Negative Binomial Generalized Linear Model for Count Data

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Description

A generarlized linear model for count data.

Implementation

The file nb.glm.sim.R simulates data according to the model statement presented below, and nb.glm.mcmc.R contains the MCMC algorithm for model fitting.

Model statement

Let z_i, for $i = 1, \ldots, n$, be observed count data (i.e., z_i are integers greater than or equal to 0). Also let $\mathbf{x}_i$ be a vector of covariates associated with z_i for which inference is desired, and the vector $\boldsymbol{\beta}$ be the corresponding coefficients.

$$
\begin{aligned}
z_i &\sim \text{NB}\left(\lambda_i, \alpha\right) \\
\log\left(\lambda_i\right) &= \mathbf{x}_i' \boldsymbol{\beta} \\
\boldsymbol{\beta} &\sim \mathcal{N}(\mathbf{0}, \sigma_\beta^2 \mathbf{I}) \\
\alpha &\sim \text{Gamma}(a, b),
\end{aligned}
$$

where $\text{E}\left[z_i\right] = \lambda_i$ and $\text{Var}\left[z_i\right] = \lambda_i + \frac{\lambda_i^2}{\alpha}$.

Full conditional distributions

Regression coefficients ($\boldsymbol{\beta}$):

$$
\begin{aligned}
\left[\boldsymbol{\beta} \mid \cdot\right] &\propto \prod_{i=1}^{n} \left[z_i \mid \boldsymbol{\beta}, \alpha\right] \left[\boldsymbol{\beta}\right] \\
&\propto \prod_{i=1}^{n} \text{NB}\left(z_i \mid \mathbf{x}_i' \boldsymbol{\beta}, \alpha\right) \mathcal{N}\left(\boldsymbol{\beta} \mid \mathbf{0}, \sigma_\beta^2 \mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\beta}$ proceeds using Metropolis-Hastings.

Dispersion (i.e., size) parameter (α):

$$
\begin{aligned}
\left[\alpha \mid \cdot\right] &\propto \prod_{i=1}^{n} \left[z_i | \boldsymbol{\beta}, \alpha\right] \left[\alpha\right] \\
&\propto \prod_{i=1}^{n} \text{NB}\left(z_i \mid \mathbf{x}_i' \boldsymbol{\beta}, \alpha\right) \text{Gamma}\left(\alpha \mid a, b\right).
\end{aligned}
$$

The update for α proceeds using Metropolis-Hastings.