

# Homework

We solve the image reconstruction problem, but this time using `JuMP`. We are given a noisy image, and we want to clean the image up.

```
using Images
lenna = load("lenna128missing.png")
# put the image lena128missing.png in the same folder, where your julia
file or ipynb file resides
```



```
# convert to real matrices
Y = Float64.(lenna);
```

```
observed_entries_Y = findall(x->x!=0.0, Y);
```

We want to solve:

minimize  $\|X\|_*$   
subject to  $X_{i,j} = Y_{i,j}, \quad (i,j) \in \text{observed pixels of } Y$  (IMGOPT)

The problem above can be formulated as an SDP. This time we will use `JuMP`.

To that goal, use the result from [Lemma 1 Fazel et. al. (2001)] ([paper here](#))

$$(\|X\|_* \leq t) \Leftrightarrow \begin{bmatrix} U & X \\ X^\top & V \end{bmatrix} \succeq 0, \text{tr}(U) + \text{tr}(V) \leq 2t.$$

By introducing a new variable, write the optimization problem (IMGOPT) in a way so that you can apply the result above directly.

Next, use the `JuMP` syntax to encode positive-semidefiniteness of a matrix  $X \succeq 0$  as:

$X \succeq 0 \equiv \text{Symmetric}(X)$  in `PSDCone()` (put this in a Constraint)

## But, I never solved an SDP in JuMP before 😞

Consider the following SDP:

$$\begin{aligned} &\text{minimize} && \text{tr}(CX) \\ &\text{subject to} && \text{tr}(AX) = b \\ &&& X \succeq 0, \end{aligned}$$

where

$$A = \begin{bmatrix} 1 & 5 \\ 5 & 2 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix},$$
$$b = 4.$$

You can solve this in JuMP with the following code:

```
using JuMP, Mosek, MosekTools, LinearAlgebra
# if M1 chip, then
# using COSMO, JuMP, LinearAlgebra

C = [1. 2; 2 2]
A = [1. 5; 5 2]
b = 4.0;

m = Model(with_optimizer(COSMO.Optimizer));
@variable(m, X[1:2, 1:2], PSD)
@objective(m, Min, tr(C * X));
@constraint(m, tr(A * X) == b);
JuMP.optimize!(m);

status = JuMP.termination_status(m)
X_sol = JuMP.value(X)
```

## Now solve (IMGOPT) using JuMP

- Provide the JuMP code
- After solving the problem using JuMP, please provide the trace of the optimal solution. For example, if `X_sol` is the solution matrix, then what is `tr(X_sol)`?