UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Monday 30th April 2012

09:30 to 11:30

MSc Courses

Convener: B. Franke
External Examiners: T. Attwood, R. Connor, R. Cooper, D. Marshall, M. Richardson

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

CALCULATORS MAY NOT BE USED IN THIS EXAMINATION

1. (a) Define what it means for a finite 2-player normal form (i.e., strategic form) game to be *symmetric*. Then define an *evolutionarily stable strategy* (ESS) in a finite symmetric 2-player normal form game. [*5 marks*]

(b) Does every finite symmetric 2-player normal form game have an ESS? Justify your answer. [*3 marks*]

(c) Give an example of a 2-player normal form game where iterated elimination of *weakly* dominated strategies may lead to two distinct reduced games (depending on which pure strategy is eliminated in each iteration), where each of the two reduced games have a unique and pure NE, but such that the original game had at least two distinct pure NEs. Show why your example has these properties. [*6 marks*]

(d) Consider the following bimatrix (i.e., 2-player) game:

$$\begin{bmatrix} (3,5) & (8,2) & (2,8) & (9,5) \\ (9,5) & (2,9) & (9,2) & (6,5) \\ (5,5) & (5,5) & (5,6) & (8,3) \end{bmatrix}$$

Compute a Nash equilibrium in this game, and furthermore compute the expected payoffs to both players under that NE. [*7 marks*]

(e) Does the game given in the previous part of this question (part (d)) have any other NEs besides the one you have computed?

If not, explain why not, and if so, compute a different NE. Fully justify your answer in either case. [*4 marks*]

2. (a) Describe the the Fourier-Motzkin elimination algorithm for solving LPs. (You don't need to give pseudo-code for the algorithm. Just describe it in sufficient enough detail to make clear how it works.) Also explain why the method can, in the worst case, be very slow. [*5 marks*]

(b) Consider the following LP:

Minimize x_4
Subject to:
$3x_1 + 7x_2 + 8x_3 \leq x_4$
$9x_1 + 5x_2 + 7x_3 \leq x_4$
$5x_1 + 6x_2 + 7x_3 \leq x_4$
$x_1 + x_2 + x_3 = 1$
$x_1, x_2, x_3 \geq 0$

Use any method you wish in order solve this LP, meaning find an optimal feasible solution for this LP, and compute the optimal objective value.

Show your work. [*7 marks*]

(c) Find the dual for the LP given in the previous question (part (b)). [*5 marks*]

(d) Define what is a *pure strategy* for a player i in an n-player extensive form game of perfect information. [*3 marks*]

(e) Recall that an extensive form game of perfect information is called a win-lose game if it is a 2-player zero-sum game, with no random nodes, and where the only payoffs (utilities) that players can receive under combinations of pure strategies is either 1 (win) or -1 (lose).

Describe what it means for a win-lose extensive form game of perfect information to be *determined*. Are all finite extensive form win-lose games of perfect information determined? [*5 marks*]

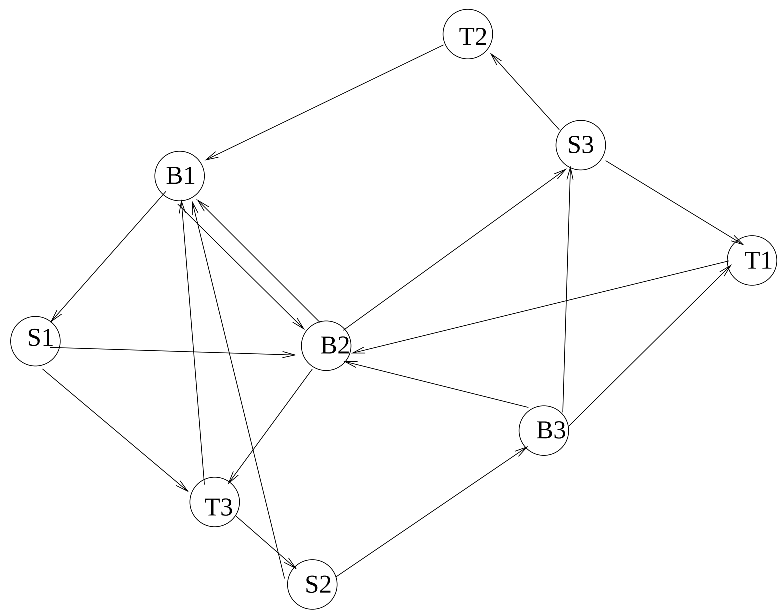

Figure 1: Atomic network congestion game

3. (a) Consider the atomic *network congestion game*, with three players, described by the directed graph in Figure 1.

   In this game, player i (for every $i = 1, 2, 3$) needs to obtain as a "resource" a directed path from source Si to target Ti (any path will do). Thus, player i's set of possible actions (i.e., its set of pure strategies) are all possible directed paths from Si to Ti.

   Given a profile (s_1, s_2, s_3) of strategies for all the players, the *cost* to player i is the *total* number of players that have chosen that edge in their chosen path (i.e., in their chosen strategy). (Note that directed edges going in opposite directions between two nodes are considered distinct edges.) The total cost to player i is the sum of the costs (in the given profile of strategies) of *all* the edges that it has chosen in its path s_i from Si to Ti.

   Each player of course wants to minimize their own total cost.
   Find a pure strategy NE in this atomic network congestion game. Also say what the total cost is to each player under this pure strategy NE. [*6 marks*]

   (b) Is there any other pure strategy NE in the congestion game described in part (a)? Explain your answer. [*3 marks*]

   (c) Define what a *pure strategy* is for a player in a *Bayesian game*. [*3 marks*]

   (d) Suppose we want to model a second-price single-item sealed bid auction for a single used iPad tablet computer as a Bayesian game (where the bidders, known in advance, are the players). What would be a reasonable definition of the set of possible types, T_i, associated with each player (bidder) i, and

what would be a reasonable prior probability distribution over types? Justify your answer.
(I'm looking for any reasonable answers, with adequate justification.) [*5 marks*]

(e) Suppose there are two items A and B, being auctioned, and two bidders, 1 and 2. The auction will use the VCG mechanism to calculate both the payments that each bidder will pay and the allocations of the items to the two bidders. We assume each bidder i has a valuation function $v_i(X)$ which defines how much (in British pounds) getting the subset $X \subseteq \{A, B\}$ is worth to player i. Suppose $v_1(\emptyset) = 0$, $v_1(\{A\}) = 6$, $v_1(\{B\}) = 5$, and $v_1(\{A, B\}) = 12$. Suppose $v_2(\emptyset) = 0$, $v_2(\{A\}) = 9$, $v_2(\{B\}) = 3$, and $v_2(\{A, B\}) = 10$.

Calculate the VCG allocations, and VCG payments by the two bidders, for this auction. Explain why your answers are correct.

(You can assume that the bidders will bid their true valuation functions since, as we know from class, the VCG mechanism is incentive compatible.) [*8 marks*]