AGTA Tutorial Sheet 1 (Week 2)

Please attempt all these questions before your tutorial.

1. Consider the "Guess Half the average game" described as the "food for thought" question on the last slide of Lecture 1. (Each player (indepedently) guesses a whole number between 1 to 1000. The goal of each of the $n > 1$ players is to guess a number that is closest to half the average of all the guessed numbers.)

   the all-1.

   We described two versions of that game, depending on which tie-breaking rule is used: in the first tie-breaking rule, all players who are closest to half the average split the payoff of 1 equally amongst themselves; in the second tie-break rule, all who are closest to half the average get the full payoff of 1.

   game 1: pareto optimal for any case as the total sum payoff is always 1

   game 2: pareto optimal if everyone chooses same

   Firstly, discuss how you would play each of these games, if you were playing them against the rest of the students in the AGTA course, and why. Are your assumptions realistic?

   Second, find all pure Nash Equilibria (NEs) in both versions of this game. In all cases, argue why the pure NEs that you have found are the only pure NEs in the game.

2. Consider the following 2-player finite strategic form game, G:

$$\begin{bmatrix} (7,3) & (6,3) & (5,5) & (4,7) \\ (4,2) & (5,7) & (8,6) & (5,8) \\ (6,1) & (3,8) & (2,4) & (5,9) \end{bmatrix}$$

   This is a "bimatrix", to be read as follows: Player 1 is the row player, and Player 2 is the column player. If the content of the bimatrix at row i and column j is the pair (a, b), then the payoff to Player 1, under the combination of pure strategies (i, j) is $u_1(i, j) = a$, likewise for Player 2 the payoff is $u_2(i, j) = b$.

   (1/4)*(2/3 * 7 + 1/3 * 6) + (1/2) * (2/3 * 4 + 1/3 * 5) + (1/4) * (2/3*6 + 1/3 * 3)
   =
   (1/4) * (20/3) + 1/2 * 13/3 + 1/4 * 15/3
   =
   20/12 + 26/12 + 15/12 = 61/12

   (a) Consider the mixed strategies $x_1 = (1/4, 1/2, 1/4)$ and $x_2 = (2/3, 1/3, 0, 0)$, for player 1 and 2, respectively. Here, e.g., player 1 is playing row 2 with probability 1/2, etc.
   What is the *expected payoff* to Player 1 under profile $x = (x_1, x_2)$?

   (b) Using what you have learned so far in lectures, see if you can find all the Nash Equilbria (pure or mixed) of the game G.