Lecture03-Wind_Wave

September 9, 2025

1 Lecture03 - Wind waves

Learning Objectives: wave length, wave period, wave speed, sea wave vs swell, significant wave height, wave spectrum, wave generation and prediction

Before class: - Read news about Hurricane Kiko that is happening now! Hurricane Kiko may bring life-threatening waves to Hawaii

After class: - Assignment02.ipynb (due 09/15/2025 23:59PM) - Watch video, read waves and possibly learn surfing? How to Catch More Waves with Less Effort | Positioning

Reference:

- Textbook chapter 4, skip 4.3.2
- optional: Textbook 4.4.2

1.1 1. Wave Characteristics

1.1.1 1) basics

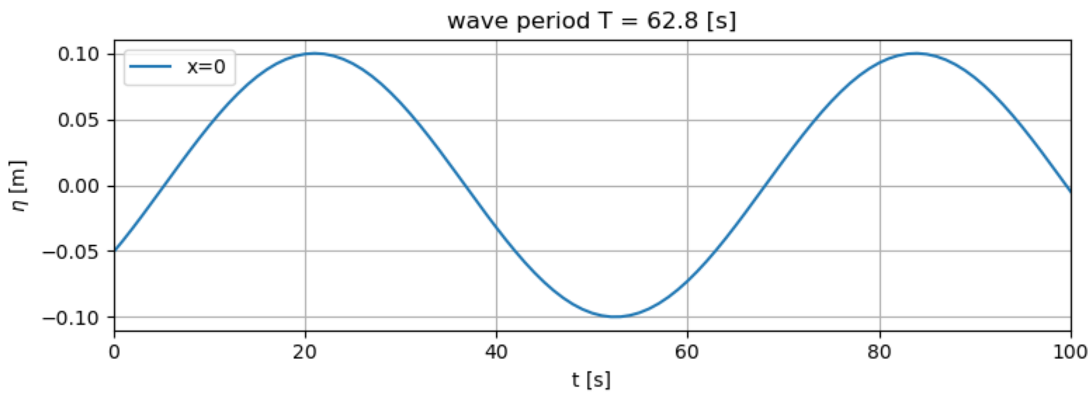
The surface elevation   of a plane wave can be described by: $\eta = a\sin(\sigma t - kx)$,
where a is the wave amplitude, wave height $H = 2a$,
σ is the angular frequency (or radian frequency, unit=rad/s) and k is the wavenumber,
wave period and wave length are $T = \frac{2\pi}{\sigma}$ and $L = \frac{2\pi}{k}$,
We can define the variable phase of the wave as $\theta = \sigma t - kx$
We define the phase speed or celerity to be the speed of propagation of phase in the direction of the wave vector $c = \frac{L}{T} = \frac{\sigma}{k}$.

[1]:
```
# %%capture

# # Install necessary packages and restart the kernel.
# !pip install netcdf4
# !pip install xarray
# !pip install numpy
# !pip install matplotlib
# !pip install ipympl
# # resetart kernel
# import IPython
# IPython.Application.instance().kernel.do_shutdown(True) #automatically
 ↪restarts kernel
```

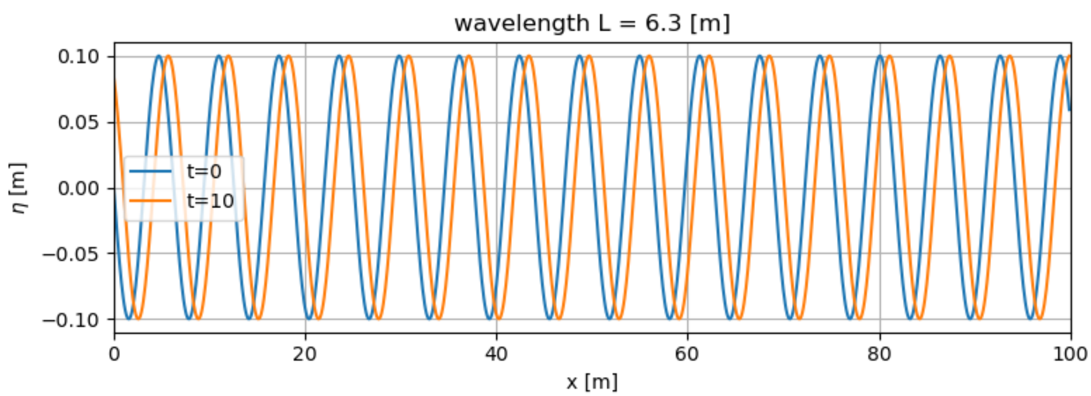
[2]:
```python
# # to enable the jupyter widgets so that you can plot interactive figures
# from google.colab import output
# output.enable_custom_widget_manager()
```

[1]:
```python
# import python libraries
import xarray as xr
import numpy as np
from matplotlib import cm
import matplotlib.pyplot as plt
%matplotlib widget

a = 0.1 # wave amplitude, unit: m
sigma = 0.1 #wave frequency, unit: rad/s, To convert from revolutions per␣
↪second to radians per second, multiply by 2
k = 1 # wavenumber, unit: rad/n

t = np.arange(0, 1000) # define time range, unit: s
x = np.arange(-100, 100, .1) # define space range, unit: m
eta = np.zeros((len(t), len(x)))

for i in range(len(t)):
    for j in range(len(x)):
        eta[i,j] = a*np.sin(sigma*t[i] - k*x[j]) # calculate surface elevation␣
↪for wave
```

[2]:
```python
# plot wave at x=x0; see temporal variation at one location
fig, ax = plt.subplots(nrows=1, ncols=1, figsize=(8,3))
ax.plot(t, eta[:,0], label = 'x=0')
# ax.plot(t, eta[:,2], label = 't=2')
ax.set_xlabel('t [s]')
ax.set_ylabel(r'$\eta$ [m]')
ax.set_xlim([0,100])
ax.legend()
ax.set_title(f'wave period T = {np.round(2*np.pi/sigma,1)} [s]') # wave period,␣
↪unit: s
ax.grid(True)
plt.tight_layout()
```



[3]:
```
# plot wave at t=t1, t=t2; see wave propagation
fig, ax = plt.subplots(nrows=1, ncols=1, figsize=(8,3))
ax.plot(x, eta[0,:], label = 't=0')
ax.plot(x, eta[10,:], label = 't=10')
ax.set_xlim([0,100])
ax.set_xlabel('x [m]')
ax.set_ylabel(r'$\eta$ [m]')
ax.legend()
ax.set_title(f'wavelength L = {np.round(2*np.pi/k,1)} [m]') # wave length, unit:
↪ m
ax.grid(True)
plt.tight_layout()
```



[4]:
```
# this is just a fancy moving version of the previous plot, no need to study␣
↪the code

from ipywidgets import AppLayout, FloatSlider
```

```
plt.ioff()

slider = FloatSlider(
    orientation='horizontal',
    description='t=',
    value=0,
    min=0,
    max=1000
)

slider.layout.margin = '0px 30% 0px 5%'
slider.layout.width = '60%'

fig = plt.figure(figsize=(8,3))
fig.canvas.header_visible = False
fig.canvas.layout.min_height = '400px'
plt.title(r'$\eta$=a*sin($\sigma$t-kx) at t={} s'.format(np.round(slider.
 ↪value,1)))
plt.grid(True)
plt.xlabel('x [m]')
plt.ylabel(r'$\eta$ [m]')
plt.xlim([0,100])
plt.ylim([-.12, .12])
plt.tight_layout()

lines = plt.plot(x, a*np.sin(sigma*slider.value - k*x))

def update_lines(change):
    plt.title(r'$\eta$=a*sin($\sigma$t-kx) at t={} s'.format(np.round(change.
 ↪new,1)))
    lines[0].set_data(x, a*np.sin(sigma*change.new - k*x))
    fig.canvas.draw()
    fig.canvas.flush_events()

slider.observe(update_lines, names='value')

AppLayout(
    center=fig.canvas,
    footer=slider,
    pane_heights=[0, 6, 1]
)
```

[4]:
```
AppLayout(children=(FloatSlider(value=0.0, description='t=',
layout=Layout(grid_area='footer', margin='0px 30%…
```

1.1.2 2) wave dispersion

Wave dispersion refers to the phenomenon where the phase velocity of a wave changes with its frequency or wavelength.

[5]:
```
# import python libraries
import numpy as np
from matplotlib import cm
import matplotlib.pyplot as plt
%matplotlib widget

a = 0.1 # wave amplitude, unit: m
sigma1 = 0.1 #wave frequency, unit: rad/s, To convert from revolutions per␣
↪second to radians per second, multiply by 2
k1 = 1 # wavenumber, unit: rad/n
dk = .2
dsigma = .05
k2 = k1 + dk #define wave number and wave frequency of wave 2
sigma2 = sigma1 + dsigma

t = np.arange(0, 1000) # define time range, unit: s
x = np.arange(-100, 100, 0.1) # define space range, unit: m
eta1 = np.zeros((len(t), len(x)))
eta2 = np.zeros((len(t), len(x)))

for i in range(len(t)):
    for j in range(len(x)):
        eta1[i,j] = a*np.sin(sigma1*t[i] - k1*x[j]) # calculate surface␣
↪elevation for wave1
        eta2[i,j] = a*np.sin(sigma2*t[i] - k2*x[j]) # calculate surface␣
↪elevation for wave2
```

[6]:
```
# compute and show wave period and phase speed for wave 1
T1 = 2*np.pi/sigma1
c1 = sigma1/k1
T1, c1
```

[6]: (62.83185307179586, 0.1)

[7]:
```
# compute and show wave period and phase speed for wave 2
T2 = 2*np.pi/sigma2
c2 = sigma2/k2
T2, c2
```

[7]: (41.8879020478639, 0.12500000000000003)

[8]:

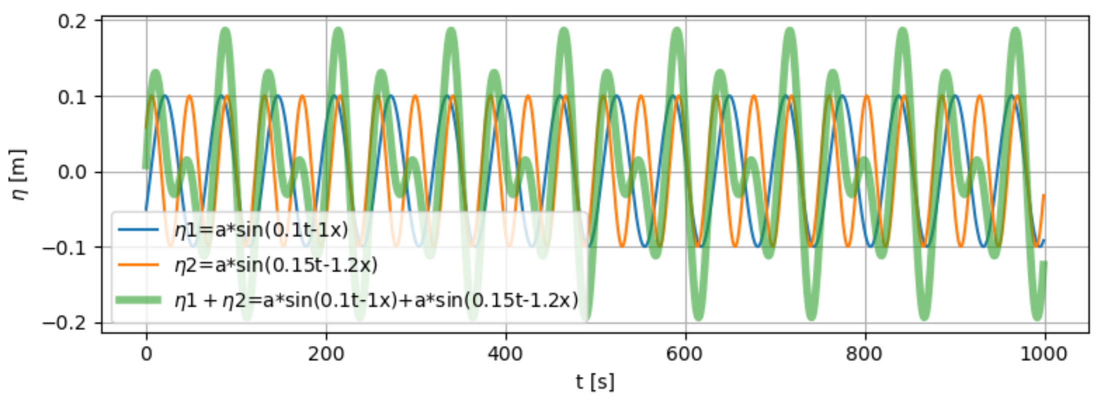
```python
# this is just a fancy moving version of the previous plot, no need to study
    ↪the code

from ipywidgets import AppLayout, FloatSlider

plt.ioff()

slider = FloatSlider(
    orientation='horizontal',
    description='t=',
    value=0,
    min=0,
    max=800
)

slider.layout.margin = '0px 30% 0px 5%'
slider.layout.width = '60%'

fig = plt.figure(figsize=(8,3))
fig.canvas.header_visible = False
fig.canvas.layout.min_height = '400px'
plt.title(r'$\eta1$ and $\eta2$ at t={} s'.format(np.round(slider.value,1)))
plt.grid(True)
plt.xlabel('x [m]')
plt.ylabel(r'$\eta$ [m]')
plt.xlim([0,100])
plt.ylim([-.12, .12])
plt.tight_layout()

# only show wave propagation from x=0, t=0
mask1 = np.where(x<=c1*slider.value)
mask2 = np.where(x<=c2*slider.value)
lines1 = plt.plot(x[mask1], a*np.sin(sigma1*slider.value - k1*x[mask1]))
lines2 = plt.plot(x[mask2], a*np.sin(sigma2*slider.value - k2*x[mask2]))

def update_lines(change):
    plt.title(r'$\eta1$ and $\eta2$ at t={} s'.format(np.round(change.new,1)))
    mask1 = np.where(x<=c1*change.new)
    mask2 = np.where(x<=c2*change.new)
    lines1[0].set_data(x[mask1], a*np.sin(sigma1*change.new - k1*x[mask1]))
    lines2[0].set_data(x[mask2], a*np.sin(sigma2*change.new - k2*x[mask2]))
    fig.canvas.draw()
    fig.canvas.flush_events()

slider.observe(update_lines, names='value')

AppLayout(
```

```
    center=fig.canvas,
    footer=slider,
    pane_heights=[0, 6, 1]
)
```

[8]: AppLayout(children=(FloatSlider(value=0.0, description='t=', layout=Layout(grid_area='footer', margin='0px 30%…

1.1.3 3) wave packet/envolope

[9]:
```
# plot wave at x=x0; see temporal variation at the same location for two waves␣
↪of different frequency and the "wave packet"
plt.ion()
fig, ax = plt.subplots(nrows=1, ncols=1, figsize=(8,3))
ax.plot(t, eta1[:,0], label = r'$\eta1$'+f'=a*sin({np.round(sigma1,2)}t-{k1}x)')
ax.plot(t, eta2[:,0], label = r'$\eta2$'+f'=a*sin({np.round(sigma2,2)}t-{k2}x)')
ax.plot(t, eta1[:,0]+ eta2[:,0], label = r'$\eta1+\eta2$'+f'=a*sin({np.
↪round(sigma1,2)}t-{k1}x)+a*sin({np.round(sigma2,2)}t-{k2}x)', linewidth = 4,␣
↪alpha = .6)
ax.set_xlabel('t [s]')
ax.set_ylabel(r'$\eta$ [m]')
ax.legend()
ax.grid(True)
plt.tight_layout()
```

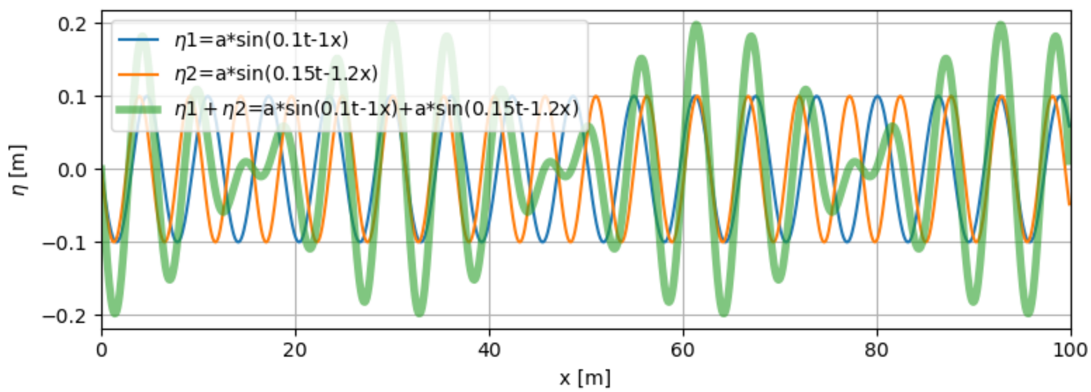


[10]:
```
dT = 2*np.pi/dsigma
dT
```

[10]: 125.66370614359172

[11]:
```
# plot wave at t=t0; see spatial variation at the same location for two waves␣
↪of different frequency and the "wave packet"
```

```
fig, ax = plt.subplots(nrows=1, ncols=1, figsize=(8,3))
ax.plot(x, eta1[0,:], label = r'$\eta1$'+f'=a*sin({np.round(sigma1,2)}t-{k1}x)')
ax.plot(x, eta2[0,:], label = r'$\eta2$'+f'=a*sin({np.round(sigma2,2)}t-{k2}x)')
ax.plot(x, eta1[0,:]+ eta2[0,:], label = r'$\eta1+\eta2$'+f'=a*sin({np.
↪round(sigma1,2)}t-{k1}x)+a*sin({np.round(sigma2,2)}t-{k2}x)', linewidth = 4,␣
↪alpha = .6)
ax.set_xlim([0,100])
ax.set_xlabel('x [m]')
ax.set_ylabel(r'$\eta$ [m]')
ax.legend()
ax.grid(True)
plt.tight_layout()
```



[12]:
```
dL = 2*np.pi/dk
dL
```

[12]: 31.41592653589793

1.1.4 4) group velocity vs phase velocity

While the phase velocity represents the propagation of phase (signal), the group velocity is associated with the speed of energy propagation.

a. $c_g > 0$, $c > 0$; $c_g \neq c$

[13]:
```
# compute group speed cg = dL/dT = dsigma/dk
cg = dsigma/dk
print(f"cg = {cg}, c1 = {c1}, c2 = {c2} [m/s]")
```

```
cg = 0.25, c1 = 0.1, c2 = 0.12500000000000003 [m/s]
```

[14]:
```
# this is just a fancy moving version of the previous plot, no need to study
↪the code

from ipywidgets import AppLayout, FloatSlider

plt.ioff()

slider = FloatSlider(
    orientation='horizontal',
    description='t=',
    value=0,
    min=0,
    max=800
)

slider.layout.margin = '0px 30% 0px 5%'
slider.layout.width = '60%'

fig = plt.figure(figsize=(8,3))
fig.canvas.header_visible = False
fig.canvas.layout.min_height = '400px'
plt.title(r'at t={} s'.format(np.round(slider.value,1)))
plt.grid(True)
plt.xlabel('x [m]')
plt.ylabel(r'$\eta$ [m]')
plt.xlim([0,100])
plt.ylim([-.22, .22])
plt.tight_layout()

# only show wave propagation from x=0, t=0
mask1 = np.where(x<=c1*slider.value)
mask2 = np.where(x<=c2*slider.value)
lines1 = plt.plot(x[mask1], a*np.sin(sigma1*slider.value - k1*x[mask1]),
↪label=r'$\eta1$')
lines2 = plt.plot(x[mask2], a*np.sin(sigma2*slider.value - k2*x[mask2]),
↪label=r'$\eta2$')
# wave group
mask = np.where(x<=cg*slider.value)
lines = plt.plot(x[mask], a*np.sin(sigma1*slider.value - k1*x[mask]) + a*np.
↪sin(sigma2*slider.value - k2*x[mask]), linewidth = 4, alpha = .6,
↪label=r'$\eta1+\eta2$')
plt.legend(loc='upper right')

def update_lines(change):
    plt.title(r'at t={} s'.format(np.round(change.new,1)))
    mask1 = np.where(x<=c1*change.new)
    mask2 = np.where(x<=c2*change.new)
```

```
    lines1[0].set_data(x[mask1], a*np.sin(sigma1*change.new - k1*x[mask1]))
    lines2[0].set_data(x[mask2], a*np.sin(sigma2*change.new - k2*x[mask2]))
    mask= np.where(x<=cg*change.new)
    lines[0].set_data(x[mask], a*np.sin(sigma1*change.new - k1*x[mask]) + a*np.
 ↪sin(sigma2*change.new - k2*x[mask]))
    fig.canvas.draw()
    fig.canvas.flush_events()

slider.observe(update_lines, names='value')

AppLayout(
    center=fig.canvas,
    footer=slider,
    pane_heights=[0, 6, 1]
)
```

[14]:
```
AppLayout(children=(FloatSlider(value=0.0, description='t=',
layout=Layout(grid_area='footer', margin='0px 30%…
```

b. $c_g > 0$, $c > 0$; $c_g = c$

[15]:
```
# In this example, we reset dsigma and dk so that c1 = c2
dk = .2
dsigma = .02 # instead of .05 in the previous example
k2 = k1 + dk #define wave number and wave frequency of wave 2
sigma2 = sigma1 + dsigma
c2 = sigma2/k2

# compute group speed cg = dL/dT = dsigma/dk
cg = dsigma/dk
print(f"cg = {cg}, c1 = {c1}, c2 = {c2} [m/s]")
```

```
cg = 0.09999999999999999, c1 = 0.1, c2 = 0.1 [m/s]
```

[16]:
```
# this is just a fancy moving version of the previous plot, no need to study␣
 ↪the code

from ipywidgets import AppLayout, FloatSlider

plt.ioff()

slider = FloatSlider(
    orientation='horizontal',
    description='t=',
    value=0,
    min=0,
    max=800
)
```

```
slider.layout.margin = '0px 30% 0px 5%'
slider.layout.width = '60%'

fig = plt.figure(figsize=(8,3))
fig.canvas.header_visible = False
fig.canvas.layout.min_height = '400px'
plt.title(r'at t={} s'.format(np.round(slider.value,1)))
plt.grid(True)
plt.xlabel('x [m]')
plt.ylabel(r'$\eta$ [m]')
plt.xlim([0,100])
plt.ylim([-.22, .22])
plt.tight_layout()

# only show wave propagation from x=0, t=0
mask1 = np.where(x<=c1*slider.value)
mask2 = np.where(x<=c2*slider.value)
lines1 = plt.plot(x[mask1], a*np.sin(sigma1*slider.value - k1*x[mask1]),␣
 ↪label=r'$\eta1$')
lines2 = plt.plot(x[mask2], a*np.sin(sigma2*slider.value - k2*x[mask2]),␣
 ↪label=r'$\eta2$')
# wave group
mask = np.where(x<=cg*slider.value)
lines = plt.plot(x[mask], a*np.sin(sigma1*slider.value - k1*x[mask]) + a*np.
 ↪sin(sigma2*slider.value - k2*x[mask]), linewidth = 4, alpha = .6,␣
 ↪label=r'$\eta1+\eta2$')
plt.legend(loc='upper right')

def update_lines(change):
    plt.title(r'at t={} s'.format(np.round(change.new,1)))
    mask1 = np.where(x<=c1*change.new)
    mask2 = np.where(x<=c2*change.new)
    lines1[0].set_data(x[mask1], a*np.sin(sigma1*change.new - k1*x[mask1]))
    lines2[0].set_data(x[mask2], a*np.sin(sigma2*change.new - k2*x[mask2]))
    mask= np.where(x<=cg*change.new)
    lines[0].set_data(x[mask], a*np.sin(sigma1*change.new - k1*x[mask]) + a*np.
 ↪sin(sigma2*change.new - k2*x[mask]))
    fig.canvas.draw()
    fig.canvas.flush_events()

slider.observe(update_lines, names='value')

AppLayout(
    center=fig.canvas,
    footer=slider,
    pane_heights=[0, 6, 1]
```

```
)
```

[16]:
```
AppLayout(children=(FloatSlider(value=0.0, description='t=',
layout=Layout(grid_area='footer', margin='0px 30%…
```

c. $c_g < 0$, $c > 0$; $c_g \neq c$

[17]:
```
# Let's look at another example, where we set dsigma<0, dk>0 and recalculate
 ↪eta2
dk = .2
dsigma = -.05 # instead of .05 in the previous example
k2 = k1 + dk #define wave number and wave frequency of wave 2
sigma2 = sigma1 + dsigma
c2 = sigma2/k2

# now let's compute group speed cg = dsigma/dk, note that this time cg<0, in
 ↪which direction will the wave packet move?
cg = dsigma/dk
print(f"cg = {cg}, c1 = {c1}, c2 = {c2} [m/s]")
```

```
cg = -0.25, c1 = 0.1, c2 = 0.041666666666666667 [m/s]
```

[18]:
```
# this is just a fancy moving version of the previous plot, no need to study
 ↪the code

from ipywidgets import AppLayout, FloatSlider

plt.ioff()

slider = FloatSlider(
    orientation='horizontal',
    description='t=',
    value=0,
    min=0,
    max=800
)

slider.layout.margin = '0px 30% 0px 5%'
slider.layout.width = '60%'

fig = plt.figure(figsize=(8,3))
fig.canvas.header_visible = False
fig.canvas.layout.min_height = '400px'
plt.title(r'at t={} s'.format(np.round(slider.value,1)))
plt.grid(True)
plt.xlabel('x [m]')
plt.ylabel(r'$\eta$ [m]')
plt.xlim([-100,100])
```

```python
plt.ylim([-.22, .22])
plt.tight_layout()

# only show wave propagation from x=0, t=0
mask1 = np.where(x<=c1*slider.value)
mask2 = np.where(x<=c2*slider.value)
lines1 = plt.plot(x[mask1], a*np.sin(sigma1*slider.value - k1*x[mask1]),␣
 ↪label=r'$\eta1$')
lines2 = plt.plot(x[mask2], a*np.sin(sigma2*slider.value - k2*x[mask2]),␣
 ↪label=r'$\eta2$')
# wave group
mask = np.where(x<=cg*slider.value)
lines = plt.plot(x[mask], a*np.sin(sigma1*slider.value - k1*x[mask]) + a*np.
 ↪sin(sigma2*slider.value - k2*x[mask]), linewidth = 4, alpha = .6,␣
 ↪label=r'$\eta1+\eta2$')
plt.legend(loc='upper right')

def update_lines(change):
    plt.title(r'at t={} s'.format(np.round(change.new,1)))
    mask1 = np.where(x<=c1*change.new)
    mask2 = np.where(x<=c2*change.new)
    lines1[0].set_data(x[mask1], a*np.sin(sigma1*change.new - k1*x[mask1]))
    lines2[0].set_data(x[mask2], a*np.sin(sigma2*change.new - k2*x[mask2]))
    mask= np.where(x<=cg*change.new)
    lines[0].set_data(x[mask], a*np.sin(sigma1*change.new - k1*x[mask]) + a*np.
 ↪sin(sigma2*change.new - k2*x[mask]))
    fig.canvas.draw()
    fig.canvas.flush_events()

slider.observe(update_lines, names='value')

AppLayout(
    center=fig.canvas,
    footer=slider,
    pane_heights=[0, 6, 1]
)
```

[18]:
```
AppLayout(children=(FloatSlider(value=0.0, description='t=',
layout=Layout(grid_area='footer', margin='0px 30%…
```

The wave packet doesn't have to move in the same direction as individual wave constituants! CRAZY!!!

1.1.5 5) dispersion relation

Dispersion relationship $\sigma(k)$ is essentially the relationship between wave frequency σ and wave number k.

$c = \sigma/k$ (phase speed) $c_g = \frac{\partial \sigma}{\partial k}$ (group speed)

Non-dispersive waves are waves where the phase speed of the wave is the same for all frequencies or wavelengths. In other words, the shape of a wave packet (a group of waves) doesn't change as it propagates through a non-dispersive medium. This means the group velocity (speed of the wave packet) is equal to the phase velocity (speed of individual waves within the packet): $>c = c_g$

If wave components *do not* have the same phase speed, then the waves are called **dispersive**. In this case, a pulse will break up. Very long waves and very short waves will form low amplitude trailing edges, and the bulk of the energy from the generation event will travel at a speed called the group speed.
$>c \neq c_g$

1.2 2. Wind-generated waves

Let's watch video of wind waves experienced by RV Knorr at Irminger Sea together!

Surface waves on the ocean, seas and lakes can be generated by several mechanisms, including the wind, earthquakes, the gravitational force of the sun and moon (tides) and pressure fluctuations. It is useful to identify the characteristics of waves generated by each of these mechanisms in terms of their characteristic period or frequency and relative energy and restoring force (Figure 4.2).

Wind-generated waves generally have the **shortest** period, ranging from < 0.25 seconds for capillary waves to a maximum of about 30 seconds for the longest swell waves in the open ocean.

Capillary waves are the little ripples, with a wave length <10 cm which can be seen on the surface as a gust of wind passes over a still body of water (Figure 4.3) and they can be found on the surface of much larger waves when the wind is strong and blowing continuously. They are affected by **surface tension** and quickly diminish if the wind dies down.

Figure 4.3 Winds blowing across a pond (toward the camera) leading to the generation of capillary waves close to the edge of the vegetation and the growth of these in a downwind direction. Near the upwind boundary there is a zone of calm water where wind forces are insufficient to overcome surface tension.

Waves with a period longer than about 0.25 s are too large to be greatly affected by surface tension and the main restoring force is **gravity**; hence they are termed ordinary gravity waves. In the area where waves are being actively generated by winds they are confused, irregular, short-crested, and often characterised by the appearance of white caps where there is local wave breaking. Such conditions are termed **sea** and they result from the fact that new waves are constantly being generated by the wind.

Waves continue to grow in height and period as the wind blows, giving rise to the superposition of waves with a wide range of period, height, and length. Conditions within the area of wave generation contrast with the much more regular, long-crested **swell waves** that propogate outward from the area of wave generation and may travel thousands of kilometres across the ocean.

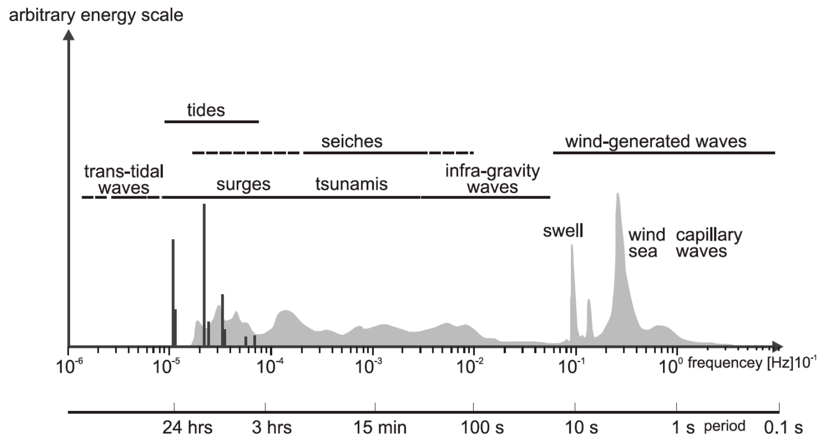


Figure 4.2 Types of wave motion in the oceans as a function of frequency and an indication of the relative ranking of energy associated with the each frequency grouping (Holthuijsen 2007 – after Munk, 1950).

1.3 3. Sea wave and swell

Wind-generated waves are important as energy-transfer agents-they first obtain their energy from the wind, then transfer it across the expanse of the ocean, and finally deliver it to the coastal zone where the energy can be the primary cause of erosion or may generate a variety of nearshore currents and sediment-transport patterns. This transfer of energy is depicted schematically in the below figure.

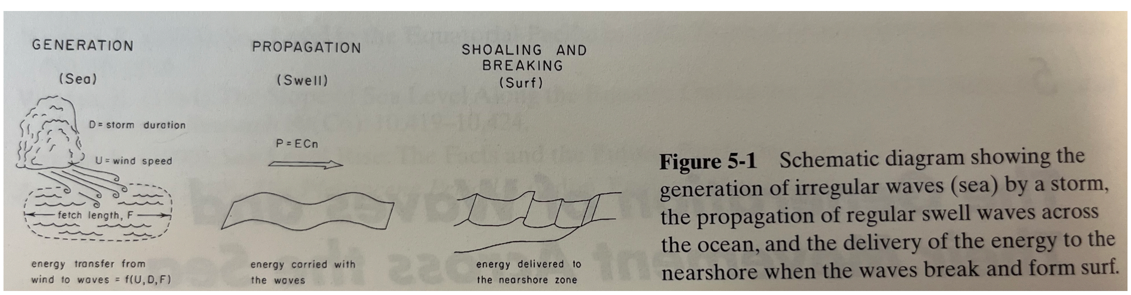


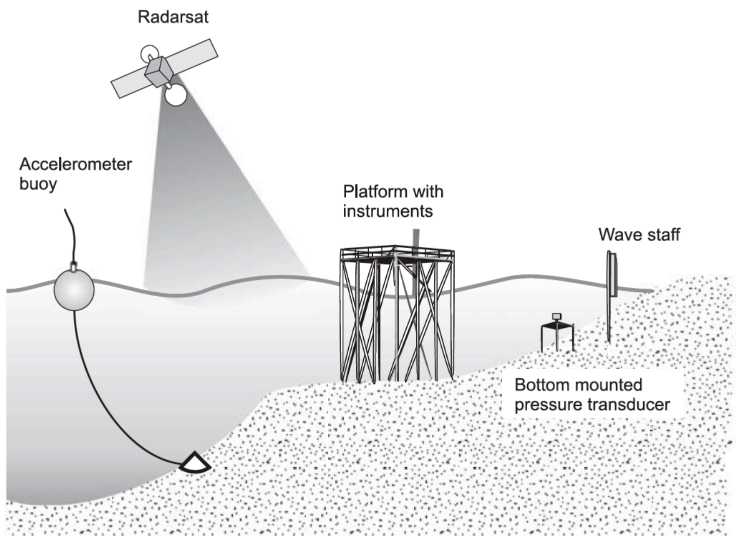
Figure 5-1 Schematic diagram showing the generation of irregular waves (sea) by a storm, the propagation of regular swell waves across the ocean, and the delivery of the energy to the nearshore when the waves break and form surf.

Sea waves generated in storm areas are highly complex. There are many different heights all at once, and waves seem to suddenly appear and then as abruptly disappear. This complex pattern results because a storm does not simply generate one set of waves but instead a whole spectrum of waves having a range of periods and heights. However, as the waves leave the storm area, they become more regular and develop into **swell**, having more uniform heights and distances between crests. This is due to the **dispersion** of waves. With this regularity, one can follow individual waves for considerable distances as they travel across the sea. It is the **swell** that transfers energy across the ocean and delivers it to the coastal zone where the waves finally break and expend their energy in the surf.

Check out local swell predictions here via surf map

1.4 4. Wave measurements and analysis

Figure 4.4 Schematic illustrating a variety of approaches to the measurement of waves.



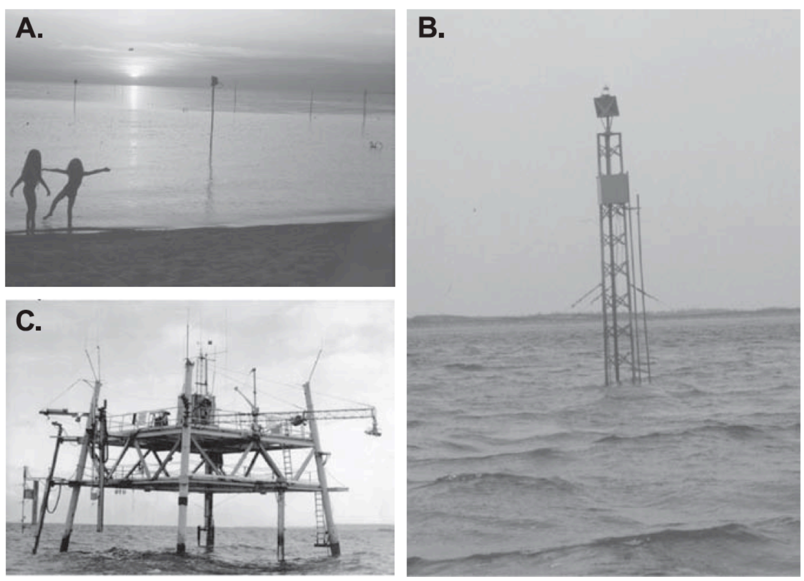


Figure 4.5 Photographs of instrumentation for measuring waves and wave generation: (A) resistance wave staffs deployed in Georgian Bay, Lake Huron as part of a study of wave transformation over nearshore bars (Davidson-Arnott and Randall, 1984); (B) mounted on a tower deployed in 6 m water depth (Greenwood and Davidson-Arnott, 1975; and (C) mounted on a platform as part of a study of wind wave generation and wave direction in Lake Ontario (Donelan *et al.*, 1985, 1996; photo courtesy Mark Donelan).

1.4.1 1) height distribution

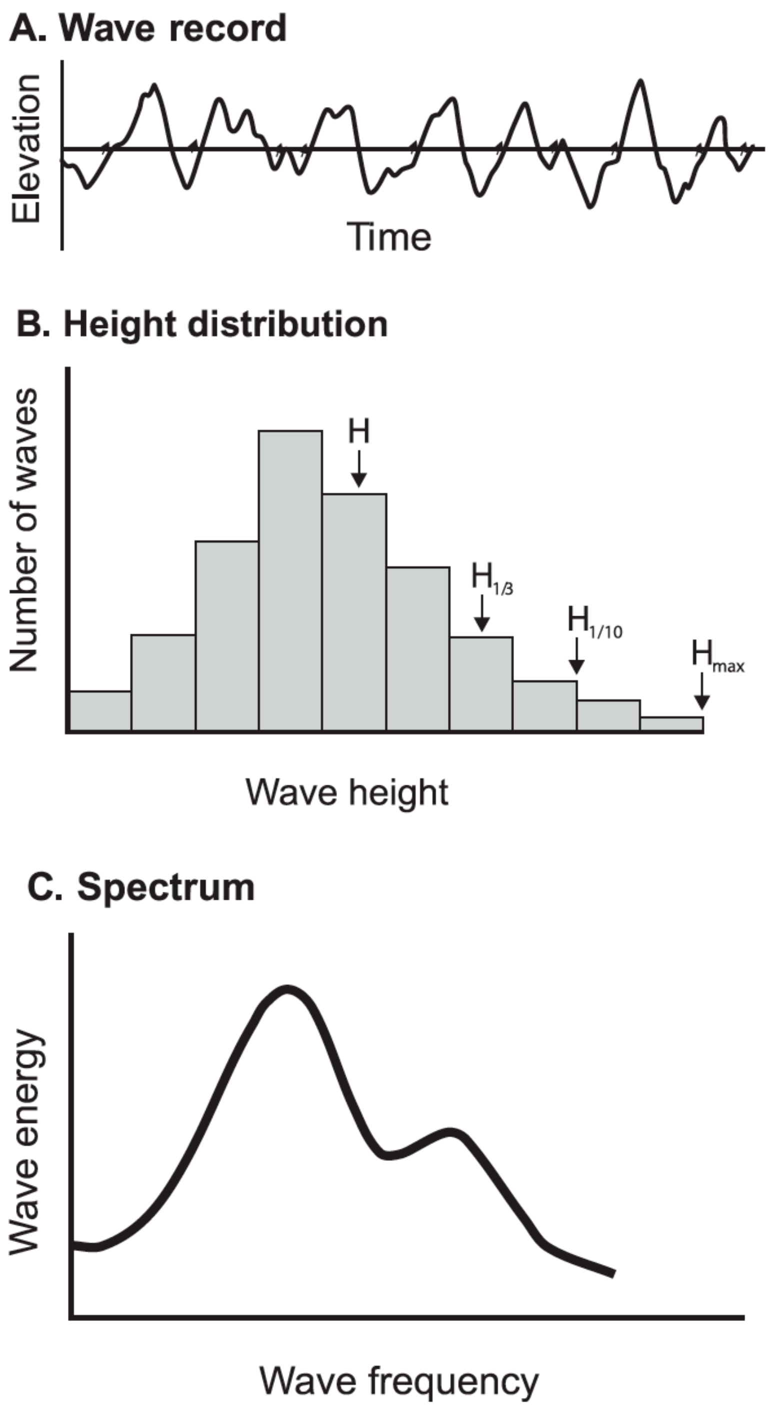


Figure 4.8 Description of a wave record: (A) graph of a portion of the record; (B) derivation of the statistical properties of the distribution of wave height H and period T; (C) the spectrum for a similar record relating wave energy (H^2) to wave frequency ($1/T$).

A commonly used statistical wave height is the **significant wave height**, defined as the average of the highest one third of the waves measured over a stated interval of time, usually 17 or 20 minutes (Thompson and Vincent, 1985). It is designated by H_s or $H_{1/3}$. The number of waves to be averaged is determined by dividing one-third of the time duration of wave observations by the significant wave period, in turn defined as the average period of the highest one-third of the waves determined from large, well-defined groups of waves. The use of significant-wave parameters was based on the impression that in many applications the larger waves are more “significant” than the small waves, and H_s, thereby provides a more representative measure of wave heights than, for example, the average wave height. It also has been shown that H_s, roughly corresponds to a visual estimate of a representative wave height in that the observer naturally tends to weight his observation toward the larger waves.

We use significant wave height a lot in navigation and to advise surfing! surf map

1.4.2 2) wave spectrum

The above approach to derive statistics for the wave heights and periods can be useful in many applications, but more generally an analysis technique is required that unravels the waves generated by different storms and better describes the complete distributions of wave energies and periods. Basically, the problem is to work backward from the complexity of measured waves to determine the simple components whose summation yields that complexity. The procedure by which this is done is known as harmonic or **spectral analysis**, based on the mathematics of Fourier, who, in 1807, showed that any curve can theoretically be broken down into a series of sine waves having different lengths and amplitudes.

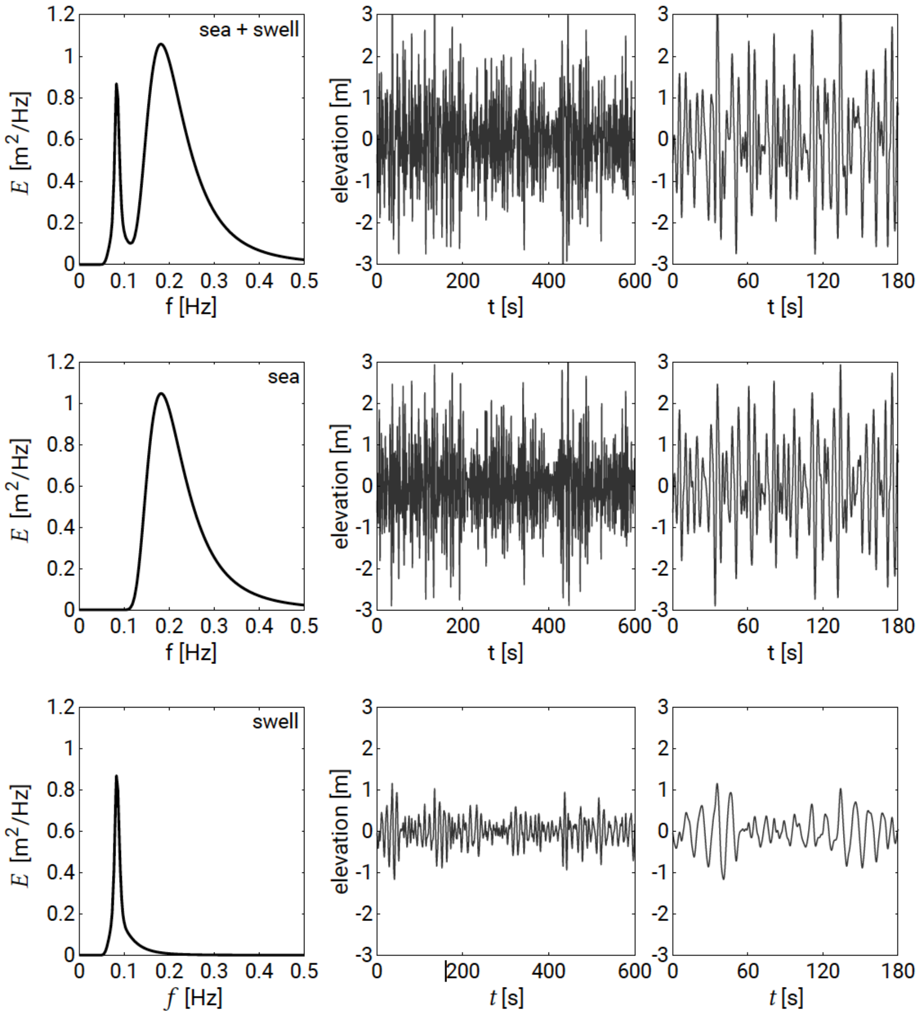


Figure 3.7: Surface elevation time series and corresponding spectra for sea and swell combined (upper panel), sea (middle panel) and swell (lower panel). To generate the sea component, a so-called JONSWAP spectrum (a typical spectrum for sea, see Sect. 3.5.1) is used with $H_S = 1.8$ m, $T_p = 5.5$ s and a peak enhancement factor $\gamma = 1$ (see eq. (6.3.15) in Holthuijsen, 2007). For the swell component, $H_S = 0.5$ m, $T_p = 12$ s and $\gamma = 5$ are used. For both sea and swell, the peak-width parameter is $\sigma = 0.07$ for $f \leq f_p$ and $\sigma = 0.09$ for $f > f_p$. The sea and swell spectra are combined in a bi-modal spectrum of swell and sea. The time series are generated from the spectra at a sampling frequency of 25 Hz assuming random phases. Note that the timeseries in the right panels correspond to Fig. 3.6.

Let's explore data from a wave buoy near Maui, Hawaii together and

Class Discussion: Hurricane Chacing! Discuss wave statistics and plot the wave spectrum using online tool.

See Hurricane Kiko forecast

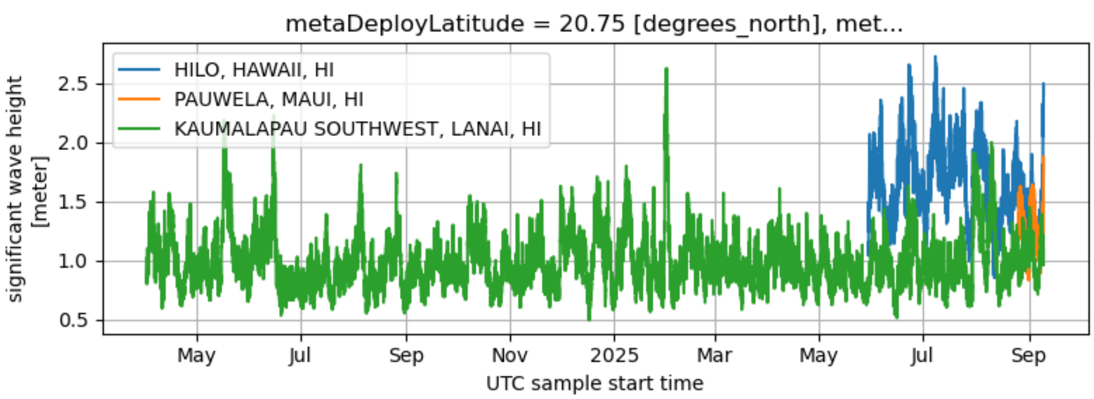
[19]:
```python
# load wave data (https://cdip.ucsd.edu/m/products/?stn=187p1)
opendap_url_hilo = 'http://thredds.cdip.ucsd.edu/thredds/dodsC/cdip/realtime/
↪188p1_rt.nc'
opendap_url_maui = 'http://thredds.cdip.ucsd.edu/thredds/dodsC/cdip/realtime/
↪187p1_rt.nc'
opendap_url_lanai = 'http://thredds.cdip.ucsd.edu/thredds/dodsC/cdip/realtime/
↪239p1_rt.nc'
ds_hilo = xr.open_dataset(opendap_url_hilo, engine='netcdf4')
ds_maui = xr.open_dataset(opendap_url_maui, engine='netcdf4')
ds_lanai = xr.open_dataset(opendap_url_lanai, engine='netcdf4')
ds_maui
```

[19]:
```
<xarray.Dataset> Size: 2MB
Dimensions:                     (sourceCount: 717, waveTime: 706,
                                 metaBoundsCount: 2, waveFrequency: 64,
                                 sstTime: 706, gpsTime: 706, dwrTime: 706)
Coordinates:
  * waveTime                    (waveTime) datetime64[ns] 6kB 2025-08-25T23:0…
  * waveFrequency               (waveFrequency) float32 256B 0.025 0.03 … 0.58
  * sstTime                     (sstTime) datetime64[ns] 6kB 2025-08-25T23:28…
  * gpsTime                     (gpsTime) datetime64[ns] 6kB 2025-08-25T23:21…
  * dwrTime                     (dwrTime) datetime64[ns] 6kB 2025-08-25T23:00…
    metaDeployLatitude          float32 4B …
    metaDeployLongitude         float32 4B …
Dimensions without coordinates: sourceCount, metaBoundsCount
Data variables: (12/51)
    sourceFilename              (sourceCount) object 6kB …
    waveTimeBounds              (waveTime, metaBoundsCount) datetime64[ns] 11kB
…
    waveFlagPrimary             (waveTime) float32 3kB …
    waveFlagSecondary           (waveTime) float32 3kB …
    waveHs                      (waveTime) float32 3kB …
    waveTp                      (waveTime) float32 3kB …
    …                            …
    waveA2Value                 (waveTime, waveFrequency) float32 181kB …
    waveB2Value                 (waveTime, waveFrequency) float32 181kB …
    waveCheckFactor             (waveTime, waveFrequency) float32 181kB …
    waveSpread                  (waveTime, waveFrequency) float32 181kB …
    waveM2Value                 (waveTime, waveFrequency) float32 181kB …
    waveN2Value                 (waveTime, waveFrequency) float32 181kB …
Attributes: (12/68)
    naming_authority:                edu.ucsd.cdip
    keywords_vocabulary:             Global Change Master Directory (GCMD) Ea…
    date_created:                    2025-09-09T15:59:53Z
    date_issued:                     2025-09-09T15:59:53Z
    date_modified:                   2025-09-09T15:59:53Z
    creator_name:                    Coastal Data Information Program, SIO/UCSD
```

```
…                                    …
instrument:                          metaInstrumentation
platform:                            wave_buoy
platform_vocabulary:                 http://mmisw.org/ont/ioos/platform
platform_name:                       PAUWELA, MAUI, HI BUOY - 187p1
DODS.strlen:                         0
DODS.dimName:                        metaStationNameLength
```

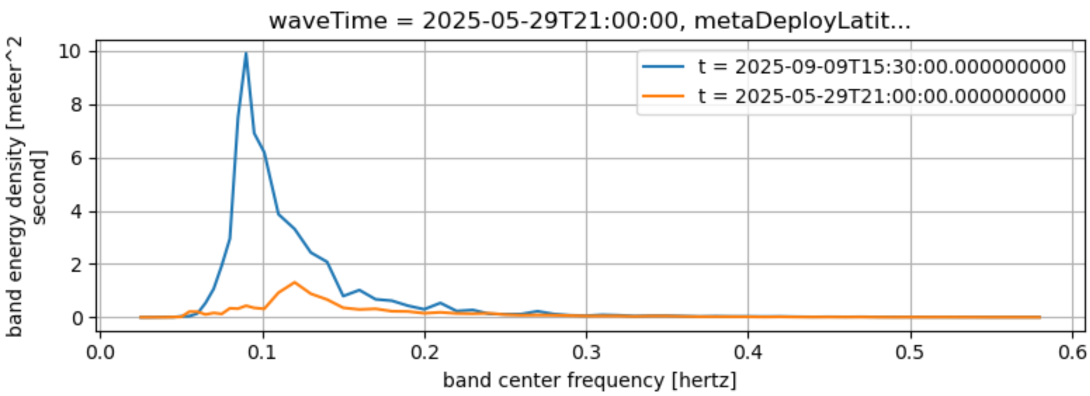
[20]:

```
# plot significant wave height
fig, ax = plt.subplots(nrows=1, ncols=1, figsize=(8,3))
ds_hilo.waveHs.plot(ax=ax, label = 'HILO, HAWAII, HI')
ds_maui.waveHs.plot(ax=ax, label = 'PAUWELA, MAUI, HI')
ds_lanai.waveHs.plot(ax=ax, label = 'KAUMALAPAU SOUTHWEST, LANAI, HI')
ax.grid(True)
ax.legend()
plt.tight_layout()
plt.show()
```



[21]:

```
# plot wave spectrum of Hilo at last time step
fig, ax = plt.subplots(nrows=1, ncols=1, figsize=(8,3))
ds_hilo.waveEnergyDensity[-1,:].plot(ax=ax, label = f"t = {ds_hilo.waveTime[-1].
  ↪data}")
ds_hilo.waveEnergyDensity[0,:].plot(ax=ax, label = f"t = {ds_hilo.waveTime[0].
  ↪data}")
ax.legend()
ax.grid(True)
plt.tight_layout()
plt.show()
```



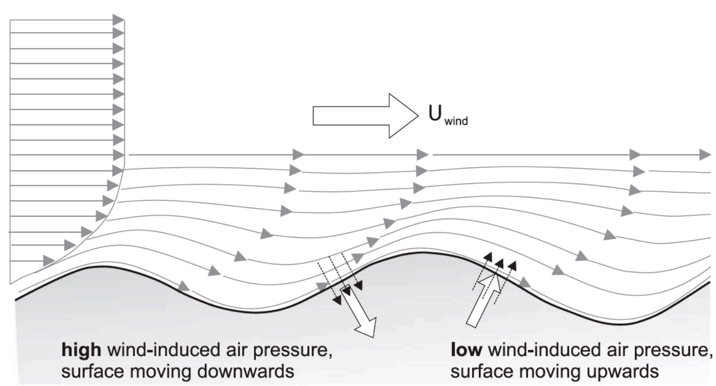
1.5 5. Wave generation and prediction

1.5.1 1) wave generation

The actual mechanism of wave generation by wind is highly complex and still poorly understood. At present, the generally accepted theory to account for the growth of waves is the combined Miles-Phillips mechanism that attributes the wave growth to the resonant interaction between the growing waves and the **pressure fluctuations** associated with air movement across them.

Air pressure is at a maximum on the windward side of the crest and a minimum on the leeward slope which thus reinforces the upward movement as the crest approaches and the downward movement after it has passed. The result is a transfer of energy from the wind to the wave (Figure 4.11).

Figure 4.11 The wave-induced wind-pressure variation over a propagating harmonic wave (Holthuijsen, 2007).



For a given wind speed, the wind wave spectrum evolved to the point where it became invariable ("fully developed sea" - the rate of energy transfer from the wind to the water surface balanced by the rate of energy dissipation). The **energy in the spectrum increases** and the **peak frequency shifts towards lower frequencies as the distance over which a constant wind blows increases** (Figure 5-17, developing sea with increasing fetch and time). The total **energy in the spectrum increased** with increasing wind speed and the **peak frequency shifted toward the lower frequency** (Figure 4.12b), longer period waves for a fully developed sea.

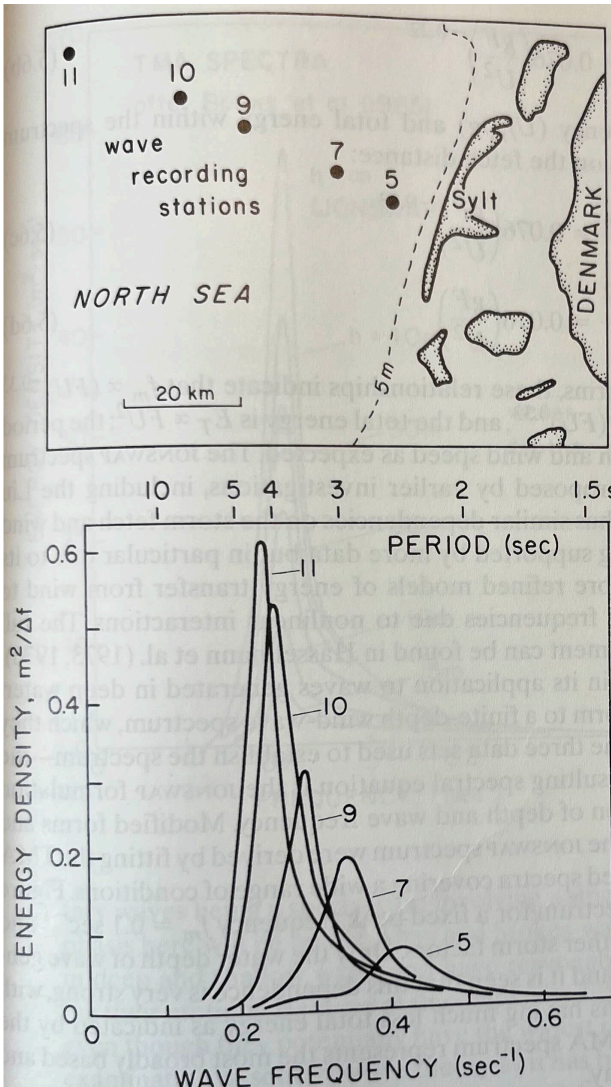


Figure 5-17 A series of spectra of waves measured at recording stations offshore from the island of Sylt on the North Sea coast of Denmark during the JONSWAP experiments. At the time of measurement, the winds were directed from land toward the offshore, resulting in the growth of the spectra from stations 5 through 11. [From K. Hasselmann, et al., Measurements of Wind-wave Growth and Swell Delay During the Joint North Sea Wave Project, *Deutsche Hydrographische Zeitschrift*, Supplement A8, 1973.]

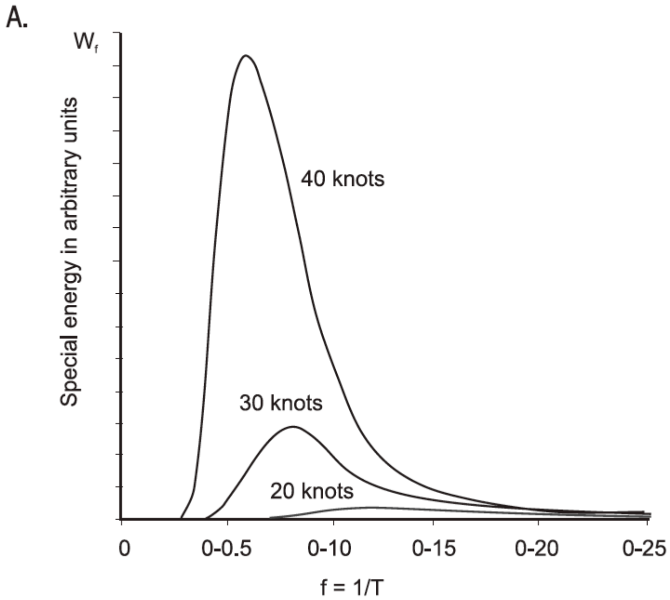


Figure 4.12 Spectral evolution: (A) wave spectra for a fully aroused sea for winds of 20, 30 and 40 knots; (B) evolution of wave spectra for constant wind speed with distance offshore (km) along a profile in the North Sea as part of the JONSWAP experiments (Hasselman *et al.*, 1973).

1.5.2 2) wave prediction

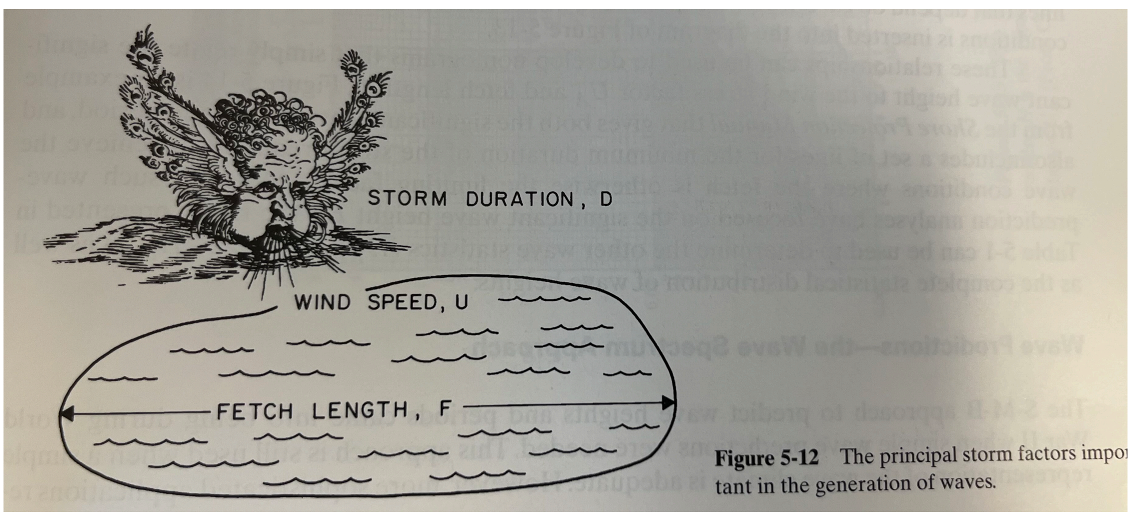


Figure 5-12 The principal storm factors impor tant in the generation of waves.

Similarly, the total **energy in the spectrum increased** with increasing fetch and the **peak frequency shifted toward the lower frequency** (Figure 5-16), longer period waves for a fully developed sea.

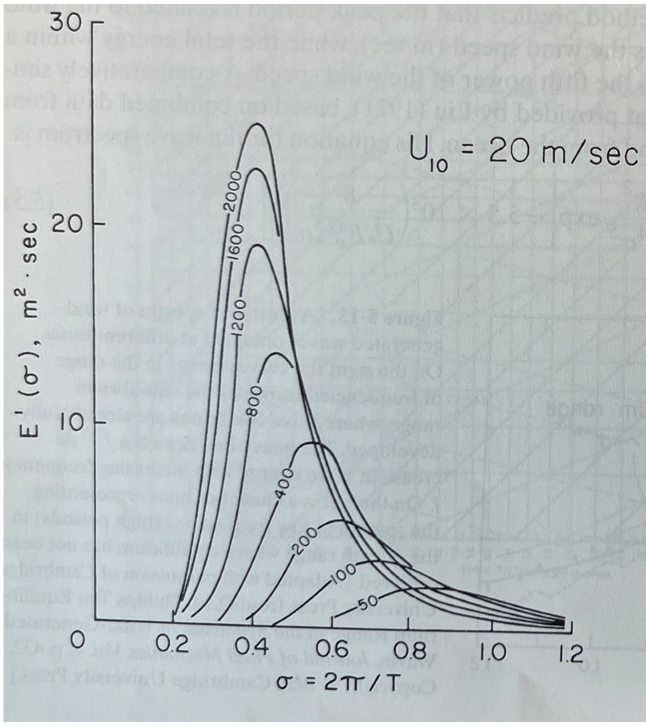
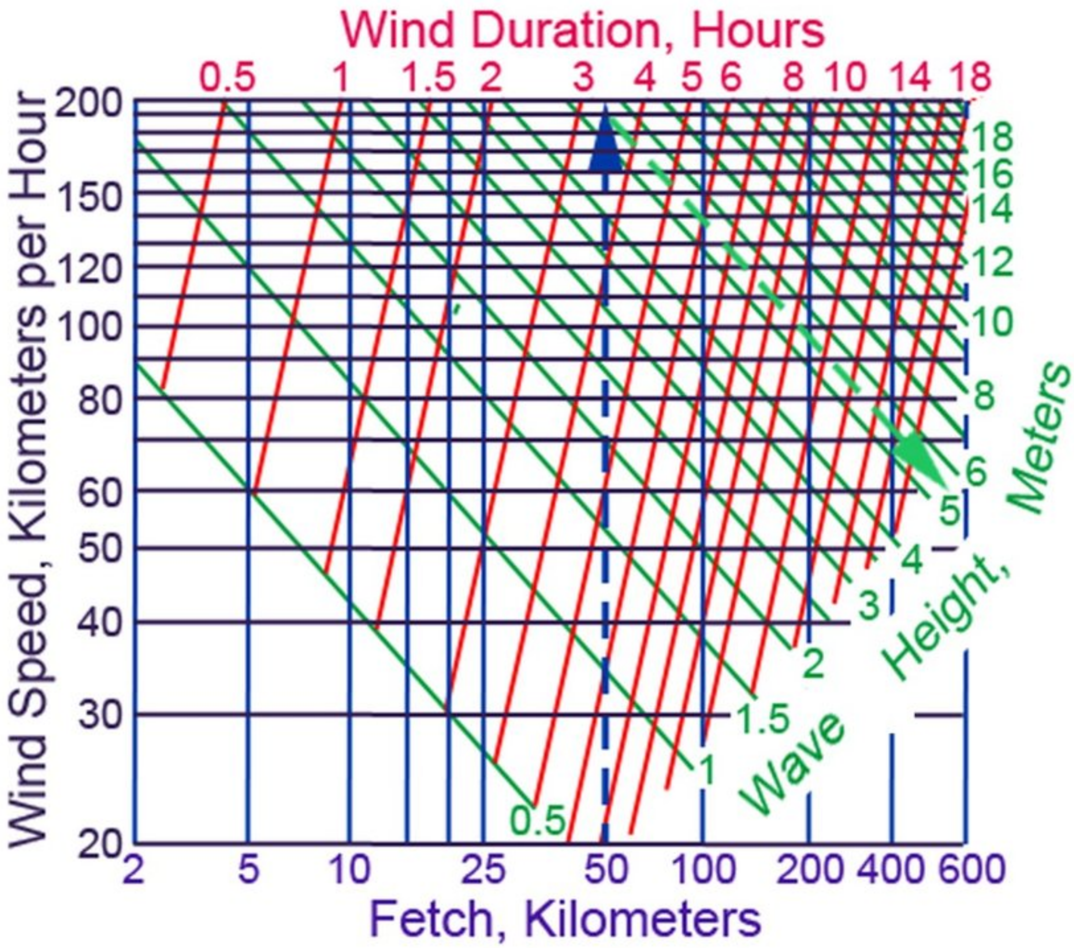


Figure 5-16 The fetch-limited spectra of Liu (1971) calculated with Equation (5.5) for a wind speed of 20 m/sec and for a series of fetch lengths in kilometers.

The Sverdrup-Munk-Bretschneider (S-M-B) nomogram The S-M-B method predicts the significant wave height H_S, and significant wave period T_S, from the storm conditions: wind velocity U, fetch distance F, and storm duration D.



Rodolfo, Kelvin. (2014). On the geological hazards that threaten existing and proposed reclamations of Manila Bay. Philippine Science Lertters. 7. 228-240.

[ ]: