UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Friday 15 May 2009

14:30 to 16:30

MSc Courses

Convener: A Smaill
External Examiners: R Connor, R Cooper, D Marshall, I Marshall

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

CALCULATORS MAY NOT BE USED IN THIS EXAMINATION

1. (a) Define what it means for a general n-player game to be *zero-sum*. [*3 marks*]

(b) Define what a *minmaximizer* mixed strategy is for a player (who is trying to maximize its own payoff) in a finite 2-player zero-sum game. [*4 marks*]

(c) Consider the following bimatrix game:

$$\begin{bmatrix} (1,9) & (7,1) & (4,0) & (1,3) \\ (5,2) & (2,3) & (2,4) & (2,5) \\ (6,1) & (4,7) & (3,1) & (4,0) \end{bmatrix}$$

Compute a Nash equilibrium in this game. Show all your work, i.e., show how you computed it. [*8 marks*]

(d) For a finite normal form game, state what it means for a mixed strategy profile x to be *Pareto efficient* (or *Pareto optimal*). [*3 marks*]

(e) Give an example of a finite 2-player game which has infinitely many Nash Equilibria (NE), but such that none of its NEs are Pareto optimal. Justify why your example satisfies both these conditions. [*5 marks*]

(f) For normal form games, is there a known polynomial time algorithm for deciding whether a given pure strategy is strictly dominated by a mixed strategy? [*2 marks*]

2. (a) Consider the LP in primal form given by:

Maximize $x_0 + 5x_1 + 4x_2$
Subject to:
$3x_0 + 2x_1 \leq 9$
$x_1 + 3x_2 \leq 4$
$x_0, x_1, x_2 \geq 0$

By adding slack variables, provide the *feasible dictionary* associated with this LP. [*4 marks*]

(b) On the resulting feasible dictionary from part (a), perform a single pivoting step that leads to a new basis whose corresponding basic feasible solution has an improved value for the objective function, better than the original basis. Show the resulting feasible dictionary. [*6 marks*]

(c) Give the dual LP for the LP given in Part (a). [*6 marks*]

(d) By adding a new variable, z, convert the LP in part (a) to an equivalent LP where the objective function is z. [*3 marks*]

(e) Use Fourier-Motzkin elimination to eliminate the variable x_0 from the following LP, and show the resulting LP.

Maximize x_3
Subject to:
$x_2 + 3x_1 \leq x_3$
$x_2 + x_1 \geq x_3$
$3x_0 + 2x_1 \leq 9$
$x_0 + 3x_2 \leq 4$
$x_0, x_1, x_2, x_3 \geq 0$

[*6 marks*]

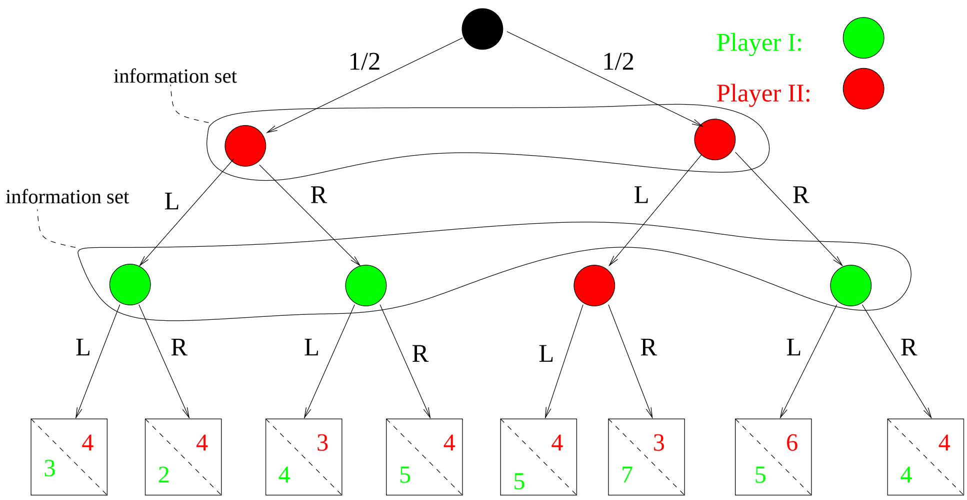

Figure 1: Game tree for part 3(c)

3. (a) Define what is meant by a memoryless strategy in a game on a graph. [*3 marks*]

(b) Consider the following winning condition for a win-lose game on a graph: there are 2 specially marked vertices $\{a, b\}$ (which are not dead ends). Player 1's goal is to either hit the vertex a infinitely often, or to not hit b infinitely often, during the play (i.e., in either case it wins). Player 2's goal is to make sure neither of these happens (i.e., it wins in all other cases).

Are all 2-player such win-lose games on graphs with this winning condition memorylessly determined? Justify your answer. [*5 marks*]

(c) Find a Nash equilibrium for the extensive form game described in Figure 1, and compute the expected payoff to each player under this Nash Equilibrium. [*9 marks*]

(d) Does every finite extensive form game of perfect information have a pure strategy Nash equilibrium? Justify your answer. [*3 marks*]

(e) Consider the 2-player zero-sum win-lose game graph depicted in Figure 2. In this game graph two nodes are marked as "Goal" states. Player II's aim is to infinitely often hit one of the two goal states $\{4, 7\}$, during play, and player I's goal is to not allow this to happen. Compute the set of vertices of the graph starting from which player II has a winning strategy in this game. Does player II always have a memoryless optimal strategy in such 2-player zero-sum win-lose games on graphs (where the goal is to infinitely often hit one of a specified set of goal nodes)? [*5 marks*]

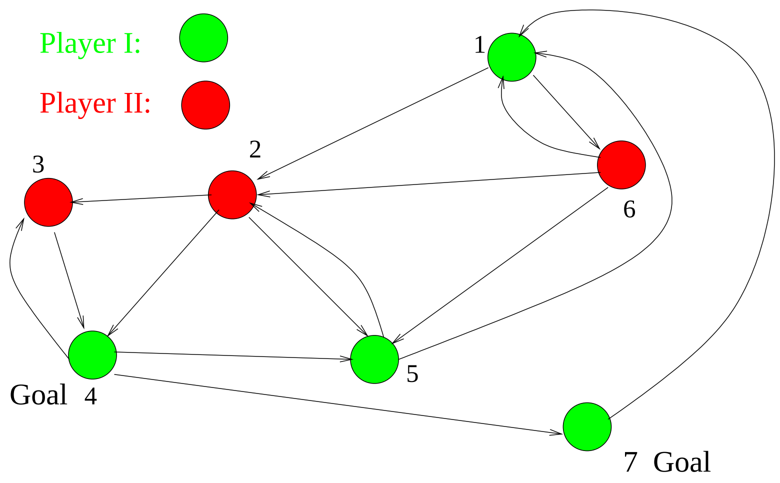

Figure 2: Game graph for part 3(e)