

1. A body of weight 300 N is lying on a rough horizontal plane having coefficient of friction 0.3. Find the magnitude of the force which can move the body, while acting at an angle of 25° with the horizontal. (**$P=87.1$ N**)

Solution. Given: Weight of the body (W) = 300 N; Coefficient of friction (μ) = 0.3 and angle made by the force with the horizontal (α) = 25°

Let P = Magnitude of the force, which can move the body, and

F = Force of friction.

Resolving the forces horizontally,

$$F = P \cos \alpha = P \cos 25^\circ = P \times 0.9063$$

and now resolving the forces vertically,

$$\begin{aligned} R &= W - P \sin \alpha = 300 - P \sin 25^\circ \\ &= 300 - P \times 0.4226 \end{aligned}$$

We know that the force of friction (F),

$$0.9063 P = \mu R = 0.3 \times (300 - 0.4226 P) = 90 - 0.1268 P$$

$$\text{or } 90 = 0.9063 P + 0.1268 P = 1.0331 P$$

$$\therefore P = \frac{90}{1.0331} = 87.1 \text{ N} \quad \text{Ans.}$$

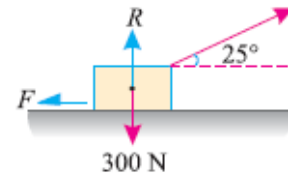


Fig. 8.2.

2. A body resting on a rough horizontal plane required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of the friction.
(W=991.2 N and coefficient of friction=0.173)

Solution. Given: Pull = 180 N; Push = 220 N and angle at which force is inclined with horizontal plane (α) = 30°

Let W = Weight of the body
 R = Normal reaction, and
 μ = Coefficient of friction.

First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction (F_1) will act towards left as shown in Fig. 8.3. (a).

Resolving the forces horizontally,

$$F_1 = 180 \cos 30^\circ = 180 \times 0.866 = 155.9 \text{ N}$$

and now resolving the forces vertically,

$$R_1 = W - 180 \sin 30^\circ = W - 180 \times 0.5 = W - 90 \text{ N}$$

We know that the force of friction (F_1),

$$155.9 = \mu R_1 = \mu (W - 90) \quad \dots(i)$$

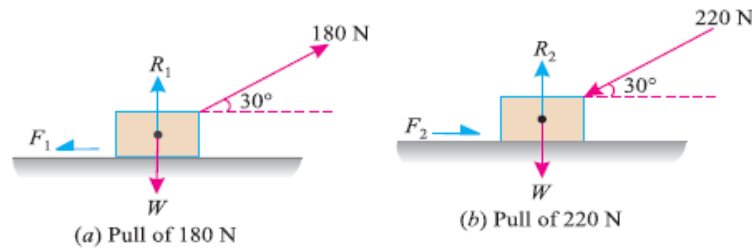


Fig. 8.3.

Now consider a push of 220 N acting on the body. We know that in this case, the force of friction (F_2) will act towards right as shown in Fig. 8.3 (b).

Resolving the forces horizontally,

$$F_2 = 220 \cos 30^\circ = 220 \times 0.866 = 190.5 \text{ N}$$

and now resolving the forces horizontally,

$$R_2 = W + 220 \sin 30^\circ = W + 220 \times 0.5 = W + 110 \text{ N}$$

We know that the force of friction (F_2),

$$190.5 = \mu R_2 = \mu (W + 110) \quad \dots(ii)$$

Dividing equation (i) by (ii)

Dividing equation (i) by (ii)

$$\frac{155.9}{190.5} = \frac{\mu(W - 90)}{\mu(W + 110)} = \frac{W - 90}{W + 110}$$

$$155.9 W + 17\,149 = 190.5 W - 17\,145$$

$$34.6 W = 34\,294$$

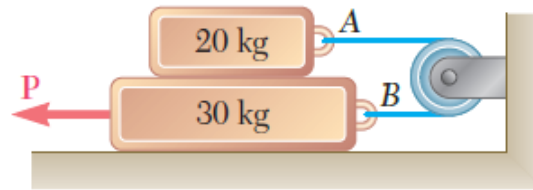
$$\text{or } W = \frac{34\,294}{34.6} = 991.2 \text{ N} \quad \text{Ans.}$$

Now substituting the value of W in equation (i),

$$155.9 = \mu (991.2 - 90) = 901.2 \mu$$

$$\therefore \mu = \frac{155.9}{901.2} = 0.173 \quad \text{Ans.}$$

3. The Coefficients of friction are $\mu_s=0.4$ and $\mu_k=0.3$ between all the surfaces of contact. Determine the smallest force P required to start the 30 kg block moving if the cable AB (a) is attached as shown (b) is removed. (**$P=353.2$ N and $P=196.2$ N**)



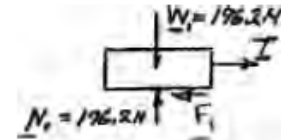
SOLUTION

(a) Free body: 20-kg block

$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$

$$\leftarrow \Sigma F = 0: T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$



Free body: 30-kg block

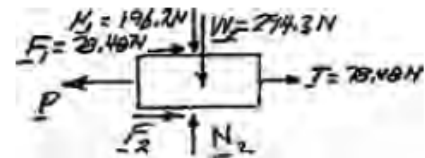
$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$

$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

$$\leftarrow \Sigma F = 0: P - F_1 - F_2 - T = 0$$

$$P = 78.48 \text{ N} + 196.2 \text{ N} + 78.48 \text{ N} = 353.2 \text{ N}$$



$$P = 353 \text{ N} \leftarrow \blacktriangleleft$$

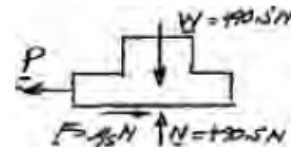
(b) Free body: Both blocks

Blocks move together

$$W = (50 \text{ kg})(9.81 \text{ m/s}^2) = 490.5 \text{ N}$$

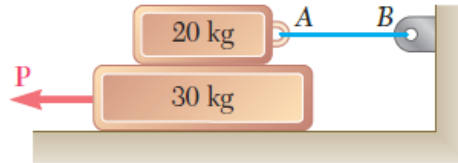
$$\leftarrow \Sigma F = 0: P - F = 0$$

$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$



$$P = 196.2 \text{ N} \leftarrow \blacktriangleleft$$

4. The Coefficients of friction are $\mu_s=0.4$ and $\mu_k=0.3$ between all the surfaces of contact. Determine the smallest force P required to start the 30 kg block moving if the cable AB (a) is attached as shown (b) is removed (**$P=275\text{ N}$ and $P=196.2\text{ N}$**)



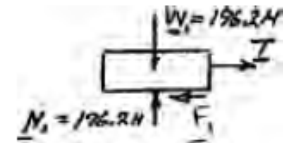
SOLUTION

(a) Free body: 20-kg block

$$W_1 = (20\text{ kg})(9.81\text{ m/s}^2) = 196.2\text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2\text{ N}) = 78.48\text{ N}$$

$$\rightarrow \Sigma F = 0: T - F_1 = 0 \quad T = F_1 = 78.48\text{ N}$$



Free body: 30-kg block

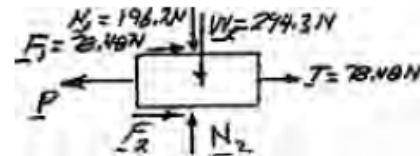
$$W_2 = (30\text{ kg})(9.81\text{ m/s}^2) = 294.3\text{ N}$$

$$N_2 = 196.2\text{ N} + 294.3\text{ N} = 490.5\text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5\text{ N}) = 196.2\text{ N}$$

$$\leftarrow \Sigma F = 0: P - F_1 - F_2 = 0$$

$$P = 78.48\text{ N} + 196.2\text{ N} = 274.7\text{ N}$$



$$P = 275\text{ N} \leftarrow \blacktriangleleft$$

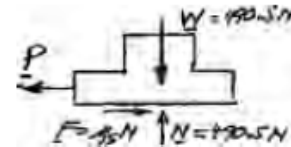
(b) Free body: Both blocks

Blocks move together

$$W = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N}$$

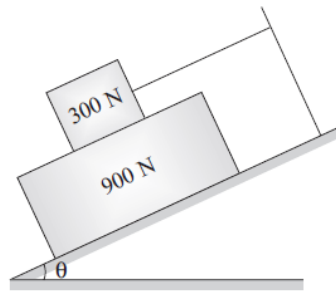
$$\leftarrow \Sigma F = 0: P - F = 0$$

$$P = \mu_s N = 0.4(490.5\text{ N}) = 196.2\text{ N}$$



$$P = 196.2\text{ N} \leftarrow \blacktriangleleft$$

5. What should be the value of θ which will make the motion of 900 N block down the block to impend? The coefficient of friction for all contact surfaces is $1/3$. ($\theta=29.1^\circ$)



Solution: 900 N block is on the verge of moving downward. Hence frictional forces F_1 and F_2 [Ref. Fig. 5.6(b)] act up the plane on 900 N block. Free body diagrams of the blocks are as shown in Fig. 5.6(b).

Consider the equilibrium of 300 N block.

Σ Forces normal to plane = 0 \rightarrow

$$N_1 - 300 \cos \theta = 0 \quad \text{or} \quad N_1 = 300 \cos \theta \quad \dots(i)$$

From law of friction,

$$F_1 = \frac{1}{3} N_1 = 100 \cos \theta$$

For 900 N block:

Σ Forces normal to plane = 0 \rightarrow

$$N_2 - N_1 - 900 \cos \theta = 0$$

$$\begin{aligned} N_2 &= N_1 + 900 \cos \theta \\ &= 300 \cos \theta + 900 \cos \theta \\ &= 1200 \cos \theta. \end{aligned}$$

From law of friction,

$$F_2 = \mu_2 N_2 = \frac{1}{3} \times 1200 \cos \theta = 400 \cos \theta.$$

Σ Forces parallel to the plane = 0 \rightarrow

$$\begin{aligned} F_1 + F_2 - 900 \sin \theta &= 0 \\ 100 \cos \theta + 400 \cos \theta &= 900 \sin \theta \end{aligned}$$

$$\tan \theta = \frac{500}{900}$$

$$\theta = 29.05^\circ$$

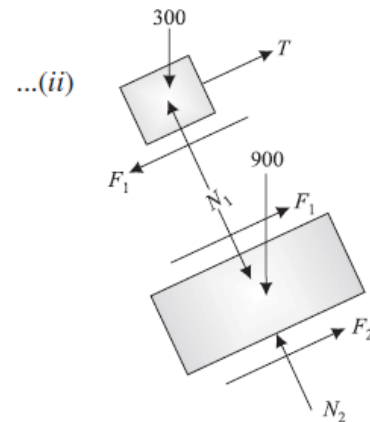
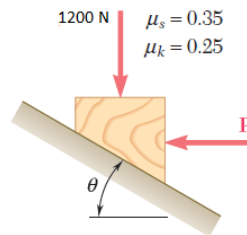


Fig. 5.6(b)

6. Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta=25^\circ$ and $P=750\text{ N}$. **(in equilibrium & $F=172.6\text{ N}$)**



SOLUTION

Assume equilibrium:

$$\sum F_x = 0: F + (1200\text{ N}) \sin 25^\circ - (750\text{ N}) \cos 25^\circ = 0$$

$$F = +172.589\text{ N}$$

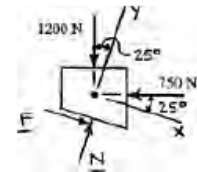
$$F = 172.589\text{ N} \searrow$$

$$\sum F_y = 0: N - (1200\text{ N}) \cos 25^\circ - (750\text{ N}) \sin 25^\circ = 0$$

$$N = +1404.53\text{ N}$$

$$N = 1404.53\text{ N} \nearrow$$

$$\text{Maximum friction force: } F_m = \mu_s N = 0.35(1404.53\text{ N}) = 491.587\text{ N}$$

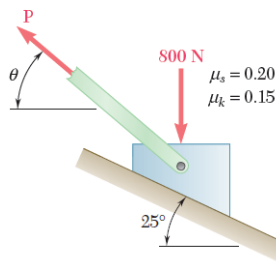


Block is in equilibrium ◀

Since $F < F_m$,

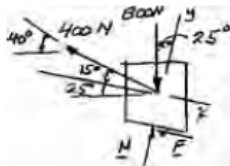
$$F = 172.6\text{ N} \searrow \blacktriangleleft$$

7. Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 40^\circ$ and $P=400\text{ N}$. **(in equilibrium & $F=48.3\text{ N}$)**



SOLUTION

Assume equilibrium:



$$\sum F_y = 0: N - (800\text{ N}) \cos 25^\circ + (400\text{ N}) \sin 15^\circ = 0$$

$$N = +621.5\text{ N}$$

$$N = 621.5\text{ N} \uparrow$$

$$\sum F_x = 0: -F + (800\text{ N}) \sin 25^\circ - (400\text{ N}) \cos 15^\circ = 0$$

$$F = +48.28\text{ N}$$

$$F = 48.28\text{ N} \searrow$$

Maximum friction force:

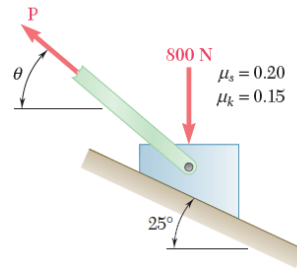
$$\begin{aligned} F_m &= \mu_s N \\ &= 0.20(621.5\text{ N}) \\ &= 124.3\text{ N} \end{aligned}$$

Block is in equilibrium ◀

Since $F < F_m$,

$$F = 48.3\text{ N} \searrow \blacktriangleleft$$

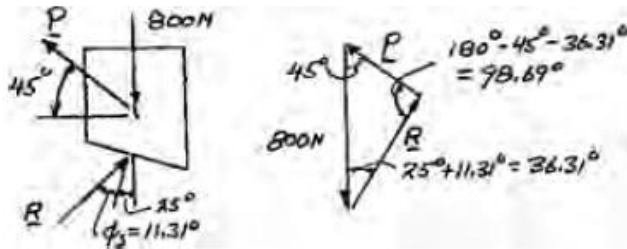
8. Knowing that $\theta=45^\circ$, determine the range of values of P for which equilibrium is maintained ($222\text{N} \leq P \leq 479\text{N}$)



To start block up the incline:

$$\mu_s = 0.20$$

$$\phi_s = \tan^{-1} 0.20 = 11.31^\circ$$

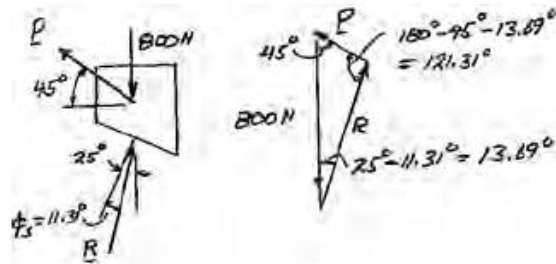


Force triangle:

$$\frac{P}{\sin 36.31^\circ} = \frac{800 \text{ N}}{\sin 98.69^\circ}$$

$$P = 479.2 \text{ N} <$$

To prevent block from moving down:



Force triangle:

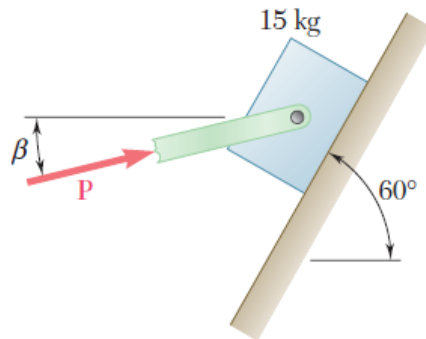
$$\frac{P}{\sin 13.69^\circ} = \frac{800 \text{ N}}{\sin 121.31^\circ}$$

$$P = 221.61 \text{ N} <$$

Equilibrium is maintained for

$$222 \text{ N} \leq P \leq 479 \text{ N} \blacktriangleleft$$

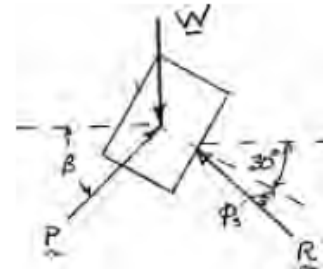
9. Knowing that the coefficient of friction between the 15-kg block and the incline is $\mu_s = 0.25$, determine (a) the smallest value of P required to maintain the block in equilibrium, (b) the corresponding value of β . ($P=108.8 \text{ N}$, $\beta=46^\circ$)



SOLUTION

FBD block (Impending motion downward):

$$\begin{aligned} W &= (15 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 147.150 \text{ N} \\ \phi_s &= \tan^{-1} \mu_s \\ &= \tan^{-1}(0.25) \\ &= 14.036^\circ \end{aligned}$$



(a) Note: For minimum P ,

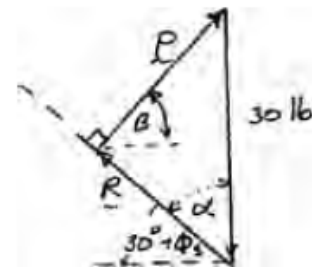
$$\mathbf{P} \perp \mathbf{R}$$

So

$$\begin{aligned} \beta &= \alpha \\ &= 90^\circ - (30^\circ + 14.036^\circ) \\ &= 45.964^\circ \end{aligned}$$

and

$$\begin{aligned} P &= (147.150 \text{ N}) \sin \alpha \\ &= (147.150 \text{ N}) \sin(45.964^\circ) \end{aligned}$$



$$P = 108.8 \text{ N} \blacktriangleleft$$

(b)

$$\beta = 46.0^\circ \blacktriangleleft$$

10. A block of weight $W_1=100\text{N}$ rests on an inclined plane and another weight W_2 is attached to the first weight through a string as shown in fig below. If the coefficient of friction between the block and plane is 0.3, determine the maximum and minimum values of W_2 so that equilibrium can exist ($24 \leq W \leq 76$)

(i) When weight $W_1 = 100\text{ N}$ is about to slide down the plane

(ii) When weight $W_1 = 100\text{ N}$ is about to slide up the plane

Case (i): Refer Fig. 9.38 (a) for the free body diagram for impending motion down the plane.

Under equilibrium conditions,

$$\Sigma F (\text{along the plane}) = 0$$

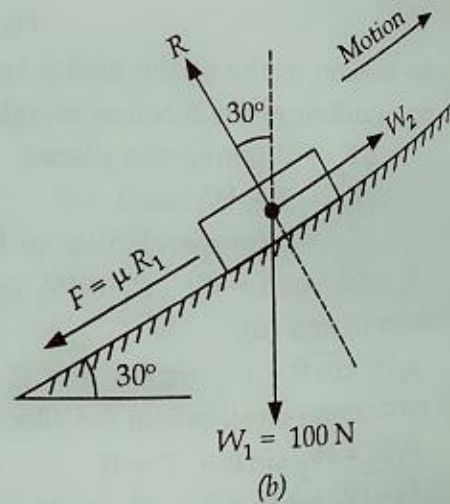
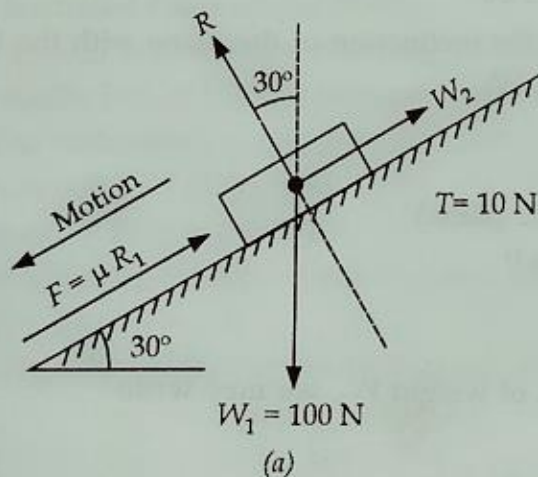
$$W_2 + \mu R - W_1 \sin 30^\circ = 0$$

$$W_2 = W_1 \sin 30^\circ - \mu R = 100 \sin 30^\circ - 0.3 R = 50 - 0.3 R$$

$$\Sigma F (\text{perpendicular to the plane}) = 0$$

$$R = 100 \cos 30^\circ = 86.6\text{ N}$$

$$\therefore W_2 = 50 - 86.6 \times 0.3 = 24.02\text{ N} \approx 24\text{ N}$$



Case (ii): Refer Fig. 9.38 (b) for the free body diagram for impending motion up the plane.

Under equilibrium conditions,

$$\Sigma F (\text{along the plane}) = 0$$

$$W_2 - W_1 \sin 30^\circ - \mu R = 0$$

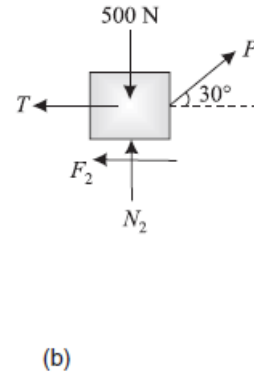
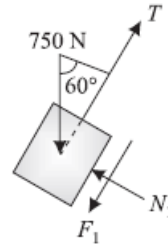
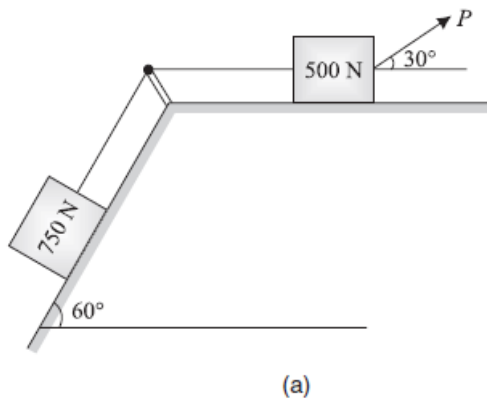
$$W_2 = \mu R + W_1 \sin 30^\circ = 0.3 R + 100 \sin 30^\circ = 0.3 R + 50$$

$$\Sigma F (\text{perpendicular to plane}) = 0$$

$$R = 100 \cos 30^\circ = 86.6$$

$$\therefore W_2 = 0.3 \times 86.6 + 50 = 75.98\text{ N} \approx 76\text{ N}$$

11. What is the value of P in the system shown in figure to cause the motion to impend?
Assume the pulley is smooth and coefficient of friction between the other contact surfaces is 0.2. (**$P=853.5 \text{ N}$**)



Solution: Free body diagrams of the blocks are as shown in Fig. 5.9(b). Consider the equilibrium of 750 N block.

$$\Sigma \text{ Forces normal to the plane} = 0 \rightarrow$$

$$N_1 - 750 \cos 60 = 0 \quad \therefore N_1 = 375 \text{ newton} \quad \dots(i)$$

Since the motion is impending, from law of friction,

$$F_1 = \mu N_1 = 0.2 \times 375 = 75 \text{ newton} \quad \dots(ii)$$

$$\Sigma \text{ Forces parallel to the plane} = 0 \rightarrow$$

$$T - F_1 - 750 \sin 60 = 0$$

$$\therefore T = 75 + 750 \sin 60 = 724.5 \text{ newton.} \quad \dots(iii)$$

Consider the equilibrium of 500 N block.

$$\Sigma F_V = 0 \rightarrow$$

$$N_2 - 500 + P \sin 30 = 0$$

$$\text{i.e.,} \quad N_2 + 0.5P = 500 \quad \dots(iv)$$

From law of friction,

$$F_2 = \mu N_2 = 0.2 (500 - 0.5P) = 100 - 0.1P \quad \dots(v)$$

$$\Sigma F_H = 0 \rightarrow$$

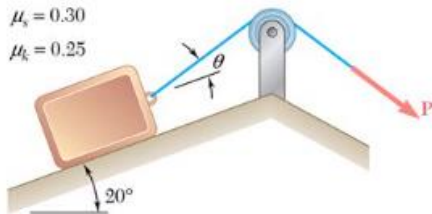
$$P \cos 30 - T - F_2 = 0$$

$$\text{i.e.,} \quad P \cos 30 - 724.5 - 100 + 0.1P = 0$$

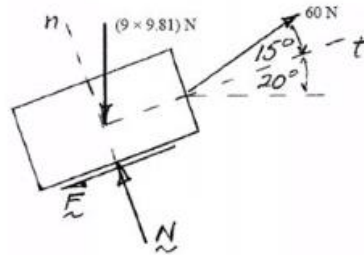
$$\therefore P = 853.5 \text{ N}$$

12. Determine whether the 9 Kg block shown is in equilibrium, find the magnitude and direction of the friction force when $P=60\text{ N}$ and $\theta = 15^\circ$? **(Not in equilibrium and $F=16.86\text{ N}$)**

Solution



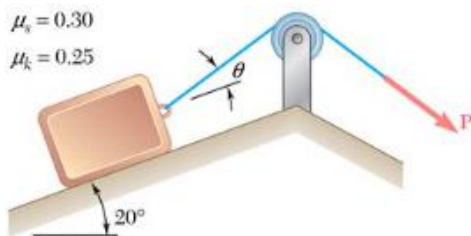
Free Body Diagram



$$\sum F_n = 0: N - (9 \times 9.81\text{ N}) \cos 20^\circ + (60\text{ N}) \sin 15^\circ = 0$$

$$N = 67.436\text{ N}$$

$$F_{\max} = \mu_s N = (0.3) (67.436\text{ N}) = 20.231\text{ N}$$



$$\sum F_n = 0: N - (9 \times 9.81\text{ N}) \cos 20^\circ + (60\text{ N}) \sin 15^\circ = 0$$

$$N = 67.436\text{ N}$$

$$F_{\max} = \mu_s N = (0.3) (67.436\text{ N}) = 20.231\text{ N}$$

Assume equilibrium:

$$\sum F_t = 0: (60\text{ N}) \cos 15^\circ - (9 \times 9.81\text{ N}) \sin 20^\circ - F = 0$$

$$F = 27.759\text{ N} = F_{\text{eq.}}$$

but $F_{\text{eq.}} > F_{\max}$ impossible, so block slides up ◀

and

$$F = \mu_k N = (0.25) (67.436\text{ N})$$

$$F = 16.86\text{ N} \swarrow \blacktriangleleft$$

