UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR11020 ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Tuesday 8th May 2018

14:30 to 16:30

INSTRUCTIONS TO CANDIDATES

Answer any TWO of the three questions. If more than two questions are answered, only QUESTION 1 and QUESTION 2 will be marked.

All questions carry equal weight.

CALCULATORS MAY NOT BE USED IN THIS EXAMINATION

Year 4 Courses

Convener: I. Murray
External Examiners: S. Rogers, A. Donaldson, S. Kalvala

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) Consider the finite 2-player zero-sum game specified by the following (3×2) payoff matrix, A, for player 1 (the row player):

$$A = \begin{bmatrix} 5 & 8 \\ 5 & 5 \\ 8 & 4 \end{bmatrix}$$

The entry $A_{i,j}$ of this matrix specifies the payoff to player 1 if it plays its i'th pure strategy (i'th row), and player 2 plays its j'th pure strategy (j'th column).

i. Specify a corresponding linear program (LP), such that the objective value of the LP in any optimal feasible solution is precisely the "minimax value" of this 2-player zero-sum game, and such that any optimal feasible solution for the LP also provides an optimal "maxminimizer" mixed strategy for player 2 (the column player), whose goal is to minimize the expected payoff of player 1. [*4 marks*]

ii. Calculate the minimax value for this game, and also calculate a Nash equilibrium for this game. [*7 marks*]

(b) Consider the following linear programming problem, given as a "feasible dictionary", with "basis" $\{x_1, x_2\}$:

Maximize: $5 + 6x_3 + 2x_4$

Subject to:

$$x_1 = 8 - 2x_3 - 3x_4$$
$$x_2 = 6 - 3x_3 - 7x_4$$

$$x_1 \geq 0 \ , \ x_2 \geq 0 \ , \ x_3 \geq 0 \ , \ x_4 \geq 0$$

i. What is the "basic feasible solution" (BFS) corresponding to the basis $\{x_1, x_2\}$ of this feasible dictionary? [*2 marks*]

ii. Apply one "pivot" step (of the simplex algorithm) to this feasible dictionary, in order to move the variable x_3 into the basis and move the variable x_2 out of the basis.
Show the resulting dictionary after this pivot step, including the resulting new objective function. [*6 marks*]

iii. Is the new dictionary obtained via this pivot step a *feasible* dictionary? Explain your answer. [*2 marks*]

iv. What is the optimal (maximum) feasible value for the objective of this LP? Explain your answer. [*4 marks*]

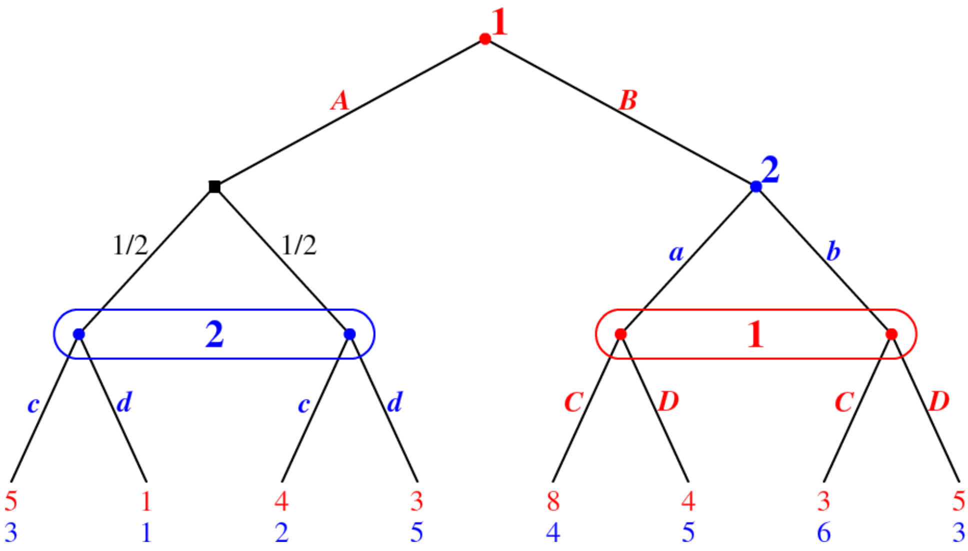

Figure 1: extensive form game

2. (a) Consider the following 2-player normal form (bimatrix) game:

$$\begin{bmatrix} (9,2) & (4,9) & (3,8) \\ (2,2) & (5,5) & (5,5) \\ (8,6) & (5,5) & (7,7) \end{bmatrix}$$

Compute all Nash equilibria (pure or mixed) in this game. Justify why what you have computed constitutes all NEs. [*5 marks*]

(b) Prove that for any positive integer $n \geq 1$, there exists a 2-player finite game with exactly n pure Nash equilibria and no other (mixed) Nash equilibria. [*10 marks*]

(c) Consider the 2-player extensive form game (of imperfect information) depicted in Figure 1. (At leaves, the top payoff is for player 1, and the bottom payoff is for player 2.) Compute a subgame-perfect equilibrium for this game. [*10 marks*]

3. (a) Recall that if C is a finite set of outcomes, and L is the set of all possible linear orderings of C, a *social choice function*, $f : L^n \rightarrow C$, aggregates the orderings on C given by n different individuals (voters) into a "social choice" outcome.

Explain what it means to say that social choice function f can be "strategically manipulated" by voter $i \in \{1, \ldots, n\}$. [*3 marks*]

(b) State the Gibbard-Satterthwaite Theorem (regarding the existence/non-existence of incentive-compatible social choice functions with certain properties). [*4 marks*]

(c) In a VCG-based auction, three distinct Faberge eggs are all being auctioned simultaneously. Let us denote the set of these three eggs as $E = \{1, 2, 3\}$. Suppose there are three potential buyers (bidders), A, B, and C, who provide their claimed valuation functions v_A, v_B, and v_C for this auction. For each subset $E' \subseteq E$ of eggs, $v_x(E')$ denotes the valuation in (millions of) British pounds that player x has for obtaining the subset E' of the eggs:

	valuation							
bidder x	$v_x(\{\})$	$v_x(\{1\})$	$v_x(\{2\})$	$v_x(\{3\})$	$v_x(\{1,2\})$	$v_x(\{1,3\})$	$v_x(\{2,3\})$	$v_x(\{1,2,3\})$
$x := A$	0	9	6	10	16	20	17	24
$x := B$	0	5	8	9	14	21	15	23
$x := C$	0	1	11	1	12	2	12	13

A VCG allocation outcome for this auction is specified by a partition of the eggs $\{1, 2, 3\}$ into three disjoint subsets $E_A, E_B, E_C \subseteq \{1, 2, 3\}$, such that E_A (E_B and E_C) is the set of eggs allocated to bidder A (respectively, to bidder B and to bidder C). Note that some of these sets may be empty. Each bidder will also be asked to pay a certain amount (in millions of British pounds), p_A, p_B, and p_C, respectively, for their allocation, by the VCG mechanism.

(You can assume the bidders will bid their true valuation functions, because the VCG mechanism is incentive compatible.)

What are the VCG allocations, and VCG payments, for this auction? In other words, which items will each bidder get, and what price will each pay for the items they get, if the VCG mechanism is used? [*15 marks*]

(d) Suppose instead that a simultaneous auction is being conducted for a set E of hundreds of items, with hundreds of bidders. Suppose that bidders are all "single-minded", meaning that for each bidder x, there is a particular subset $E_x \subseteq E$, and a positive real value $w_x > 0$, such that for all $E' \subseteq E$, if $E_x \subseteq E'$, then $v_x(E') = w_x$, and otherwise (if $E_x \not\subseteq E'$) then $v_x(E') = 0$. Thus, in order to completely specify their valuation functions, each bidder x simply provides their set E_x and their value $w_x > 0$ for that set.

Explain why for such a large auction, even if all bidders are known to be single-minded and will specify their valuations compactly as above, it may not be a good idea to use the VCG mechanism. [*3 marks*]