UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Friday 12 May 2006

09:30 to 11:15

MSc Courses

Convener: D K Arvind
External Examiners: J Carroll, M Hepple, E Hull, I Marshall

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

1. (a) For an n-player strategic form game, define what is meant by a *pure Nash Equilibrium* (a.k.a., a Nash equilibrium in pure strategies). [*3 marks*]

   (b) Suppose Γ is a finite n-player game in strategic form, where player i has m_i pure strategies.

   What is the greatest number of distinct pure Nash equilibria that can exist in such a game? What is the least number? Justify your answers. [*5 marks*]

   (c) Can more than one player have a strictly dominant strategy in a strategic form game? Justify your answer. [*3 marks*]

   (d) Consider the following bimatrix game (i.e., 2 player strategic form game):

$$\begin{bmatrix} (2,5) & (1,1) & (1,0) \\ (4,3) & (2,3) & (0,2) \\ (4,1) & (3,1) & (3,0) \end{bmatrix}$$

   Suppose player 1 plays mixed strategy $x_1 = (1/2, 1/2, 0)$ and player 2 plays mixed strategy $x_2 = (1/4, 0, 3/4)$. What is the expected payoff to each player under this mixed strategy profile? [*3 marks*]

   (e) Find a Nash equilibrium in the above game. [*3 marks*]

   (f) Are there finitely many Nash equilibria in the above game? Justify your answer. [*3 marks*]

   (g) Given an example of a finite two player zero-sum strategic game that does not have any pure Nash equilibrium and has exactly one Nash equilibrium. Justify your answer, showing why these conditions hold for the game you've described. [*5 marks*]

2. (a) Consider the following LP:

Maximize $x_1 + x_2 - 2x_3$
Subject to:
$x_1 + 3x_2 \leq 5$
$x_1 + x_3 \leq 6$
$x_2 + x_3 \leq 8$
$x_1, x_2, x_3 \geq 0$

Give the dual LP for this primal LP. [*5 marks*]

(b) Convert the LP given in part (a) to a feasible dictionary by adding slack variables. [*3 marks*]

(c) Give the *basic feasible solution* corresponding to the dictionary in the previous problem. [*2 marks*]

(d) Solve the LP from part (a) with any method you wish, providing an optimal solution or showing why none exists. Show your work. [*5 marks*]

(e) Describe how iterative elimination of strictly dominated strategies is done for a strategic form game. [*4 marks*]

(f) Explain what the "rationality of all players is common knowledge" assumption is, and why it justifies iteratively eliminating strictly dominated strategies. [*3 marks*]

(g) Can a pure strategy in a strategic form game that is not strictly dominated by any other pure strategy be strictly dominated by a mixed strategy? Justify your answer. [*3 marks*]

3. (a)

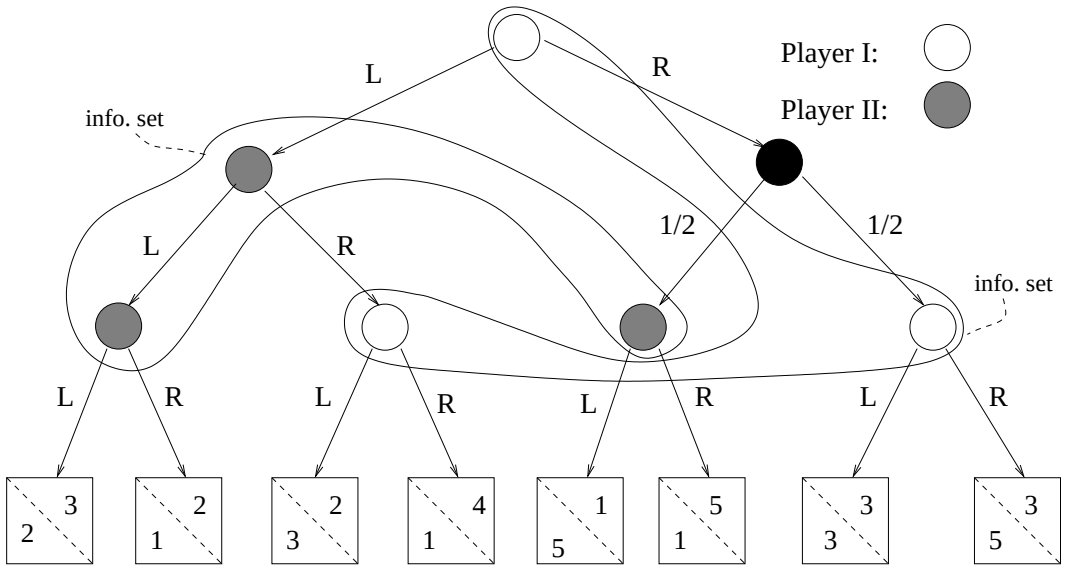

Figure 1:

Consider the extensive form game provided in the Figure 1. Construct a bimatrix represention of the strategic form game corresponding to this extensive form game. [*6 marks*]

(b) Find a Nash Equilibrium for the above extensive form game. Specify the strategy of each player in the Nash Equilibrium, and the expected payoff to each player under that NE. [*5 marks*]

(c) State Kuhn's Theorem about finite extensive form games. [*4 marks*]

(d) For an extensive form game with an infinite game tree, describe what a history oblivious payoff function is. [*3 marks*]

(e) Define what it means for a win-lose game on a graph to be memorylessly determined. [*4 marks*]

(f) Give an example of a history oblivious win-lose game on a graph which is not memorylessly determined. Explain your answer. [*3 marks*]