UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR11020 ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Tuesday 16 th May 2017

14:30 to 16:30

INSTRUCTIONS TO CANDIDATES

Answer any TWO of the three questions. If more than two questions are answered, only QUESTION 1 and QUESTION 2 will be marked.

All questions carry equal weight.

CALCULATORS MAY NOT BE USED IN THIS EXAMINATION

Year 4 Courses

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External Examiners: A. Cohn, A. Donaldson, S. Kalvala

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. Let A be the $(n \times m)$ integer payoff matrix that defines the payoffs for player 1 (the row player, who has n pure strategies) in a finite 2-player zero-sum matrix game.

   (a) Give an expression for the expected payoff for player 2 in this game, if player 1 plays a mixed strategy given by an n-dimensional vector x and player 2 plays a mixed strategy given by an m-dimensional vector y. [*3 marks*]

   (b) Define mathematically what a *minimax profile* for such a game is, and what the *minimax value* of such a game is. [*5 marks*]

   (c) Consider now the following specific (2×3) payoff matrix, A, for player 1 (row player) in a 2-player zero-sum game:

   $$A = \begin{bmatrix} 5 & 2 & 4 \\ 3 & 6 & 5 \end{bmatrix}$$

   Compute a minimax profile, and the minimax value, for this game. (Show your calculations.) [*7 marks*]

   (d) Consider the following linear program (LP):

   Maximize $5x_1 + 3x_2$
   Subject to:
   $2x_1 + 3x_2 \leq 8$
   $x_1 + x_2 \leq 6$
   $x_1 + 4x_2 = 4$
   $x_1, x_2 \geq 0$

   Compute the dual LP for this LP. [*5 marks*]

   (e) Solve the LP given in part (d) of this question, meaning, compute an optimal solution for it (if one exists), and (if so) compute the optimal value of the objective. (You can use any method you like to solve this LP.) [*5 marks*]

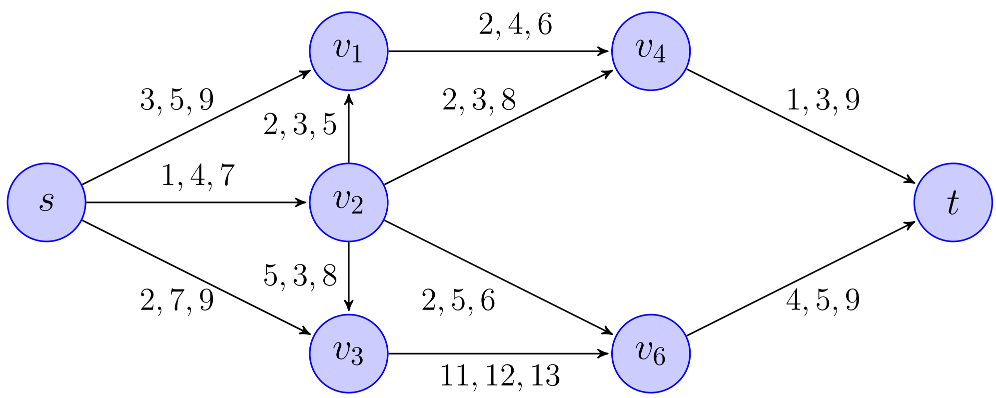

Figure 1: An atomic network congestion game

2. (a) Consider the following 2-player normal form (bimatrix) game:

$$\begin{bmatrix} (8,1) & (2,3) & (10,4) & (4,6) \\ (3,12) & (8,6) & (5,3) & (9,1) \\ (5,9) & (4,24) & (11,2) & (6,11) \end{bmatrix}$$

Compute all Nash equilibria in this game. Justify why what you have computed constitutes all Nash equilibria. [*8 marks*]

(b) Compute the price of anarchy for the game given in part (a) of this question. (It suffices for you give an explicit expression using fractions for this quantity, without computing the exact value of the expression.) [*7 marks*]

(c) Consider the *atomic network congestion game*, with three players, described by the directed graph in Figure 1.

In this game, every player i (for $i = 1, 2, 3$) needs to choose a directed path from the source s to the target t. Thus, every player i's set of possible actions (i.e., its set of pure strategies) is the set of all possible directed paths from s to t. Each edge e is labeled with a sequence of three numbers (c_1, c_2, c_3). Given a profile $\pi = (\pi_1, \pi_2, \pi_3)$ of pure strategies (i.e., s-t-paths) for all three players, the *cost* to player i of each directed edge, e, that is contained in player i's path π_i, is c_k, where k is the total number of players that have chosen edge e in their path. The total cost to player i, in the given profile π, is the sum of the costs of *all* the edges in its path π_i from s to t. Each player of course wants to minimize its own total cost.

Compute a pure strategy Nash Equilibrium in this atomic network congestion game. Compute the total cost to each player in the Nash equilibrium you have computed. [*10 marks*]

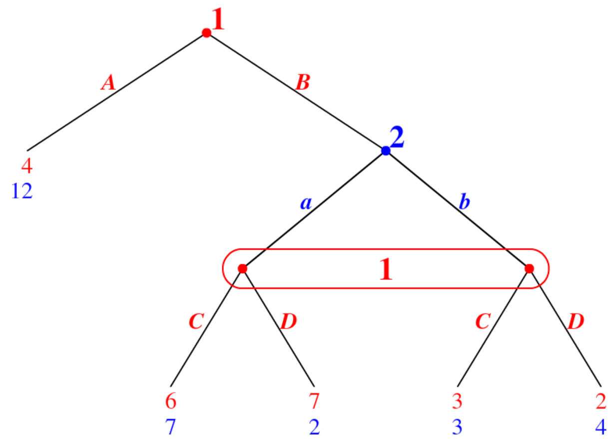

Figure 2: extensive form game

3. (a) Consider the 2-player extensive form game (of imperfect information) depicted in Figure 2. (At leaves, the top payoff is for player 1, and the bottom payoff is for player 2.) Compute both: (i) a subgame-perfect equilibrium for this game, and (ii) a Nash equilibrium which is *not* subgame perfect. [*10 marks*]

(b) In a VCG-based auction, four *identical* items are being auctioned simultaneously. Suppose there are two buyers (bidders), A and B, who provide their claimed valuation functions v_A and v_B, as follows; $v_x(j)$ denotes the value, in pounds, that player x has for receiving j items:

	valuation				
bidder x	$v_x(0)$	$v_x(1)$	$v_x(2)$	$v_x(3)$	$v_x(4)$
$x := A$	0	12	15	19	31
$x := B$	0	7	18	29	35

An allocation outcome for this auction is specified by a pair of numbers $j_A, j_B \in \{0, 1, 2, 3, 4\}$, such that j_A (j_B) is the number of (identical) items allocated to bidder A (respectively, bidder B), and such that $j_1 + j_2 \leq 4$. Each bidder will also be asked to pay a certain amount (in British pounds), p_A and p_B, respectively, for their allocation.

What are the VCG allocations, and VCG payments, for this auction? In other words, how many items will each bidder get, and what price will each pay for the items they get, if the VCG mechanism is used. [*8 marks*]

(c) Suppose that player utilities are given by their true valuation for the outcome minus the price they have to pay for that outcome. Explain why it is then reasonable to assume that both bidders will provide their true valuation functions when they know the VCG mechanism is being used to determine the allocation and payment outcome in such an auction. [*3 marks*]

(d) Give an example of a 2-player normal form game with exactly 3 Nash equilibria. Provide the game's payoff table, and all three of its different NEs. [*4 marks*]