Lecture09-Sediment_properties_movement

November 6, 2025

1 Lecture09 - Sediment properties and movement

Learning Objectives: sedimentation properties, sediment grain diameter, classification by grain size, force balance for particle motion, fall velocity dependence, Reynolds number, Shields curve

In-class Lab:

- We will sieve and weigh sediments of different diameter classes collected from Eastern Point Beach, and plot a particle size distribution.

After class:

- Read about mapping sediment in the Long Island Sound here

Reference:

- classnote taken from CEE262G Sediment Transport Physics and Modeling by Oliver Fringer, Stanford University
- B.S. 6.1, 6.2, 6,3

1.1 1. Basic sediment properties

1.1.1 1). Notation and basic properties

- Sediment mass and volume: m_s, V_s
- Sediment density: $\rho_s = \frac{m_s}{V_s}$
- Specific weight (=weight per unit volume): $\gamma_s = \frac{W_s}{V_s} = \frac{\rho_s g V_s}{V_s} = \rho_s g$
- Specific gravity: $s = \frac{\rho_s g}{\rho_0 g} = \frac{\gamma_s}{\gamma_0}$, e.g. Quartz: s = 2.65 times more dense than water at 24°C.
- Porosity p: the ratio of pore space (voids) to the whole sediment volume. Natural sands have porosities in the range of 0.25 to 0.50; a frequently applied figure is 0.40 (or 40 %); for maximum possible sphere packing: p=0.64; for M&M candies, p=0.68.
- Sediment concentration c can be defined in two ways: mass concentration and volume concentration.
  - The mass concentration is the mass of the solid particles per volume (c in kg/m^3 or equivalently g/L) and is often used when measuring sediment concentrations.
  - Volume concentration is defined as the ratio of the volume of solid particles to the whole volume (c in m^3/m^3 or in %). Sediment in a sediment bed has a volume concentration of $n = 1 - p$. For sediment in suspension, the volume concentration c indicates the volume of sediment per volume of the mixture. If the sediment in such a mixture settles to the bed, 1 m^3 of solid particles will occupy $1/(1 - p)$ (p: porosity) m^3 at the bed. Volume concentrations are obtained from mass concentrations by multiplication by $1/\rho_s$;

1.1.2 2). Defining the sediment grain diameter d_s

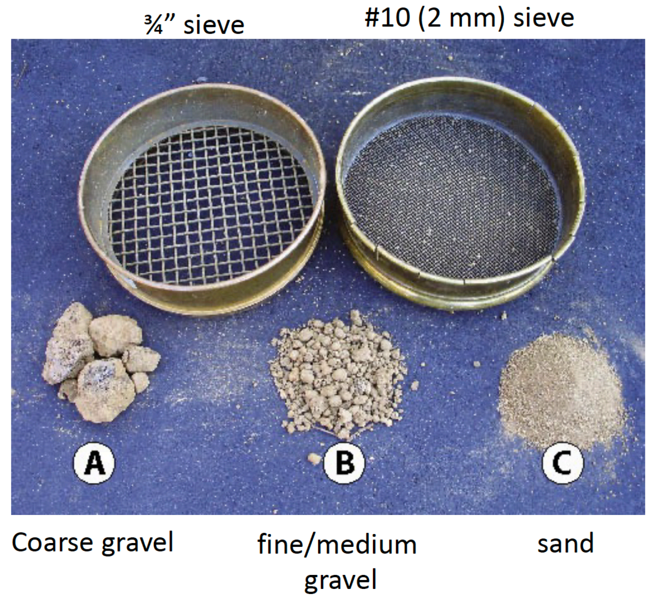
http://www.sand-atlas.com/en/shape-of-sand-grains/

a. Equivalent sphere or nominal diameter

$$V_s = \frac{4}{2}\pi r_s^3 = \frac{1}{6}\pi d_s^3$$

$$d_s = (\frac{6V_s}{\pi})^{1/3}$$

Minimum side length of square sieve opening through which a particle will fall.



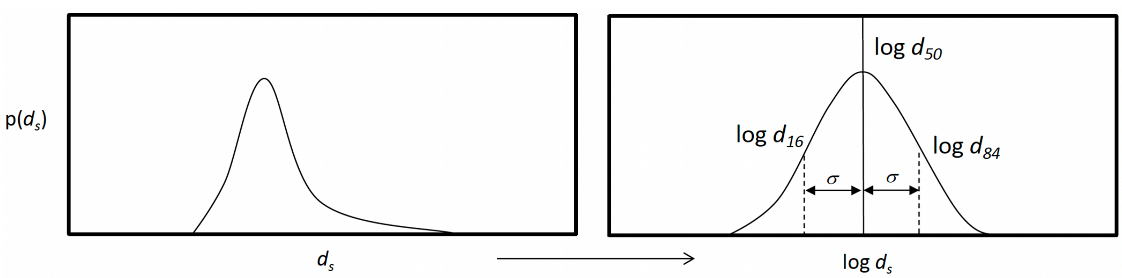
#5/4000 micron = 4 mm
#10/2000 micron = 2 mm
#35/500 micron = 0.5 mm
#60/250 micron = 0.25 mm
#120/125 micron = 0.125 mm
#230/63 micron = 0.063 mm

1.1.3 3). Sediment distribution

a. Particle size distribution (PSD or GSD) Sediment grain sizes distributions are typically approximated as lognormal with mean μ and standard deviation σ.

d_N = diameter for which $N\%$ of grains by mass are smaller.

e.g. d_{50} = median grain size; 50% of grains by mass are smaller. d_{50}=median grain size

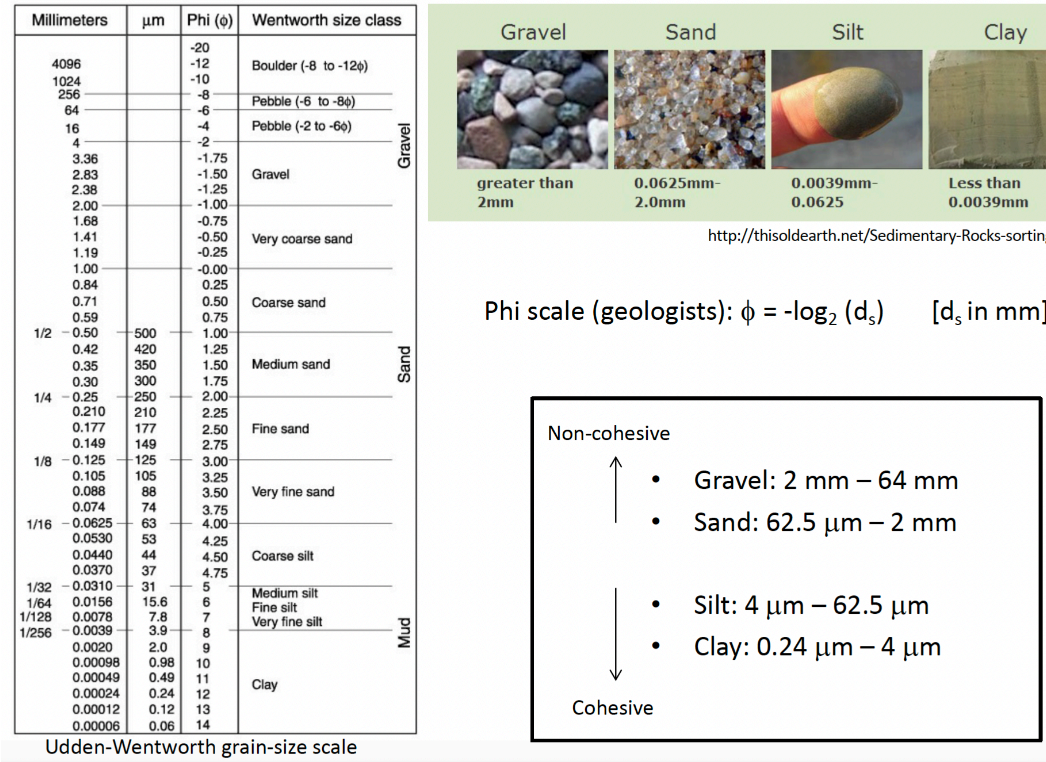


$log(d_{84}) = log(d_{50}) + \sigma \rightarrow d_{84} = d_{50}\exp(\sigma)$

$log(d_{16}) = log(d_{50}) - \sigma \rightarrow d_{16} = d_{50}/\exp(\sigma)$

If distribution is lognormal, then $\exp(\sigma) = d_{50}/d_{16} = d_{84}/d_{50}$

b. Cumulative particle size distribution Sediment is called **well-sorted** if d_{84}/d_{16} is small (say < 1.5, although there is no formal classification); for large values of d_{84}/d_{16} (for instance > 3) we speak of **poorly sorted** or **well-graded** sediment.

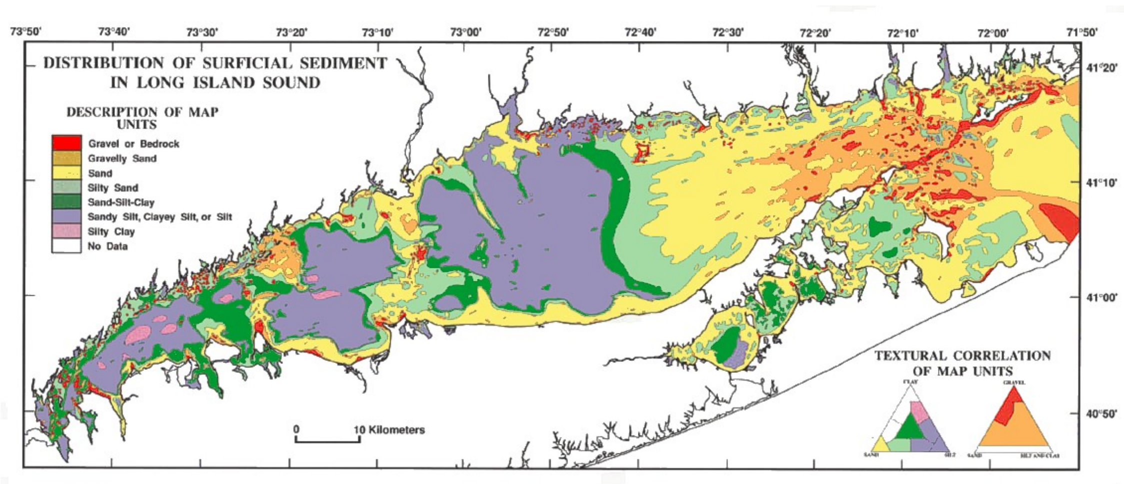


Millimeters	μm	Phi (ϕ)	Wentworth size class
		-20	
4096		-12	Boulder (-8 to -12ϕ)
1024		-10	
256		-8	
64		-6	Pebble (-6 to -8ϕ)
16		-4	Pebble (-2 to -6ϕ)
4		-2	
3.36		-1.75	
2.83		-1.50	Gravel
2.38		-1.25	
2.00		-1.00	
1.68		-0.75	
1.41		-0.50	Very coarse sand
1.19		-0.25	
1.00		-0.00	
0.84		0.25	
0.71		0.50	Coarse sand
0.59		0.75	
1/2 0.50	500	1.00	
0.42	420	1.25	
0.35	350	1.50	Medium sand
0.30	300	1.75	
1/4 0.25	250	2.00	
0.210	210	2.25	
0.177	177	2.50	Fine sand
0.149	149	2.75	
1/8 0.125	125	3.00	
0.105	105	3.25	
0.088	88	3.50	Very fine sand
0.074	74	3.75	
1/16 0.0625	63	4.00	
0.0530	53	4.25	
0.0440	44	4.50	Coarse silt
0.0370	37	4.75	
1/32 0.0310	31	5	Medium silt
1/64 0.0156	15.6	6	Fine silt
1/128 0.0078	7.8	7	Very fine silt
1/256 0.0039	3.9	8	
0.0020	2.0	9	
0.00098	0.98	10	
0.00049	0.49	11	Clay
0.00024	0.24	12	
0.00012	0.12	13	
0.00006	0.06	14	

Udden-Wentworth grain-size scale

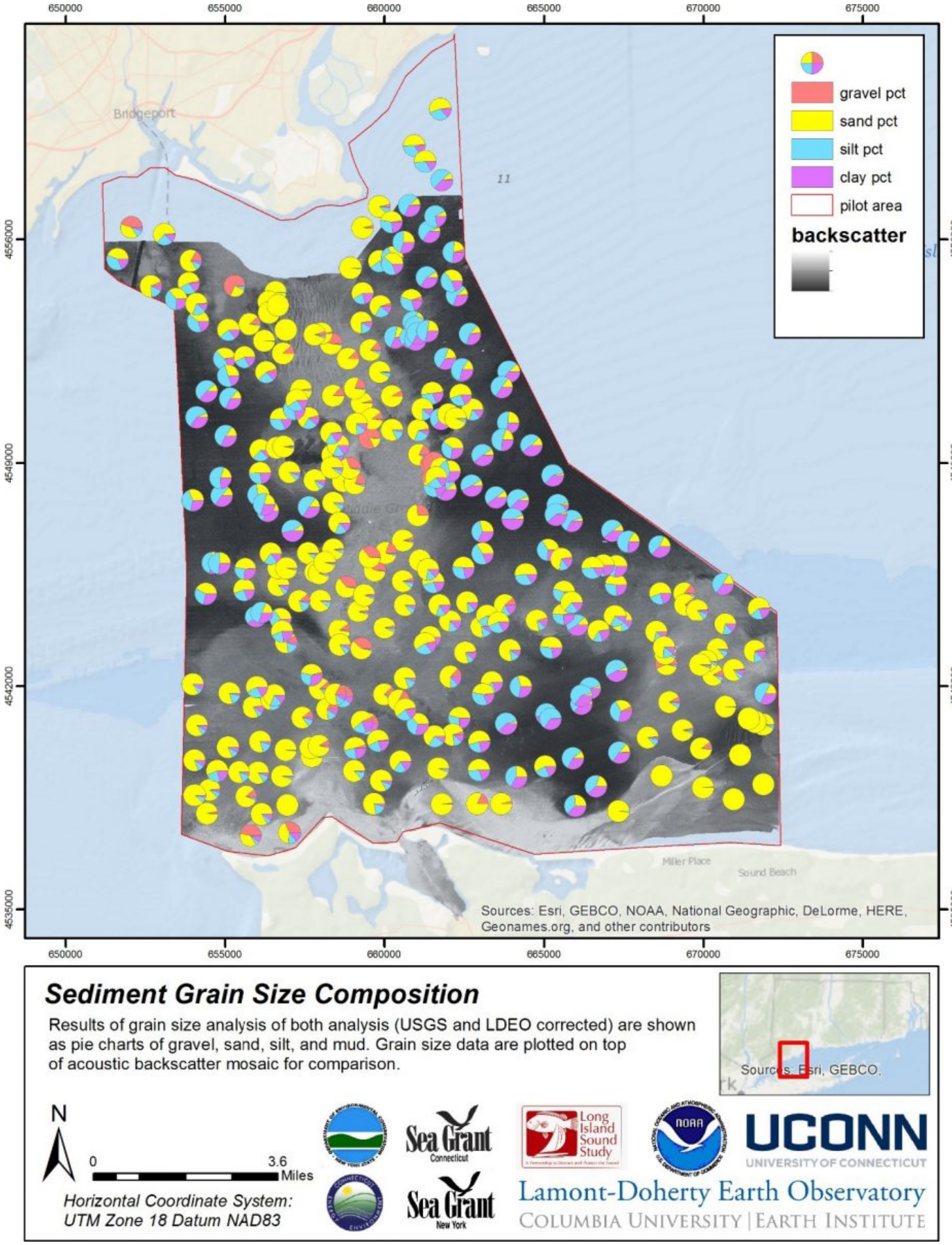
c. Grain size classification

1.1.4 4). Sediment distributions in Long Island Sound



source: [https://pubs.usgs.gov/of/2000/of00-304/htmldocs/chap04/index.htm], see the Shepard's sediment classification diagram here.

Along the central axis of the Sound, the grain size progressively decreases from gravel prevalent near the Race to clayey silt on the flat floor of the central basin. This progression reflects the general east-to-west succession of sedimentary environments (**from erosion to transport to sorting to deposition**) caused by the **decreasing gradient of tidal-current speeds coupled with the net westward estuarine bottom drift**.



source: [https://lismap.uconn.edu/sediment-texture-and-grain-size-distribution-2/]

1.2 2. Particle motion

1.2.1 1). Governing equation for a single particle

$$m_s \frac{d\vec{u}_s}{dt} = \overrightarrow{W} + \overrightarrow{F}_{pressure} + \overrightarrow{F}_{viscous}$$

where the different terms are given by: > Weight: $W = -m_s g\vec{k}$ > > Pressure force: $\overrightarrow{F}_{pressure} = -\int_A P\vec{n}dA$ > > Viscous force: $\overrightarrow{F}_{viscous} = \int_A T \cdot \vec{n}dA$ > where T is the viscous stress tensor, representing viscous stress τ in all three directions.

1.2.2 2). Suspended sediment transport

Discussion: Watch sediment transport in a river here and discuss 1) Do sediments remain suspended and transported with water indefinitely? 2) What might happen when moving water flows over a sediment bed? 3) Under what conditions do we expect sediment transport, erosion, and deposition? Tip1: Lab experiment: Low head dam installation effects on coarse sediment transport; Tip2: Lab experiment: Flow speed relation with sediment erosion and deposition.

1.2.3 3). The fall velocity

When a particle falls in still and clear water, it accelerates until it reaches a constant vertical velocity that is called fall velocity or settling velocity. This velocity can be assessed from the balance between the downward-directed gravity force F_G (weight minus buoyancy, which is a pressure force that results from the difference in pressure exerted by a fluid on the particle) and the upward-directed drag force F_D.

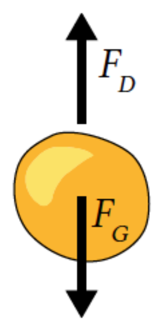


Figure 6.1: Forces on a 'sphere' in clear water.

For a perfect sphere ($V_s = \frac{\pi}{6}d_s^3$; cross-sectional area of the sphere $A_s = \frac{\pi}{4}d_s^2$):

$$F_G = (\rho_s - \rho_0)g(\tfrac{\pi}{6}d_s^3),\ F_D = \tfrac{1}{2}C_D\rho_0 w_s^2(\tfrac{\pi}{4}d_s^2),$$

where C_D is the unitless drag coefficient and w_s is the particle fall velocity.

In equilibrium, both forces are in balance and the fall velocity w_s (in m/s) is given by: > $w_s = \sqrt{\frac{4(s-1)gd_s}{3C_D}}$,

where s is the specific density.

- A particle's fall velocity thus depends on **its size, its density and the magnitude of the drag coefficient** C_D.
- This drag coefficient depends on the shape of the particle and its roughness, but mainly on the grain's Reynolds number: > $Re = \frac{w_s d_s}{\nu}$,

where ν is the kinematic viscosity coefficient (unit m^2/s), which is defined as the dynamic viscosity divided by the water density $\nu = \mu/\rho_0$. A characteristic value for ν is 10^{-6} m^2/s. The dynamic viscosity represents the fluid's internal resistance to flow ('thickness') and is a function of the temperature and to a smaller extent of the density.

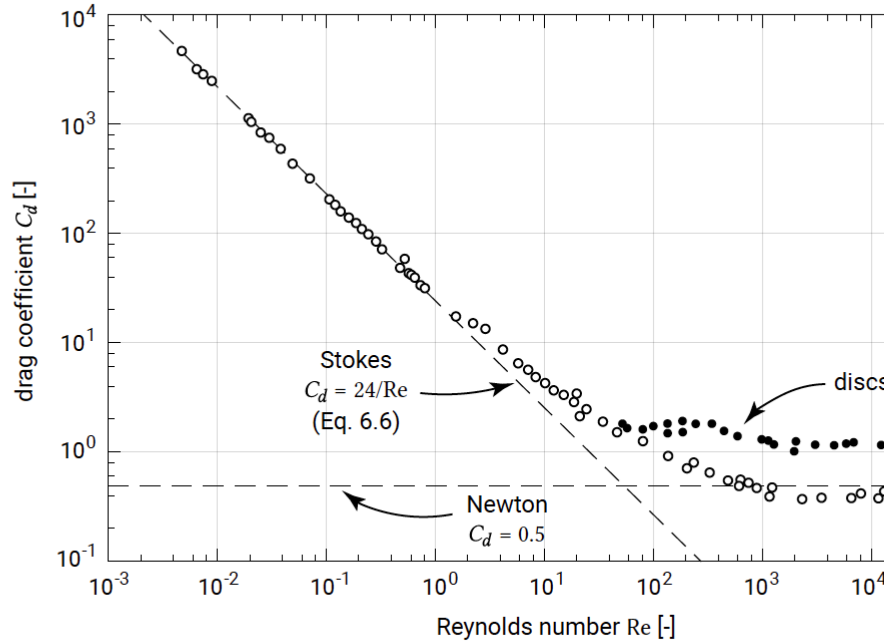


Figure 6.2: Drag coefficient as a function of Reynolds Number (Vanoni, 1975

a. Dependence on Reynolds number

For low grain Reynolds numbers (Re < 0.1 to 0.5) in the so-called **Stokes range**, the drag coefficient can be described by: $> C_D = 24/Re$

yielding $> w_s = \frac{(s-1)gd_s^2}{18\nu}$

In this range, the fall velocity depends on **the square of the grain diameter, the relative density and the kinematic viscosity coefficient**.

For high grain Reynolds numbers (400 < < 2 × 105), in the so-called **Newton range**, the drag coefficient becomes a constant (C_D 0.5). In that case: $> w_s = 1.6\sqrt{gd_s(s-1)}$

In this range, the fall velocity depends on **the square root of the grain diameter and the relative density** and is **independent of the kinematic viscosity coefficient**. This is also the case for extremely high Reynolds numbers (> 2 × 105), where the drag coefficient is (constant) around 0.2.

Chalk talk and Discussion: Watch the video on How to design your own airplanes, Reynolds Number Explained For quartz spheres falling in still water, a 0.08 mm diameter particle corresponds to a Reynolds number of 0.5, while a diameter of about 1.9 mm particle corresponds to a Reynolds number of 400. For very small particles (silt, clay) does the fall velocity scale with d_s^2 or $\sqrt{d_s}$; what about gravels?

b. Hindered settling In high-concentration mixtures, the fall velocity of a single particle is reduced due to the presence of other particles. This can be explained as follows: with each downward grain movement, a similar fluid volume must flow upward; this upward flow slows down the other grains.

movie and modeled by Yinuo Yao, Stanford

1.2.4 4). Initiation of motion

Sediment can only be transported if the water movement exerts a large enough shear stress τ_b on the grains. The so-called critical shear stress $\tau_{b,cr}$ describes the point of initiation of motion. If this condition is exceeded, grains move, roll or are brought into suspension.

Watch the video on Underwater video of sediment transport in Akutan, Alaska by High Tide Exploration and see the sediment at the bed initiate motion.

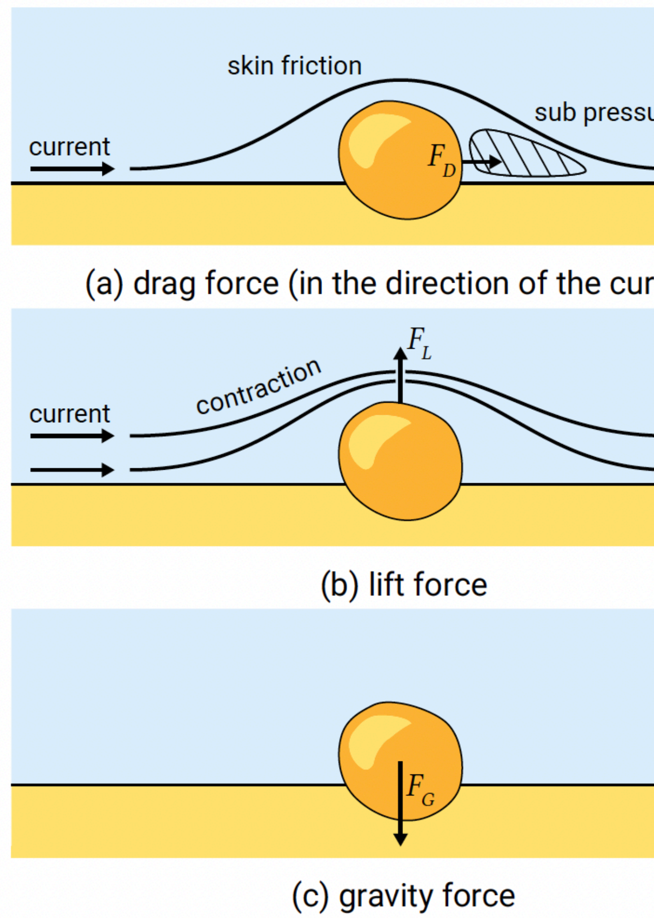


Figure 6.4: Forces on an individual grain in a stationary situation.

a. Forces on a single grain on bed

- **Drag force** is a combination of skin friction acting on the surface of the grain and a pressure difference on the up- and downstream sides of the grain because of flow separation at the downstream end of the particle. Similar to the drag on a free-falling particle, **drag force** in moving water is proportional to:
  - the square u^2 of a typical upstream horizontal flow velocity;
  - the particle's surface area and hence for spheres to d_s^2;

- the water density ρ_0.

- **Lift force** results from the flow separation, as well as from the flow contraction above the grain. A higher local flow velocity results in a lower local pressure (Bernoulli law). The difference in vertical pressure causes an upward-directed lift force. Similarly to the drag force, the lift force is proportional to
  - the particle's surface area (and thus to d_s^2 in the case of a sphere)
  - and to u^2.
- **Gravity force** is proportional to $(\rho_s - \rho_0)gd_s^3$

b. Critical Shields parameter The total driving force (drag and lift combined) is therefore proportional to $\rho_0 u^2 d_s^2$. The resisting gravity force is proportional to $(\rho_s - \rho_0)gd_s^3$. Equilibrium of forces, whether horizontal, vertical or rotational, therefore, is expressed through a formula of the following type: > $(\rho_s - \rho_0)gd_s^3 \propto \rho_0 u_{cr}^2 d_s^2$, > where u_{cr} is the critical velocity of the water at which grains start moving.

The bed shear stress is proportional to the velocity squared times the water density (recall wind stress at the sea surface). The above equation can be expressed by: > $(\rho_s - \rho_0)gd_s^3 \propto \rho_0 \tau_{b,cr} d_s^2$, > where $\tau_{b,cr} = \rho_0 u_{cr}^2$ is the critical bottom shear stress.

Critical in the sense that higher bottom shear stresses lead to the initiation of motion.

Finally, the so-called **critical Shields parameter** θ_{cr} can be deduced: > $\theta_{cr} = \frac{\tau_{b,cr}}{(\rho_s - \rho_0)gd_s} = C$, > The constant C has to be determined experimentally. The experiments of Shields, performed on a flat bed, are the most widely used. He defined the critical bed shear stress as the bed shear stress at which the (extrapolated) measured transport rates were just zero. **For sand placed smoothly on this flat bed, C was found to be around 0.05.** The critical Shields parameter can be used to assess the stability of stones and structural damage risks for slopes of loose rock and breakwater elements.

c. Shields curve Shields found experimentally that the 'constant' $C \approx 0.05$ is a weak function of the grain Reynolds number > $Re_* = \frac{u_* d_s}{\nu}$

Figure 6.5 shows measured values of C as a function of Re_*. The shaded band separates two zones: movement of sediment particles was observed in the zone above this shaded band, whereas no movement was observed in the zone underneath the shaded band. **The shaded band therefore indicates initiation of motion.** Sometimes, the shaded band is represented by a single line, which is then referred to as the **Shields curve**. The average value can be seen to be approximately 0.05.

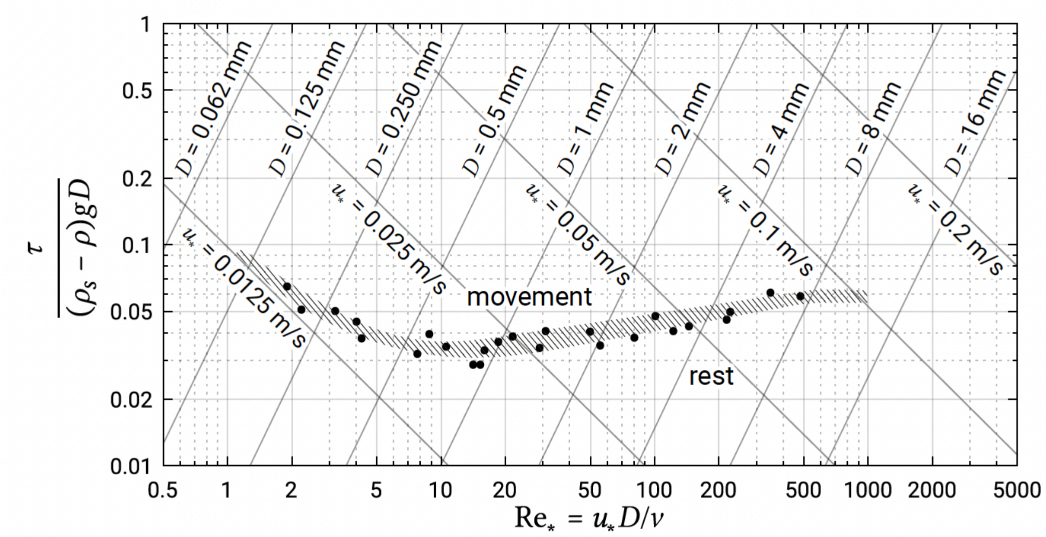


Figure 6.5: Shields curve (Shields, 1936). Note that the axes are drawn on a log-log scale. The lines of constant D and u_* do not originate from Shields and are valid for constant density $\rho_s = 2650\ \mathrm{kg/m^3}$ and kinematic viscosity $\nu = 1.25 \times 10^{-6}\ \mathrm{m^2/s}$ at a water temperature of 12 °C.

However, reality is more complex in terms of: - The Shields curve is valid for uniform flow on a flat bed. The effect of bed ripples and the effect of the combination of unidirectional and oscillatory flow on initiation of motion are largely unknown; - Gradation of the bed material may play a role, especially for poorly sorted sediment. In these cases, the smaller particles will be hidden in the voids between the larger particles, while the larger particles are more exposed. After exposed smaller particles are washed out, a top layer of coarser particles (with higher critical flow velocities) remains and prevents movement of the underlying smaller particles. This is called **bed armouring**; - For a sloping bed in the flow direction, it can be argued that the critical flow velocity will be somewhat smaller for downward-sloping beds and somewhat higher for upward-sloping beds; - Cohesive forces between the grains – due to the presence of cohesive sediment in the bed – may drastically increase resistance against erosion. Biological activity and consolidation may be important in this respect as well.

```
[ ]:
```