UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Thursday 15 May 2008

09:30 to 11:30

MSc Courses

Convener: A Smaill
External Examiners: M Hepple, E Hull, D Marshall, I Marshall

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

CALCULATORS MAY NOT BE USED IN THIS EXAMINATION

1. (a) Define what it means for a 2-player strategic form game to be *symmetric.* [*3 marks*]

   (b) Define what an *evolutionarily stable strategy* is for a 2-player symmetric strategic form game. [*4 marks*]

   (c) Consider the following bimatrix game:

$$\begin{bmatrix} (3,1) & (1,6) & (4,0) & (1,0) \\ (1,2) & (2,3) & (2,4) & (2,1) \\ (2,3) & (4,1) & (3,1) & (4,0) \end{bmatrix}$$

   Compute a Nash Equilibrium (NE) in this game. Show all your work, i.e., show how you computed it. [*7 marks*]

   (d) How many NEs does the game in the previous question have? Justify your answer. [*3 marks*]

   (e) Suppose you are given a 2-player strategic form game, Γ, and you are told that there exists a Nash Equilibrium in this game with support sets (i.e., the sets of strategies that are played with non-zero probability for each player) $\texttt{support}_1$ and $\texttt{support}_2$ for the two players. Describe a Linear Payoff (LP) (without an object function) for this game, such that a feasible solution to this LP gives both a Nash Equilibrium with these support sets, and also gives the expected payoffs to each player under the NE. (In other words, there will be variables in this LP corresponding to the probabilities that each player puts on each strategy, as well as variables for the expected payoff of each player.) [*5 marks*]

   (f) Suppose (x_1^*, x_2^*) is a Nash Equilibrium in a 2-player zero-sum strategic form game. Suppose that the value of this game is v^*, and suppose that there is a pure strategy, j, for player 2, such that if player 2 switches unilaterally to this pure strategy, $\pi_{2,j}$, then under the profile $(x_1^*, \pi_{2,j})$ the expected payoff to player 1 would increase from v^* to a value $v' > v^*$. What is the probability $x_2^*(j)$ with which player 2 plays its j'th strategy in the minimax profile (x_1^*, x_2^*)? Explain your answer. [*3 marks*]

2. (a) How many different (mixed) minimax profiles can there be in a 2-player zero-sum strategic form game where both players have 3 pure strategies? [*3 marks*]

(b) Consider the following LP:

Maximize x_0
Subject to:
$x_0 + 2x_1 \leq 9$
$x_0 + x_2 \leq 8$
$x_1 + x_2 \geq 4$
$x_0, x_1, x_2 \geq 0$

Use Fourier-Motzkin elimination to eliminate the variable x_1 from this LP. Show your work and show the resulting LP from which x_1 is eliminated. [*4 marks*]

(c) Solve the LP given in part (b). Show show you solved it. [*4 marks*]

(d) Construct the dual LP to the LP given in part (b). . [*5 marks*]

(e) Define what a *strictly dominated strategy* in a strategic form game is. [*4 marks*]

(f) Explain how one can use an LP to determine whether a given mixed strategy x_i for player i is strictly dominated in a given strategic game Γ. [*5 marks*]

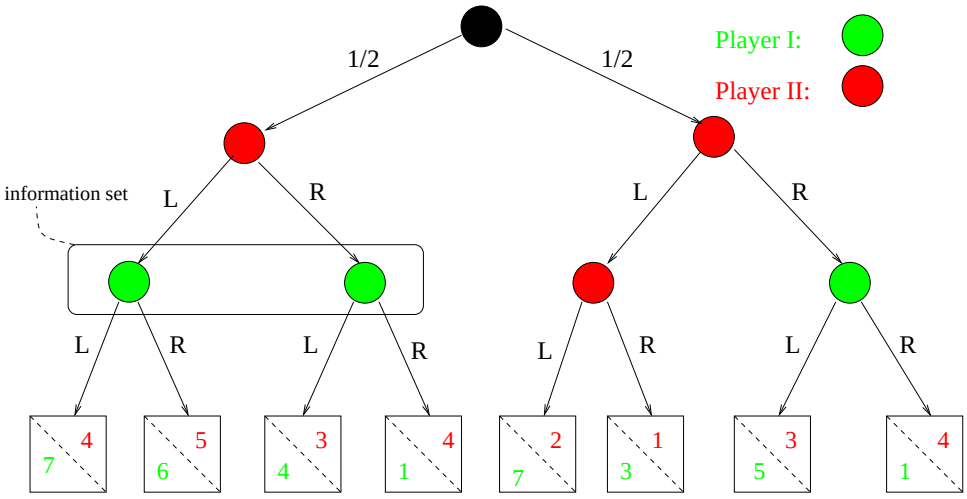

Figure 1: Game tree for part 3(a)

3. (a) Find a Nash Equilibrium for the extensive form game described in Figure 1, and compute the expected payoff to each player under this Nash Equilibrium. [*8 marks*]

   (b) Define what is meant by a memoryless strategy in a game on a graph. [*3 marks*]

   (c) Describe an example of a game of perfect information on a graph which is determined, but not memorylessly determined. [*3 marks*]

   (d) Define what a history-oblivious payoff function for a player in an infinite duration extensive form game on a graph is. [*3 marks*]

   (e) Consider the zero-sum game graph depicted in Figure 2 overleaf, where player I's goal is to reach the goal state, and player II's is to avoid it. Compute the set Win_2 of vertices of the graph starting from which player II has a winning strategy in this game, and give a memoryless winning strategy for player II starting from these vertices. [*5 marks*]

   (f) Suppose that for some primal LP its dual LP is infeasible. What can you conclude about the primal LP? [*3 marks*]

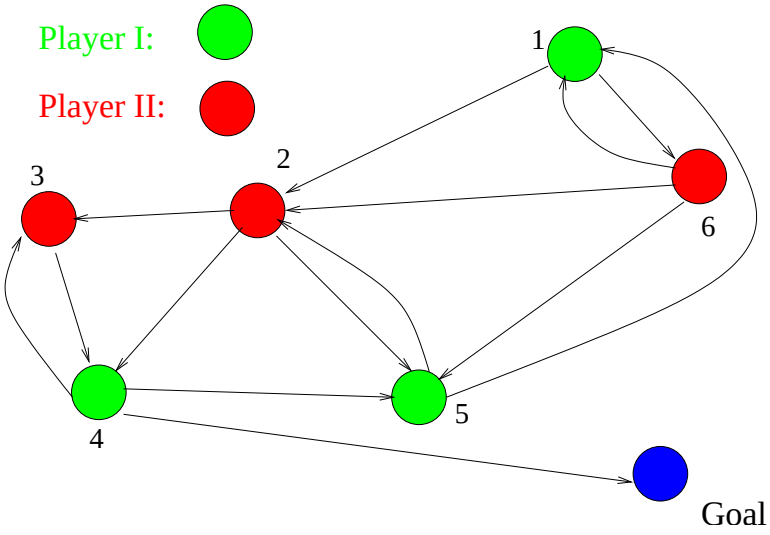

Figure 2: Game graph for part 3(e)