

知识点

一. 无理数

- modulus $r = |z| = |x+iy| = \sqrt{x^2+y^2}$

- argument

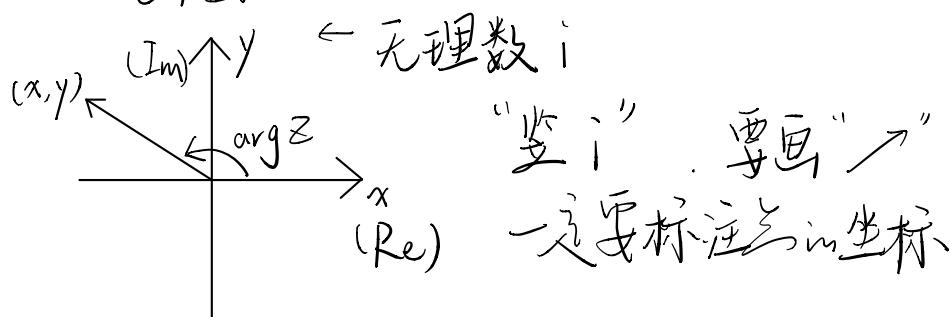
$$z = x+iy \quad \arg z = \tan^{-1}\left(\frac{y}{x}\right) \quad (-\pi < \theta < \pi)$$

若x或y为负数，则要画图解决

- conjugate pair \bar{z}^*

"i" 前系数得互颠倒

- 画图



- 已知根式 equation

和: $\alpha + \beta$ 积: $\alpha\beta$

$$\alpha^2 + (\alpha+\beta)\alpha + \alpha\beta = 0$$

$$\alpha^3 + \beta^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$$

$$\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{\alpha^3\beta^3}$$

答完题回头检查一下
是否按要求做答。

矩阵

$$R_1 \xrightarrow{PQ} R_2$$

$$\det PQ = \frac{A_{R_2}}{A_{R_1}}$$

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$$

$a+bi$ 在数轴上表示为 (a, b)
不用带 i

$f'(x)$ 计算

计算高加平方 & 计算梯形

"exact number"

计算机高敲错

计算解方程

巧妙运用平方差公式

$$\text{e.g. } \frac{12}{P} - 12P^3$$

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二. 找根

- IB interval bisection

$$a \ f(a) \quad b \ f(b) \quad \frac{a+b}{2} \ f\left(\frac{a+b}{2}\right)$$

- LI linear interpolation

$$\frac{a}{b} = \frac{c}{d}$$

- NR

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

三. 参数方程

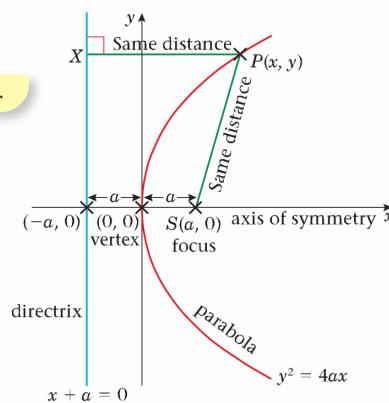
- This curve is symmetrical about the x-axis.

- A general point P on this curve has coordinates $P(x, y)$ or $P(at^2, 2at)$.

A locus of points is a set of points which obey a certain rule.

- A parabola is the **locus of points** where every point $P(x, y)$ on the parabola is the same distance from a fixed point S , called the focus, and a fixed straight line called the directrix.

- The parabola is the set of points where $SP = PX$.
The **focus**, S , has coordinates $(a, 0)$.
The **directrix** has equation $x + a = 0$.
The **vertex** is at the point $(0, 0)$.



- The curve opposite is an example of a **rectangular hyperbola** which has parametric equations:

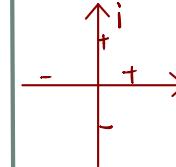
$$x = ct, y = \frac{c}{t}, t \in \mathbb{R}, t \neq 0$$

where c is a positive constant.

- The Cartesian equation of this curve is $xy = c^2$, where c is a positive constant.

- The curve has asymptotes with equations $x = 0$ (the y-axis) and $y = 0$ (the x-axis).

- A general point P on this curve has coordinates $P(x, y)$ or $P(ct^2, \frac{c}{t})$.



数轴 +/- 注意

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$$\sum_{r=1}^k r^r = \frac{n(n^k - 1)}{n - 1}$$

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数字畅销

4

证明方法不熟

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计算解方程

4

计算级数

5

计算数列

6

$$y^2 = 4ax \text{ in directrix } x = -a$$

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1. 矩阵变换中 m 带 in 正确

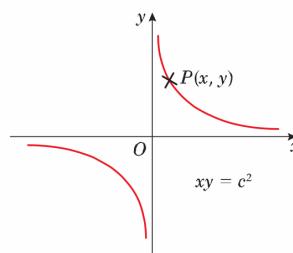
应用示意图解决

2. 利用 180°

$$\sin 75^\circ = \sin 105^\circ$$

$$\cos 75^\circ = -\cos 105^\circ$$

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四. 矩阵

• 运算

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$\neq 0$ non-singular

$$1. \det A = ad - bc \quad \leftarrow \neq 0 \text{ singular}$$

2. 乘法不符合交换律

$$3. \text{self-inverse 逆} \quad A^{-1} = A$$

• 变换

1. 旋转 rotation of n° (anti)clockwise about $(0,0)$

$$M = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

$$\cos(\pi - \theta) = \cos \theta$$

$$\sin(\pi - \theta) = \sin \theta$$

4. M^n 问题, 360° in 整数倍 $\div \theta = n$

2. 放大 enlargement of scale factor k and centre $(0,0)$

• 单方向的伸缩变换

1. a stretch, scale factor k , parallel to x -axis $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$

2. a stretch, scale factor k , from x -axis $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$

3. 对称 reflection

in y -axis $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

画图辅助理解

in x -axis $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

in $y=x$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

in $y=-x$ $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

4. self-inverse

$$T^{-1} = T$$

5. transformation T maps $\begin{pmatrix} a \\ b \end{pmatrix}$ to itself

$$T \times \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

• 面积 $T \xrightarrow{\text{B}} T' \quad BT = T'$

$$S_T = S_{T'} \div |\det T| \quad \text{绝对值}$$

竖 1

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计算问题. 例 3

3

图像变换表述—放大
enlargement with scale
factor 2, centre $(0,0)$

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证明总结表述

Assume that T_0 is $U_k \cdot U_{k+1}$
总结 U_n is true when $U_k \cdot U_{k+1}$

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$$A + B = -\frac{b}{a}$$

得出 a & b 的解时, 注意负号

5

无理数计算

未画图辅助理解

6

$$\left(\begin{matrix} a & c \\ b & d \end{matrix} \right)^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$$

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matrix parallel to y -axis

仅 x 轴数变化

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五. 数列

$$\sum_{r=0}^n r^0 = n$$

$$\sum_{r=1}^n r = \frac{n}{2} (n+1)$$

$$\sum_{r=1}^n r^2 = \frac{n}{6} (n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{n^2}{4} (n+1)^2$$

$$\sum_{r=1}^k n^r = \frac{n(n^k-1)}{n-1}$$

六. 数学归纳法

证明总结表述最后一句

Then it's true for all $n \in \mathbb{Z}^+$

未知数

计算步骤

2

证明题最后之题

3

$$\sum_{r=k}^n U_r = \sum_{r=1}^n U_r - \sum_{r=1}^{k-1} U_r$$

$$\text{e.g. } \sum_{r=3}^{49} 2 = \sum_{r=1}^{49} 2 - \sum_{r=1}^2 2$$

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$$\cos 75^\circ = -\cos 105^\circ$$

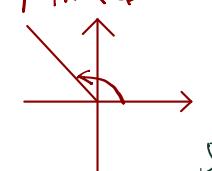
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计算移项变号

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$\arg z$ 计算中，坐标中有负数

要画数轴解决



5

证明 x 为 $f(x)$ 的根 to 3 decimal

$$f(x+0.005) > 0 \quad f(x-0.005) < 0$$

6

matrix 变换: parallel to x 轴

只有 x 轴变换

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