UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Friday 13 May 2005

09:30 to 11:15

MSc Courses

Convener: D Arvind
External Examiners: J Carroll, I Marshall, M Hepple, E Hull

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

1. (a) Suppose Γ is a finite n-player game in strategic form, with pure strategy sets $S_i = \{1, \ldots, m_i\}$, for $i = 1, \ldots, n$, and with payoff functions $u_i : S \mapsto \mathbb{R}$, for each player i. Define the *expected payoff (utility)* $U_i(x)$ for player i, under a profile of mixed strategies $x = (x_1, \ldots, x_n) \in X$. (I'm asking for the mathematical expression for $U_i(x)$ in terms of x and u_i.) [*3 marks*]

(b) Define what a *Nash Equilibrium* is for such a game. [*3 marks*]

(c) Define what it means for a mixed strategy for player i to be *strictly dominant*. [*3 marks*]

(d) Can player i have a strictly dominant strategy which is not a pure strategy? Justify your answer. [*4 marks*]

(e) For the remaining parts of the question, consider the following "bimatrix", which specifies a 2-player game in strategic form.

$$\begin{bmatrix} (3,2) & (1,3) & (2,1) & (5,1) \\ (4,6) & (2,1) & (1,2) & (6,1) \\ (3,2) & (1,8) & (3,3) & (7,0) \\ (3,8) & (0,8) & (1,4) & (3,9) \end{bmatrix}$$

Find a Nash Equilibrium (NE) in this game, and give the expected payoffs that each player gets under this NE. Explain how you found the NE, and justify why it is an NE. [*8 marks*]

(f) How many different Nash Equilibria does this game have? Justify your answer. [*4 marks*]

2. Consider a Primal Form LP of the form:

Maximize $c_1x_1 + c_2x_2 + c_3x_3$
Subject to:
$a_{1,1}x_1 + a_{1,2}x_2 + a_{1,3}x_3 \leq b_1$
$a_{2,1}x_1 + a_{2,2}x_2 + a_{2,3}x_3 \leq b_2$
$a_{3,1}x_1 + a_{3,2}x_2 + a_{3,3}x_3 \leq b_3$
$x_1, x_2, x_3 \geq 0$

(a) Define an *optimal feasible solution* for such an LP. [*3 marks*]

(b) By adding slack variables, rewrite this LP as an equivalent *dictionary*, where all constraints are equality constraints other than the non-negativity constraints given by $x_i \geq 0$, i=1,2,3. [*3 marks*]

(c) Describe under what condition this dictionary would be a *feasible* dictionary, and if it is a feasible dictionary, describe the *basic feasible solution* that it generates. [*3 marks*]

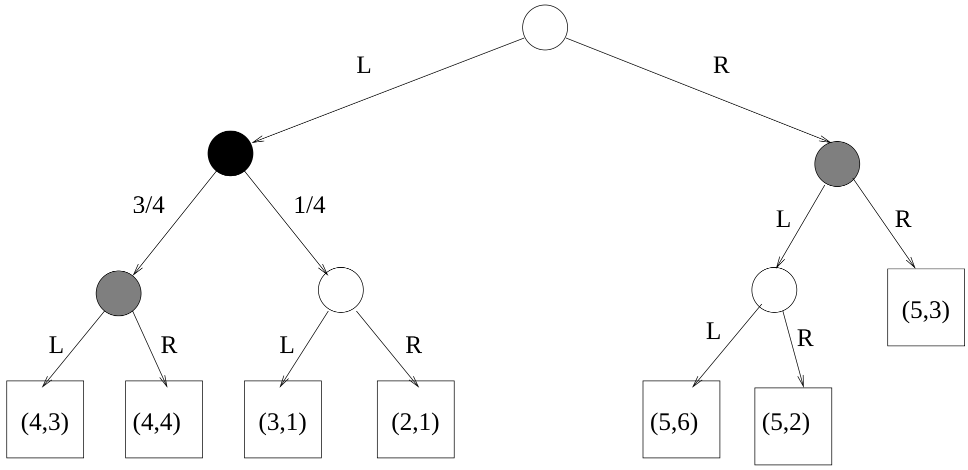
(d) Consider a finite 2-player zero-sum game in strategic form given by the following payoff matrix (where player 1 is the row player):

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

Write an LP whose optimal solution will give a minmaximizing strategy for player 1, and whose optimal value will be the mimimax value of this game. [*5 marks*]

(e) Solve this game by whatever means you wish, giving the resulting minmaximizer for player 1 and the minimax value of the game. Show your work. [*8 marks*]

(f) Write an LP which is dual to the LP in your answer for part (d.). . [*3 marks*]



3. (a) Consider the finite 2-player extensive game of perfect information depicted in the Figure above. The pair of numbers (a, b) in each leaf denote the payoffs a to player 1, and b to player 2. Unshaded nodes belong to player 1, nodes shaded grey belong to player 2, and nodes shaded black are chance nodes. Using the algorithm derived from Kuhn's theorem, compute a pure Nash Equilibrium (NE) for this game, as well as the payoffs to each player under this pure NE. [*7 marks*]

(b) Does the game above have only one pure NE? Justify your answer. [*3 marks*]

(c) For a game on a graph, define what a *memoryless strategy* for some player i is. [*4 marks*]

(d) Consider the following kind of game on a finite graph $G = (V, E, \mathtt{p}l)$. We have a finite game graph, each of whose vertices belongs to either player 1 or to player 2 (there are no chance nodes). Assume every vertex has at least one edge coming out of it. There are two subsets of the vertices, $F_1 \subseteq V$ and $F_2 \subseteq V$, and these two sets are disjoint meaning $F_1 \cap F_2 = \emptyset$. The game is a win-lose game with the following winning condition: the plays that are winning for player 1 are precisely those where either every vertex in F_1 occurs only finitely often in the play or some node in F_2 occurs infinitely often in the play, or both.

Reformulate this game as a Parity Game with three distinct priorities. In other words, considering the same game graph, partition the vertices V into three disjoint priority sets, V_i, $i = 0, 1, 2$, such that $V = V_0 \cup V_1 \cup V_2$, and such that player 1 wins iff the lowest priority that occurs infinitely often is Even. [*6 marks*]

(e) Describe what it means for a win-lose game on a finite graph to be *memorylessly determined.* Is the game described in the previous question memorylessly determined? Explain your answer. [*5 marks*]