THE SPIN TEMPERATURE OF INTERGALACTIC NEUTRAL HYDROGEN

George B. Field
Princeton University Observatory
Received July 30, 1958; revised December 5, 1958

ABSTRACT

Three mechanisms (collisions, radio-frequency radiation, and Lyman-α radiation) which affect the spin temperature (T_S) of intergalactic hydrogen are studied in the hope of being able to draw conclusions concerning density from the upper limit on the 21-cm opacity derived in the preceding paper (Field 1959*a*), hereinafter referred to as "Paper I." In Section II it is shown that the density of material can be limited enough from the observations themselves to insure that collisions cannot raise T_S above about $\frac{1}{2}°$ K, a lower limit imposed by radio-frequency radiation. A strong upper limit on density (1.3×10^{-30} gm cm^{-3}) would follow for a mostly neutral gas if this were the whole story. In Appendix A a study of the transfer of Lyman-α radiation in an expanding gas is carried out. Using the intensity of Lyman-α predicted by this study, we show in Section III that Lyman-α from galaxies alone is sufficient to raise T_S to 32° K. In Section IV we consider recombinations in a partially ionized medium—it is found that if the ionization is greater than 13 per cent, T_S rapidly approaches the kinetic temperature as a result of the copious Lyman-α generated by recombinations. We conclude in Section V that ionization greater than 13 per cent makes it impossible to draw deductions from the observations of opacity, while if it is less than 13 per cent, the total density is not likely to exceed 5.9×10^{-29} gm cm^{-3}. This may be compared with a value of 3×10^{-28} needed to explain a sizable deceleration of the expansion. Some discussion of ways to detect ionized gas which may be present is also given.

I. INTRODUCTION

In Paper I (Field, 1959*a*) we concluded that the density of a homogeneous distribution of neutral hydrogen atoms between the galaxies has the upper limit

$$n_H < 8.1 \times 10^{-7} T_S \text{ cm}^{-3} , \qquad (1)$$

where the "spin temperature," T_S, is just the excitation temperature of the hyperfine levels defined by the Boltzmann equation (T_S enters eq. [1] because our observations concerned the *absorption* by intergalactic atoms). Paper I indicated that emission observations would have the advantage of being independent of T_S. If, in the future, emission observations give an indication of n_H, equation (1) may then be viewed as giving a lower limit on T_S. It seems, therefore, that a theory connecting T_S with other physical conditions between the galaxies is desirable both for the interpretation of the present absorption observation in terms of density and possibly future emission observations in terms of temperature.

The theory of T_S was stated in a general way by Field (1958). There it was shown that its value follows from a consideration of statistical equilibrium of three mechanisms which excite and de-excite the upper (triplet) hyperfine level. It is believed that the important mechanisms are as follows:

a) *Radio-frequency processes at 21-cm wave length.*—A triplet atom radiates spontaneously with probability $A_{10} = 2.85 \times 10^{-15}$ sec^{-1}; correspondingly, there are absorptions and stimulated emissions which are proportional to the mean brightness of 21-cm radiation over the sky (expressed in temperature units by $J_\nu = 2kT_R/\lambda^2$).

b) *Collisions with other particles.*—The most important type of collision is with a particle having an electron of spin opposite to that of the hydrogen atom concerned. Here other hydrogen atoms and free electrons play the biggest role. We denote the de-excitation probabilities by $P_{10}(H)$ and $P_{10}(e)$, respectively; they are proportional to the particle densities and to cross-sections which have been calculated by Purcell and Field (1956) and Field (1959*a*) for kinetic temperatures between 1° and 10000° K.

c) Light-quantum processes.—Here, typically, a triplet atom is excited by absorption of a Lyman photon to the $n = 2$ level, from which it returns to the ground-state hyperfine singlet in a certain fraction of the cases. Thus a de-excitation of the triplet effectively takes place via the intermediary of the $n = 2$ level. Although any Lyman photon is capable of this process, only Lyman-α need be considered because of the tendency of other lines to be converted by fluorescence to Lyman-α. Obviously, the de-excitation probability for Lyman-α, $P_{10}(\alpha)$, depends on the number of such photons. A theory for it was given by Field (1958), but we shall find it necessary to reconsider the problem in Section III for the case of an expanding medium.

With these mechanisms in mind, T_S is written as follows:

$$T_S = \frac{T_R + \tau_c + \tau_\alpha}{1 + (\tau_c + \tau_\alpha) / T_K}, \tag{2}$$

where T_K is the kinetic temperature, and the τ's, referring to collisions and Lyman-α, are normalized de-excitation probabilities defined by[1]

$$\tau = \frac{h\nu}{k} \frac{P_{10}}{A_{10}}. \tag{3}$$

Equation (2) shows that if both τ's are small, $T_S \to T_R$, signifying equilibrium with the 21-cm radiation field, while if either of the τ's is large, $T_S \to T_K$. This seems natural enough for collisions, but perhaps not so for Lyman-α. Actually, if Lyman-α dominates, the spin temperature depends on the Lyman-α line profile. For perfect symmetry of this profile, the "light-temperature" is infinite, and the Lyman-α term in the denominator of equation (2) is zero. But it can be shown (Field 1959*b*) that if the Lyman-α photons are scattered many times in a static gas, recoil effects redden them so that there is a shift of the profile to the red. It may further be shown that the "light-temperature" is then close to T_K, the kinetic temperature (see Field 1958). We should therefore have no hesitation about using T_K as in equation (2), except that a discussion of the scattering process in an expanding medium (Appendix A) casts some doubt as to the conditions under which this will give the same result as for a static medium. We shall, however, carry out the analysis with T_K, comforted by the fact that the critical values in the following analysis fall in a region of density and temperature in which the difference between the denominator and unity are negligible, even on the assumption that T_K is correct for Lyman-α. The real temperature we ascribe to the profile is evidently larger, if anything, than T_K, because the profile relaxes to T_K from infinite temperatures, so that the denominator is still unity and no harm is done.

Equation (2), together with equation (1), implies an upper limit on the density of neutral hydrogen between the galaxies. We cannot extract the upper limit by a straightforward estimate of T_S, as it depends on the density. Hence it is necessary to calculate n_H/T_S theoretically as a function of n_H, and then to compare this with the observed upper limit for n_H/T_S, 8.1×10^{-7}. This leads to an upper limit on n_H.

For clarity we shall carry out the process in three stages. In the first stage (Sec. II) we ignore Lyman-α and show that equations (1) and (2) imply that collisions have a negligible effect on T_S, so that $T_S \simeq T_R$. The reason for this is that any density at which collisions would be important would make n_H/T_S larger than the observed upper limit.

In the second stage (Sec. III) we introduce Lyman-α and show how its effects may be estimated. We then specialize this to Lyman-α emanating from known sources—the galaxies. An estimate of T_S based on our knowledge of the radiation from galaxies shows that T_S will not much exceed 32° K.

[1] In the notation of Field (1958), $\tau = yT^{-1}$.

Finally, in Section IV we consider the effect of Lyman-α radiation emitted by the intergalactic medium itself, supposing a ratio of ions to neutrals, x. We find that even modest ionization generates sufficient Lyman-α by recombination to drive up T_S significantly. In particular, we estimate a critical value of x which, if exceeded, would make $T_S \rightarrow T_K$.

T_R is known, but T_K and x are not, and so the three stages are in order of decreasing certainty of results. In Section V we present our conclusions concerning the density of neutral hydrogen between the galaxies. Because of our uncertainty concerning x, the resulting upper limit on density is qualified by conditions on the ionization. We also consider briefly other observations bearing on the problem of diffuse matter between galaxies.

II. SPIN TEMPERATURE IN THE ABSENCE OF LYMAN-α

If Lyman-α is altogether absent, we have the simpler equation,

$$T_S = \frac{T_R + \tau_c}{1 + \tau_c / T_K} . \tag{4}$$

We shall show that, in the absence of Lyman-α, the observation of Paper I itself limits the density sufficiently to make τ_c negligible.

First we need an estimate of T_R. In principle, we could put an upper limit on T_R by measuring the brightness temperature of continuum radiation near 21 cm at the coldest spot in the sky. However, its value is expected to be less than 1° K, and the difficulties inherent in absolute flux measurements have so far prevented such a direct determination.

Piddington and Trent (1956) estimated the corresponding upper limit at 600 Mc/s by extrapolating temperatures at the coldest spots of the sky determined at eight lower frequencies from 10 to 400 Mc/s. This is possible because the background spectrum rises smoothly toward lower frequencies. A similar procedure for 1420.4 Mc/s yields the figure

$$T_R \leq 0.^\circ 4 .$$

By the nature of the derivation, 0°4 is evidently the sum of galactic and extragalactic radiation temperatures at the coldest spot. The upper limit, therefore, arises because the fraction constituted by extragalactic sources (the portion of interest) is unknown. One might wonder whether thermal (free-free), and 21-cm line radiation from galaxies might depart from the extrapolated spectrum and hence not be included in the above estimate. Here we must employ theory; the result is that galaxies like our own contribute only 0°002 thermal radiation to T_R, while the effect of line radiation is only 0°02. This last depends on the fact that, while individual galaxies may have brightness temperatures in the 21-cm line in the range 5°–10° K (van de Hulst, Raimond, and van Woerden 1957; Heeschen 1957), the effective solid angle subtended by such galaxies is small when one considers only the galaxies which have red shifts less than their own line widths. We shall adopt 0°4 for T_R; the upper limit on n_H implied by equations (1) and (4) is evidently smaller, if anything, than analysis based on this assumption indicates.

The theory of τ_c is given by Field (1958). It may be written

$$\tau_c = (\tau_H' + x \tau_e') n_H , \tag{5}$$

where the τ''s are τ/n's in each case (they are denoted by y/nT_K and given for various T_K's by Field [1958]) and x is the ratio of electrons to neutrals (n_e/n_H) and is related to the ionization $i = (n_e/n_{\text{total}})$ by

$$i = \frac{x}{1 + x} . \tag{6}$$

A glance at Table II of Field (1958) reveals that $\tau_e' \simeq 20\tau_H'$, a consequence of the higher thermal velocities of the electrons. It is apparent that for $x > 1/20$ the electrons dominate the picture, so that our results depend critically on x. This being the case, we select a value of x which is particularly significant. If x is taken to be small (say 5 per cent), we perhaps would be underestimating the effect of collisions with free electrons—the density estimates we would obtain could then be much too low. If, on the other hand, we assume x is large (say 95 per cent), the neutral hydrogen would be the residual of a much larger amount of ionized material; therefore, we would not expect to see it anyway, if the over-all density is low. The dividing line, $x = 1$, or 50 per cent ionization, is particularly significant, and we shall use it in further calculations. Our results then apply to any medium that is *predominantly neutral*, since the upper limits still apply to less ionized media, while more ionized media should be studied by means other than the present one.

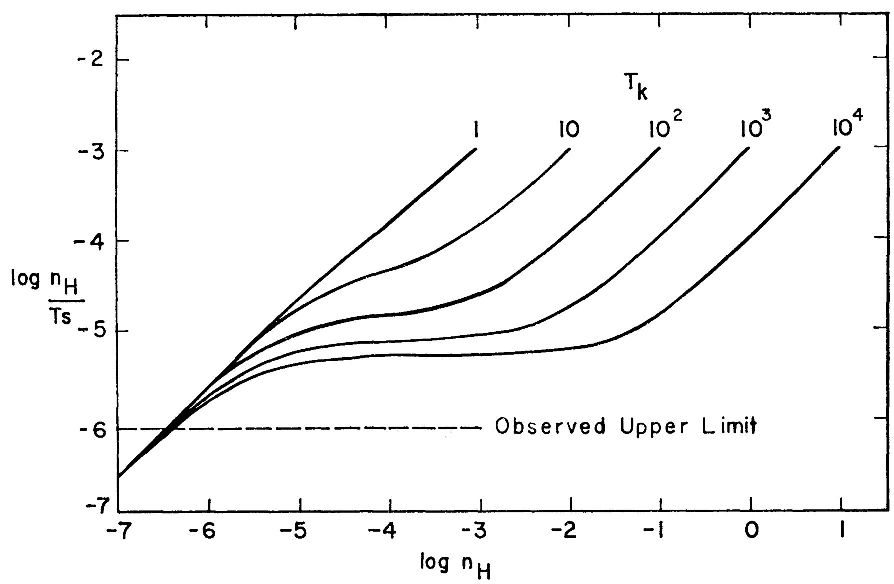

FIG. 1.—The relation between spin temperature and density for various kinetic temperatures; no Lyman-α.

Using previous results for τ', we have calculated

$$\frac{n_H}{T_S} = \frac{n_H + T_K^{-1}(\tau_H' + \tau_e')\, n_H^2}{T_R + (\tau_H' + \tau_e')\, n_H} \tag{7}$$

for $T_R = 0^\circ\!.4$ and with various T_K's between 1° and 10000°, as a function of n_H; the result is plotted in Figure 1. We note the following behavior: for small n_H, n_H/T_S approaches n_H/T_R because collisions have little effect when the density is low. For large n_H, collisions dominate and $n_H/T_S = n_H/T_K$. The quadratic behavior of the numerator compared with the linear behavior of the denominator assures that n_H/T_S is a monotonic increasing function of n_H; so, at worst, the transition between small and large densities is horizontal. This means that the line

$$\frac{n_H}{T_S} = C = 8.1 \times 10^{-7} \tag{8}$$

has precisely one root for the density for each T_K. These roots are plotted against T_K in Figure 2. It is evident, as one expects from Figure 1, that the roots are very close to

CT_R, as would be the case if there were no collisions. It is interesting that the observations themselves thus limit the densities to regions where collisions are unimportant.

We note that n_H is certainly less than the critical value plotted in Figure 2; so these values represent upper limits as functions of T_K. These upper limits evidently continue to increase with T_K, but we may suppose on physical grounds that temperatures in excess of 10000° will ionize the medium. Therefore, we would be entitled to conclude that, if it were not for Lyman-α, a mostly neutral gas would have $n_H \leq 3.8 \times 10^{-7}$ (the value for 10000°). Inasmuch as the total mass density would then be less than 1.3×10^{-30} gm cm^{-3}, we see that Lyman-α is the only alternative to supposing either predominantly ionized gas or very rare neutral hydrogen gas in intergalactic space.

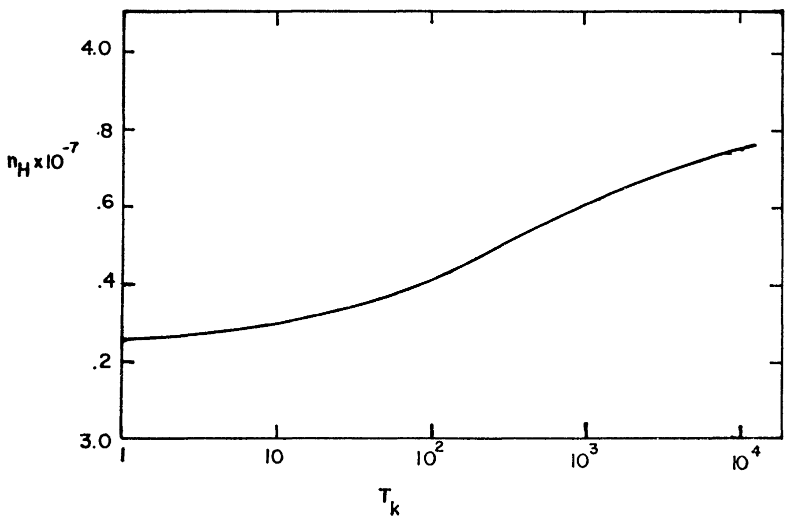

FIG. 2.—Critical densities which cannot be exceeded according to the observations of 21-cm opacity, plotted against kinetic temperature, ignoring Lyman-α.

III. INTRODUCTION OF LYMAN-α

Now τ_α is just the Lyman-α de-excitation probability by equation (3). A study of the energy-level diagram of $n = 2$ and $n = 1$ in hydrogen, given by Field (1958), shows that

$$P_{10}(\alpha) = \frac{4}{27} P(\alpha) , \tag{9}$$

where the latter quantity is the mean probability per second that an atom will scatter a Lyman-α photon and $P(\alpha)$ depends on the density of Lyman-α photons; formulae for it are given by Field (1958). Here, however, we are concerned with a steady-state process in which Lyman-α photons are created and destroyed in the gas, and it seems advantageous to compute the scattering rate directly in terms of the rate of creation of Lyman-α photons. Reflection shows that if the creation rate is $\dot{n}_\alpha$ and the number of times a photon is scattered before being destroyed is N, then

$$n_H P(\alpha) = \dot{n}_\alpha N , \tag{10}$$

each side being the number of scatterings per cubic centimeter per second. This form is advantageous because N is directly calculable for an expanding medium. The calculation is given in detail in Appendix A, but the argument may be summarized thus: $\dot{n}_\alpha$ photons are generated per cubic centimeter per second by two processes. First, photons shortward of Lyman-α are emitted by galaxies and then commence scattering at a remote point where the red shift makes $\lambda = 1216$ A. But they stop scattering, once they are thrown by Doppler motions into a wave length sufficiently red that they never again encounter atoms which can scatter them. These "galactic" photons are "created" and "destroyed" at the rate $(\dot{n}_a)_G$. It is shown in Appendix A that, under these conditions,

$$N = \tau_0 = 2.4 \times 10^{10} n_H \,, \tag{11}$$

independent of the random motions of the gas, where τ_0 is the optical depth to infinity for a very blue photon. Crudely, this statement results from the fact that a photon escapes when it is so red as to have optical depth unity to infinity. This happens at a frequency x (in units of the Doppler width), defined by $e^{-x^2} = \tau_0^{-1}$. The chance to be scattered to such a frequency is of order e^{-x^2}; so τ_0 scatterings are required.

In another process, Lyman-α photons are generated at the line center by recombinations at the rate $(\dot{n}_a)_R$. Because they are created in the line center, the analysis of Appendix A shows that, in this case,

$$N \simeq 1.5\, \tau_0 = 3.6 \times 10^{10} n_H \,. \tag{12}$$

Introducing equations (11) and (12) into equations (9) and (10), we find

$$\text{Galaxies:} \qquad P_{10}(\alpha) = \tfrac{4}{27} \times 2.4 \times 10^{10} (\dot{n}_a)_G \,, \tag{13}$$

$$\text{Recombination:} \qquad P_{10}(\alpha) = \tfrac{4}{27} \times 3.6 \times 10^{10} (\dot{n}_a)_R \,,$$

independent of the hydrogen density. (This is because a higher density traps the photons longer but also presents more targets; the rate of scattering per atom is constant.) After normalizing, we find that

$$\begin{aligned} (\tau_a)_G &= 8.5 \times 10^{22} (\dot{n}_a)_G \,, \\ (\tau_a)_R &= 1.3 \times 10^{23} (\dot{n}_a)_R \,. \end{aligned} \tag{14}$$

The problem is therefore reduced to computing $\dot{n}_a$ for the two sources of Lyman-α photons. In the remainder of this section we shall attend to $(\dot{n}_a)_G$, the contribution to $\dot{n}_a$ by galaxies. A detailed derivation given in Appendix B is summarized below. From the argument of Appendix A one sees that every photon arriving at a point which has precisely the critical capture frequency is captured. Bluer ones are captured before reaching the point, redder ones proceed farther. Thus the total rate of capture of photons per cubic centimeter per second is $F_\lambda \Delta\lambda / h\nu_a$ summed over all galaxies, where $\Delta\lambda$ is the red shift in 1 cm or λ_a/R_0, with R_0 the inverse Hubble constant in centimeters; F_λ is the ultraviolet flux for a galaxy with $v/c = z$—it reflects the intrinsic spectra of the galaxies at wave lengths $\lambda_a/(1+z)$ and the energy and number effects. The quantity F_λ is written as a ratio of ultraviolet to photographic light *times* a photographic flux, which is then put in terms of an apparent magnitude by referring to the sun. Finally, the numbers of galaxies of each apparent magnitude are summed over, using Hubble's counts, and the sum terminated at a recession velocity equal to $c/3$, at which point the photons have a wave length of 912 A at the source. We terminate the sum then because the continuum shortward of 912 A is presumably much weaker than longward of 912 A.

The final equation is, from Appendix B,

$$\begin{aligned} (\dot{n}_a)_G &= \frac{5}{4 \times 2.3} \frac{\lambda_a}{R_0} \frac{F_\lambda(\lambda_a)}{F_\lambda(pg)} \frac{F_\lambda(pg, \odot)}{h\nu_a} \, 10^{0\ 4m_\odot + 0\ 2B + C + 10\ 24} \\ &= 3.77 \times 10^{-22} \text{ cm}^{-3} \text{ sec}^{-1} \,. \end{aligned} \tag{15}$$

The most uncertain quantity in equation (15) is $F_\lambda(\lambda_a)/F_\lambda(pg)$; a value 3.9 is estimated for it in Appendix B.

Putting the derived value into equation (14), we find that

$$(\tau_a)_G = 32^\circ \text{ K} . \tag{16}$$

Let us discuss the consequences of introducing galactic Lyman-α into the picture, still leaving out the recombination Lyman-α. From equation (2) we see that

$$T_S = \frac{[T_R + (\tau_a)_G] + \tau_c}{1 + T_K^{-1}(\tau_a)_G + T_K^{-1}\tau_c} . \tag{17}$$

In this equation, the bracket is 32°.4. As before, $T_S \rightarrow T_K$ for high densities, while this time $T_S \rightarrow [T_R + (\tau_a)_G]/1 + T_K^{-1}(\tau_a)_G$ for low densities, rather than $T_S \rightarrow T_R$. Thus for low kinetic temperatures ($\ll$32°.4) one finds $T_S \rightarrow T_K[1 + T_R^{-1}(\tau_a)_G] = 1.01 T_K$. This results from the high efficiency for a given amount of Lyman-α in bringing T_S into equilibrium with T_K. But, as the kinetic temperature rises, the efficiency of Lyman-α goes down: T_S cannot rise much above $T_R + (\tau_a)_G = 32°.4$. The resulting behavior of n_H/T_S is shown in Figure 3 for $x = 1$, and for various T_K. As before, we note the intersections of the observed upper limit, $n_H/T_S = C$ with the curves of various T_K. Evidently, the density (for a given T_K) must be less than the value at the intersection. This critical value is plotted against T_K in Figure 4. From Figure 4 it is evident that, if we knew that T_K was low, we could conclude strong upper limits on n_H; in the absence of such evidence, we can conclude only that it is less than the critical value for $T_K = 10000°$, or 3.1×10^{-5}, for a predominantly neutral gas. Again collisions are unimportant, and the appropriate T_S is evidently quite close to 32°.4. Thus for a predominantly neutral gas including the effects of galactic Lyman-α, we conclude that

$$n_H < 3.1 \times 10^{-5} \text{ cm}^{-3} . \tag{18}$$

IV. RECOMBINATION LYMAN-α

Following equation (14), we need the value of $(\dot{n}_a)_R$, the number of photons per cubic centimeter per second created by recombination in the line center. Evidently this is just the total number of recombinations excepting those on the 1S and 2S states; the 1S recaptures are ineffective because they emit Lyman continuum, which then ionizes another atom. The 2S recaptures, owing to the metastability of the 2S state, lead to two-photon emission rather than to Lyman-α. Excluding the 1S and 2S recaptures, we have (Allen 1955)

$$(\dot{n}_a)_R = 2.08 \times 10^{-13} x^2 n_H^2 (10^{-4} T_K)^{-0.8} . \tag{19}$$

We must now specify x and T_K. It is possible that $(\dot{n}_a)_R$ will be important for $x > 1$, so we shall consider $x = 0.1$ and $x = 0.2$. As for T_K, we may suppose that it will be $\geq$10000° K, owing to the high energy of ejected photoelectrons and subsequent cooling by ionic collisions. Hence we shall consider $T_K = 10000°$ K.

From equation (14) we have

$$(\tau_a)_R = 2.7 \times 10^{10} x^2 n_H^2 , \tag{20}$$

and thus, from equation (2),

$$T_S = \frac{32.4 + (1.3 \times 10^4 + 1.8 \times 10^5 x)\, n_H + 2.7 \times 10^{10} x^2 n_H^2}{1 + (1.3 + 18x)\, n_H + 2.7 \times 10^6 x^2 n_H^2} , \tag{21}$$

where we have used the previously established values for T_R and $(\tau_a)_G$ and the τ_c' appropriate to 10000°. Owing to the presence of the $(\tau_a)_R$ term, which is quadratic in n_H,

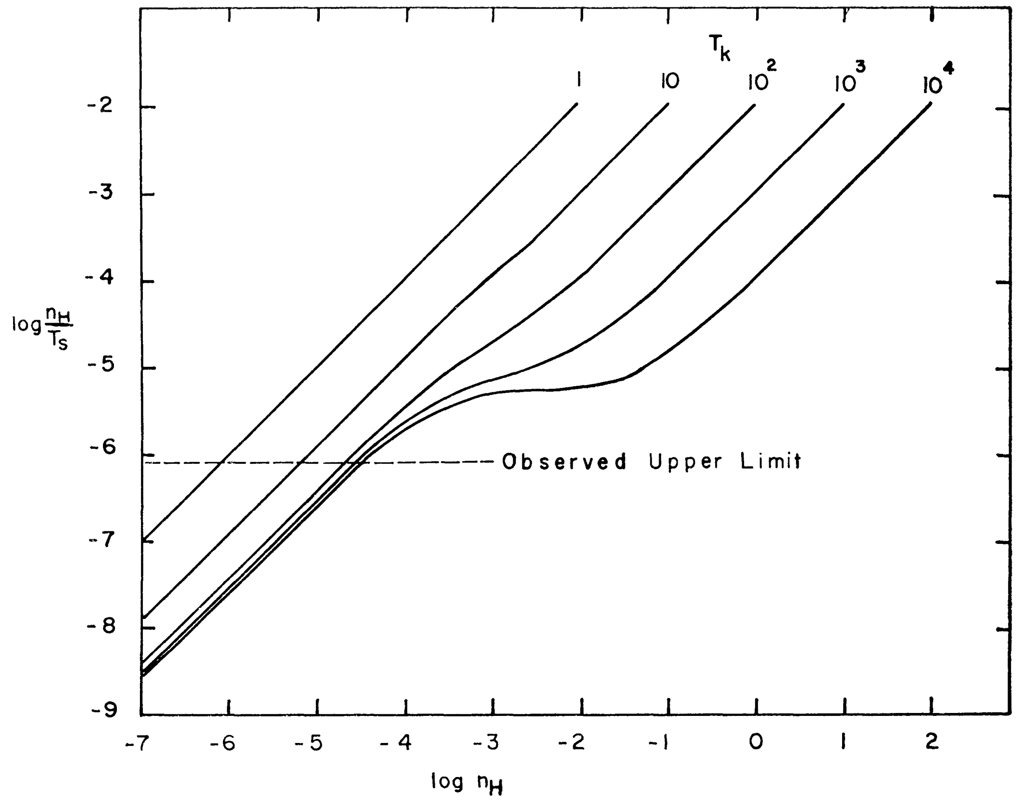

FIG. 3.—The relation between spin temperature and density for various kinetic temperatures, taking account of collisions and galactic Lyman-α.

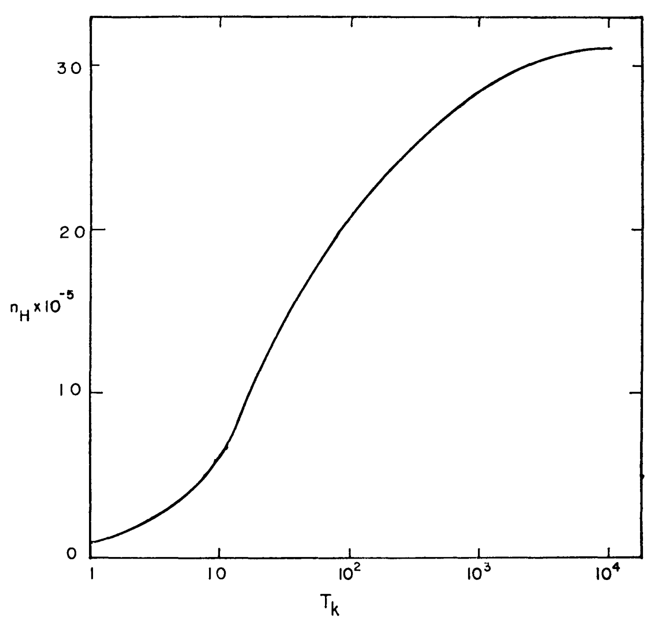

FIG. 4.—Critical densities which cannot be exceeded according to the observation of 21-cm opacity plotted against kinetic temperature, taking account of collisions and galactic Lyman-α.

the function n_H/T_S no longer necessarily increases with n_H; at sufficiently large n_H it can decrease for a while before finally increasing like n_H/T_K. The values of n_H/T_S from equation (21) are plotted against n_H in Figure 5, for $x = 0.1$ and 0.2. For low densities this quantity varies like $n_H/32.4^\circ$, but for higher densities it exhibits the predicted dip. Inspection shows that if there is considerable ionization, T_S is coupled strongly to T_K for densities consistent with $n_H/T_S < 8.1 \times 10^{-7}$; one can conclude very little from our observations. For $x = 0.1$, however, the dip is not deep enough, and $T_S < 32.4^\circ$. Clearly, there is a critical value of x separating these regions. By manipulations on equation (21) it is found to be

$$x_c = 0.15 . \tag{22}$$

The curve is drawn for $x = 0.15$ in Figure 5: clearly, if $x > 0.15$, there are solutions consistent with $n_H/T_S < 8.1 \times 10^{-7}$ which have high density and hence, by the resulting large amount of Lyman-α, high spin temperature.

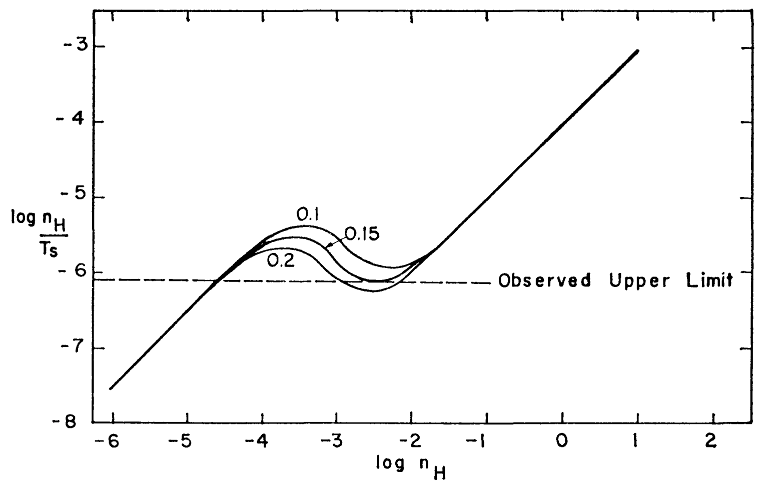

FIG. 5.—The relation between spin temperature and density for the kinetic temperature = 10000°, taking account of collisions, galactic Lyman-α, and Lyman-α resulting from recombinations in a partially ionized medium. The ratio of ions to neutrals identifies each curve.

Note that the main effect of increasing T_K is to reduce the recombination rate and hence $(\tau_\alpha)_R$; the term $(\tau_\alpha)_R/T_K$ is negligible anyway at points in the neighborhood of the coincidence of the minimum in the n_H/T_S-curve with $n_H/T_S = C$. This is the basis for our conclusion in Section I that the precise value of the light-temperature does not affect the argument as long as it exceeds T_K.

Evidently, the upper limit on n_H if $x < 0.15$ is given by the intersection of the line $n_H/T_S = C$ with the $x = 0.15$ curve; its value, 3.1×10^{-5} cm^{-3}, does not differ measurably from the value obtained in the absence of recombination Lyman-α.

V. CONCLUSION

We have seen that if Lyman-α is absent, we must choose between ionization or a low density. If Lyman-α is present, a critical level of ionization,

$$i_c = \frac{x_c}{1 + x_c} = 0.13 , \tag{23}$$

is established such that recombinations are ineffective for $i < i_c$. If $i < i_c$, we are left primarily with galactic Lyman-α; its estimated effect is to raise T_S to about 38°, and so give an upper limit for $i < i_c$:

$$n_H < 3.1 \times 10^{-5} \text{ cm}^{-3} ; \qquad \rho < 5.9 \times 10^{-29} \text{ gm cm}^{-3} , \tag{24}$$

where ρ includes the contribution of the protons. If, on the other hand, $i > i_c$, we can conclude very little concerning densities from the observations of Paper I.

It seemed in the early stages of this investigation that recombination Lyman-α would play a role only for almost complete ionization. In that case we would have the present upper limits applying to predominantly neutral material, and a predominantly ionized medium would have to be studied by other means in any case. The present analysis shows that we are not so fortunate—a "poisoning" of only 13 per cent ionization is sufficient to make the remaining 87 per cent of neutral hydrogen almost transparent. Hence our upper limits apply only if the ionization is relatively small, and there is still a range of ionizations, 13–95 per cent, which we cannot rule out theoretically at present and which would make transparent even a fairly large amount of neutral gas.

Before closing, we shall make a few general comments about the problem of diffuse intergalactic matter. First, there is the question of the distance scale. In this paper we have used an inverse Hubble constant of 5.4×10^9 years. If this should be increased by a factor f, we find that T_R, T_K, and τ_c are unaffected, while all opacities are increased by a factor f. Therefore, the Lyman-α photons scatter a factor f more times, while $(\dot{n}_a)_G$ decreases by a factor f and $(\dot{n}_a)_R$ remains the same. The result is that $(\tau_a)_G$ remains the same, while $(\tau_a)_R$ increases by a factor of f. The observed n_H/T_S varies as f^{-1}. One finds, then, that

$$x_c = 0.15\ f^{1/2}\ , \tag{25}$$

while the upper limit on the density, if $x < x_c$, is

$$n_H < 3.1 \times 10^{-5} f^{-1}\ \mathrm{cm}^{-3}\ . \tag{26}$$

If, as Sandage (1958) suggests, f is about 2.4, x_c is increased to 0.23, or 20 per cent ionization, while the upper limit on n_H is lowered to $1.3 \times 10^{-5}\ \mathrm{cm}^{-3}$.

Second, we consider the possible values of brightness temperature emitted by the neutral hydrogen. From the relation $T_B = \tau T_S$ and our measured opacity, we see that

$$T_B < 0.29°\ \mathrm{K} \tag{27}$$

if the ionization is low. This would be just at the limit of present techniques to detect. If, on the other hand, the ionization is greater than 13 per cent, the brightness temperature T_B, could be much higher. In any case, a combination of emission and absorption observations would yield T_S; this could be interpreted by means of the present theory.

Finally, one is interested in ways to detect any ionized gas which might be present. Electron scattering would affect the surface brightness of galaxies, but the effect (686 n_e mag. per billion light-years) seems too small to detect if the present distance scale is correct. Three other possibilities do have some hope—low-frequency free-free absorption, Lyman-α emission, and H and K line absorption. Hey and Hughes (1954) showed that Cyg A is absorbed by free-free transitions at 22.6 Mc., which can be shown to imply

$$n_e < 1.5 \times 10^{-6} T_K^{3/4}\ . \tag{28}$$

The upper limit arises because we are uncertain as to how much of the effect to ascribe to galactic H II. Hence studies of H II in the neighborhood of Cyg A would push down the upper limit. Next, Lyman-α photons resulting from recombination will cause a broad emission line to the longward of Lyman-α. We can put an upper limit on this effect from the observations in the region 1225–1350 A made by NRL rockets (Kupperian *et al.* 1958). The result, $F_\lambda = 5 \times 10^{-5}\ \mathrm{erg\ cm^{-2}\ sec^{-1}}$ in this band, can be shown to imply that

$$n_e < 4 \times 10^{-5} T_K^{0.4}\ . \tag{29}$$

Finally, a refinement of Whitford's (1954) suggestion indicates that if α is the abundance of Ca relative to the galaxy and β is the ratio of electrons to all nuclei, (n_{total}),

$$(\alpha\beta)^{1/2} n_{\text{total}} < 2.3 \times 10^{-6} T_K^{1/3} \tag{30}$$

follows from the lack of visible broad absorption H and K lines in spectra of galaxies.

It is to be noticed that equations (28)–(30) depend on the unknown T_K and that equation (30) depends on unknown abundances as well. It seems, therefore, that satellite experiments on Lyman-α, because of the low temperature sensitivity, hold most promise for advance in the question of ionized diffuse matter between the galaxies.

The earlier phases of the work were carried out while the author was a Junior Fellow of the Harvard Society of Fellows and a guest of the Harvard College Observatory. He is grateful to them for their support.

APPENDIX A

Scattering of Lyman-α in an Expanding Universe

We consider a homogeneous gas of H I participating in the expansion. At each point the gas has a most probable speed about the local standard, V; we assume the velocity distribution to be Maxwellian, although it could be either turbulent or thermal in origin.

Follow the path of a particular photon. It becomes redder as it proceeds, relative to the local frame of reference. A photon sufficiently longward of Lyman-α at a particular point will never scatter, because even the atoms in the wings of the local velocity distribution will be too few in number to scatter it. A photon anywhere shortward of Lyman-α, however, is bound somewhere to be red-shifted enough so that its wave length is about 1216 A in the local frame. When it reaches that point (which could be billions of light-years from the starting point for a very blue photon) its "opacity to infinity," τ_0, is independent of its origin and of the precise velocity distribution of the hydrogen atoms. This latter is because the opacity is proportional to the number of atoms per unit wave-length interval; the local velocity distribution has only the effect of distributing scattering atoms over different regions of space. It is easy to show that, for $H_0^{-1} = 5.4 \times 10^9$ years (as we shall assume throughout),

$$\tau_0 = 1.38 \times 10^{10} \frac{S}{g_1} n , \tag{A1}$$

where S is the line strength in atomic units, g is the weight of the ground state, and n is the atomic density. (This shows that τ_0/n for optical and radio-frequency lines is independent of wave length, though not of the transition type.) For Lyman-α,

$$\tau_0 = 2.40 \times 10^{10} n_H . \tag{A2}$$

Define τ_x as the optical depth, from a particular point to infinity, for a photon of normalized frequency

$$x = \frac{\nu - \nu_0}{V\nu_0 / c} \tag{A3}$$

at that point (x is zero in the line center and positive and $\simeq 1$ for blue photons in the Doppler wings). One can show, for our hypothesis of Maxwellian local velocities, that

$$\tau_x = \tau_0 \pi^{-1/2} \int_{-\infty}^{x} e^{-v^2} d v . \tag{A4}$$

Therefore, τ_x is τ_0 for blue photons ($x \gg 1$), and zero for red ones ($x \ll -1$), as explained above. The opacity already encountered by a photon coming from infinity and having local frequency x is

$$\tau_x' = \tau_0 - \tau_x = \tau_0 \pi^{-1/2} \int_{x}^{\infty} e^{-v^2} d v ,$$

which is τ_0 for very red photons and zero for very blue.

We shall treat the two methods of generating Lyman-α photons; first, consider blue photons from galaxies. We say blue because the red ones never scatter, as shown above. But a blue photon will scatter when τ'_x, the opacity it has traversed, is about unity, independent of its wave length at its origin. We postulate that all photons are scattered for the first time when $\tau'_x = 1$; this defines a blue frequency, x_c, by

$$\pi^{-1/2}\int_{x_c}^{\infty} e^{-v^2} d\, v = \tau_0^{-1} . \tag{A5}$$

Physically, the photon is first scattered by an atom which is moving at it with speed $x_c V$ relative to the local frame. Such an atom, if it scatters isotropically, can therefore impart, with equal probability, any frequency between $+x_c$ and $-x_c$ to the scattered photon ($-x_c$ for emission in the direction of incidence) with a smaller probability of scattering outside that interval. Thus, following the first scattering, the photons are distributed uniformly over $(-x_c, +x_c)$. As successive scatterings occur, the Doppler effect redistributes the photons over this interval. But what happens if $x \to x' < -x_c$, as it will be from time to time? According to equation (A4), the opacity to infinity is

$$\tau_{x'} < \tau_0 \pi^{-1/2}\int_{-\infty}^{-x_c} e^{-v^2} d\, v = \tau_0 \pi^{-1/2}\int_{x_c}^{\infty} e^{-v^2} d\, v = 1 , \tag{A6}$$

where we have used equation (A5). Hence, if $x' < -x_c$, the photon effectively never scatters again. We may view the process as capture at $+x_c$, scattering in $(-x_c, +x_c)$, and escape at $-x_c$. The prime question is, How many scatterings occur between capture and escape? It turns out that this can be answered precisely for the hypotheses we have used, namely, isotropic scattering with Doppler effect the only broadening mechanism, and the capture and escape as described. Field (1959*b*) solves the time-dependent scattering problem in a static gas. We assume that the problem is the same as ours, the effect of the expansion being taken into account by the conditions of capture and escape. The mean intensity per normalized frequency interval is $J(x)$; the probability per unit time (in units of the mean free time of a photon for which $x = 0$) that an x-photon will scatter into $(x', x' + dx')$ is $r(x, x')$. It may be shown that $J(x)$ satisfies the integral equation

$$\int_{-\infty}^{\infty} r\,(x,\, x')\, J\,(x)\, d\, x' - \int_{-\infty}^{\infty} r\,(x',\, x)\, J\,(x')\, d\, x' = C\,(x) \ , \tag{A7}$$

where $C(x)$ is the rate of creation of mean intensity. Detailed results depend on $r(x, x')$, which, for isotropic Doppler scattering, is

$$r\,(x,\, x') = \int_{|\bar{x}|}^{\infty} e^{-v^2} d\, v\, ; \tag{A8}$$

it is seen that $r(x, x') = r(x', x)$, because the symbol $|\bar{x}|$, means "the greater of $|x|$ and $|x'|$." This function for a given x is constant between $-x$ and $+x$; it decreases very rapidly outside that interval. Hence an x-photon is equally likely to be scattered anywhere between $-x$ and $+x$, and there is a small chance that it will be scattered to larger absolute frequencies.

We now formulate $C(x)$ for our case. First, each scattering at $+x_c$ results in a creation of a photon in $(-x_c, +x_c)$; $C(x)$ by equation (A8) is independent of x. Escape takes place by scattering to any frequency $x < -x_c$; scatterings to $x > x_c$ may be neglected, since, on the next scattering, they are uniformly distributed in $(-x_c, +x_c)$ again with overwhelming probability. Hence the creation rate caused by escapes is

$$-J\,(x)\int_{-\infty}^{-x_c} r\,(x,\, x')\, d\, x' .$$

The equation resulting by putting these contributions into equation (A9) is immediately seen to have the solution $J(x) =$ Constant in $(-x_c, +x_c)$. This is proved as follows: the capture rate is independent of x, while if $J(x)$ is constant, so is the escape rate (because the integral extends over $|x'| > x_c$, for which $r[x, x']$ is independent of x because $|x| < x_c$). Hence the right-hand side of equation (A7) becomes zero for some proper choice of the constant $J(x)$. But, by the symmetry of $r(x, x')$, the left-hand side vanishes for $J(x)$ constant. We do not need to know the exact value of $J(x)$ to proceed to our results. Consider the probability of escape per unit time from frequency x. This is

$$\int_{-\infty}^{-x_c} r(x, x')\, dx' = \frac{e^{-x_c^2}}{4x_c^2}\left[1 + O(x_c^{-2})\right] \tag{A9}$$

independent of x, as we have noted. Now integrating equation (A8) over x', one can show that the total probability of scattering per unit time is e^{-x^2}. Using the fact that $J(x)$ is constant, the average rate of scattering is $(\sqrt{\pi}/2x_c)[1 + O(x_c^{-2})]$, so the fraction of scatterings leading to escape, F, is $\exp(-x_c^2)/2x_c\sqrt{\pi}$ (to terms of order x_c^{-2}), and the mean number of scatterings before escape is

$$N = F^{-1} = 2\sqrt{\pi}\, x_c e^{x_c^2} = \tau_0 , \tag{A10}$$

using the definition of x_c (eq. [A5]) and expanding to terms of order x_c^{-2}. We may inquire as to the value of the neglected terms. From equation (A10) we see that

$$x_c^{-2} \simeq (\log \tau_0)^{-1} ; \tag{A11}$$

whence, if $n_H > 10^{-8}$ cm^{-3}, $x_c^{-2} < 20$ per cent. Equation (A10) then represents the number of scatterings with sufficient accuracy. It should be noted that a more precise theory, taking into account recoil effects, shows that the profile slopes down toward the blue, so that $hd\nu/hd \log J_\nu$ approaches T_K rather than infinity, as it is here.

To turn to the case of recombinations, we note that the creation is now proportional to e^{-x^2} (because of the Doppler motions of the protons) and the steady-state profile is evidently somewhere between this and the flat one described above. This will make the number of scatterings larger because the photons must diffuse to the wings before escaping. It is easy to show that, for a Doppler profile, the escape probability is unaltered, but the mean scattering rate is higher by the factor $(2/\pi)^{1/2}x_c$. Hence N is $(2/\pi)^{1/2}x_c$ times the previous value. Using the definition of x_c, one finds that this factor can be in the range 1.7–2.7 for reasonable H I densities. In view of the fact that this results from the extreme assumption of a Doppler profile, it seems reasonable that the factor for the true and hence more flattened profile might be 1.5. Hence we adopt $N = 1.5\tau_0$ for recombinations.

APPENDIX B

The Rate of Capture of Lyman-α from Galaxies

If the apparent photographic magnitude of a galaxy is m and that of the sun is $m_\odot$, the ratio of photographic fluxes is $10^{0.4(m_\odot - m)}$; we take the photographic flux to be proportional to the flux per unit wave length at 4400 A. Evidently, that flux per unit wave length at 1216 A in a galaxy at rest is $F_\lambda(1216)/F_\lambda(4400)$ times this. Therefore, the number flux of Lyman-α photons from a nearby galaxy is

$$10^{0.4(m_\odot - m)} \frac{F_\lambda(\lambda_a)}{F_\lambda(pg)} \frac{F_\lambda(pg, \odot)}{h\nu_a} . \tag{B1}$$

For a red-shifted galaxy, a photon which is Lyman-α at the point in question was emitted at wave length $\lambda_a/1 + z$), where z is the recession velocity divided by c. This has four effects: first, the proper flux at the galaxy now is $F_\lambda(\lambda_a/1 + z)$—this introduces a factor, $F_\lambda(\lambda_a/1 + z)/F_\lambda(\lambda_a)$ into equation (B1); second, the wave-length intervals transform according to $d\lambda = d\lambda_a/(1 + z)$—this introduces a factor $(1 + z)^{-1}$; third, the "number effect" reduces F_λ by $(1 + z)^{-1}$; fourth, the energy effect reduces F_λ by $(1 + z)^{-1}$. The energy flux is thus reduced by the factor $(1 + z)^{-3}F_\lambda(\lambda_a/1 + z)/F_\lambda(\lambda_a)$, and the photon flux (energy flux divided by $h\nu_a$) by the same number. Hence the photon flux is

$$(1 + z)^{-3} 10^{0.4(m_\odot - m)} \frac{F_\lambda(\lambda_a/1 + z)}{F_\lambda(\lambda_a)} \frac{F_\lambda(\lambda_a)}{F_\lambda(pg)} \frac{F_\lambda(pg, \odot)}{h\nu_a} . \tag{B2}$$

The photon flux corresponding to $d\lambda_a$ is captured in a distance $dr = (R_0/\lambda_a)d\lambda_a$, so $(\dot{n}_a)_G$ due to a single galaxy is the photon flux per unit wave length times $d\lambda_a/dr$, or a factor λ_a/R_0 times (B2). Now we must add the contributions of various galaxies. Let $\mathfrak{N}_m$ be the number of galaxies over the whole sky between m and $m + dm$. Then

$$(\dot{n}_a)_G = \frac{\lambda_a}{R_0} \frac{F_\lambda(\lambda_a)}{F_\lambda(pg)} \frac{F_\lambda(pg, \odot)}{h\nu_a} 10^{0.4m_\odot} \int_{m_1}^{m_2} \frac{\mathfrak{N}_m 10^{-0.4m}}{(1+z)^3} \frac{F_\lambda(\lambda_a/1+z)}{F_\lambda(\lambda_a)} dm\ . \quad \text{(B3)}$$

To go further, we need a relation between m and z—the velocity-distance relation. We take this as

$$m = 5 \log cz + B = 5 \log z + 27.4 + B\ . \quad \text{(B4)}$$

Similarly, we adopt Hubble's galaxy counts per square degree as sufficiently accurate:

$$\log \mathfrak{N}_m = \log N_m + 4.76 = 0.6m + 4.76 + C\ . \quad \text{(B5)}$$

We find that

$$10^{-0.4m} \mathfrak{N}_m dm = \frac{5}{2.3} 10^{10.24+0.2B+C} dz\ . \quad \text{(B6)}$$

We now must consider a model from the ultraviolet light from a galaxy. We start with the compound light from our own Galaxy. According to recent calculations of Lambrecht and Zimmermann (1954), the flux rises sharply toward the Lyman limit, beyond which it drops rapidly to practically zero. This behavior is due to the heavy Lyman absorption in the early-type stars, which contribute most to the ultraviolet. Recent rocket studies (Kupperian *et al.* 1958) of the radiation at these wave lengths tends to confirm these values rather than the much smaller ones deduced by Dunham, assuming black-body radiation. We find that $(a) F_\lambda = 0$ for $\lambda < 912$A; (b) $F_\lambda \sim \lambda^{-1}$ for 912 A $< \lambda <$ 1216 A; and $(c) F_\lambda(1216 \text{ A})/F_\lambda(pg) = 5.6$ in our Galaxy. We shall assume *a* and *b* to be true for every galaxy; we shall modify assumption *c* as to the absolute value of the flux by considering galaxies of various types. Using assumptions *a* and *b*, we find that $z(m_1) = 0$, that $z(m_2) = \frac{1}{3}$, and that $F_\lambda(\lambda_a/1 + z)\ F_\lambda(\lambda_a) = 1 + z$. Hence

$$(\dot{n}_a)_G = \frac{5}{2.3} \frac{\lambda_a}{R_0} \frac{F_\lambda(\lambda_a)}{F_\lambda(pg)} \frac{F_\lambda(pg, \odot)}{h\nu_a} (10^{0.4m_\odot + 0.2B + C + 10\ 24}) (\tfrac{1}{4})\ , \quad \text{(B7}$$

the last factor being integral of $(1 + z)^{-2}$ from 0 to $\frac{1}{3}$.

We may correct $F_\lambda(\lambda_a)/F_\lambda(pg)$ from its value for an Sb galaxy such as our own as follows. According to Holmberg (1958), the color indices of galaxies indicate the following percentages of population I in various galaxies: Sc, 90; Sb, 60; Sa, 44; E, 0. If we assume that the ultraviolet flux is proportional to these percentages and if we take account of numbers of galaxies of various types when selected by apparent magnitude (Hubble 1936), we find that the mean ratio of 1216 A light to 4400 A light is 3.9. Other data used in equation (B7) are

λ_a	$= 1216$ A,
R_0	$= 5.1 \times 10^{27}$ cm (Humason, Mayall, and Sandage 1956),
$F_\lambda(pg, \odot)$	$= 2.02 \times 10^{10}$ erg cm^{-3} sec^{-1} (Allen 1955) ,
$h\nu_a$	$= 1.63 \times 10^{-11}$ erg ,
$m_\odot$	$= -26.41$ (pg) (Allen 1955) ,
B	$= -4.24$ (Humason, Mayall, and Sandage 1956) ,
C	$= -9.05$ (Hubble 1936) .

When these are inserted, we find the number quoted in the text.

REFERENCES

Allen, C. W. 1955, *Astrophysical Quantities* (London: Athlone Press).
Field, G. B. 1958, *Proc. I.R.E.*, **46**, 240.
———. 1959*a*, *Ap. J.*, **129**, 525.
———. 1959*b*, *ibid.*, p. 551.
Heeschen, D. S. 1957, *Ap. J.*, **126**, 471.

Hey, J. S., and Hughes, V. A. 1954, *Nature*, **173**, 819.
Holmberg, E. 1958, *Lund Med.*, Ser. II, No. 136.
Hubble, E. 1936, *Ap. J.*, **84**, 517.
Hulst, H. C. van de, Raimond, E., and Woerden, H. van. 1957, *B.A.N.*, **14**, 1.
Humason, M. L., Mayall, N. U., and Sandage, A. R. 1956, *A.J.*, **61**, 97.
Kupperian, J. E., Boggess, A., and Milligan, J. E., *Ap. J.*, **128**, 453.
Lambrecht, H., and Zimmermann, H. 1955, *Wissensch. Zr. Fr.-Schiller U.* (Jena), **5**, 217.
Piddington, J. H., and Trent, G. H., *Australian J. Phys.*, **9**, 481.
Purcell, E. M., and Field, G. B., *Ap. J.*, **124**, 542.