PS2

April 30, 2018

1. 2D kernel density estimator

```
In [1]: import numpy as np
        bq_data = np.loadtxt('BQmat_orig.txt', delimiter=',')
```

```
In [19]: # (a)
         import matplotlib.pyplot as plt
         from mpl_toolkits.mplot3d import Axes3D
         %matplotlib notebook

         ages_vec = np.arange(18, 96)
         abils = np.array([0.25, 0.25, 0.20, 0.10, 0.10, 0.09, 0.01])
         abils_mdpts = np.array([0.125, 0.375, 0.60, 0.75, 0.85, 0.94, 0.995])
         abils_mat, ages_mat = np.meshgrid(abils_mdpts, ages_vec)

         fig = plt.figure()
         ax = fig.gca(projection='3d')
         ax.plot_surface(ages_mat, abils_mat, bq_data)
         ax.set_title('Distribution of bequest recipient proportion')
         ax.set_xlabel('Age')
         ax.set_ylabel('Lifetime Income Group')
         ax.set_zlabel('Percent of bequest received')
         plt.show()
```

```
<IPython.core.display.Javascript object>
```

```
<IPython.core.display.HTML object>
```

```
In [3]: # (b)
        from scipy.stats import gaussian_kde

        def get_scaled(bandwidth):
            prop_mat_inc = np.sum(bq_data, axis=0)
            prop_mat_age = np.sum(bq_data, axis=1)
            lrg_samp = 70000
```

```
        age_probs = np.random.multinomial(lrg_samp, prop_mat_age)
        income_probs = np.random.multinomial(lrg_samp, prop_mat_inc)
        age_freq = np.array([])
        inc_freq = np.array([])

        for age, num_s in zip(ages_vec, age_probs):
            vec_age_s = np.ones(num_s)
            vec_age_s *= age
            age_freq = np.append(age_freq, vec_age_s)

        for abil, num_j in zip(abils_mdpts, income_probs):
            vec_abil_j = np.ones(num_j)
            vec_abil_j *= abil
            inc_freq = np.append(inc_freq, vec_abil_j)

        data = np.vstack((age_freq, inc_freq))
        density = gaussian_kde(data, bw_method = bandwidth)

        coords = np.vstack([item.ravel() for item in [ages_mat, abils_mat]])
        BQkde = density(coords).reshape(ages_mat.shape)
        BQkde_scaled = BQkde / np.sum(BQkde)

        return BQkde_scaled
```

```
In [4]: def draw_scaled(data):
            fig = plt.figure()
            ax = fig.gca(projection='3d')
            ax.plot_surface(ages_mat, abils_mat, data)
            ax.set_title('Scaled distribution of bequest recipient proportion')
            ax.set_xlabel('Age')
            ax.set_ylabel('Lifetime Income Group')
            ax.set_zlabel('Scaled percent of bequest received')
            plt.show()
```

```
In [20]: for bandwidth in np.arange(0.05, 0.2, 0.05):
             draw_scaled(get_scaled(bandwidth))
```

```
<IPython.core.display.Javascript object>


<IPython.core.display.HTML object>


<IPython.core.display.Javascript object>


<IPython.core.display.HTML object>
```

```
<IPython.core.display.Javascript object>
```

```
<IPython.core.display.HTML object>
```

```
<IPython.core.display.Javascript object>
```

```
<IPython.core.display.HTML object>
```

I will choose the bandwidth parameter as 0.1, as it reserves the unique pattern within each age group, and also it smooths the noise to some extent. The result is shown as below:

```
In [21]: draw_scaled(get_scaled(0.1))
```

```
<IPython.core.display.Javascript object>
```

```
<IPython.core.display.HTML object>
```

```
In [8]: BQkde_scaled = get_scaled(0.1)
        print('The estimated density for bequest recipients who are age 61 in the 6th lifetime
        is', BQkde_scaled[43][6])
```

```
The estimated density for bequest recipients who are age 61 in the 6th lifetime income category
```

2. Interaction terms

```
In [9]: import pandas as pd

        biden = pd.read_csv('biden.csv')
        biden.dropna(inplace=True)
        biden.head()
```

Out[9]:

	biden	female	age	educ	dem	rep
0	90.0	0	19.0	12.0	1.0	0.0
1	70.0	1	51.0	14.0	1.0	0.0
2	60.0	0	27.0	14.0	0.0	0.0
3	50.0	1	43.0	14.0	1.0	0.0
4	60.0	1	38.0	14.0	0.0	1.0

```
In [10]: from statsmodels.formula.api import ols
         model = ols(formula = "biden ~ age + educ + age * educ", data = biden)
         result = model.fit()
         print(result.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  biden   R-squared:                       0.018
Model:                            OLS   Adj. R-squared:                  0.016
Method:                 Least Squares   F-statistic:                     10.74
Date:                Mon, 30 Apr 2018   Prob (F-statistic):           5.37e-07
Time:                        10:47:37   Log-Likelihood:                -8249.3
No. Observations:                1807   AIC:                         1.651e+04
Df Residuals:                    1803   BIC:                         1.653e+04
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     38.3735      9.564      4.012      0.000      19.617      57.130
age            0.6719      0.170      3.941      0.000       0.337       1.006
educ           1.6574      0.714      2.321      0.020       0.257       3.058
age:educ      -0.0480      0.013     -3.723      0.000      -0.073      -0.023
==============================================================================
Omnibus:                       64.246   Durbin-Watson:                   1.975
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               70.414
Skew:                          -0.481   Prob(JB):                     5.13e-16
Kurtosis:                       3.094   Cond. No.                     1.19e+04
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.19e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

In [11]:
```
result.cov_params()
```

Out[11]:

	Intercept	age	educ	age:educ
Intercept	91.461810	-1.545276	-6.725883	0.114416
age	-1.545276	0.029067	0.114149	-0.002159
educ	-6.725883	0.114149	0.509785	-0.008739
age:educ	0.114416	-0.002159	-0.008739	0.000166

Please find the coefficient parameters and standard error of the fitted model above.

In [12]:
```
b1 = 0.6719
b2 = 1.6574
b3 = -0.0480
var_b1 = 0.029067
var_b2 = 0.509785
var_b3 = 0.000166
cov_13 = -0.002159
```

```
cov_12 = 0.114149
cov_23 = -0.008739
```

(a)

$Y = \beta_0 + \beta_1 age + \beta_2 educ + \beta_3 age * educ$

The marginal effect of age on Joe Biden thermometer rating, conditional on education $= \beta_1 + \beta_3 educ$, and the standard error of the marignal effect $= \sqrt{(Var(\beta_1) + educ^2 * Var(\beta_3) + 2 * educ * Cov(\beta_1, \beta_3))}$

```
In [13]: marginal_age = pd.DataFrame(columns = ['educ', 'mar', 'std', 't'])
         marginal_age['educ'] = np.arange(0, 18)
         marginal_age['mar'] = b1 + marginal_age['educ'] * b3
         marginal_age['std'] = np.sqrt(var_b1 + marginal_age['educ']** 2 * var_b3 + 2 * margina
         marginal_age['t'] = marginal_age['mar'] / marginal_age['std']
```

```
In [14]: marginal_age
```

Out[14]:

	educ	mar	std	t
0	0	0.6719	0.170490	3.940983
1	1	0.6239	0.157845	3.952615
2	2	0.5759	0.145241	3.965129
3	3	0.5279	0.132691	3.978405
4	4	0.4799	0.120212	3.992104
5	5	0.4319	0.107829	4.005432
6	6	0.3839	0.095577	4.016649
7	7	0.3359	0.083516	4.021961
8	8	0.2879	0.071743	4.012958
9	9	0.2399	0.060424	3.970309
10	10	0.1919	0.049870	3.848018
11	11	0.1439	0.040682	3.537218
12	12	0.0959	0.033985	2.821809
13	13	0.0479	0.031417	1.524674
14	14	-0.0001	0.033926	-0.002948
15	15	-0.0481	0.040583	-1.185218
16	16	-0.0961	0.049749	-1.931683
17	17	-0.1441	0.060291	-2.390076

The magnitude of the marginal effect is decreasing with the increasing of education, and direction of the marginal effect changes from positive to negative. The statistical significance of marginal effect is pretty strong according to the t value we calculated.

(b)

$Y = \beta_0 + \beta_1 age + \beta_2 educ + \beta_3 age * educ$

The marginal effect of education on Joe Biden thermometer rating, conditional on age $= \beta_2 + \beta_3 age$, and the standard error of the marignal effect $= \sqrt{(Var(\beta_2) + age^2 * Var(\beta_3) + 2 * age * Cov(\beta_2, \beta_3))}$

```
In [15]: marginal_educ = pd.DataFrame(columns = ['age', 'mar', 'std', 't'])
         marginal_educ['age'] = np.arange(18, 94)
         marginal_educ['mar'] = b2 + marginal_educ['age'] * b3
         marginal_educ['std'] = np.sqrt(var_b2 + marginal_educ['age']** 2 * var_b3 + 2 * margin
         marginal_educ['t'] = marginal_educ['mar'] / marginal_educ['std']

In [16]: marginal_educ
```

Out[16]:

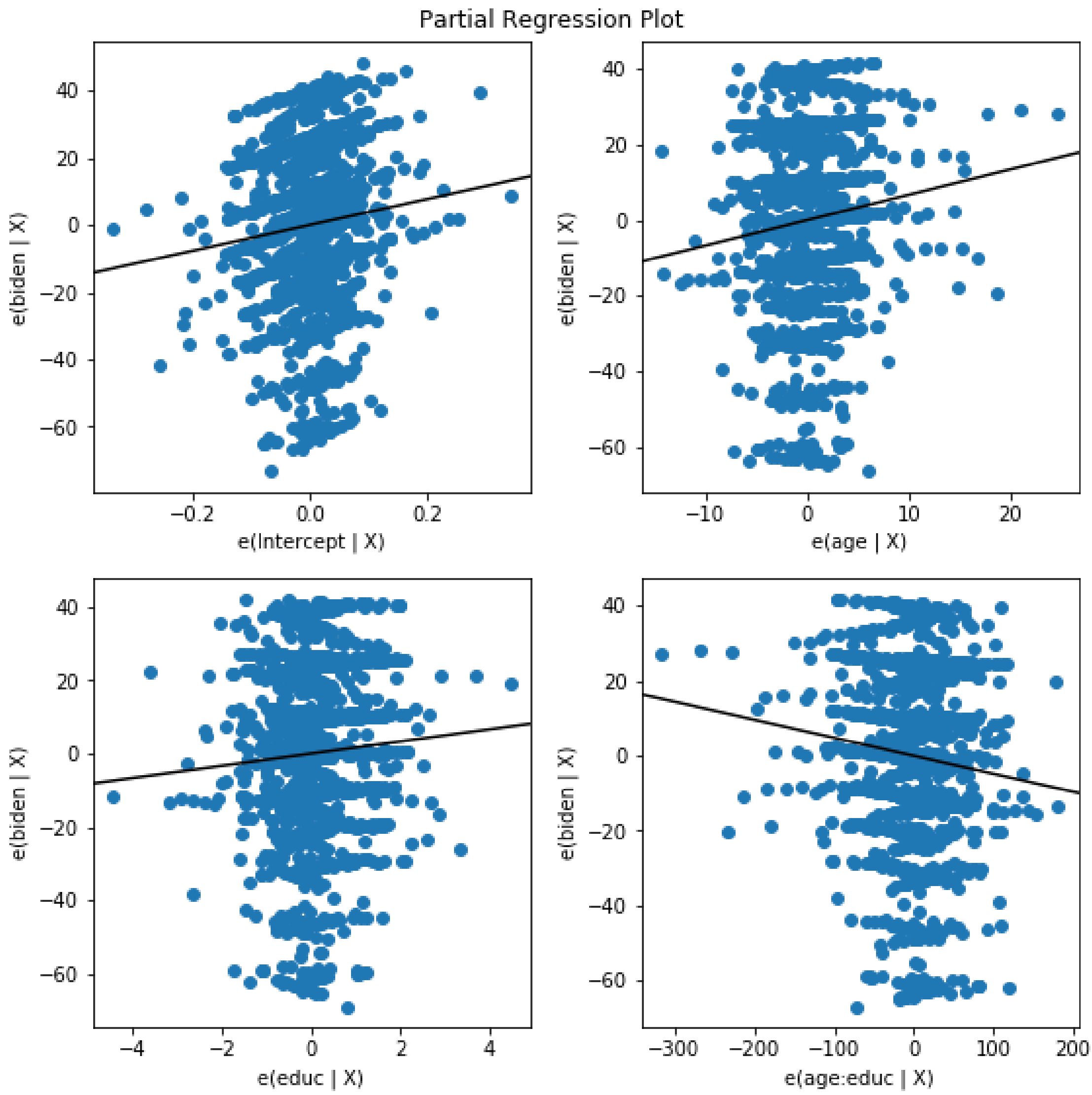
	age	mar	std	t
0	18	0.7934	0.498964	1.590095
1	19	0.7454	0.487472	1.529113
2	20	0.6974	0.476051	1.464968
3	21	0.6494	0.464707	1.397438
4	22	0.6014	0.453446	1.326289
5	23	0.5534	0.442273	1.251265
6	24	0.5054	0.431195	1.172092
7	25	0.4574	0.420220	1.088477
8	26	0.4094	0.409357	1.000106
9	27	0.3614	0.398614	0.906642
10	28	0.3134	0.388001	0.807729
11	29	0.2654	0.377530	0.702990
12	30	0.2174	0.367212	0.592028
13	31	0.1694	0.357062	0.474428
14	32	0.1214	0.347092	0.349763
15	33	0.0734	0.337320	0.217597
16	34	0.0254	0.327764	0.077495
17	35	-0.0226	0.318442	-0.070971
18	36	-0.0706	0.309375	-0.228202
19	37	-0.1186	0.300588	-0.394560
20	38	-0.1666	0.292104	-0.570344
21	39	-0.2146	0.283952	-0.755760
22	40	-0.2626	0.276161	-0.950894
23	41	-0.3106	0.268762	-1.155669
24	42	-0.3586	0.261788	-1.369810
25	43	-0.4066	0.255274	-1.592796
26	44	-0.4546	0.249257	-1.823821
27	45	-0.5026	0.243772	-2.061759
28	46	-0.5506	0.238858	-2.305138
29	47	-0.5986	0.234549	-2.552137
..	...	...	...	...
46	64	-1.4146	0.266700	-5.304083
47	65	-1.4626	0.273980	-5.338347
48	66	-1.5106	0.281661	-5.363182
49	67	-1.5586	0.289712	-5.379827
50	68	-1.6066	0.298102	-5.389424
51	69	-1.6546	0.306804	-5.393011
52	70	-1.7026	0.315793	-5.391512
53	71	-1.7506	0.325043	-5.385748

```
54    72 -1.7986  0.334534 -5.376434
55    73 -1.8466  0.344246 -5.364194
56    74 -1.8946  0.354160 -5.349566
57    75 -1.9426  0.364260 -5.333011
58    76 -1.9906  0.374530 -5.314923
59    77 -2.0386  0.384958 -5.295637
60    78 -2.0866  0.395531 -5.275436
61    79 -2.1346  0.406238 -5.254560
62    80 -2.1826  0.417067 -5.233210
63    81 -2.2306  0.428011 -5.211554
64    82 -2.2786  0.439059 -5.189733
65    83 -2.3266  0.450206 -5.167862
66    84 -2.3746  0.461442 -5.146039
67    85 -2.4226  0.472763 -5.124342
68    86 -2.4706  0.484162 -5.102836
69    87 -2.5186  0.495634 -5.081573
70    88 -2.5666  0.507174 -5.060595
71    89 -2.6146  0.518776 -5.039936
72    90 -2.6626  0.530438 -5.019621
73    91 -2.7106  0.542156 -4.999669
74    92 -2.7586  0.553925 -4.980096
75    93 -2.8066  0.565743 -4.960911

[76 rows x 4 columns]
```

The magnitude of the marginal effect is first decreasing, then increasing with the increasing of age, and direction of the marginal effect changes from positive to negative. The statistical significance of marginal effect is pretty strong according to the t value we calculated, when age is larger than 30.

```
In [18]: import statsmodels.api as sm
         from pandas.core import datetools
         fig = plt.figure(figsize = (8,8))
         fig = sm.graphics.plot_partregress_grid(result, fig = fig)
```




```
In [24]: from statsmodels.graphics.factorplots import interaction_plot
         fig = interaction_plot(biden['age'], biden['educ'], biden['biden'])
```

```
<IPython.core.display.Javascript object>
```

```
<IPython.core.display.HTML object>
```

The partial regression plot and the interaction plot shown above serve as the graphical support of the answers of question 2. We can see that the interaction term is actally changing the overall pattern of regression model, and it varies with the level of age and education.