PS4

May 28, 2018

```
In [1]: import scipy.stats as sts

        T = 500
        sigma = 0.7
        unif_vec = sts.uniform.rvs(loc=0, scale=1, size=T, random_state=25)
        eps_vec = sts.norm.ppf(unif_vec, loc=0, scale=sigma)
```

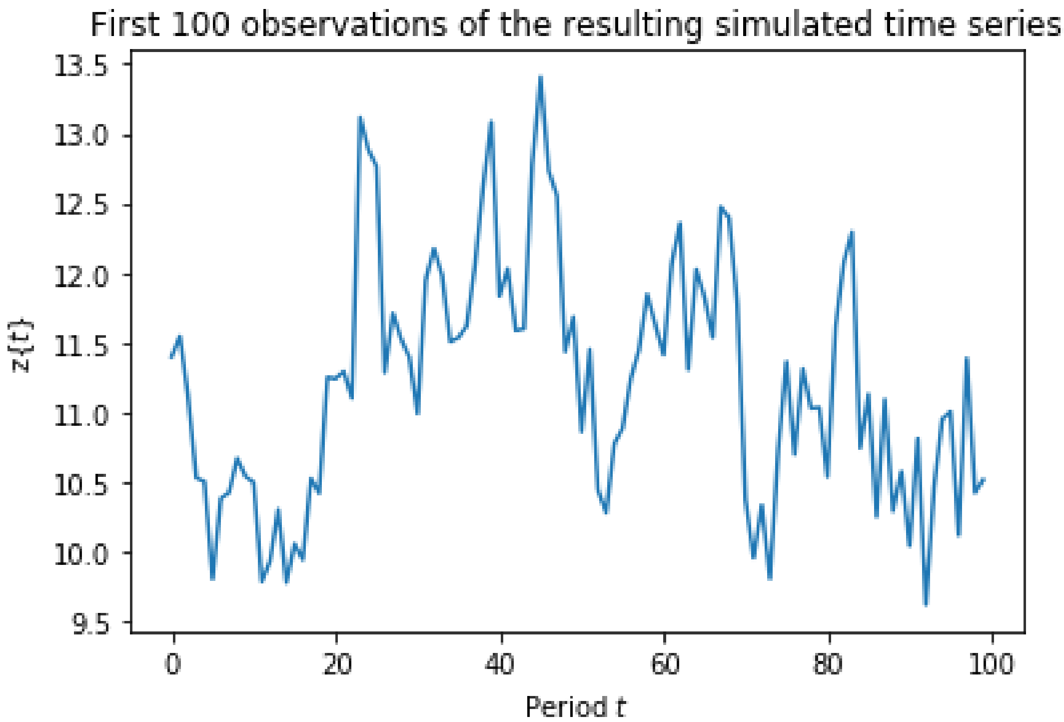
(a)

```
In [2]: z = []
        ro = 0.85
        mu = 11.4

        for i in range(T):
            if i == 0 :
                z_t = mu
                z += [z_t]
            else:
                z_t = ro * z_t + (1 - ro) * mu + eps_vec[i]
                z += [z_t]
```

```
In [3]: import matplotlib.pyplot as plt

        %matplotlib inline
        plt.plot(range(100), z[:100])
        plt.title('First 100 observations of the resulting simulated time series')
        plt.xlabel(r'Period $t$')
        plt.ylabel(r'z{$t$}')
        plt.show()
```



(b)

```
In [4]: import numpy as np
        z_vals = np.array([mu + i * 1.5 * sigma for i in range(-2, 3)])

In [5]: z_vals

Out[5]: array([ 9.3 , 10.35, 11.4 , 12.45, 13.5 ])
```

(c)

```
In [6]: z_cuts = 0.5 * z_vals[:-1] + 0.5 * z_vals[1:]

In [7]: z_bin = []

        for i in range(T):
            if z[i] <= z_cuts[0]:
                z_bin += [1]
            elif z[i] <= z_cuts[1]:
                z_bin += [2]
            elif z[i] <= z_cuts[2]:
                z_bin += [3]
            elif z[i] <= z_cuts[3]:
                z_bin += [4]
            else:
                z_bin += [5]
```

```
In [8]: markov_matrix = [[0] * 5 for i in range(5)]

        for i in range(T - 1):
            pre = z_bin[i]
            now = z_bin[i + 1]
            markov_matrix[pre - 1][now - 1] += 1

In [9]: markov_matrix = markov_matrix / (np.sum(markov_matrix, axis = 1).reshape(5,1))
        markov_matrix

Out[9]: array([[0.65384615, 0.30769231, 0.03846154, 0.        , 0.        ],
               [0.11666667, 0.53333333, 0.31666667, 0.03333333, 0.        ],
               [0.0308642 , 0.21604938, 0.48765432, 0.22839506, 0.03703704],
               [0.        , 0.04273504, 0.34188034, 0.5042735 , 0.11111111],
               [0.        , 0.        , 0.04166667, 0.35416667, 0.60416667]])
```

(d)

```
In [10]: import numpy.linalg as LA
         X_now = np.array([0, 0, 1, 0, 0])
         X_next = np.dot(LA.matrix_power(markov_matrix.T, 3), X_now)
         print('The probability of zt+3 being in bin 5 given that zt is in bin 3: ', X_next[4]

The probability of zt+3 being in bin 5 given that zt is in bin 3:  0.08152797732142057
```

(e)

```
In [11]: threshold = 0.00001
         diff = 1
         X_now = np.array([0, 0, 1, 0, 0])

         while diff > threshold:
             X_next = np.dot(markov_matrix.T, X_now)
             X_diff = X_next - X_now
             diff = np.sum(X_diff)
             X_now = X_next

In [12]: print('The stationary distribution of zt is ', X_now)

The stationary distribution of zt is  [0.0308642  0.21604938 0.48765432 0.22839506 0.03703704]
```

(f)

```
In [13]: z_compare = []
         z_t = z_vals[2]
         zt_ind = 2

         for i in range(T):
```

```
            ztp_ind_now = np.argwhere(unif_vec[i] <= np.cumsum(markov_matrix[zt_ind, :])).min
            z_tp_now = z_vals[ztp_ind_now]
            z_compare += [z_tp_now]

            z_t = z_tp_now
            zt_ind = ztp_ind_now
```

In [14]:
```
plt.plot(range(500), z, label = 'continuous')
plt.plot(range(500), z_compare, label = 'discretized')
plt.title('The time series of discretized series versus the continuous version')
plt.xlabel(r'Period $t$')
plt.ylabel(r'z{$t$}')
plt.legend()
plt.show()
```