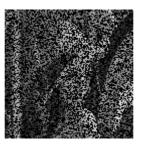
## Homework

We solve the image reconstruction problem, but this time using Jump. We are given a noisy image, and we want to clean the image up.

```
using Images
lenna = load("lena128missing.png")
# put the image lena128missing.png in the same folder, where your julia
file or ipynb file resides
```



```
# convert to real matrices
Y = Float64.(lenna);
```

```
observed_entries_Y = findall(x \rightarrow x! = 0.0, Y);
```

We want to solve:

```
egin{array}{ll} & \min_X & \|X\|_\star \ & 	ext{subject to} & X_{i,j} = Y_{i,j}, \quad (i,j) \in 	ext{observed pixels of } Y \end{array} (IMGOPT)
```

The problem above can be formulated as an SDP. This time we will use Jump.

To that goal, use the result from [Lemma 1 Fazel et. al. (2001)] (paper here)

$$(\|X\|_\star \leq t) \Leftrightarrow egin{bmatrix} U & X \ X^ op & V \end{bmatrix} \succeq 0, \mathbf{tr}(U) + \mathbf{tr}(V) \leq 2t.$$

By introducing a new variable, write the optimization problem (IMGOPT) in a way so that you can apply the result above directly.

Next, use the Jump syntax to encode positive-semidefiniteness of a matrix  $X \succeq 0$  as:

```
X \succeq 0 \equiv \frac{\text{Symmetric}(X) \text{ in PSDCone}()}{\text{(put this in a Constraint)}}
```

## But, I never solved an SDP in JuMP before 😟

Consider the following SDP:

```
minimize \operatorname{tr}(CX)
subject to \operatorname{tr}(AX) = b
X \succeq 0,
```

where

$$A=egin{bmatrix}1&5\5&2\end{bmatrix},\ C=egin{bmatrix}1&2\2&2\end{bmatrix},\ b=4.$$

You can solve this in Jump with the following code:

```
using JuMP, Mosek, MosekTools, LinearAlgebra
# if M1 chip, then
# using COSMO, JuMP, LinearAlgebra

C = [1. 2; 2 2]
A = [1. 5; 5 2]
b = 4.0;

m = Model(with_optimizer(COSMO.Optimizer));
@variable(m, X[1:2, 1:2], PSD)
@objective(m, Min, tr(C * X));
@constraint(m, tr(A * X) = b);
JuMP.optimize!(m);

status = JuMP.termination_status(m)
X_sol = JuMP.value.(X)
```

## Now solve (IMGOPT) using Jump

- Provide the Jump code
- After solving the problem using JuMP, please provide the trace of the optimal solution. For example, if  $X_{sol}$  is the solution matrix, then what is  $tr(X_{sol})$ ?