UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Thursday 28$^{\underline{\text{th}}}$ April 2011

14:30 to 16:30

MSc Courses

Convener: C. Stirling
External Examiners: T. Attwood, R. Connor, R. Cooper, D. Marshall, M. Richardson

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

CALCULATORS MAY NOT BE USED IN THIS EXAMINATION

1. (a) Define what (both a *pure* and a *mixed*) *Nash Equilibrium* (NE) is in a finite n-player normal form game. [*4 marks*]

   (b) Two friends, Alice and Brenda, each have two party dresses (a *Red* dress and a *Blue* dress) which happen to be identical to the same two dresses owned by their friend. They are both invited to a party, but they are desperate not to wear the same dress as each other to the party. Given the choice, if they knew that their friend would not wear the same color, they would both prefer to wear the red dress over the blue dress. For unknown reasons, they can't communicate with each other to coordinate before going to the party.

   Let us model this as a *symmetric* 2-player normal form game as follows: If both wear Red or both wear Blue, then the payoff to both is -1. If one of them wears Red and the other Blue, then the payoff to the one wearing Red is 2, and the payoff to the one wearing blue is 1.

   Compute <u>all</u> of the NEs in this game, and say what the expected payoff to each player is under each NE. [*6 marks*]

   (c) Which of the NEs in the above Alice-Brenda game are *pareto optimal* strategy profiles? Explain your answer. [*4 marks*]

   (d) In the above Alice-Brenda symmetric bimatrix game, is there an *evolutionarily stable strategy*? If so, give one. [*5 marks*]

   (e) In a finite 2-player zero-sum normal form game, is it possible for there to exist two different NEs in which the payoff to player 1 is different? Explain your answer. [*3 marks*]

   (f) Answer the same question as (e) for a finite 3-player zero-sum normal form game instead. Explain your answer. [*3 marks*]

2. (a) Consider the following LP in primal form:

Maximize $3x_1 - x_2 + x_3$
Subject to:
$x_1 - x_2 + x_3 \leq 5$
$3x_1 + 2x_2 - x_3 \leq 10$
$x_1 + 4x_2 + 3x_3 \leq 3$
$x_1, x_2, x_3 \geq 0$

By adding variables, describe a *feasible dictionary* corresponding to this primal LP. Furthermore, describe the basis feasible solution (BFS) corresponding to that dictionary, and a basis associated with that BFS. What is the value of the objective function at this BFS? [*5 marks*]

(b) On the dictionary obtained in (a), perform one improving pivoting step (of the simplex algorithm) to move to an improved BFS, by moving the variable x_1 into the basis, and moving some other variable out of the basis. Show the resulting feasible dictionary, and the BFS corresponding to it. What is the value of the objective function at this new BFS? [*6 marks*]

(c) Compute the dual LP for the LP described in (a). [*6 marks*]

(d) Define what is meant by a finite extensive form game of *perfect information*. [*3 marks*]

(e) Give an example of a frequently played finite extensive form game that is not a perfect information game. [*2 marks*]

(f) State Kuhn's Theorem for finite extensive form games of perfect information. [*3 marks*]

3. (a) Describe what it means for a 2-player zero-sum win-lose extensive form game of perfect information (not necessarily finite) and without any chance nodes, to be *determined*. [*3 marks*]

(b) Are all such games described in (a) determined? [*2 marks*]

(c) Consider the 2-player zero-sum matrix game specified by the following payoff matrix for player 1's (the row player's) payoff's:

$$\begin{bmatrix} 7 & 3 & 5 & 8 \\ 6 & 9 & 2 & 7 \\ 2 & 5 & 3 & 12 \end{bmatrix}$$

Compute a minimax profile of optimal (minmaximizer and maxminimizer, respectively) mixed strategies for the two players in this game. And furthermore, say what the precise game value is (for player 1). Show your work. [*7 marks*]

(d) Consider the following 2-player game: the two players first play a single zero-sum win-lose strategic game of "matching pennies" against each other. This game has the following payoff table

$$\begin{bmatrix} (1,-1) & (-1,1) \\ (-1,1) & (1,-1) \end{bmatrix}$$

After playing this game once, if player i wins (where i is either 1 or 2), then they play the following 2-player bimatrix game, given by the following payoff table:

$$\begin{bmatrix} (i,i) & (i-2,i+1) \\ (i+1,i-2) & (i-1,i-1) \end{bmatrix}$$

For each player, their total payoff in the entire game is then calculated as the sum total of their payoffs in the two consecutive games they played.

i. Describe how you can specify this game as a finite extensive form game of imperfect information. [*4 marks*]

ii. Find a *subgame-perfect* Nash Equilibrium for this game, <u>and</u> provide the expected payoff to each player under this Nash equilibrium. [*5 marks*]

(e) Consider an election with 3 candidates. Suppose voters are asked to order the 3 candidates, in a strict total order (no ties), according to their own preference. Suppose that we consider the following *majority* voting rule: if a candidate A, beats every other candidate C head-to-head, meaning that a strict majority of the voters prefer A to C in their own ordering, then we call such an A the *head-to-head majority winner* of the election.

Give an example of a set of voter preference orderings for the 3 candidates, under which there is no majority winner. [*4 marks*]