UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR11020 ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Saturday 4th May 2013

09:30 to 11:30

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

CALCULATORS MAY NOT BE USED IN THIS EXAMINATION

MSc Courses

Convener: B. Franke
External Examiners: T. Attwood, R. Connor, R. Cooper, S. Denham, T. Norman

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) Consider an n-player finite normal form game Γ, consisting of pure strategy sets S_i, $i = 1, \ldots, n$, and utility functions $u_i : S \to \mathbb{R}$, where $S = S_1 \times \ldots \times S_n$. For a mixed strategy profile $x = (x_1, \ldots, x_n)$, define in precise mathematical terms, what the *expected payoff* to player i is, under the mixed profile x. Then define in precise mathematical terms what a (mixed) *Nash equilibrium* of the game Γ is. [*5 marks*]

(b) Describe a way to use linear programming in order to determine, given a finite n-player normal form game, Γ, and a mixed strategy x_i for player i, whether x_i is strictly dominated by some other mixed strategy. [*5 marks*]

(c) Consider the following bimatrix (i.e., 2-player) game:

$$\begin{bmatrix} (2,3) & (0,0) & (5,-1) \\ (0,0) & (4,1) & (6,0) \\ (1,5) & (-1,4) & (7,1) \end{bmatrix}$$

Compute *all* of the NEs of this game, compute the expected payoffs for both players in each NE, and explain why there are no other NEs. [*9 marks*]

(d) Recall that if C is a finite set of outcomes, and L is the set of all possible linear orderings of C, a *social choice function*, $f : L^n \to C$, aggregates the orderings on C given by n different individuals into a "social choice" outcome.

The Gibbard-Satterthwaite theorem says that if there are at least 3 choices, i.e., if $|C| \geq 3$, then any *incentive compatible* social choice function $f : L^n \to C$, $n \geq 1$, which is a *surjective* (i.e, onto) function is a *dictatorship.*

Consider the social choice function, that picks the outcome c_i from $C = \{c_1, \ldots, c_k\}$, with $k \geq 3$, from the preferences of $n > 2$ agents, using the following method: for each outcome $c_j \in C$, it counts the number of agents, $N_j \leq n$, who rank outcome c_j at the top of their linear order, and it chooses the outcome c_i that maximizes N_i, and if there are ties, it chooses the smallest index i such that $N_i = \max_j N_j$.

Explain which properties given in the description of the Gibbard-Satterthwaite theorem this social choice function satisfies, and which ones it does not satisfy, and discuss any strategic implications that this has. Justify your answer. [*6 marks*]

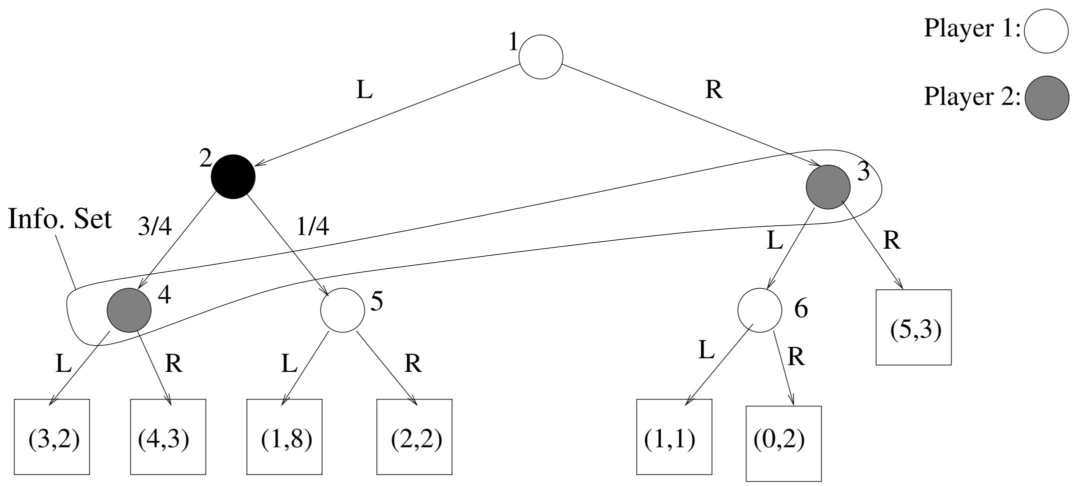

Figure 1:

2. (a) Consider the LP in primal form given by:

    Maximize $2x_1 + x_2$
    Subject to:
    $x_1 \leq 2$
    $x_1 + 3x_2 \leq 4$
    $x_1, x_2 \geq 0$

    Compute the dual LP for this primal LP. [*5 marks*]

   (b) Find an optimal feasible solution, and the optimal value of the objective, for the LP given in part (a). Prove that it is an optimal solution by using the dual LP. [*8 marks*]

   (c) Compute a Nash Equilibrium for the finite extensive form game given in Figure 1, and give the expected payoff for each player in the NE you compute. (The numbers next to each internal node are only the names of the nodes.) [*8 marks*]

   (d) State Kuhn's Theorem about finite extensive form games. [*4 marks*]

3. (a) Explain what a *Bayesian game* is, and describe why, when the space of possible types associated with every player is finite and the action spaces are finite, a Bayesian game can be viewed as a particular kind of finite extensive form game of imperfect information. [*6 marks*]

(b) In Edinburgh, flats are typically auctioned using a first-price (single-item) sealed bid auction run by ESPC. Suppose we want to model such an auction for a 2-bedroom flat in Edinburgh as a Bayesian game (where the bidders, who are known in advance, are the players). What would be a reasonable definition of the set of possible types, T_i, and actions A_i, associated with each player i, and what would be a reasonable prior joint probability distribution over types? Justify your answer. [*5 marks*]

(c) Suppose there are two items A and B, being auctioned, and two bidders, 1 and 2. The auction will use the VCG mechanism to calculate both the payments that each bidder will pay and the allocations of the items to the two bidders. We assume each bidder i has a valuation function $v_i(X)$ which defines how much (in British pounds) getting the subset $X \subseteq \{A, B\}$ is worth to player i. Suppose $v_1(\emptyset) = 0$, $v_1(\{A\}) = 6$, $v_1(\{B\}) = 5$, and $v_1(\{A, B\}) = 12$. Suppose $v_2(\emptyset) = 0$, $v_2(\{A\}) = 4$, $v_2(\{B\}) = 7$, and $v_2(\{A, B\}) = 15$.

Calculate the VCG allocations, and VCG payments by the two bidders, for this auction. Justify your answers.

(You can assume that the bidders will bid their true valuation functions since, as we know from class, the VCG mechanism is strategy proof.) [*8 marks*]

(d) For a finite normal form game, Γ, define a continuous function $F_\Gamma : X \to X$, which maps the set X of mixed strategy profiles of Γ to itself, and which has the property that the Nash equilibria of the game Γ are precisely the set of profiles $x \in X$ that are fixed points of F_Γ. [*6 marks*]