METHODS FOR CAUSAL INFERENCE: ASSIGNMENT

This is a summative assignment (marked and evaluated, counting towards 20% of the student's final mark)

1. [**7 marks**] This exercise concerns some basic notions of probability theory.

   (a) Using the definition of covariance and Pearson correlation, prove in general that covariance and Pearson correlation vanish between two random variables X and Y that are independent. [**3 mark**]

   (b) In the lectures, we stated that two random variables X and Y can be dependent but have zero Pearson correlation. Show this for the following probability distribution [**4 mark**]:

   $$P(X = x) = 1/3 \ , \ \text{for } x \in \{-1, 0, 1\} \text{ and } Y = X^2. \tag{1}$$

   Remark: It is possible to construct the joint distribution $P(X, Y)$ uniquely in this case.

2. [**10 marks**] Create a simulation in Python or R for the two scenarios in Fig. 1. On the left hand side, the treatment variable is dependent of the confounder (simulation 1), and on the right hand side the treatment variable is independent of the confounder (simulation 2). For simplicity, in both cases consider (i) a normal distribution to generate continuous data for the confounder, (ii) a binomial distribution to generate binary data from the treatment variable (note, in simulation 2 the values of the treatment variable have to depend on the confounder in some way), and (iii) simulate the outcome as a linear combination of treatment and confounder $Y = \alpha + \beta_t t + \beta_x x$. Without loss of generality, choose $\alpha = 2$, $\beta_t = 6$, $\beta_x = -3$. Choose any level of noise you wish for each of the variables.

   (a) In simulation 1, fit the generated data to a linear model and show that ignoring the confounder leads to an incorrect estimate of the causal effect size of treatment t on outcome y. Show that including the confounder yields the correct causal effect (*i.e.* , 6 within 2σ). [**5 marks**]

   (b) In simulation 2, fit the generated data to a linear model with an without W and show that the causal effect size of treatment t on outcome y is not affected by it. [**5 marks**]

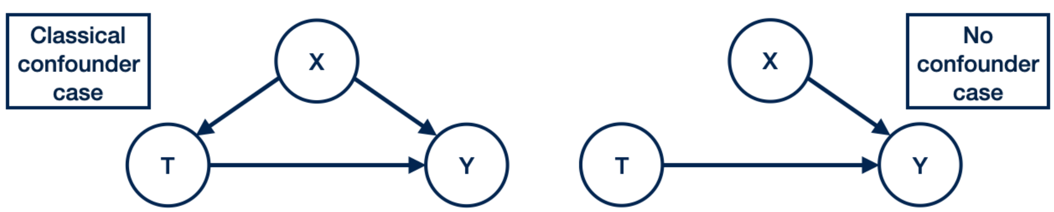

FIG. 1: Left: Confounding case. Right: No confounding case (treatment is randomised).

3. [**13 marks**] Consider the causal graph, which is directed and acyclic, in Fig. 2 below.

   (a) Give all sets of variables that d-separate X_0 and Y, and explain why this is the case [**3 marks**].
   Note: Answers without explanations will not receive any marks.

   (b) Are X_0 and X_2 independent conditional on X_3? Why (not)? [**2 marks**]

   (c) Which arrows in the graph can be reversed without being detected by any statistical test? [**3 marks**]
   Hint: think about equivalence classes of causal graphs.

   (d) Write down and explain all adjustment sets for the causal effect of T on Y. [**4 marks**]

   (e) Suppose X_1 is unobserved. What is the smallest adjustment set for the causal effect of T on Y? [**1 mark**]

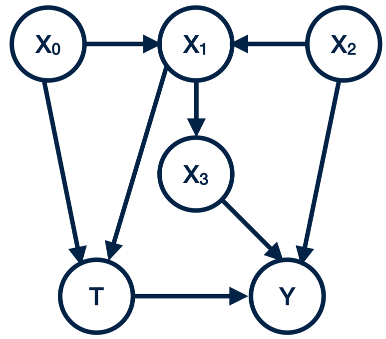

FIG. 2: DAG for question 3