge. Then update the in-degree.

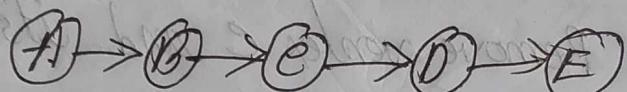


in Case 2: Remove vertex D and its associated edge. Then update the in-degree.

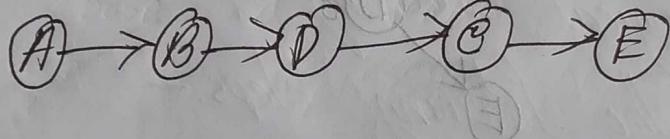


Step 5: Now there is 1 vertex E, remove  
the remaining vertex E.

Case 1:



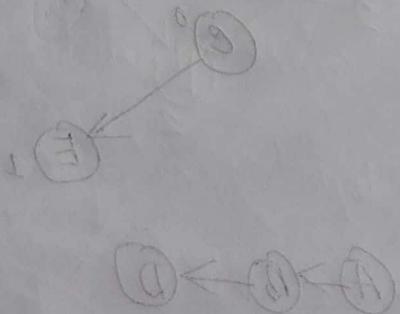
Case 2:



Conclusion: For the given graph, following 2 different topological orderings are possible.

① ABCDE

② ABDCE



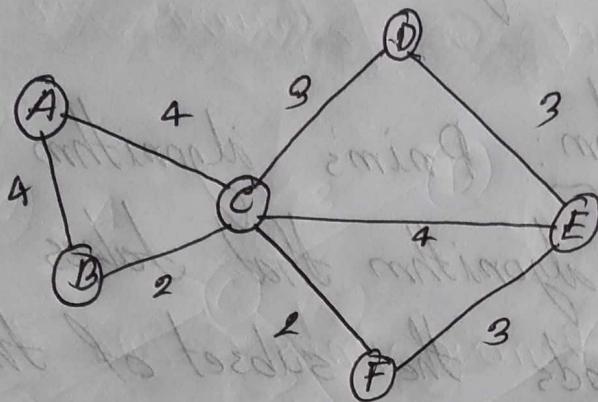
### 3.1 Minimum Spanning Tree

Prims Algorithm: Prims algorithm is a minimum spanning tree algorithm that takes a graph as input and finds the subset of the edges of that graph.

The steps for implementing Prims algorithm are as follows:

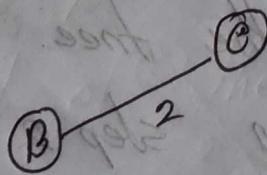
- ① Initialize the minimum spanning tree with a vertex chosen at random.
- ② Find all the edges that connect the tree to new vertices, find the minimum and add it to the tree.
- ③ Keep repeating step 2 until we get a minimum spanning tree.

Example:



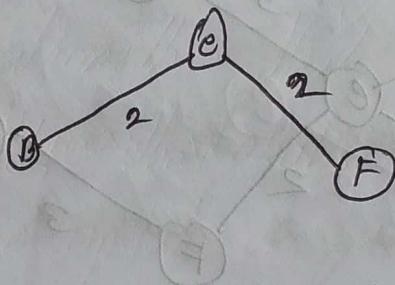
step 1: choose a vertex

step 2: choose the shortest edge from  
this vertex and add it.

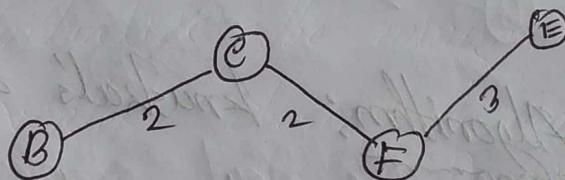


(10)

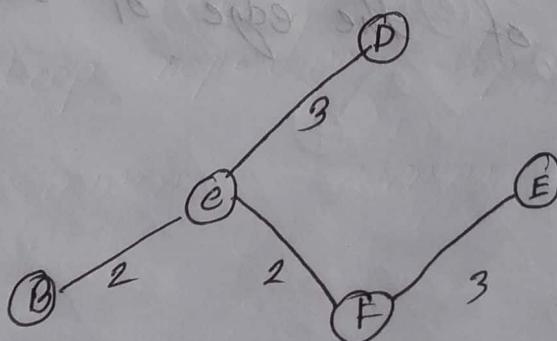
step 3: choose the nearest vertex not yet in the solution.



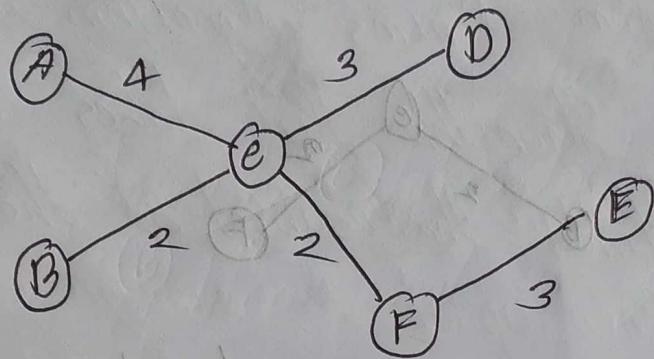
step 4: choose the nearest edge not yet in the solution, if there are multiple choices, choose one at random



step 5: choose the nearest edge.

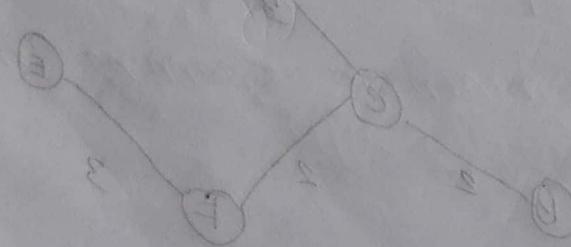


Step 6: choose the nearest edge



This graph is represent the minimum spanning tree.

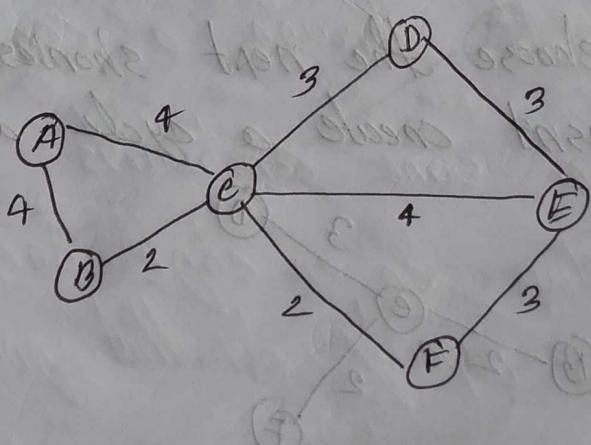
Kruskal's Algorithm: Kruskal's algorithm is a minimum spanning tree algorithm that takes a graph as input and find the subset of the edge of that graph.



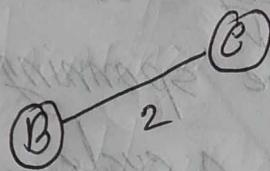
(12) The steps for implementing Kruskal's algorithm are as follows:

- ① Sort all the edges from low to high
- ② Take the edge with the lowest weight and add it to the spanning tree. If adding the edge created a cycle, then reject this edge.
- ③ keep adding edges until we reach all vertices.

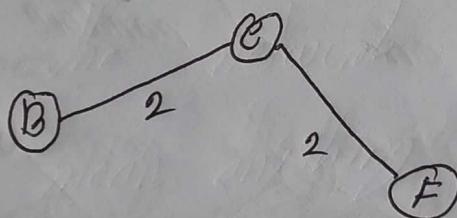
Example:



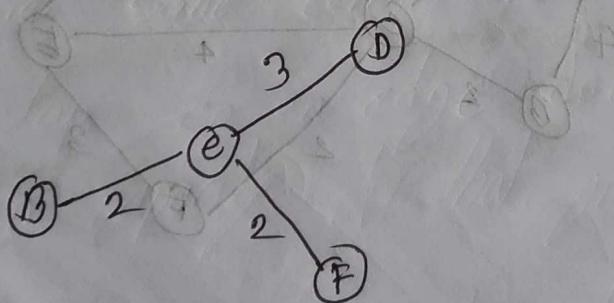
Step 1: choose the edge with least weight  
if there are more than 1 choose any



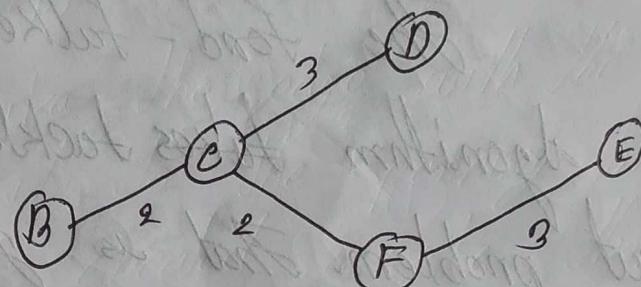
Step 2: choose the next shortest edge and add it.



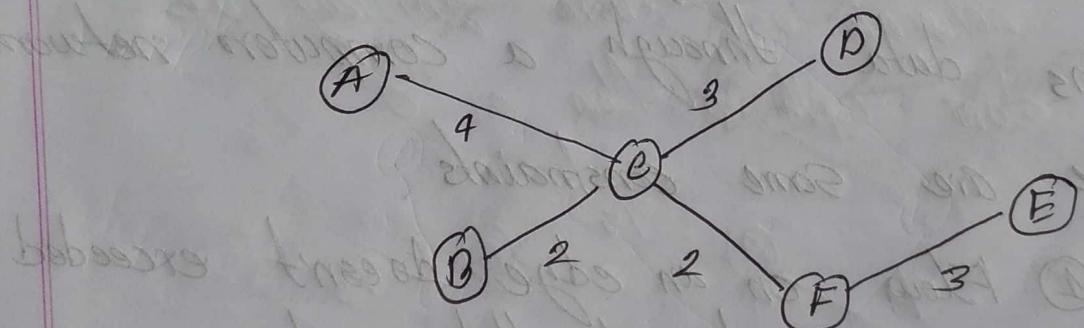
Step 3: choose the next shortest edge that doesn't create a cycle and add it.



Step 4: Choose the next shortest edge that doesn't create a cycle and add it.



Step 5: choose the next shortest edge that doesn't create a cycle and add it.



Thus graph is represent as a minimum spanning tree.

## 4.1

### Ford-Fulkerson Algorithm:

The Ford-Fulkerson algorithm is an algorithm that tackles the max-flow min-cut problem. That is, given a network with vertices and edges between those vertices that have certain weights, how much flow can the network process at a time? Flow can mean anything, but typically it means data through a computer network.

There are some constraints:

- ① Flow on an edge doesn't exceed the given capacity of that graph.
- ② Incoming flow and outgoing flow will also equal for every edge, except the source and the sink.

(2b)

## Ford-Fulkerson Algorithm:-

Ford-Fulkerson ( $G, s, t$ )

for each edge  $(u, v) \in E[G]$

do  $f[u, v] \leftarrow 0$

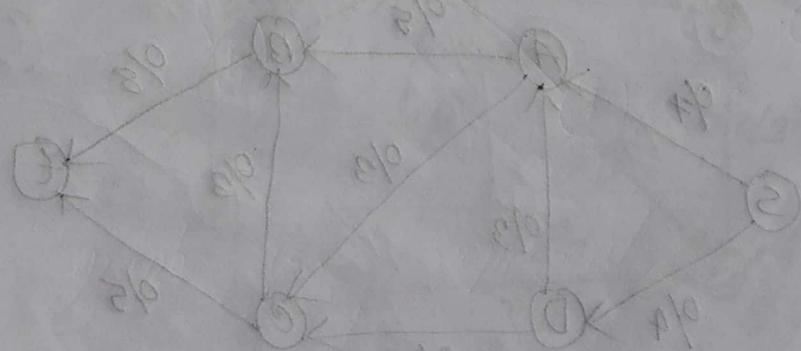
$f[v, u] \leftarrow 0$

while there exists a path  $p$  from  $s$  to  $t$  in the residual network  $G_F$

do  $c_p(p) \leftarrow \min \{c_p(u, v) : (u, v) \text{ is in } p\}$

for each edge  $(u, v)$  in  $p$

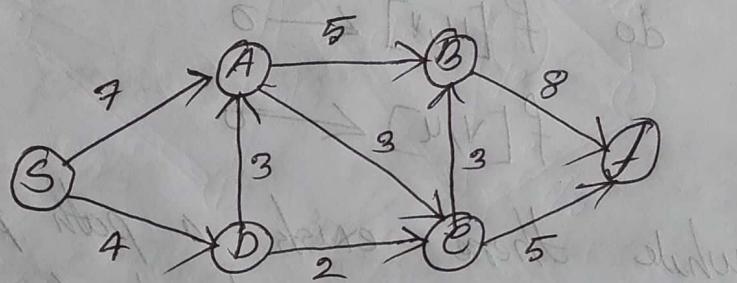
do  $f[u, v] \leftarrow f[u, v] + c_p(p)$



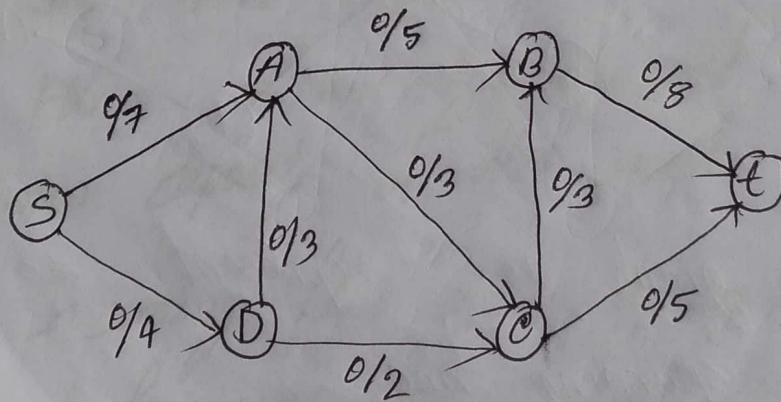
(17)

5.1 Find the maximum flow with Ford Fulkerson's algorithm.

The Given graph-



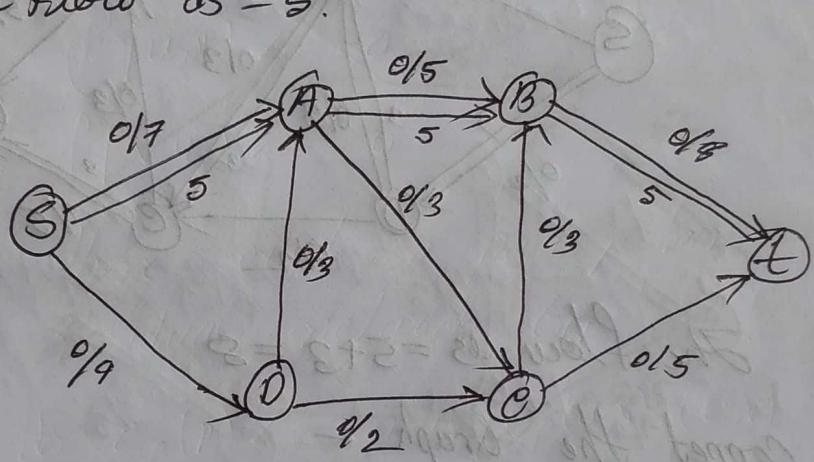
Step 1: we use the same flow network as above. Initially we start with a flow of 0.



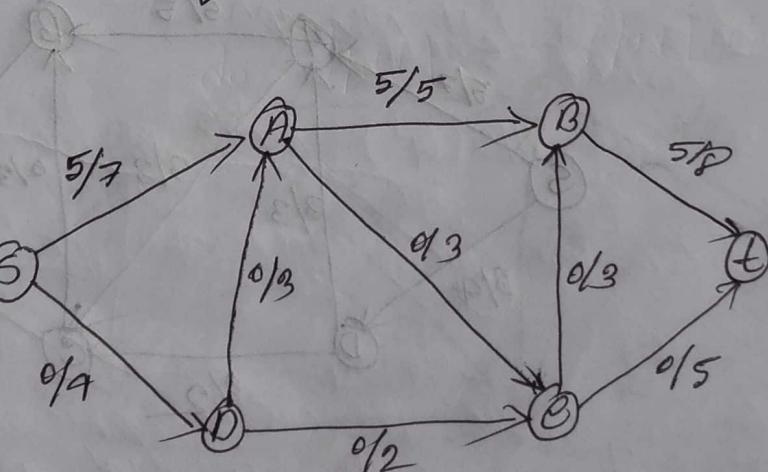
(18)

Step 2: We can find the path  $S \rightarrow A \rightarrow B \rightarrow t$  with the residual capacities 7, 5, and 8.

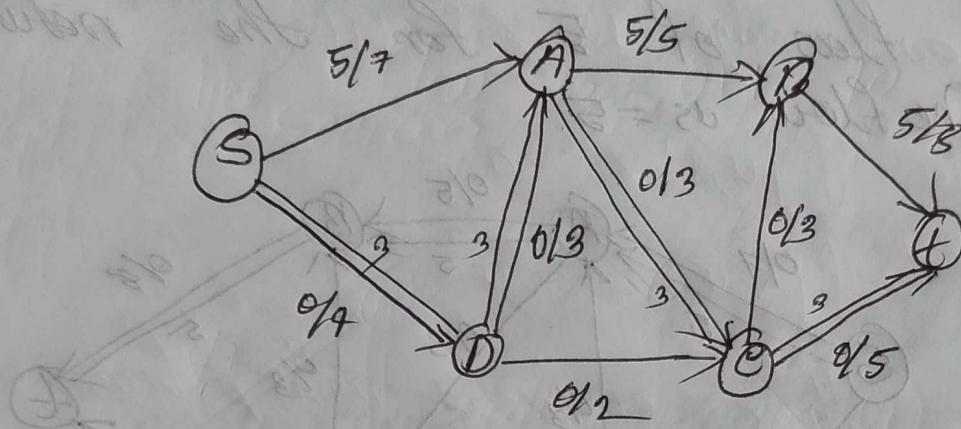
Their minimum is 5. So this gives a flow of 5 for the network.  
The flow is  $= 5$ .



~~Step 3:~~ connect the graph -

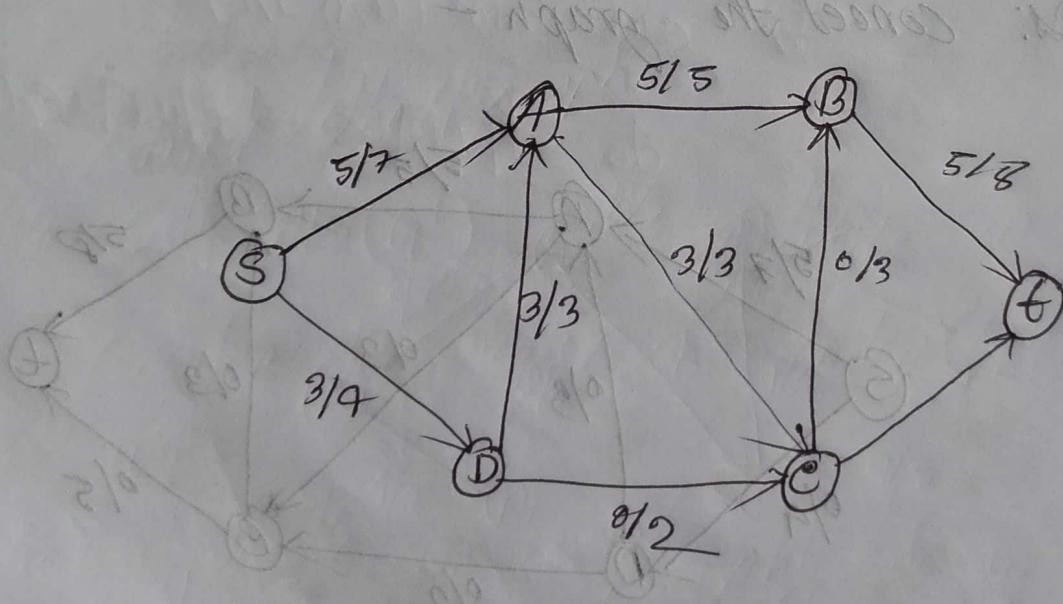


Step 3: Again we look for an augmenting path, this time we find  $S \rightarrow D \rightarrow A \rightarrow C$  and the minimum flow is 3.



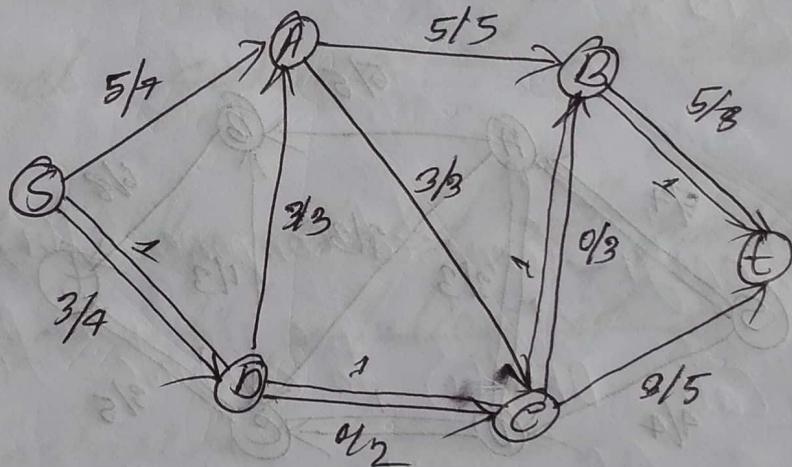
$$\text{The flow is } 5 + 3 = 8$$

connect the graph -

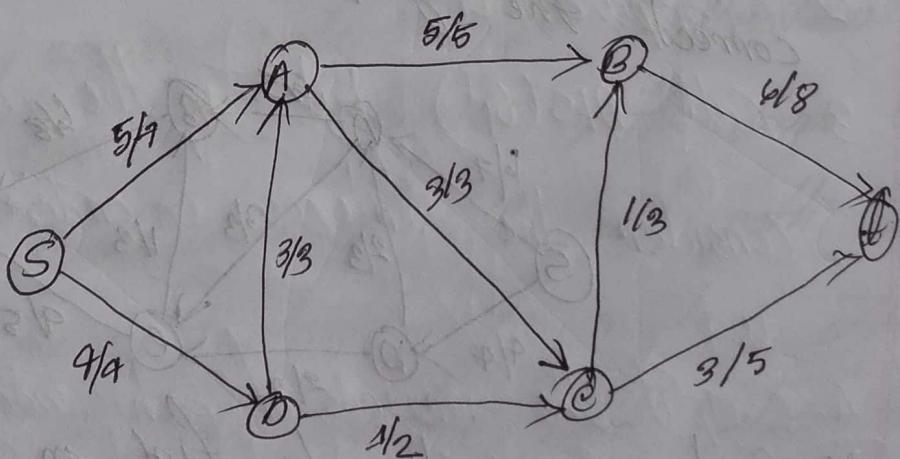


Step-4: This time we find the path  $S \rightarrow D \rightarrow C \rightarrow E$  with minimum flow 1.

The flow is  $= 8 + 1 = 9$



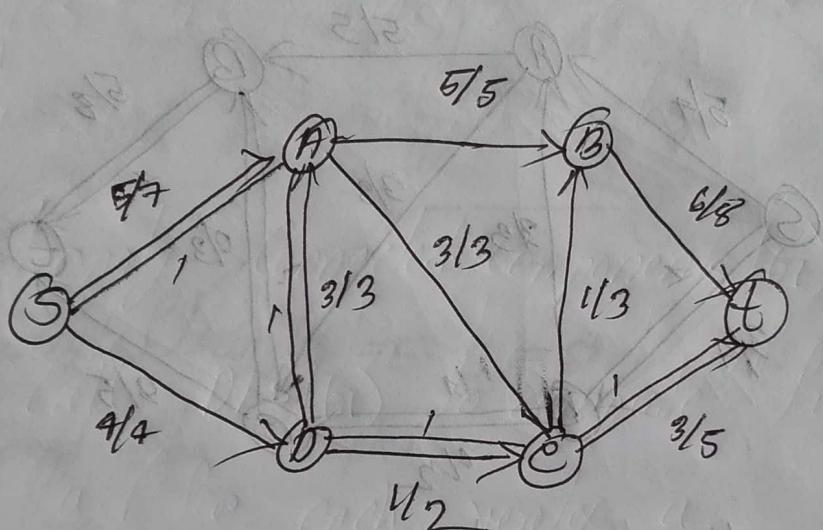
connect the graph -



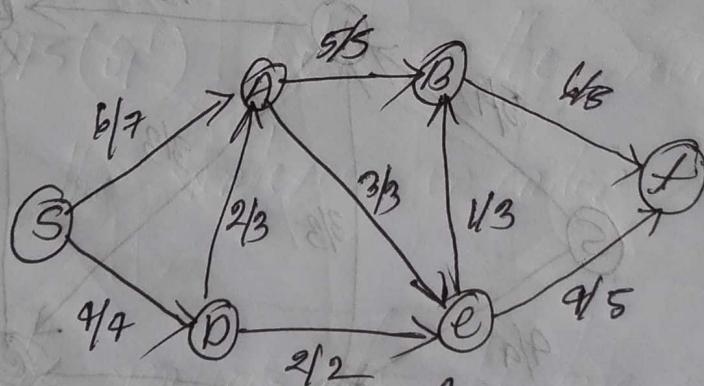
(2)

Step 5: This time we find the augmenting path  $S \rightarrow A \rightarrow D \rightarrow e \rightarrow t$  and the minimum flow is 1.

$$\text{The flow is } g+1 = 9+1 = 10$$



connect the graph-



Now it is impossible to find an augmenting path between  $s$  and  $t$ , therefore the flow of  $\Phi$  is the maximal possible.

## 6) Optimal Substructure property:

A given problem has optimal Substructure property if optimal solution of the given problem can be obtained by using optimal solutions of its subproblems.

For example, the shortest path problem has following optimal solution of its subproblems: Substructure property -

If a node  $x$  lies in the shortest path from a source node  $u$  to destination node  $v$  then the shortest path from  $u$  to  $v$  is combination of shortest path from  $u$  to  $x$  and shortest from  $x$  to  $v$ . The standard all pair shortest path algorithms like Floyd-Warshall and Bellman-Ford are typical example of optimal substructure.

7.1 The following linear-time ( $V+E$ ) algorithm compute the strongly connected components of a directed graph  $G=(V,E)$  using two depth-first-searches. One on  $G$  and one on  $G^T$ .

### Strongly-Connected-Components (or)

1. Call  $\text{DFS}(G)$  to compute finishing times  $u.f$  for each vertex  $u$ .
2. Compute  $G^T$
3. Call  $\text{DFS}(G^T)$ , but in the main loop of  $\text{DFS}$ , consider the vertices in order of decreasing  $u.f$ .
4. Output the vertices of each tree in the depth first forest formed in line 3 as a separate strongly connected component.

## 8.1 The Floyd-Warshall algorithm:

Floyd-Warshall (W)

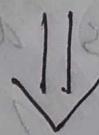
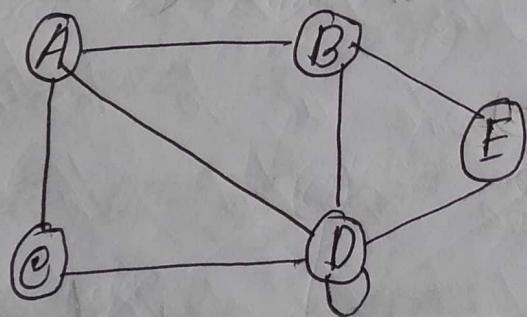
1.  $n = W.\text{rows}$
2.  $D^{(0)} = W$
3. for  $k=1$  to  $n$
4. let  $D^{(k)} = (d_{ij}^{(k)})$  be a new  $n \times n$  matrix
5. for  $i = 1$  to  $n$
6. for  $j = 1$  to  $n$   
 $d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik} + d_{kj}^{(k-1)})$
- 7.
8. Return  $D^{(n)}$

Here we have to discuss about the procedure of Floyd Warshall algorithm.

## 10.1 Adjacency Matrix :-

Adjacency matrix is a sequential representation. It is used to represent which nodes are adjacent to each other. Is there any edge connecting nodes to a graph.

Example: Consider the following graph representation.

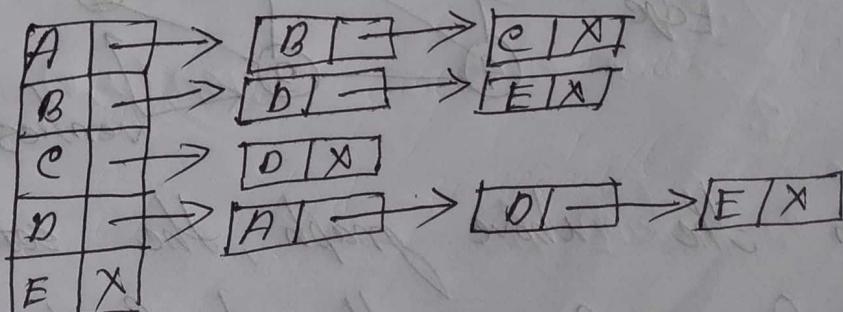
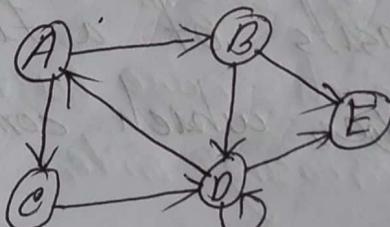


	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	1	0
D	1	1	1	1	1
E	0	1	0	1	0

## Adjacency list:-

Adjacency list is a linked representation. In this representation, for each vertex in the graph, we maintain the list of its neighbors. It means, every vertex of the graph contains list of its adjacent vertices.

Example:

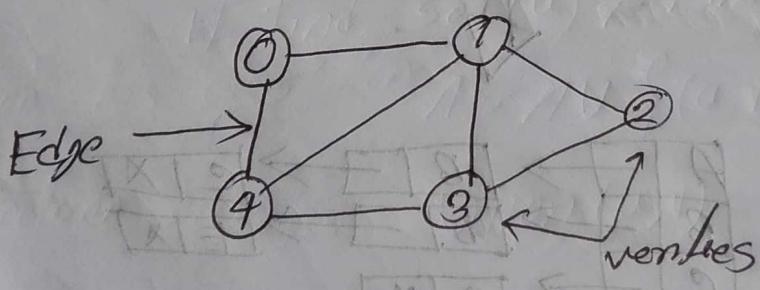


## 12.] Graph:-

A graph is a non-linear data structure consisting of nodes and edges. The nodes are sometimes also referred to as vertices and the edges are lines or arcs that connect any two nodes in the graph.

More formally a graph can be defined as,

A graph consists of a finite set of vertices and set of edges which connect a pair of nodes.



In the above graph, the set of vertices  $V = \{0, 1, 2, 3, 4\}$  and the set of edges  $E = \{01, 12, 23, 34, 04, 14, 13\}$

## 13.1 Dijkstra's Algorithm:-

Dijkstra ( $G, w, s$ )

1. Initialize\_Single\_Source ( $G, s$ )

2.  $S = \emptyset$

3.  $Q = G.V$

4. while  $Q \neq \emptyset$

5.  $u = \text{Extract-Min}(Q)$

6.  $S = S \cup \{u\}$

7. for each vertex  $v \in G.\text{adj}[u]$

Relax ( $u, v, w$ ).

$\{(u, v)\} \cup R = A$

$(u, v) \in R$

### 3. Kruskal's Algorithm:

MST\_Kruskal( $G, w$ )

1.  $A = \emptyset$
2. for each vertex  $v \in G.V$   
    Make\_Set( $v$ )
3. sort the edges of  $G.E$  into nondecreasing order by weight  $w$
4. for each edge  $(u, v) \in G.E$ , taken in nondecreasing order by weight  
    if  $\text{find\_set}(u) \neq \text{find\_set}(v)$   
         $A = A \cup \{(u, v)\}$   
        Union( $u, v$ )
- 5.
- 6.
- 7.
- 8.
9. return  $A$ .

problems Algorithm

MST\_Prim(  $G, w, r$  )

1. for each  $u \in G \cdot V$
2.  $u.key = \infty$
3.  $u.\pi = NIL$
4.  $r.key = 0$
5.  $Q = G \cdot V$
6. while  $Q \neq \emptyset$
7.  $u = Extract-Min(Q)$
8. for each  $v \in Q$  and  $w(u, v) < v.key$
9.  $v.key = w(u, v)$
10.  $v.\pi = u$
- 11.

## Topological Sort Algorithm:

$L \leftarrow$  Empty lists that will contain the sorted elements

$S \leftarrow$  Set of all nodes with no incoming edge

while  $S$  is non-empty do

remove a node  $n$  from  $S$

add  $n$  to tail of  $L$

For each node  $m$  with an edge  $e$  from  $n$  to  $m$  do

remove edge  $e$  from the graph

if  $m$  has no other incoming edges then

insert  $m$  into  $S$

if graph has edge then

return error.

else

return  $L$ .

## 1.1 Greedy Algorithm:

An algorithm is designed to achieve optimum solution for a given problem. In greedy algorithm approach, decisions are made from the given solution problem / domain. As being greedy, the closest solution that seems to provide an optimum solution is chosen.

Greedy algorithms try to find a localized optimum solution, which may eventually lead to globally optimized solutions. However, generally greedy algorithms do not provide globally optimized solutions.

Approach of greedy algorithm: Most Networking algorithm uses the greedy approach. Here

is a list of few of them -

- ① Travelling Salesman Problem
- ② Prints and Kruskal's Algorithm.
- ③ Dijkstra Minimum spanning tree Algorithm.
- ④ Graph - Map coloring
- ⑤ Graph - Vertex Cover
- ⑥ Knapsack problem
- ⑦ Job scheduling problem

There are lots of similar problems that uses the greedy approach to find an optimum solution.

## 2.1 Huffman Algorithm use:

Huffman Coding is an efficient method of compressing data without losing information. Huffman coding provides an efficient, unambiguous code by analyzing the frequencies that certain symbols appear in a message. Greedy algorithms are often easier to describe and code up than other algorithms. Greedy algorithms can be often be implemented more efficiently than other algorithms.

### 3.1 Greedy Algorithms work:

When we have a better understanding of greedy algorithm mechanism, let's explore how the core components of a greedy algorithm that sets it apart from other process:-

Candidate set: An answer is created from this set.

selection function: It selects the best candidate to be included in the solution.

Feasibility function: This section calculates if a candidate can be used to contribute to the solution.

An objective function: It assigns a value to a complete or a partial solution.

A solution function: This is used to indicate if a proper solution has been met.

e	8	4	0	0	0	0	0
8	8	2	0	0	0	0	0
11	11	0	0	0	0	0	0

Finals below

0	8	4	0	0	0	0	0
11	8	2	0	0	0	0	0
11	11	4	0	0	0	0	0

## 4.1 Activity Selection:

Given Data,

i	1	2	3	4	5	6	7	8	9	10
Si	1	2	0	5	3	5	6	8	8	12
fi	4	5	3	7	5	9	10	11	12	16

Shorted activity:

i	3	1	2	5	4	6	7	8	9	10
Si	0	4	2	3	5	5	6	8	8	12
fi	3	4	5	5	7	9	10	11	12	16

Selected activity:

i	3	5	4	8	10
Si	0	3	5	8	12
fi	3	5	7	11	16

## Algorithm:

1. Activity-Selection( $A, S, f$ ) :-
2. Sort  $A$  by finished time.
3.  $S = \{A[1]\}$
4.  $k = 1$
5.  $n = A.length$
6. for  $i = 2$  to  $n$ 
  - 7. if  $s[i] \geq f[k]$
  - 8.  $S = S \cup \{A[i]\}$
  - 9.  $k = i$
10. Return  $S$ .

### 1.1 Backtracking:

The backtracking is an algorithmic technique to solve a problem by an incremental way. It uses recursive approach to solve the problems. We can say that the backtracking is used to find all possible.

uses -

- ① Hamilton cycle
- ② M-coloring problem
- ③ N-Queen problem.

2.1 4x4 queens:

Algorithms - nQueens( $k, n$ ):

{

for  $i := 1$  to  $n$  do

{

if place( $k, i$ ) then

{

$n[k] = i$ ; ①

if ( $k == n$ )

write( $[n[1:n]]$ );

else

NQueens( $k+1, n$ );

}

{

}

Given a  $4 \times 4$  chessboard and number the rows and column of the chessboard 1 through 4

Step 1:

	1	2	3	4
1				
2		.		
3				
4				

Step 2: Now we will start by placing the first queen

	1	2	3	4
1	Q			
2				
3				
4				

Step 3: place the second queen in a safe position  
then the third queen.

	1	2	3	4
1	Q			
2			Q	
3				
4		Q		

Step-4: Now we can see that there is no safe place where we can put the last queen. So we will just change the position of the previous position of the previous queen. we will continue this process until we get the solution.

	1	2	3	4
1		Q		
2				Q
3	Q			
4			Q	

1	2	3	4
			Q

### 3. Graph Coloring:

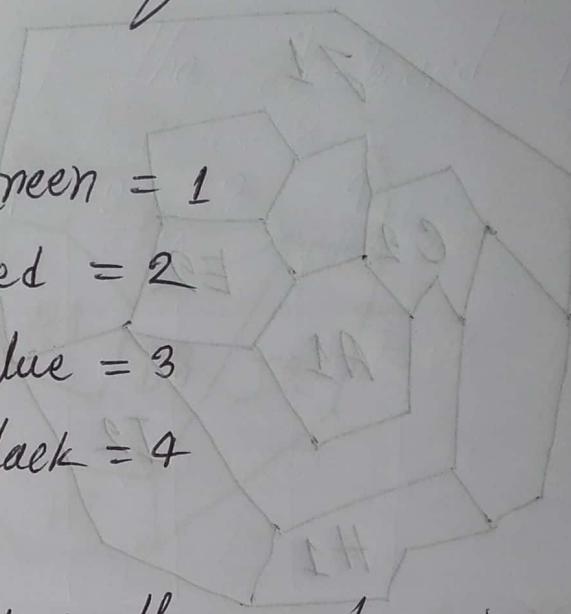
Let,

Green = 1

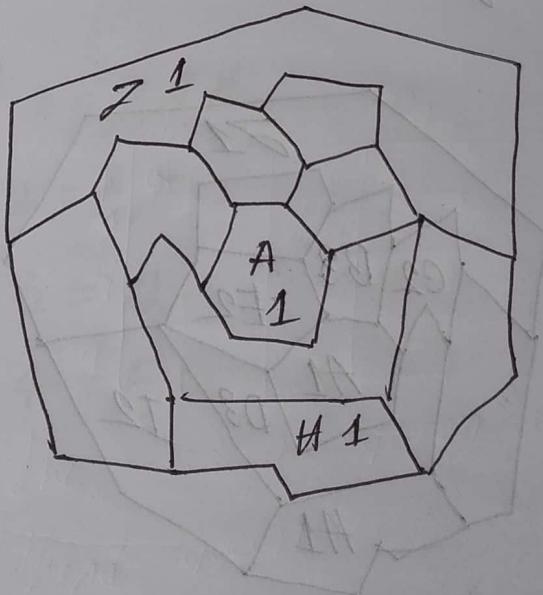
Red = 2

Blue = 3

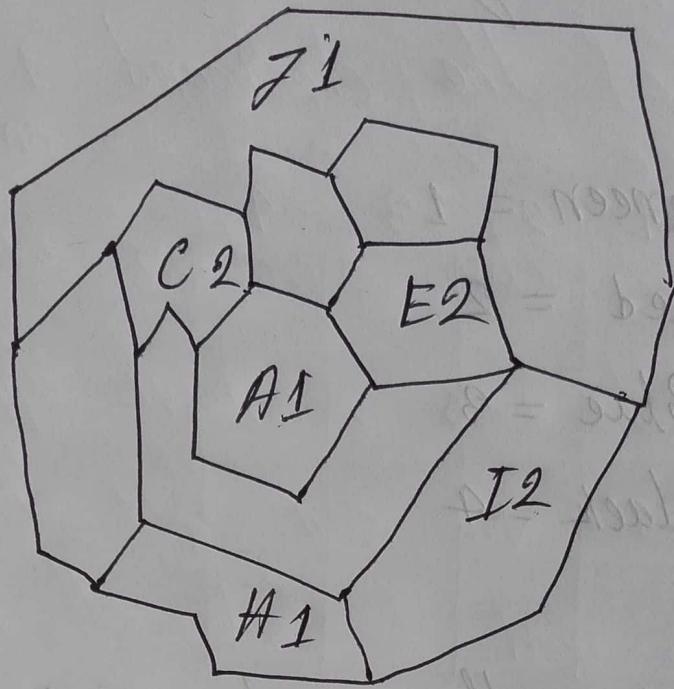
Black = 4



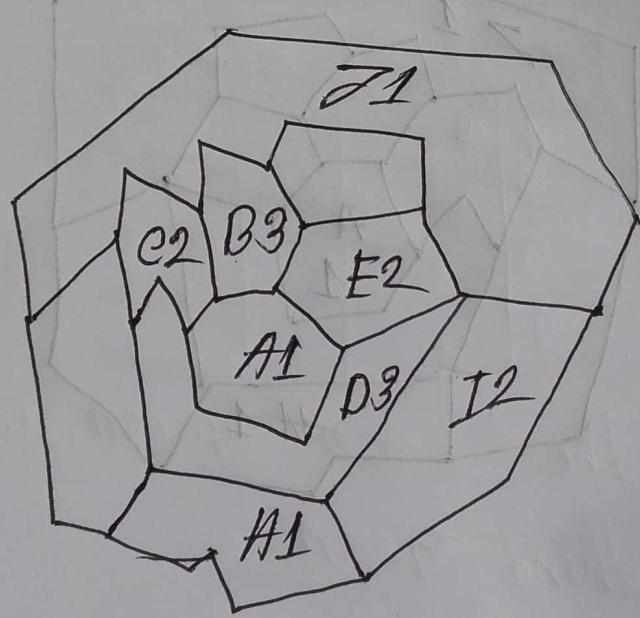
Step 1: coloring the graph using Green = 1 color.



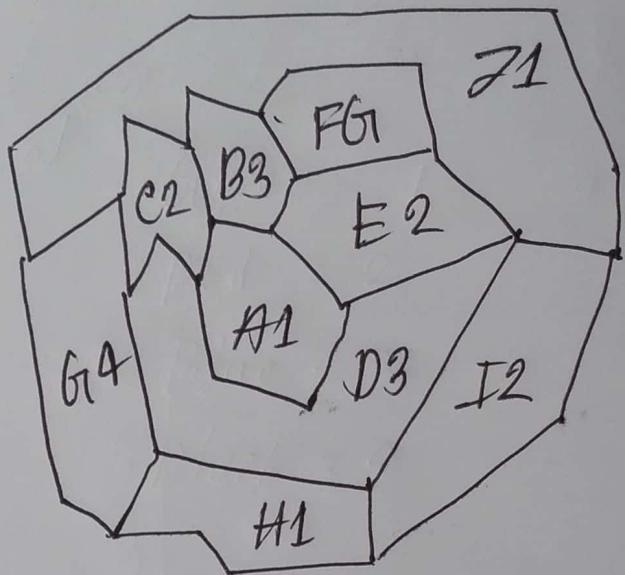
Step 2: coloring the graph using Red = 2 color



Step 3: coloring with blue = 3 colors.



Step 4: coloring the graph with black = 4 colors.



we need to minimum 4 colors to fulfill  
the graph.

$$1 = Z, A, H$$

$$2 = C, E, I$$

$$3 = B, D$$

$$4 = F, G$$