A Stackelberg Game Model for Plug-in Electric Vehicles in a Smart Grid

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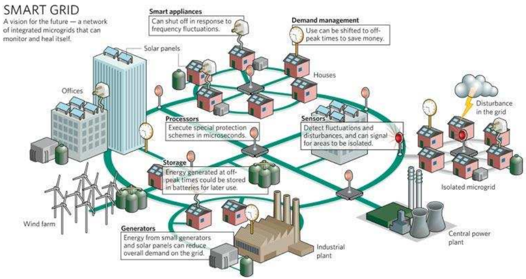
What is a PEV?

- PEV=Plugin Electric Vehicle
- Motor vehicle with electricity as its source of energy
- Recharges itself from any external source of electricity

Advantages of a PEV[2]

- Have the same speed as internal combustion engines but higher torque
- Requires less maintenance
- Responsible for much less pollution

What is a Smart-Grid **(SG)**?



- A cyber-physical electrical grid
- Uses communication technology to gather and act on information in an automated fashion
- Optimizes the power production and distribution in the presence of renewable sources[7]

General Outline

- Motivation
- Our contributions
- Game theoretic formulation
- Existence and uniqueness of the Nash Equilibrium of the game
- Closed form solution of the game for monopolistic case and general case
- Graphical representation of the results

Motivation

In Ontario

- By 2030, all of the power generation and distribution will be controlled by SG
- By 2030 there will be large scale penetration of PEVs in the market [1]

In such a situation:

- PEVs connected to the SG compete among themselves to consume as much electrical energy as possible subject to their battery capacities
- The SG sells electricity at a particular price to PEVs with the objective to maximize its revenue without overloading the grid
- Suitable pricing strategy for PEVs connected to the SG is needed

Time of use Pricing by SG[10]

- Smart Energy Meter **(SEM)** is an integral part of SG
- **Time-of-use pricing:** The price is increased when the electric energy demand is high and is decreased in the opposite situation via SEM
- Such a behaviour can be modelled by *indirect penalty approach* in Game Theory[8]

Our Contributions

- Our model attempts to account for the time-of-use pricing of the SEM by using an indirect penalty approach
- Applicable to both individual PEVs and PEV groups

Related Work

The work in [6]

- Captures the interaction between the PEVs and the SG and the corresponding decision making process in a grid-to-vehicle scenario
- A lagrangian pricing approach is used
- An algorithm based on S-S hyperplane projection method is used [13] to solve a socially stable refinement of the proposed game
- However, the mechanism of a realistic Smart Energy Meter (SEM), that implements [10], is not considered
- Charging is assumed to be provided by charging stations, but there are places (e.g Ontario) where charging will be done at home

Game Formulation

- The SG tries to maximize the total revenue, has a maximum energy of c for charging connected PEVs
- For the price p set by the SG, the PEVs tries to reach the Nash Equilibrium
- Such a problem can be formulated as a single leader - multiple Nash followers **Stackelberg** game

Notations Used

N=Number of different PEV models based on their battery capacities;

b_i=Battery capacity of any PEV belonging to the same model number $i \in \mathfrak{N}$;

n_i=Number of PEVs belonging to the model number $i \in \mathfrak{N}_\mathrm{i}$;

$\mathcal{N}$= Number of PEVs connected to the PEVs at a particular time;

u_{ij}=Consumed energy by the jth PEV of model number $i \in \Omega_{ij}$;

$\bar{u}_i = \sum_{j=1}^{n_i} u_{ij}$=Total electrical energy consumed by all the PEVs with same model i ;

Notations Used (continued)

$$\textit{Energy Capacity Constraint: } \bar{u} = u_{-i} + u_{-ij} + u_{ij} \leq c$$

where,
$\bar{u} = \sum_i^N \bar{u}_i = \sum_{i=1}^N \sum_{j=1}^{n_i} u_{ij}$=Total electrical energy consumed by all the PEVs under the SG;

$u_{-i} = \bar{u} - \bar{u}_i$=Total electrical energy consumed by all PEVs under the SG except those belonging to the model number i;

$u_{-ij} = \bar{u}_i - u_{ij}$=Total electrical energy consumed by all PEVs with same model number i except the jth one;

$\bar{\Omega} = \{\{\{u_{ij}\}_{j=1}^{n_i}\}_{i=1}^N \in \Omega : \sum_{i=1}^N \sum_{j=1}^{n_i} u_{ij} - c \leq 0\}$=Overall action space: compact, convex and coupled;

An example

- Only two models in the market, $\Rightarrow N = 2$
- Model 1 has battery capacity of 100 unit, model 2 has battery capacity of 50 unit $\Rightarrow b_1 = 100, b_2 = 50$
- There are 10 cars of model 1, and 5 cars of model 2 $\Rightarrow n_1 = 10, n_2 = 5$
- $\mathcal{N} = 10 + 5 = 15$
- The 2nd PEV belonging to model 1 is consuming 5 unit of energy, the rest of the PEVs are consuming 65 unit of energy $\Rightarrow u_{12} = 5, u_{-12} = 65, \bar{u}_1 = u_{12} + u_{-12} = 70$
- All the PEVs belonging to model 2 are consuming 50 unit energy $\Rightarrow \bar{u}_1 = 50, \bar{u} = \bar{u}_1 + \bar{u}_2 = 70 + 50 = 120$

An Assumption

Inspired by [12], we make the following assumption:

Assumption 1

Compared to the aggregate consumed energy of all the PEVs charging from the SG ($\bar{u}$), the consumed energy of any single PEV $u_{ij}(\forall j \in \mathfrak{N}_i)(\forall i \in \mathfrak{N})$ is so negligible that it will have no effect on the SE of the game and $u_{-i} + u_{-ij}$ can be considered equal to $\bar{u}$, i.e.,

$$u_{-i} + u_{-ij} \approx u_{-i} + u_{-ij} + u_{ij} = \bar{u} \;\; (\forall j \in \mathfrak{N}_i)(\forall i \in \mathfrak{N}) \tag{1}$$

Smart Energy Meter Pricing

Price set by SEM,

$$p = \frac{\alpha}{c - \bar{u}} \tag{2}$$

- The denominator penalizes the violation of the energy capacity constraint, so the price increases without any bound
- α is a positive pricing parameter set by the SG

Pricing function of the jth PEV of model number i,

$$\mathcal{P}_{ij}\left(u_{ij}, u_{-ij}, u_{-i}; \alpha\right) = p u_{ij} = \frac{\alpha}{c - \bar{u}} u_{ij} \tag{3}$$

Utility and Cost function for the PEVs

Utility function of the jth PEV of model number i,

$$\mathcal{U}_{ij}\left(u_{ij}, u_{-ij}, u_{-i}; \alpha\right) = b_i \log\left(u_{ij} + 1\right) - s_{ij} \tag{4}$$

Cost function for the jth PEV of model number i is:

$$J_{ij}\left(u_{ij}, u_{-ij}, u_{-i}; \alpha\right) = \frac{\alpha\ u_{ij}}{c - \bar{u}} - b_i \log\left(u_{ij} + 1\right) + s_{ij} \tag{5}$$

Revenue Function of the SG and the corresponding Stackelberg game

Revenue of the SG:

$$L(p, \bar{u}) = p\ \bar{u} = \frac{\alpha}{c - \bar{u}}\ \bar{u} \tag{6}$$

The **Stackelberg game** is denoted as $\mathcal{G}(\{\mathcal{N} \cup \mathrm{SG}\}, \{\{J_{ij}\}_{j=1}^{n_i}\}_{i=1}^{N}, L)$

The **Nash followers'** (PEVs') game is denoted as $\mathcal{G}(\mathcal{N}, \hat{\Omega}_{ij}, J_{ij}; \alpha)$

Two More Assumptions...

Assumption 2

Strict energy capacity constraint,

$$\bar{u}^* = u^*_{-i} + u^*_{-ij} + u^*_{ij} < c \tag{7}$$

Assumption 3

Positivity condition at NE,

$$(\forall i \in \mathfrak{N})\,(\forall j \in \mathfrak{N}_i) \quad u^*_{ij} > 0 \tag{8}$$

Nash Equilibrium of the PEVs

Definition 1

Consider the Nash followers' (PEVs') game $\mathcal{G}(\mathcal{N}, \hat{\Omega}_{ij}, J_{ij}; \alpha)$, *where* J_{ij} *is given by* (5). *For* $\forall \alpha > 0$, $\{\{u_{ij}^*\}_{i=1}^{n_i}\}_{i=1}^{N}$ *is called the NE of the game if besides* (7) *and* (8), u_{ij}^* *satisfies the following condition:*

$$\begin{aligned} &(\forall i \in \mathfrak{N})\,(\forall j \in \mathfrak{N}_i)(\forall u_{ij} \in \hat{\Omega}_{ij}) \\ &\quad J_{ij}\left(u_{ij}^*, u_{-ij}^*, u_{-i}^*; \alpha\right) \leq J_{ij}\left(u_{ij}, u_{-ij}^*, u_{-i}^*; \alpha\right) \end{aligned} \tag{9}$$

Revenue maximizing condition of the SG

Definition 2

If the Nash followers' (PEVs') game $\mathcal{G}(\mathcal{N}, \hat{\Omega}_{ij}, J_{ij}; \alpha)$ *achieves a unique NE as characterized by Definition 1, the leader's (SG's) objective is to find a pricing parameter* $\alpha^* > 0$ *such that it maximizes its revenue function* L *given by* (6), *i.e.:*

$$(\forall \alpha > 0) \qquad L(\alpha^*, \bar{u}^*) \geq L(\alpha, \bar{u}^*) \tag{10}$$

Stackelberg Equilibrium of the game

Definition 3

The pair $(\{\{u_{ij}^*\}_{j=1}^{n_i}\}_{i=1}^{N},\ p^*)$ *is called the SE of the game* $\mathcal{G}(\{\mathcal{N} \cup SG\}, \{\{J_{ij}\}_{j=1}^{n_i}\}_{i=1}^{N}, L)$*, if it satisfies* (9) *and* (10) *simultaneously.*

Preliminary lemma

Lemma 1

Under Assumption 1, $J_{ij}\left(u_{ij}, u_{-ij}, u_{-i}; \alpha\right)$ in (5) can be considered equal to (approximated by) the following equivalent augmented cost function that is identical for all PEVs:

$$
\begin{aligned}
& J\left(\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_N; \alpha\right) \\
& = \frac{\alpha \bar{u}}{c - \bar{u}} + \sum_{r=1}^{N} \sum_{k=1}^{n_r} s_{rk} - \sum_{r=1}^{N} b_r \sum_{k=1}^{n_r} \log\left(u_{rk} + 1\right) \qquad (11)
\end{aligned}
$$

and the game $\mathcal{G}(\mathcal{N}, \hat{\Omega}_{ij}, J_{ij}; \alpha)$ is equivalent to $\mathcal{G}(\mathcal{N}, \bar{\Omega}, J; \alpha)$.

Existence and uniqueness of the NE

Theorem 1

The PEVs' game $\mathcal{G}(\mathcal{N}, \bar{\Omega}, J; \alpha)$ admits a unique inner NE satisfying Assumptions 1, 2 and 3, if $0 < \alpha < \tilde{b}c$, where $\tilde{b}$=weighted mean of all battery capacities=$\frac{\sum_{i=1}^{N} b_i n_i}{\sum_{i=1}^{N} n_i}$.

Solution for monopolistic version of the game

- Each PEV has the same battery capacity parameter $b_i = b$
- Each model type has same number of PEVs $n_i = n$, thus $\mathcal{N} = Nn$

Solution of the monopolistic version of the game

Theorem 2

The monopolistic version of the Stackelberg game $\mathcal{G}_m(\{\mathcal{N} \cup SG\}, \{\{J_{ij}\}_{j=1}^{n_i}\}_{i=1}^{N}, L)$ *admits a unique SE given by*

$$(\{\{u_{ij}^*\}_{j=1}^{n_i}\}_{i=1}^{N}, p^*) =$$
$$(\{\{\frac{\sqrt{c\mathcal{N} + \mathcal{N}^2} - \mathcal{N}}{\mathcal{N}}\}_{j=1}^{n_i}\}_{i=1}^{N}, \frac{b(\sqrt{c\mathcal{N} + \mathcal{N}^2} - \mathcal{N})}{c})$$

Solution of the game for the general case

Theorem 3

Under Assumption 1, Assumption 2 and Assumption 3, the general case of the Stackelberg game $\mathcal{G}(\{\mathcal{N} \cup SG\}, \{\{J_{ij}\}_{j=1}^{n_i}\}_{i=1}^{N}, L)$ *admits a unique SE given by:*

$$(\{\{u_{ij}^*\}_{j=1}^{n_i}\}_{i=1}^{N}, p^*) =$$
$$(\{\{\frac{b_i\sqrt{c\mathcal{N}+\mathcal{N}^2}}{\mathcal{N}\tilde{b}} - 1\}_{j=1}^{n_i}\}_{i=1}^{N}, \frac{\tilde{b}(\sqrt{c\mathcal{N}+\mathcal{N}^2}-\mathcal{N})}{c})$$

Survival of a PEV model in a competitive market

From the *positivity condition*:

$$\boxed{b_i > b_{th} = \frac{\tilde{b}}{\sqrt{\tilde{c}+1}}} \tag{12}$$

- $\tilde{c} = \frac{c}{\mathcal{N}}$=Average electrical energy per PEV supplied by the SG
- $b_{th} = \frac{\tilde{b}}{\sqrt{\tilde{c}+1}}$=Threshold battery capacity
- If the battery capacity of a particular model falls below the threshold battery capacity:
  - That model can never achieve SE
  - In the long run will be out of the market

Relation between the general game and the monopolistic game

$$\tilde{u}^* = (u_{ij}^*)_{\text{monopolistic}} \quad (\forall j \in \mathfrak{N}_i)(\forall i \in \mathfrak{N}) \tag{13}$$

The average of all the PEVs' consumed energy at SE is equal to the energy consumed by a PEV at the SE in a monopolistic market.

$$\frac{p^*}{(p^*)_{\text{monopolistic}}} = \frac{L(\alpha^*, \bar{u}^*)}{(L(\alpha^*, \bar{u}^*))_{\text{monopolistic}}} = \frac{\tilde{b}}{b} \tag{14}$$

In a competitive market both the price set and the revenue earned by the SG at SE increase as the ratio of $\tilde{b}$ to b increases.

Graphical Representation of Results...

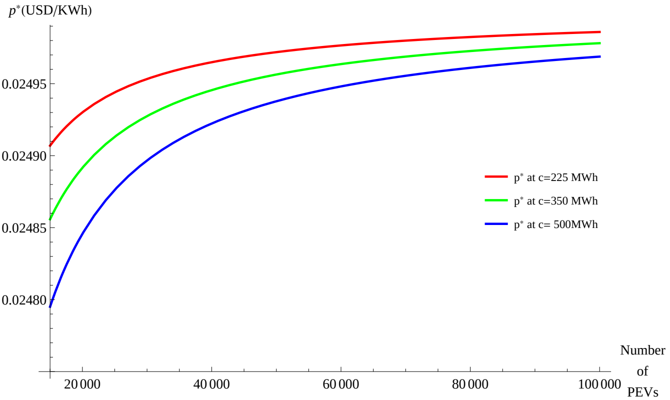


Figure 1: Change of p^* with respect to $\mathcal{N}$ for different c-s

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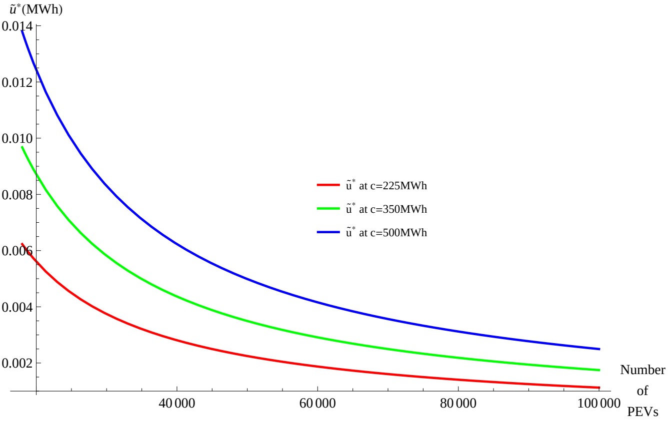

Figure 2: Change of $\tilde{u}^*$ with respect to $\mathcal{N}$ for different c-s

Graphical Representation of Results...

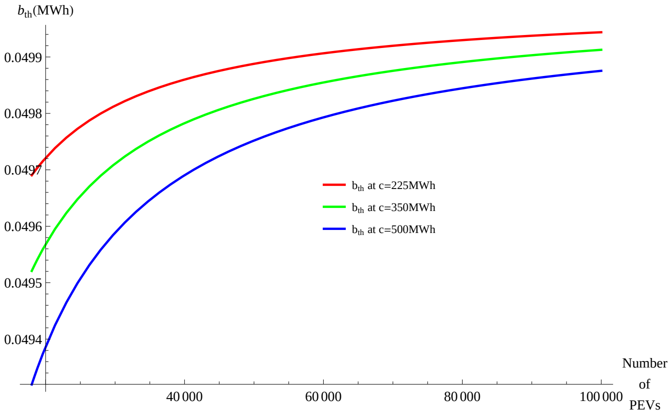

Figure 3: Change of b_{th} with respect to $\mathcal{N}$ for different c-s

Conclusion

- Existence and uniqueness of Stackelberg Equilibrium is shown
- The game is solved for a monopolistic market condition
- The game for general case is solved in explicit and tractable closed form
- Condition for survival of a PEV model in a competitive market is determined

Thank You!

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Implicit Programming Approach

- The Stackelberg game can be solved using *implicit programming* (**IMP**) approach [14] if
  - There is a one-to-one relation between α and u_{ij}^* $(\forall j \in \mathfrak{N}_i)(\forall i \in \mathfrak{N})$
  - α is Lipschitz continuous and directionally differentiable in u_{ij}^* $(\forall j \in \mathfrak{N}_i)(\forall i \in \mathfrak{N})$
- Conditions are satisfied, so IMP can be used
- Steps in IMP:
  - In the SG's revenue L, substitute the value of α in terms of $\tilde{u}^*$
  - Maximize the resultant function with respect to $\tilde{u}^*$
  - Substitute the value of the resultant maximizer $\tilde{u}^*$ in α and thus obtain the SE.