UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFR11020 ALGORITHMIC GAME THEORY AND ITS APPLICATIONS

Wednesday 30 th April 2014

09:30 to 11:30

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

CALCULATORS MAY NOT BE USED IN THIS EXAMINATION

MSc Courses

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External Examiners: A. Burns, S. Denham, P. Healey, T. Norman

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

1. (a) For an n-player finite normal form game, define what a *mixed strategy*, x_i, for player i is, and then define what a *Nash equilibrium* (NE), $x^* = (x_1^*, \ldots, x_n^*)$, in mixed strategies is. [*2 marks*]

(b) Consider the following bimatrix (i.e., 2-player) game:

$$\begin{bmatrix} (1,1) & (7,2) & (6,0) & (0,6) \\ (7,1) & (4,2) & (0,7) & (7,0) \\ (3,1) & (3,2) & (3,3) & (3,3) \end{bmatrix}$$

Compute all the Nash equilibria in this game, and justify why they constitute all NEs. Furthermore, compute the expected payoffs to both players under each NE. [*7 marks*]

(c) Consider the following Linear Program (LP), with 4 variables: x_1, x_2, x_3, v. Use any method you know of in order to solve this LP, meaning find an optimal feasible solution for this LP, and compute the optimal objective value.

Show your work.

Minimize v
Subject to:
$x_1 + 5x_2 + 6x_3 \leq v$
$7x_1 + 3x_2 + 6x_3 \leq v$
$3x_1 + 4x_2 + 5x_3 \leq v$
$x_1 + x_2 + x_3 = 1$
$x_1, x_2, x_3 \geq 0$

[*8 marks*]

(d) Find the dual for the LP given in the previous question (part (c)). [*5 marks*]

(e) Does there exist a 2-player finite normal form game which has exactly 2 Nash Equilibria? Justify your answer. [*3 marks*]

2. (a) Define what a *pure strategy* is for a player in a *Bayesian game*. [*3 marks*]

(b) Suppose there are two items A and B, being auctioned, and two bidders, 1 and 2. The auction will use the VCG mechanism to calculate both the payments that each bidder will pay and the allocations of the items to the two bidders. We assume each bidder i has a valuation function $v_i(X)$ which defines how much (in British pounds) getting the subset $X \subseteq \{A, B\}$ is worth to player i. Suppose $v_1(\emptyset) = 0$, $v_1(\{A\}) = 9$, $v_1(\{B\}) = 12$, and $v_1(\{A, B\}) = 20$. Suppose $v_2(\emptyset) = 0$, $v_2(\{A\}) = 13$, $v_2(\{B\}) = 11$, and $v_2(\{A, B\}) = 22$.

Calculate the VCG allocations, and VCG payments by the two bidders, for this auction. Explain why your answers are correct.

(You can assume that the bidders will bid their true valuation functions since, as we know from class, the VCG mechanism is incentive compatible.) [*7 marks*]

(c) Explain *Braess's Paradox*, by giving examples of routing networks whose links have delays which are a function of the link congestion, and which exhibit this "paradox". Explain what the "paradox" is. [*5 marks*]

(d) Recall that if C is a finite set of outcomes, and if L is the set of all possible linear orderings of the set C, then a *social choice function* is a function $f : L^n \to C$ that aggregates the orderings on C given by n different individuals into a "social choice" outcome.

Explain what it means to say that a social choice function f "can be strategically manipulated", and what it means to say that f is "incentive compatible" (or, equivalently, "*strategy proof*"). [*4 marks*]

(e) Does there exist a finite extensive form game of perfect information which has a pure Nash Equilibrium that is <u>not</u> a *subgame perfect Nash Equilibrium*? If so, construct an example of such a game, and show a pure NE in that game which is not subgame perfect. Otherwise, if you believe no such game exists, explain why not. [*6 marks*]

3. (a) Define what a *Evolutionarily Stable Strategy* is in a finite 2-player symmetric normal form game. [*3 marks*]

(b) Consider a 2-player "reachability" game on a graph. This is given by $G = (V = V_1 \cup V_2, E, s, t)$, where the verticies V of the graph are partitioned into two sets V_1 and V_2, belonging to players 1 and 2, respectively, and E is the edge relation $E \subseteq V \times V$, $s \in V$ is the start vertex, and $t \in V$ is the target vertex. Player 1's objective is to reach the target node, t, whereas player 2's objective is to avoid it. Suppose there are n vertices, i.e., $n = |V|$. This is clearly a finitistic win-lose game on a graph.

Describe a Linear Program (LP) with one variable x_i for each vertex i of V, such that the (unique) optimal solution x^* of this LP has for each vertex i, $x^*_i = 1$ if and only if player 1 has a winning strategy in the game starting at vertex i. [*7 marks*]

(c) (i) Define what it means for a win-lose game on a graph to be memorylessly determined. [*3 marks*]

(ii) Give an example of a history oblivious 2-player win-lose game on a graph in which neither player has a pure and memoryless winning strategy. Explain your answer. [*2 marks*]

(d) Recall that an atomic *network congestion game* with n players is specified by a directed graph, where each player i needs to obtain, as a "resource", a directed path from a given source vertex S_i to a given target vertex T_i. Thus, player i's set of possible actions (i.e., its set of pure strategies) are all possible directed paths from S_i to T_i.

Suppose there is indeed a directed path from S_i to T_i for all players i.

Suppose that, given a profile $s = (s_1, s_2, \ldots, s_n)$ of strategies for all the players, the *cost* to player i in the profile s is the sum, over all edges e that are in its chosen path s_i from S_i to T_i, of the *total* number of players who have chosen that edge e in their chosen path (i.e., in their chosen strategy). Each player of course wants to minimize their own total cost.

In the lectures we gave a classic theorem (due to Rosenthal) explaining that in such an atomic Network congestion game there always exists a pure Nash Equilibrium, and the proof also gave us an algorithm for computing a pure Nash Equilibrium.

Describe the algorithm for computing a pure NE in such an atomic network congestion game. [*7 marks*]

(e) Is there always a *unique* pure NE in an atomic network congestion game? Explain your answer. [*3 marks*]