

Lecture07-Mid-to-outer_shelf_circulation

October 16, 2025

1 Lecture07 - Mid-to-outer Shelf Circulation and Coastal Upwelling Systems

Learning Objectives: Ekman transport, Ekman layer thickness, coastal upwelling/downwelling, major ocean coastal upwelling systems

Before class:

After class:

- Read the GroupProject_guidelines.ipynb carefully and form your group of 2-3 people for the final group project by **week 8**.
- Pick a group project by **week 10** and discuss with me.

Reference:

- [Kaempf and Chapman 2016](#)
- optional: Brink 3.1, 3.2, 3.4, 3.6, 4.1

1.1 1. Geophysical Fluid Dynamics 101

1.1.1 1). Effects of Earth's Rotation - Coriolis Force

Newton's second law of motion, that the acceleration of a body is proportional to the imposed force divided by the body's mass, applies in so-called inertial frames of reference; that is, frames that are stationary or moving only with a constant rectilinear velocity relative to the distant galaxies. Now Earth spins round its own axis with a period of almost 24 hours (23h 56m, the difference due to Earth's rotation around the Sun) and so the surface of the Earth manifestly is not an inertial frame. Nevertheless, it is very convenient to describe the flow relative to Earth's surface (which in fact is moving at speeds of up to a few hundreds of metres per second), rather than in some inertial frame. - [Vallis \(2017\)](#)

Coriolis effect arises when we analyze large-scale motions in a non-inertial reference frame (the rotating Earth). In the northern hemisphere, Coriolis force deflects moving objects to the right; in the southern hemisphere to the left; Coriolis effect is proportional to speed.

Watch [this video](#) to demystify the Coriolis effect

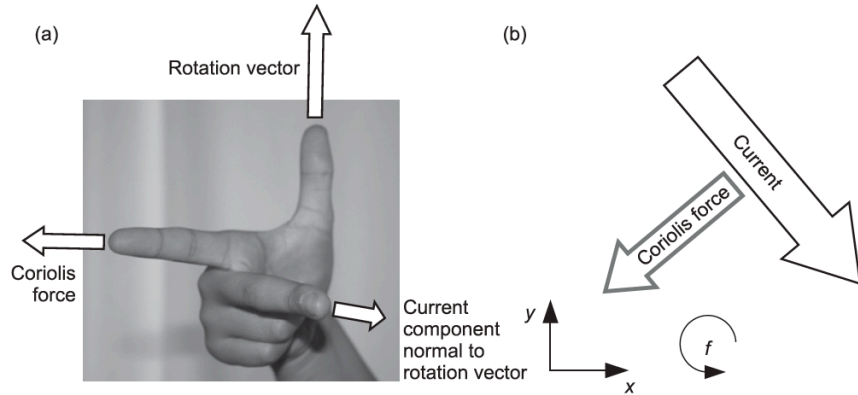


Figure 2.1

(a) Right hand rule for the Coriolis force. The vector Coriolis force equals $-2m\mathbf{\Omega} \times \mathbf{u}$ where $\mathbf{\Omega}$ is the angular velocity vector of the rotating frame, m is the mass of the moving object, and \mathbf{u} is the vector flow. (b) Coriolis force resolved in the local horizontal (x, y) plane. The force is of magnitude $mf|\mathbf{u}|$ to the right of the horizontal current \mathbf{u} . Here f is the Coriolis parameter (see Equation 2.1). These rules also work for geostrophic flow (Subsection 2.1.2): in that case an equal pressure-gradient force opposes the Coriolis force (not shown in these diagrams).

source: [\[https://www.cambridge.org/core/books/ocean-circulation-in-three-dimensions/BA67744EF2B76C3FCB239BCBF9D18271\]](https://www.cambridge.org/core/books/ocean-circulation-in-three-dimensions/BA67744EF2B76C3FCB239BCBF9D18271)

Coriolis force is given by $-2m\mathbf{\Omega} \times \mathbf{u}$

For a rotating frame of reference with a rotation period of T and an angular velocity vector with magnitude $\Omega = 2\pi/T$, an object of mass m moving with velocity \mathbf{u} relative to the rotating frame experiences a Coriolis force given by $-2m\mathbf{\Omega} \times \mathbf{u}$. The expression $-2\mathbf{\Omega} \times \mathbf{u}$ is therefore the Coriolis acceleration, or the force per unit mass.

The horizontal component of the Coriolis force depends on the vertical ($\hat{\mathbf{z}}$) component of the rotation vector. This quantity (times two) is called the Coriolis parameter. At latitude θ it is

$$f = \hat{\mathbf{z}} \cdot (2\mathbf{\Omega}) = 2\Omega \sin \theta$$

$$\text{unit} = s^{-1}$$

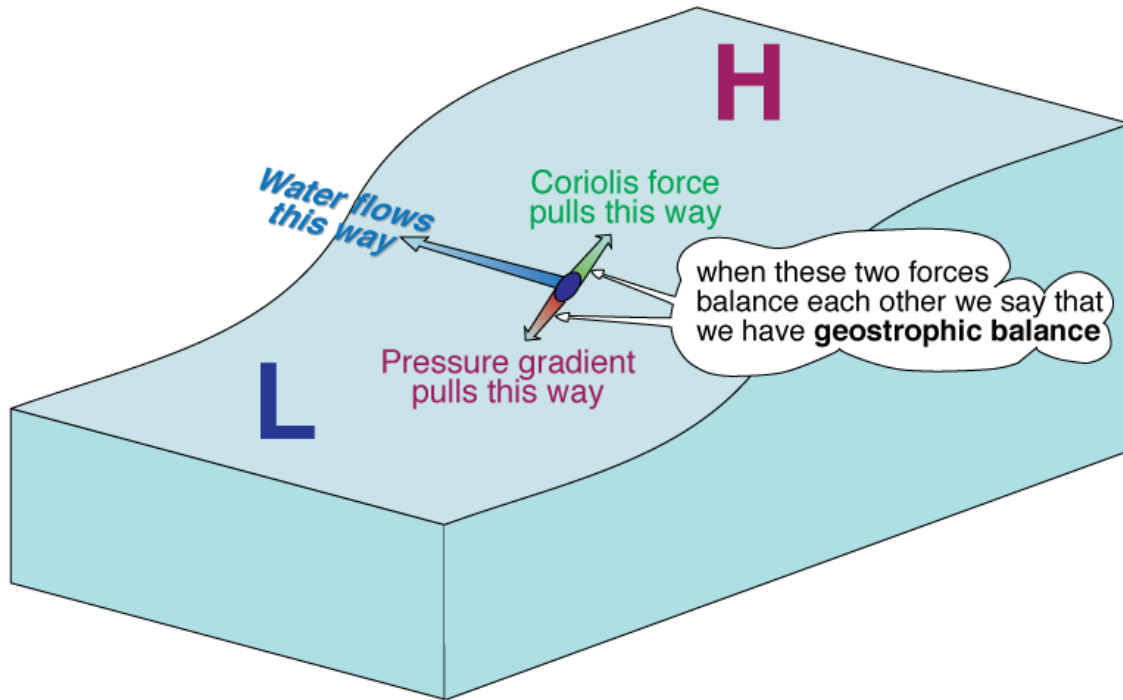
1.1.2 2) Geostrophic Balance

Assume steady ocean, without friction or any external forcing, the momentum equation for large scale $\sim O(100 \text{ km})$ flow is

$$fv = \frac{1}{\rho} \frac{\partial p}{\partial x},$$

$$fu = -\frac{1}{\rho} \frac{\partial p}{\partial y}$$

In a geostrophic flow the motion (current speed) is proportional to the pressure gradient force.



source: [<https://seos-project.eu/oceancurrents/oceancurrents-c06-s02-p01.html>]

Discussion: Observe the ocean surface height anomaly and surface velocity, and note the relationship between sea level height and the direction of ocean motion.

Tip: Copernicus Marine MyOcean Viewer - connecting the direction of Gulf stream with pressure gradient

1.2 2. Wind-driven circulation

1.2.1 1). Ekman Layer

Chalk talk and Discussion: To derive surface Ekman layer from momentum equation: 1. neglect pressure gradient forcing; 2. velocity vanishes below Ekman layer; 3. frictional stress continuous across surface and discuss typical thickness of Ekman layer.

Frictional-geostrophic balance in the horizontal momentum equation is:

$$fv = -\frac{1}{\rho} \frac{\partial \tau_x}{\partial z},$$

$$fu = \frac{1}{\rho} \frac{\partial \tau_y}{\partial z},$$

where τ is the wind stress = force / area [unit: N/m²].

If we integrate the Ekman velocities from the bottom of the Ekman layer to the surface, we obtain the > Ekman transport (volume): $U_E = \tau_{s,y} / \rho f$, $V_E = -\tau_{s,x} / \rho f$,

where τ_s is the wind stress at sea surface, which can be estimated by the air density $\rho_{air} = 1.3 \text{ kg m}^{-3}$, wind stress coefficient $C_D \approx 0.0015$, and wind speed at 10 m above the sea surface U_{10m} . $\tau_s = \rho_{air} C_D U_{10m}^2$

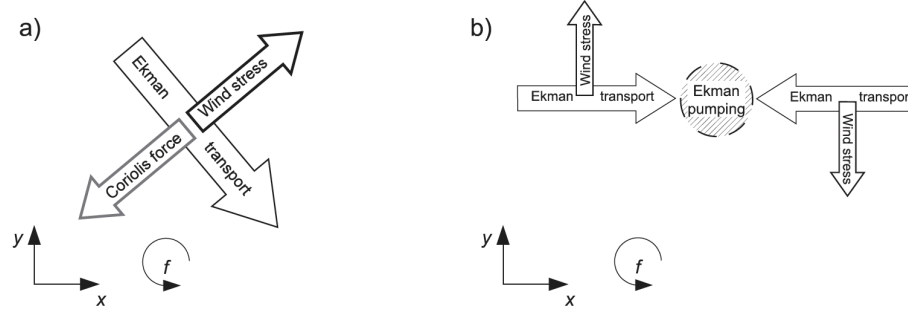


Figure 2.4

Schematic of Ekman transport in the surface wind-driven Ekman layer. (a) The wind stress on the sea τ is balanced by a Coriolis force associated with the Ekman transport at right angles \mathbf{U}_{Ek} (see Equation 2.11). (b) Where the wind stress direction changes in space, the Ekman transport can converge or diverge. This drives a weak vertical current, called Ekman pumping or suction, that is very important for the ocean circulation (see Equation 2.13). The case of the mid-latitude northern hemisphere is shown.

source: [\[https://www.cambridge.org/core/books/ocean-circulation-in-three-dimensions/BA67744EF2B76C3FCB239BCBF9D18271\]](https://www.cambridge.org/core/books/ocean-circulation-in-three-dimensions/BA67744EF2B76C3FCB239BCBF9D18271)

Note that we will need to express τ in terms of u and v in order for the above equations. We can substitute the wind stress with the flux of horizontal momentum applied by the wind through vertical mixing: $\tau_x = \rho A_z \frac{\partial u}{\partial z}$ $\tau_y = \rho A_z \frac{\partial v}{\partial z}$

into the above equations, we can then solve for the Ekman current: $u = V_0 \cdot \exp(az) \cdot \cos(-\frac{\pi}{4} + az)$, $v = V_0 \cdot \exp(az) \cdot \sin(-\frac{\pi}{4} + az)$,

where $V_0 = \frac{\tau}{\rho \sqrt{f A_z}}$ and $a = \sqrt{\frac{f}{2 A_z}}$. A_z is the vertical eddy viscosity and ranges from $10^{-5} m^2 s^{-1}$ in the deep ocean to $10^{-1} m^2 s^{-1}$ in the surface areas of a storm.

The speed magnitude decays exponentially with depth $V_e(z) = \sqrt{u(z)^2 + v(z)^2} = V_0 \cdot \exp(az)$

$$\text{Ekman layer depth: } \delta_E = \sqrt{\frac{2 A_z}{f}}$$

a. equation summary Latitude to estimate Coriolis parameter: $f = 2 |\sin \theta|$, [unit = s^{-1}]

Wind speed at 10 m above sea surface to estimate surface wind stress: $\tau_s = \rho_{air} C_D U_{10m}^2$, [unit = N/m^2]

Surface wind stress and Coriolis parameter to estimate Ekman transport: $U_E = \tau_{s,y} / \rho f$, $V_E = -\tau_{s,x} / \rho f$, [unit = m^2/s]

Vertical eddy viscosity and Coriolis parameter to estimate Ekman layer depth: $\delta_E = \sqrt{\frac{2 A_z}{f}}$, [unit = m]

b. Ekman spiral visualization

```
[1]: # %%capture

# # Install necessary packages and restart the kernel.
# !pip install numpy
```

```
# !pip install matplotlib
# !pip install ipyml
# # restart kernel
# import IPython
# IPython.Application.instance().kernel.do_shutdown(True) #automatically
↳restarts kernel
```

```
[2]: # # to enable the jupyter widgets so that you can plot interactive figures
# from google.colab import output
# output.enable_custom_widget_manager()
```

```
[3]: # import python libraries
import numpy as np
from matplotlib import cm
import matplotlib.pyplot as plt
```

```
[4]: # input necessary parameters
lat = 30
Az = 1e-3
U_10m = 10
V_10m = 0
```

```
[5]: # estimate Coriolis parameter based on latitude
T = 23*3600 + 56*60 #
omega = 2*np.pi/T # angular velocity
f = 2*omega*np.sin(lat/180*np.pi)

print(f"f at {lat} degree north = {f}")
```

f at 30 degree north = 7.292462055686613e-05

```
[6]: # estimate surface wind stress based on wind speed at 10 m above sea surface
rho_air = 1.3
C_D = 0.0015

tau_sx = rho_air*C_D*U_10m**2
tau_sy = rho_air*C_D*V_10m**2

print(f"tau_sx based on U_10m = {U_10m} m/s is {tau_sx} N/m^2")
print(f"tau_sy based on V_10m = {V_10m} m/s is {tau_sy} N/m^2")
```

tau_sx based on U_10m = 10 m/s is 0.195 N/m²
tau_sy based on V_10m = 0 m/s is 0.0 N/m²

```
[8]: # solve for Ekman current based on wind
rho = 1e3
U_E = tau_sy/rho/f
V_E = -tau_sx/rho/f
```

```

print(f"Ekman volume transport [m^2/s] at {lat} degree north with surface wind_
↳ stress tau_x={tau_sx} N/m2 and tau_y={tau_sy} N/m^2 is ")
print(f"U_E = {U_E} m^2/s")
print(f"V_E = {V_E} m^2/s")

```

Ekman volume transport [m²/s] at 30 degree north with surface wind stress
tau_x=0.195 N/m2 and tau_y=0.0 N/m² is
U_E = 0.0 m²/s
V_E = -2.6739940298755522 m²/s

```

[9]: # estimate Ekman layer depth based on vertical eddy viscosity Az
delta_E = np.sqrt(2*Az/f)

print(f"delta_E at {lat} degree north with Az = {Az} m^2/s is {delta_E} m")

```

delta_E at 30 degree north with Az = 0.001 m²/s is 5.236943745506095 m

```

[10]: # solve for Ekman current
z = np.arange(0, -100, -1) # create z coordinate
V_0 = np.sqrt(tau_sx**2+tau_sy**2)/rho/np.sqrt(f*Az) # estimate V_0
a = np.sqrt(f/2/Az) # estimate a
u = V_0*np.exp(a*z)*np.cos(-np.pi/4+a*z) # compute Ekman current at each depths
v = V_0*np.exp(a*z)*np.sin(-np.pi/4+a*z)

```

```

[11]: %matplotlib widget
ax = plt.figure().add_subplot(projection='3d')
for k in range(len(z)):
    ax.quiver(0, 0, z[k], u[k], v[k], 0, alpha=.7, color='k') # plot Ekman_
↳ current arrows at each depth

ax.quiver(0,0,0, tau_sx, tau_sy, 0, color='r', label = 'surface wind stress' )_
↳ # plot wind stress at the sea surface
ax.quiver(0,0,0, U_E, V_E, 0, color='b', label = 'Ekman transport' ) # plot_
↳ Ekman transport

x, y = np.meshgrid(np.arange(-.8, .8, .1), np.arange(-.8, .8, .1))
ax.plot_surface(x, y, np.zeros(x.shape)-delta_E, alpha=0.2, color='k') # plot_
↳ Ekman layer depth

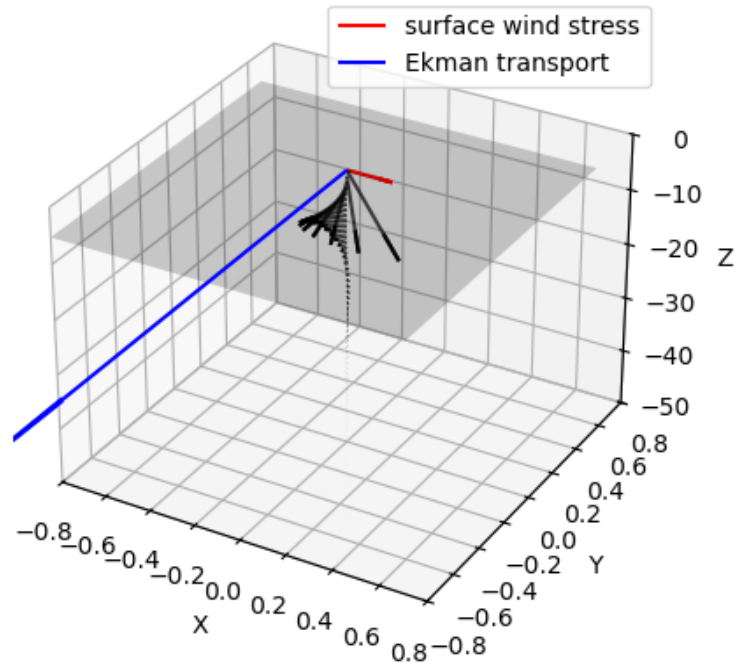
ax.set_xlim(-.8, .8)
ax.set_ylim(-.8, .8)
ax.set_zlim(-50,0)

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

```

```
plt.legend()
```

```
plt.show()
```



1.2.2 2). Coastal Upwelling

a. upwelling types The main forms of wind-driven upwelling are (i) coastal upwelling, (ii) equatorial upwelling, and (iii) ice-edge upwelling (Fig. 2.1). The Earth's rotation and its associated effects, such as the Coriolis force, play a dominant role in the dynamics of upwelling in all three systems. Coastal upwelling owes its existence to the presence of both the coast as an impermeable lateral boundary and relatively shallow water on the continental shelf. Equatorial upwelling is related to the fact that the Coriolis parameter (the constant of proportionality in the Coriolis force) changes sign across the equator. Although the Coriolis force vanishes at the equator it comes into full swing at relatively short distances (>50 km) from it. Due to this spatial variation of rotational effects, the dynamical role of the equator in the upwelling process is similar to that of coasts. Ice-edge upwelling is created via a substantial dampening of the effect of wind stresses on currents under the sea ice.

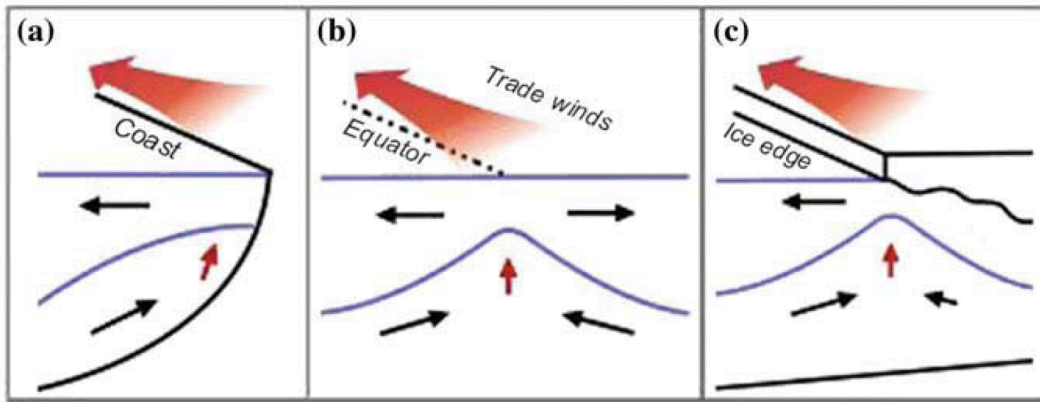
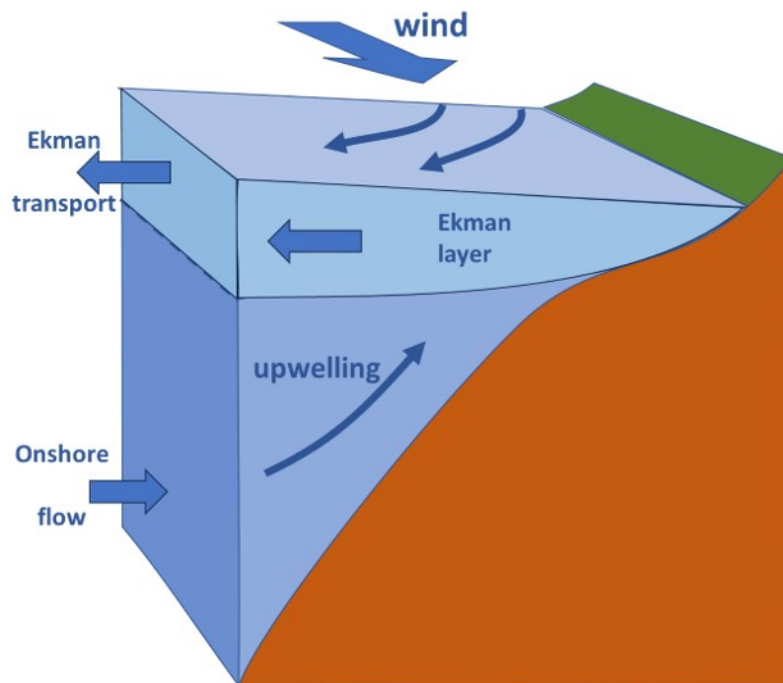


Fig. 2.1 Types of oceanic upwelling: **a** coastal upwelling, **b** equatorial upwelling, and **c** ice-edge upwelling. The *broad brown arrow* shows the direction of the prevailing wind relative to a coast, the equator, or an ice edge. In the case of the coast this shows the situation for the southern hemisphere, with the wind blowing towards the north

[source: https://www.researchgate.net/publication/307434835_The_Functioning_of_Coastal_Upwelling]

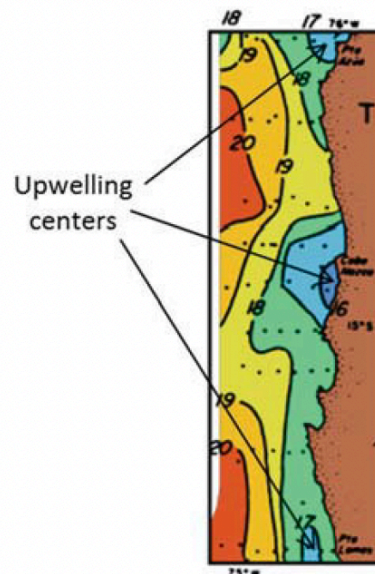
b. coastal upwelling Discussion: Why do ocean waters along the west coast of the Americas tend to cool during the summer months? Tip: Go to the Copernicus Marine MyOcean Viewer and describe what features you see in sea surface temperature on 07/29/2020.



[source: https://www.coastalwiki.org/wiki/Ekman_transport]

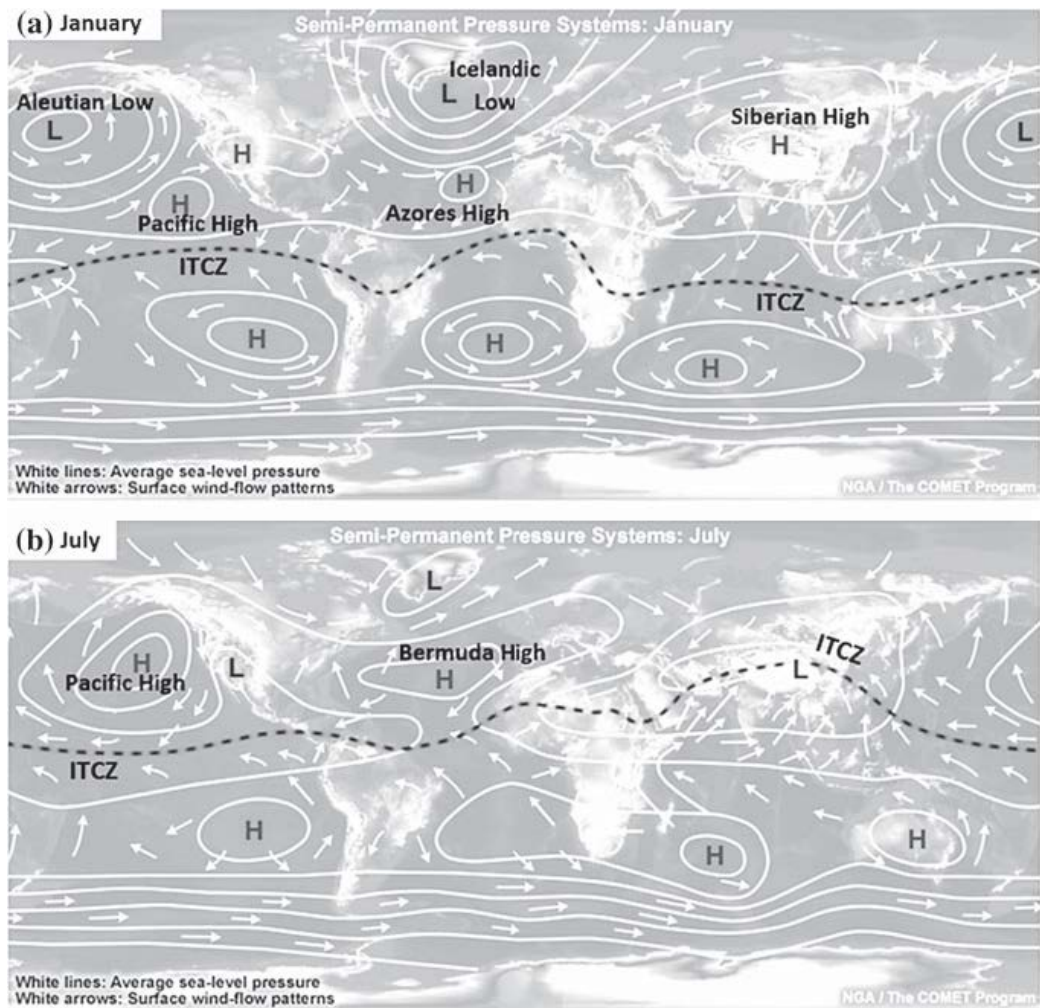
c. continuous vs seasonal coastal upwelling systems Depending on typical wind conditions in a region, coastal upwelling can be either a quasi-permanent feature in so-called major coastal upwelling systems or a seasonal feature in seasonal coastal upwelling systems. While upwelling can occur all along a straight coastline (e.g., off Oregon), since coastlines and seafloors are frequently irregular, wind-driven coastal upwelling events are generally localized, and upwelling is not at all uniform. As a consequence, upwelling is more pronounced in certain regions, called upwelling centres, than in others (Fig. 2.2). Upwelling centres are often associated with strong frontal flows associated with an upwelling jet that breaks up into mesoscale (10–20 km in coastal waters) circular circulation patterns called eddies. While most of the primary productivity takes place inside and a short distance downstream of coastal upwelling centres, more quiescent regions adjacent to upwelling centres, called upwelling shadows, are important as spawning and nursery grounds for pelagic fish.

Fig. 2.2 In warm-water regions of the ocean, coastal upwelling of colder sub-surface water leads to a decrease in sea surface temperatures. This decrease need not be spatially uniform as it can occur at specific upwelling centres. The graph shows in-situ measured sea surface temperatures for the Peru-Chile upwelling system (from Tomczak and Godfrey 2003)



[source: https://www.researchgate.net/publication/307434835_The_Functioning_of_Coastal_Upwelling_Systems]

d. major upwelling systems in the world ocean Discussion: Below figure shows the seasonal variations of surface wind patterns. Can you guess where coastal upwelling regions form?



[source: https://www.researchgate.net/publication/307434835_The_Functioning_of_Coastal_Up]

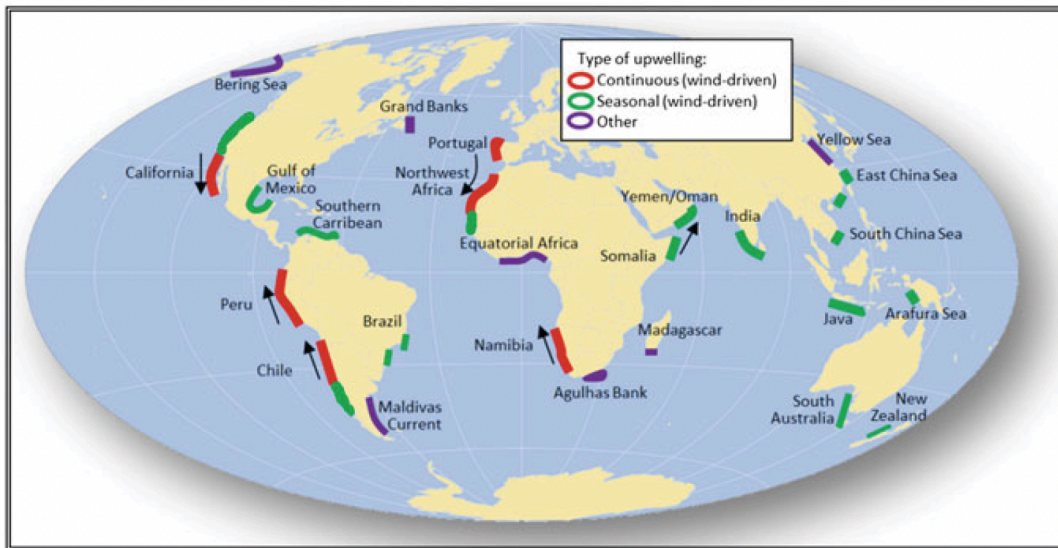


Fig. 2.14 Locations of significant coastal upwelling regions in the world ocean

[source: https://www.researchgate.net/publication/307434835_The_Functioning_of_Coastal_Upwelling_Systems]

1.2.3 3). The Ecology of Coastal Upwelling Systems

a. biological response to coastal upwelling events The upwelling process moves water and its nutrients from the bottom of the shelf into the surface mixed layer (Fig. 2.17). As nutrient-enriched water moves upward, it can lead to sub-surface phytoplankton production near the base of the euphotic zone, particularly during partial upwelling events, which cannot be detected from satellite measurements. Full upwelling, i.e., when the upwelled water intersects the surface, generally creates the strongest biological response, but the magnitude of the response depends on whether the wind stress is maintained or relaxes to allow a phytoplankton bloom to develop.

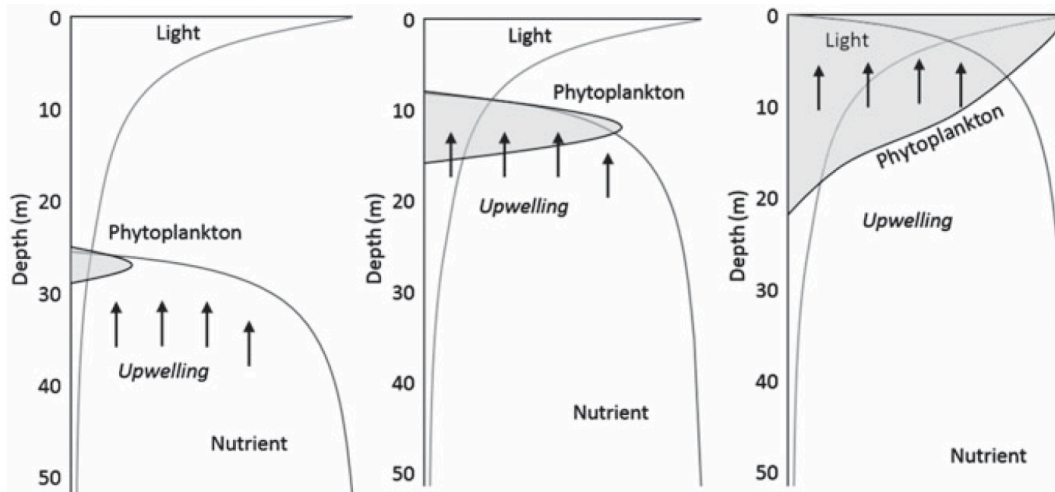
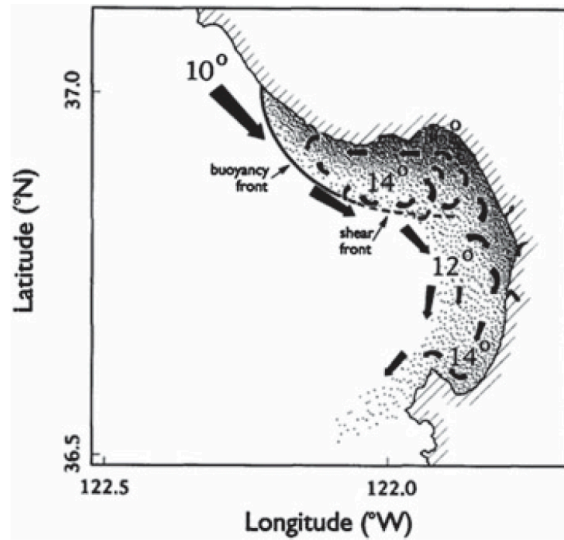


Fig. 2.17 Schematic of the response in phytoplankton production to upwelling of nutrient-rich sub-surface water. Shown is the typical summer situation for temperate waters in which thermal stratification triggers sub-surface phytoplankton production in a narrow zone near the base of the surface mixed layer. As the upwelling continues, this zone is moved upward in the water column to a level of greater light intensity, which increases production. The maximum possible production occurs for full upwelling

[source: https://www.researchgate.net/publication/307434835_The_Functioning_of_Coastal_Upwelling_Systems]

b. the significance of upwelling shadows Upwelling shadows, also known as retention zones, are relatively quiescent regions typically located downstream of upwelling centres in embayments that are sheltered from the swift and highly energetic upwelling jets (Fig. 2.19 shown an example). In addition to upwelling centres where nutrient fluxes into the euphotic zone fuel primary production, upwelling shadows form another important ecologic role in the food web dynamics of upwelling systems as these provide sheltered breeding and feeding grounds for fish species. The name “upwelling shadows” was introduced and described by Graham et al. (1992), who analyzed the zooplankton abundance in northern Monterey Bay, California, USA, although the idea of such retention zones trapping water close to the coast thus allowing blooms to develop had been considered earlier (e.g., Shannon et al. 1984; Taunton-Clark 1985).

Fig. 2.19 Hypothetical circulation in the Monterey Bay upwelling shadow. Arrows indicate currents and numbers show water temperatures. The shaded area is the retention zone where current speeds are reduced (from Graham and Largier 1997)



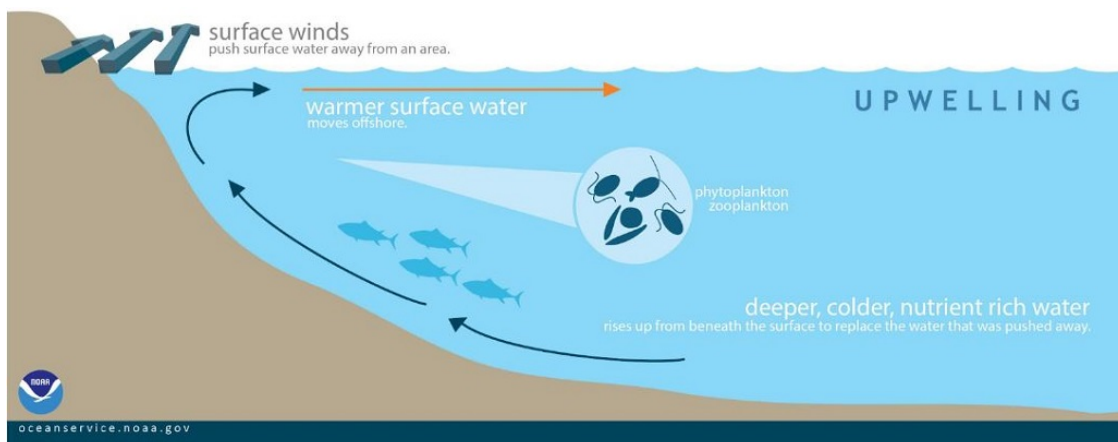
[source: https://www.researchgate.net/publication/307434835_The_Functioning_of_Coastal_Upwelling_Systems]

1.2.4 3). California upwelling system

Coastal upwelling is the dominant physical forcing affecting production in the California Current System. Strong, northerly winds that blow from April to September drive upwelling along the Oregon and Washington coasts.

Upwelling can occur year-round off the northern and central California coast. The winds transport surface waters (upper 15 m) offshore, which are replaced by the upwelling of deep (100-125 m), cold (8°C), and nutrient-rich waters from the shelf break region.

The upwelled water fuels high phytoplankton production and subsequent high biomass of copepods, euphausiids, and other zooplankton during the summer.



[source: <https://www.fisheries.noaa.gov/west-coast/science-data/oceanography-northern-california-current-study-area>]

Discussion: Connecting the summer California upwelling system with primary production. **Tip:** Go to the Copernicus Marine MyOcean Viewer and describe what features

you see off California coast in chlorophyll on 08/16/2025.

[]: