Char 1 Def. Causal effect

1.1 individual causal effect

- individual c.e   ① outcome of interest   ② action $a=1$ & $a=0$ ③ counterfactual outcome Y

  A has a causal effect on an individual's outcome Y if $Y^{a=1} \neq Y^{a=0}$ for the individual.

  $\hookrightarrow$ A has a c.e on outcome cuz $\underbrace{Y^{a=1}}_{\downarrow} \neq \underbrace{Y^{a=0}}_{\downarrow}$.

                     potential outcomes

- consistency $A_i = a \Rightarrow Y_i^a = Y_i^A = Y_i$ 个体间完全不互相影响 （理想状态）

1.2 average c.e

- average c.e （统合）   ① outcome of interest   ② action $a=1$ & $a=0$
                           ③ 明确定义in 个体群体. $Y^{a=0}$ & $Y^{a=1}$

  An average c.e of treatment A on outcome Y is present if $Pr[Y^{a=1}=1] \neq Pr[Y^{a=0}=1]$
  in the population of interest.

  average c.e in population : $E[Y^{a=1}] \neq E[Y^{a=0}]$

1.3. measures

- effect measures   (represent causal null)

  i) c. risk difference: $Pr[Y^{a=1}=1] - Pr[Y^{a=0}=1] = 0$ $\begin{cases} >0 & 导致 outcome \\ <0 & 不导致 outcome \end{cases}$

  ii) risk ratio : $\dfrac{Pr[Y^{a=1}=1]}{Pr[Y^{a=0}=1]} = 1$

  iii) odds ratio : $\dfrac{Pr[Y^{a=1}=1] / Pr[Y^{a=1}=0]}{Pr[Y^{a=0}=1] / Pr[Y^{a=0}=0]} = 1$

- # needed to treat (NNT)

  $$NNT = \frac{-1}{Pr[Y^{a=1}=1] - Pr[Y^{a=0}=1]}$$

  NNT↑ → 更不易导致 outcome

1.4 Random activity

- consistent estimator of $Pr[Y^a=1]$: $\widehat{Pr}[Y^a=1]$ (根据 sample 求出)

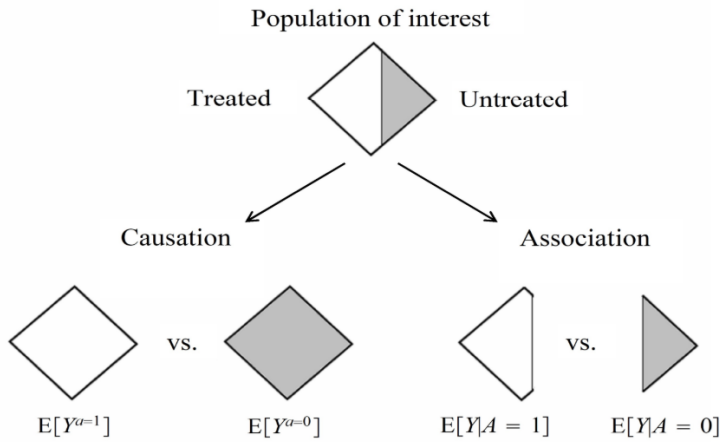   最有可能in情况 aka m.l.e

1.5 Causation versus association

- independence $Y \perp\!\!\!\perp A$ (既不是 causation, 也不是 association)

i) $Pr[Y=1 \mid A=1] - Pr[Y=1 \mid A=0] = 0$

ii) $\dfrac{Pr[Y=1 \mid A=1]}{Pr[Y=1 \mid A=0]} = 1$

iii) $\dfrac{Pr[Y=1 \mid A=1] / Pr[Y=0 \mid A=1]}{Pr[Y=1 \mid A=0] / Pr[Y=0 \mid A=0]} = 1$

Population of interest

Treated   Untreated

Causation   Association

vs.   vs.

$E[Y^{a=1}]$ $E[Y^{a=0}]$ $E[Y|A = 1]$ $E[Y|A = 0]$

- Causation   导致   夏天 ⇒ icecream 售卖量 & child 溺水
- association   相关联   夏天 icecream 售卖量 & child 溺水

Char 2 Randomized Experiments   计算 risk ratio

2.1 Randomization

- ideal randomized experiment   理想的随机事件

  no loss to follow-up

- exchangeability   (exogeneity)

  the risk under the potential treatment <u>value a</u> among the treated = ~ untreated

  $Pr[Y^a=1|A=1] = Pr[Y^a=1|A=1]$ for both a=0 & a=1

- exchangeability not hold ✗ randomized experiment

  1. 规模太小
  2. a study is a randomized experiment even if exchangeability not hold in ∞ samples.
     研究者需要不止一枚硬币来做随机实验

2.2 Conditional Randomization

- conditionally randomized experiments

  we used <u>several</u> randomization probability that <u>depend</u> <u>on the values of variable.</u>

  $Pr[Y^a=1|A=1, L=1] = Pr[Y^a=1|A=0, L=1]$ / $\underline{Y^a \perp\!\!\!\perp A | L=1}$ for all a.

  Y^a & A are indep. given L=1

- marginally randomized experiments

  we used a <u>single</u> <u>unconditional</u> (marginal) randomization probability that is common to
  all individuals   病人情况危急 treatmet? died?

  $Pr[Y^a=1|A=1] = Pr[Y^a=1|A=0]$ / $Y^a \perp\!\!\!\perp A$ for all a

2.3 Standardization   假 bias 更小

$$\frac{Pr[Y^{a=1}=1]}{Pr[Y^{a=0}=1]}$$ can be compute by standardization :

$$\frac{\sum_l \Pr[Y=1|L=l, A=1]\Pr[L=l]}{\sum_l \Pr[Y=1|L=l, A=0]\Pr[L=l]}.$$

2.4 Inverse probability weighting   ⟶ 类似

找 causal risk ratio in not randomized study

no bias

Table 2.2	L	A	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	0	1	0
Poseidon	0	1	0
Hera	0	1	0
Zeus	0	1	1
Artemis	1	0	1
Apollo	1	0	1
Leto	1	0	0
Ares	1	1	1
Athena	1	1	1
Hephaestus	1	1	1
Aphrodite	1	1	1
Polyphemus	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

Chap 3   Observational Studies

observational study 存在 bias $\implies$ conditional randomized experiment
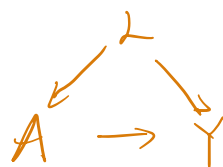
假设: 1. exchangebility   不管做什么选择, 不影响 potential outcome
      预测值: potential outcome 与真实人的选择无关

   2. positivity (3.3)   treatment 的选择都有一定有可能的
      $Pr[A=a \mid L=l] > 0$ $\forall l$. $Pr[L=l] \neq 0$

   3. consistency   不管做什么选择, 最终结果不变
      ① (3.4) 每个人其中一个 potential outcome $\implies$ $Y^{A=0}$ 与 $Y^{A=1}$

$$L$$
$$\swarrow \quad \searrow$$
$$A \longrightarrow Y$$

目的: 尽量消除 L 对 A & Y 的影响,
      直接研究 A 对 Y 的影响