Binomial Generalized Linear Model

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Description

A generalized linear model for binomially distributed data.

Implementation

The file binomial.glm.sim.R simulates data according to the model statement presented below, and binomial.glm.mcmc.R contains the MCMC algorithm for model fitting.

Model statement

Let z_i be the number of "successes" (i.e., z_i are integers greater than or equal to 0) out of N_i "trials" during event i, for $i = 1, \ldots, n$. Also let $\mathbf{x}_i$ be a vector of covariates associated with z_i for which inference is desired, and the vector $\boldsymbol{\beta}$ be the corresponding coefficients.

$$
\begin{aligned}
z_i &\sim \text{Binom}\left(N_i, p_i\right) \\
\text{logit}\left(p_i\right) &= \mathbf{x}_i' \boldsymbol{\beta} \\
\boldsymbol{\beta} &\sim \mathcal{N}(\mathbf{0}, \sigma_\beta^2 \mathbf{I})
\end{aligned}
$$

Full conditional distributions

Regression coefficients ($\boldsymbol{\beta}$):

$$
\begin{aligned}
[\boldsymbol{\beta}|\cdot] &\propto \prod_{i=1}^{n} [p_i|\boldsymbol{\beta}]\,[\boldsymbol{\beta}] \\
&\propto \prod_{i=1}^{n} \text{Binom}(z_i \mid \mathbf{x}_i' \boldsymbol{\beta}) \mathcal{N}(\boldsymbol{\beta}|\mathbf{0}, \sigma_\beta^2 \mathbf{I}).
\end{aligned}
$$

The update for $\boldsymbol{\beta}$ proceeds using Metropolis-Hastings.