UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Friday 7th May 2010

14:30 to 16:30

MSc Courses

Convener: C. Stirling
External Examiners: R. Connor, R. Cooper, D. Marshall, T. Attwood

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

CALCULATORS MAY NOT BE USED IN THIS EXAMINATION

1. (a) Define, in precise mathematical terms, what it means for a mixed strategy profile $x = (x_1, \ldots, x_n)$ to be a Nash equilibrium for a finite game. [*3 marks*]

(b) For a finite game, define mathemetically the corresponding continuous Nash function $F : X \to X$, described in class, which maps the space of mixed strategy profiles to itself, and which has the property that the Nash equilibria of the game are precisely the fixed points of F. [*4 marks*]

(c) Consider the following bimatrix (i.e., 2-player) game:

$$\begin{bmatrix} (6,4) & (3,5) & (5,3) & (9,4) \\ (1,5) & (9,4) & (8,4) & (6,3) \\ (3,3) & (4,4) & (6,8) & (10,3) \end{bmatrix}$$

Compute a Nash equilibrium in this game, and furthermore compute the expected payoffs for both players under that NE.

Show your work. [*6 marks*]

(d) Explain how one can compute whether a given mixed strategy x_i for player i in a finite game is strictly dominated by some other mixed strategy. [*5 marks*]

(e) Consider the following primal LP:

Maximize $3x_1 - x_2$
Subject to:
$x_1 - x_2 \leq 1$
$x_2 - x_1 \leq -3$
$x_1, x_2 \geq 0$

Compute the dual to this LP. [*3 marks*]

(f) Solve both the primal and dual LP in the prior problem. Show your work. [*4 marks*]

2. (a) Describe informally what is meant by the assumption that "rationality is common knowledge", in a finite game, explaining both "rationality" and "common knowledge". Name a strategy illimination procedure that is justified by this assumption. [*4 marks*]

(b) Describe (by drawing the game tree) an example of a 2-player finite game in *extensive form*, which does not have any pure Nash Equilibria, and furthermore provide a mixed Nash equilibrium for the same game. [*4 marks*]

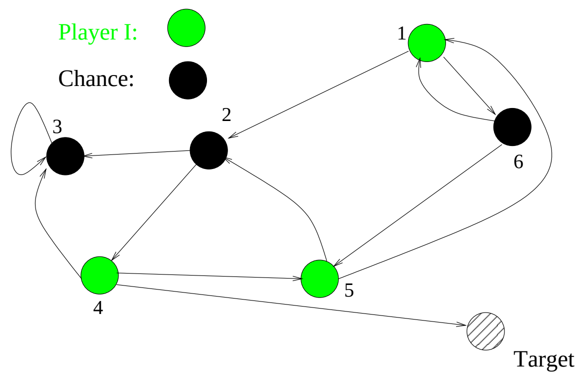
(c) Define what a (2-player, zero-sum) finitistic win-lose game on a graph is. [*3 marks*]

(d) Describe an iterative algorithm for computing who has a winning strategy starting at each vertex of a finitistic win-lose game on a graph. [*7 marks*]

(e) Are finitistic win-lose games on graphs *memorylessly determined*? Explain your answer. [*2 marks*]

(f) Consider a 2-player "reachability" game on a graph. This is given by $G = (V = V_1 \cup V_2, E, s, t)$, where the verticies V of the graph are partitioned into two sets V_1 and V_2, belonging to players 1 and 2, respectively, and E is the edge relation $E \subseteq V \times V$, $s \in V$ is the start vertex, and $t \in V$ is the target vertex. Player 1's objective is to reach the target node, t, whereas player 2's objective is to avoid it. Suppose there are n vertices, i.e., $n = |V|$. This is clearly a finitistic win-lose game on a graph.

Describe a Linear Program (LP) with one variable x_i for each vertex i of V, such that the (unique) optimal solution x^* of this LP has for each vertex i, $x_i^* = 1$ if and only if player 1 has a winning strategy in the game starting at vertex i. [*5 marks*]



3. (a) Consider the Markov Decision Process (MDP) depicted in the Figure above. This is an MDP with a reachability objective: the controller (i.e., the single Player I) wants to maximize the probability of reaching the Target state. Assume that for all "Chance" nodes, i.e., probabilistic nodes, the probability of outgoing transitions is specified by saying that there is equal probability of transitioning out of any of the outgoing edges out of each Chance node.

   Describe an LP for finding the optimal probability, starting from every state of this MDP, of reaching the Target. [*5 marks*]

   (b) Compute the optimal probability of reaching the Target starting from every state of the same MDP. [*5 marks*]

   (c) Explain *Braess's Paradox*, by giving examples of routing networks whose links have delays which are a function of the link congestion, and which exhibit this "paradox". Explain what the "paradox" is. [*5 marks*]

   (d) Recall that if C is a finite set of outcomes, and L is the set of all possible linear orderings of C, a *social choice function*, $f : L^n \rightarrow C$, aggregates the orderings on C given by n different individuals into a "social choice" outcome.

   Explain what it means to say that a social choice function f "can be strategically manipulated", and what it means to say that f is "incentive compatible" (or, equivalently, *strategy proof*). [*4 marks*]

   (e) Explain what it means for a social choice function to be a "dictatorship". [*2 marks*]

   (f) Recall that the Gibbard-Satterthwaite theorem says that if there are at least 3 choices, i.e., if $|C| \geq 3$, then any incentive compatible social choice function $f : L^n \rightarrow C$, $n \geq 1$, for which all outcomes are possible (i.e., f is a surjective (onto) function), is a dictatorship. Suppose instead there are only two outcomes possible, i.e., that $|C| = 2$, and furthermore that the number of individuals is n is odd and $n > 1$. Show that in this case there exists an incentive compatible social choice function which is not a dictatorship. [*4 marks*]