❑ Energy of the orbit of an atom:

Let us consider an atom whose nucleus has a positive charge *Ze* and let an electron of negative charge *e* move round the nucleus in an orbit of radius *r*. [For hydrogen atom, proton number *Z* = 1]

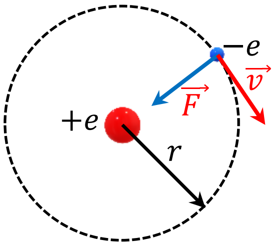
The force of attraction between the nucleus and the electron

$$= \frac{1}{4\pi\epsilon_o} \cdot \frac{Ze \cdot e}{r^2} = \frac{Ze^2}{4\pi\epsilon_o r^2}$$

As the electrons move along the circular orbit, the centripetal force ($= \frac{mv^2}{r}$) is provided by the attractive force on the electron.

$$\therefore \frac{mv^2}{r} = \frac{Ze^2}{4\pi\epsilon_o r^2}$$

$$\therefore mv^2 = \frac{Ze^2}{4\pi\epsilon_o r} \quad \dots\dots\dots\dots\dots\dots\dots \quad (1)$$



The angular momentum,

$$mvr = \frac{nh}{2\pi}, \quad \text{or,} \quad v = \frac{nh}{2\pi mr} \quad \dots\dots\dots\dots \quad (2)$$

Substituting the value of *v* in Eq. (1), we have

$$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{Ze^2}{4\pi\epsilon_o r}$$

$$\therefore \quad r = \frac{\epsilon_o n^2 h^2}{\pi m Z e^2} \quad \dots\dots\dots\dots\dots.. \quad (3)$$

Substituting the value of *r* in Eq. (2), we have

$$v = \frac{nh}{2\pi m \cdot \frac{\epsilon_o n^2 h^2}{\pi m Z e^2}} = \frac{nh}{2\pi m} \cdot \frac{\pi m Z e^2}{\epsilon_o n^2 h^2}$$

$$\therefore \ v = \frac{Ze^2}{2\epsilon_o nh} \dots\dots\dots\dots\dots. \quad (4)$$

For electron in the first orbit of Hydrogen atom,
$Z = 1$
$e = 1.6 \times 10^{-19} C$
$\epsilon_o = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$
$n = 1$
$h = 6.624 \times 10^{-34}\ Js$

$$\therefore v \cong 2.2 \times 10^6 \ \frac{m}{s}$$
$$= 2.2 \times 10^3 \frac{km}{s}$$
$$= 2200 \ \frac{km}{s}$$

Let us consider the energy of an electron in the orbit. The energy is partly potential and partly kinetic. From definition, the potential energy (P.E.) of an electron

$$\text{P.E.} = \int_{\infty}^{r} \frac{Ze^2}{4\pi\epsilon_o r^2}\, dx$$

$$\text{P.E.} = \frac{-Ze^2}{4\pi\epsilon_o r}$$

Potential energy is negative because energy must be given to the electron to bring it far away from the nucleus to zero energy level.

Kinetic energy (K.E.) $= \frac{1}{2}mv^2 = \frac{1}{2} \cdot \frac{Ze^2}{4\pi\epsilon_o r}$ [Using Eq. (1)]

$$\therefore \quad \text{K.E.} = \frac{Ze^2}{8\pi\epsilon_o r}$$

Total energy = P.E. + K.E.

$$= \frac{-Ze^2}{4\pi\epsilon_o r} + \frac{Ze^2}{8\pi\epsilon_o r}$$

$$= -\frac{Ze^2}{8\pi\epsilon_o r} \quad \dots\dots\dots\dots\dots \quad (5)$$

Putting the value of r, we get

$$E_n = \frac{-\,Ze^2}{8\pi\epsilon_o \cdot \frac{\epsilon_o n^2 h^2}{\pi m Z e^2}}$$

$$\therefore \quad E_n = \frac{-\,mZ^2e^4}{8\epsilon_o^2 n^2 h^2}\ \text{J}$$

$$\boxed{\therefore \quad E_n = \frac{-\,mZ^2e^4}{8\epsilon_o^2 n^2 h^2 \times 1.6 \times 10^{-19}}\ \ \text{eV}} \quad \dots\dots\dots\dots\dots\dots (6)$$

For Hydrogen atom,
$Z = 1$, and $e = 1.6 \times 10^{-19} C$, $m = 9.1 \times 10^{-31}$kg, $\epsilon_o = 8.85 \times 10^{-12}\, \frac{C^2}{Nm^2}$
$h = 6.624 \times 10^{-34}\, Js$

$$\therefore \quad E_n = \frac{-\,9.1 \times 10^{-31} \times (1)^2 \times (1.6 \times 10^{-19})^4}{8 \times (8.85 \times 10^{-12})^2 \times n^2 \times (6.624 \times 10^{-34})^2 \times 1.6 \times 10^{-19}}\ \ \text{eV}$$

$$\boxed{\therefore \quad E_n = \frac{-13.6}{n^2}\ \ \text{eV}} \quad \dots\dots\dots\dots\dots\dots\dots (7) \text{ [for Hydrogen atom]}$$

For first orbit of Hydrogen atom, n = 1

$$\therefore \quad E_1 = \frac{-13.6}{1^2} - 13.6\ \text{eV}$$ [Ground state energy of Hydrogen atom]

For second orbit, n = 2

$$\therefore \quad E_2 = \frac{-13.6}{2^2} = -3.4\ \text{eV}$$

$$\therefore \quad E_3 = \frac{-13.6}{3^2} = -1.511\ \text{eV}$$

$$\therefore \quad E_4 = \frac{-13.6}{4^2} = -0.85\ \text{eV}$$

$$\therefore \quad E_5 = \frac{-13.6}{5^2} = -0.544\ \text{eV}$$

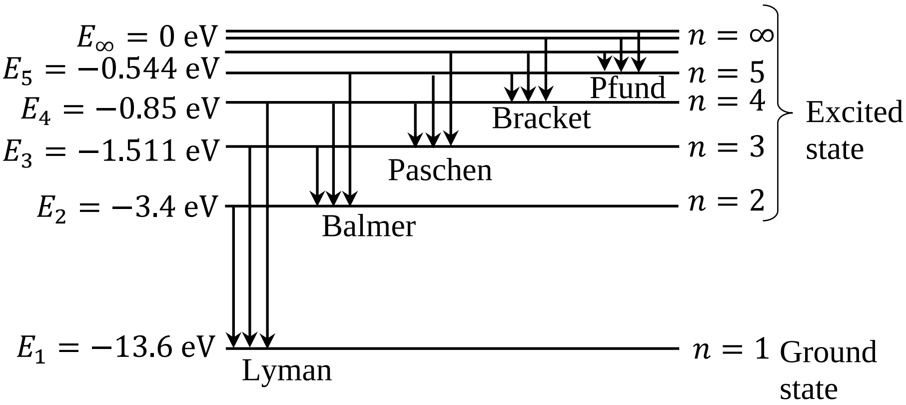


Fig.1. Energy level diagram of Hydrogen atom.

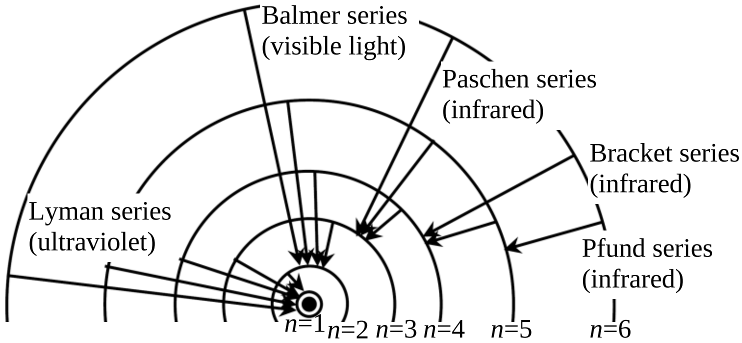


Fig.2. Spectral series of Hydrogen.

When an electron absorbs energy, it moves from the lower energy level to higher level as in the absorption spectrum. When an electron jumps from higher energy level to the lower energy level, it emits radiation as in the emission spectrum.

When an electron jumps from energy level n_2 to energy level n_1 then it will emit energy-

$$E_2 - E_1 = \frac{-mZ^2e^4}{8\epsilon_o^2 n_2^2 h^2} - \left(\frac{-mZ^2e^4}{8\epsilon_o^2 n_1^2 h^2}\right) = \frac{-mZ^2e^4}{8\epsilon_o^2 n_2^2 h^2} + \frac{mZ^2e^4}{8\epsilon_o^2 n_1^2 h^2} = \frac{mZ^2e^4}{8\epsilon_o^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

But, we know, $E_2 - E_1 = h\nu$ [ν is the frequency of spectrum/radiation]

$$\therefore \quad h\nu = \frac{mZ^2e^4}{8\epsilon_o^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore \quad \nu = \frac{mZ^2e^4}{8\epsilon_o^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For Hydrogen atom, $Z = 1$

$$\therefore \quad \nu = \frac{me^4}{8\epsilon_o^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

But $c = \nu\lambda \quad \therefore \nu = \frac{c}{\lambda}$ [c is the speed of light]

$$\therefore \quad \frac{c}{\lambda} = \frac{me^4}{8\epsilon_o^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore \quad \boxed{\frac{1}{\lambda} = \frac{me^4}{8\epsilon_o^2 ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

$$\therefore \quad \bar{\nu} = \frac{me^4}{8\epsilon_o^2 ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$ [$\frac{1}{\lambda} = \bar{\nu}$ is the wave number]

$\frac{me^4}{8\epsilon_o^2 ch^3} = R$ and is known as Rydberg constant

$R = 109678\ cm^{-1}$

$$\therefore \boxed{\bar{\nu} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$