Natural Language Understanding, Generation, and Machine Translation

Lecture 3: Conditional Language Modeling with *n*-grams

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Administrative update

- More guidance about prerequisites on piazza and in learn. **Take this seriously.**
- My office hours: Thursdays 1-2pm in Absorb Cafe (in Appleton Tower by the lecture theatres).
- Please sign up for piazza.

Revision

- Language models
- n-gram Language models

Conditional language models

Required, optional, and revision readings are listed on learn.

Revision

Predict the next word!

Predict the next word!

Summer is hot winter is _____

Predict the next word!

She is drinking a hot cup of _____

Predict the next word!

In the park I saw a _____

Predict the next word!

In the park I saw a _____

Image captioning

A language model is a probabilistic model of strings

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Example: Train a probabilistic model from CNN Business Headlines.

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- More than 1 million people subscribe to this electric toothbrush startup
- Heart drug recall expanded again

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Source: http://aiweirdness.com/

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- Star Wars Episode IX Has New Lime Blazer
- Coca-Cola is Scanning Your Messages for Big Chinese Tech
- Amazon is Recalling 1 Trillion Jobs

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Conditional language models have many uses

There are many, many applications where we want to predict strings *conditioned on some input*:

- speech recognition: condition on speech signal
- machine translation: condition on text in another language
- text completion: condition on the first few words of a sentence
- optical character recognition: condition on an image of text
- image captioning: condition on an image
- grammar checking: condition on surrounding words

Applications of Language Modeling

Machine translation:

- word ordering: $P(\text{the cat is small}) > P(\text{small the is cat})$;
- word choice: $P(\text{walking home after school}) > P(\text{walking house after school})$.

Grammar checking:

- word substitutions: $P(\text{the principal resigned}) > P(\text{the principle resigned})$;
- agreement errors: $P(\text{the cats sleep in the basket}) > P(\text{the cats sleeps in the basket})$.

DISCLAIMER: Notation is not universally consistent!

In each lecture: notation will be consistent. Variables named.

If you find something confusing or inconsistent, PLEASE ASK! Someone else also found it confusing or inconsistent.

Across lectures: notation will be similar, but may not be identical.

Expect notation to be **internally consistent** in an individual lecture or paper.

In general: there is no universally agreed upon notation for any of this stuff. Different fields and even subfields have different conventions, but even they tend to vary.

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tl;dr version: notation is a kind of language.

Language modeling as probabilistic prediction

Given a finite vocabulary V, we want to define a probability distribution $P : V^* \to \mathbb{R}_+$.

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Revision questions:

- What is the sample space? strings that have any length consisting the symbols of vocabulary V
- What might be some useful random variables?
- What constraints must P satisfy? 0<P<1, the sum of output is 1

How to derive an n-gram language model

Let w be a sequence of words. Let $|w|$ be its length and let w_i be its ith word. So, $w = w_1 \ldots w_{|w|}$.

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Use the chain rule:

$$
\begin{aligned}
P(w_1 \dots w_{|w|}) = & P(W_1 = w_1) \times \\
& P(W_2 = w_2 \mid W_1 = w_1) \times \\
& \dots \\
& P(W_{|w|} = w_{|w|} \mid W_1 = w_1, \dots, W_{k-1} = w_{|w|-1}) \\
& P(W_{|w|+1} = \langle \textsc{Stop} \rangle \mid W_1 = w_1, \dots, W_k = w_{|w|})
\end{aligned}
$$

Note: $\langle \textsc{Stop} \rangle$ is a symbol not in V.

Written more concisely

$$
\begin{aligned}
P(w_1 \ldots w_{|w|}) = & P(w_1)\times \\
& P(w_2 \mid w_1)\times \\
& \ldots \\
& P(w_{|w|} \mid w_1, \ldots, w_{|w|-1}) \\
& P(\langle \textsc{Stop} \rangle \mid w_1, \ldots, w_{|w|}) \\
= & \prod_{i=1}^{|w|+1} P(w_i | w_1, \ldots, w_{|w|})
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Defines a *joint distribution* over infinite sample space in terms of *conditional distributions*, each over finite sample spaces (but with potentially infinite history!)

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n-gram models make all terms finite with a Markov assumption

$$P(w_i \mid w_1, \ldots, w_{i-1}) \sim P(w_i \mid w_{i-n+1}, \ldots, w_{i-1})$$

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What is $P(w_i \mid w_{i-n+1}, \ldots, w_{i-1})$?

Given $w_{i-n+1}, \ldots, w_{i-1}$, P is a probability distribution, hence:

Probabilities must be non-negative $\qquad P : V \to \mathcal{R}_+$

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... and all sum to one $\quad \sum_{w \in V} P(w \mid w_{i-n+1}, \ldots, w_{i-1}) = 1$

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Any function satisfying these constraints is a probability distribution! Let's define one.

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... and all sum to one	$\sum_{w \in V} P(w \mid w_{i-n+1}, \ldots, w_{i-1}) = 1$

Any function satisfying these constraints is a probability distribution! Let's define one.

Simplest idea: Since the number of $P(w_i \mid w_{i-n+1}, \ldots, w_{i-1})$ terms is finite, let each one be a parameter (i.e. a real number) in a table indexed by $w_{i-n+1}, \ldots, w_i$.

n-gram probabilities can be estimated by counting

We can get maximum likelihood estimates for the conditional probabilities from n-gram counts in a corpus:

$$P(w_2|w_1) = \frac{n_{(w_1,w_2)}}{n_{(w_1)}} \qquad P(w_3|w_1,w_2) = \frac{n_{(w_1,w_2,w_3)}}{n_{(w_1,w_2)}}$$

But building good n-gram language models can be difficult:

- the higher the n, the better the performance
- but most higher-order n-grams will never be observed—are these *sampling zeros* or *structural zeros*? most is sampling zero
- good models need to be trained on billions of words
- this entails large memory requirements
- smoothing and backoff techniques are required.

sampling zero: it appears but not sampled
structural zero: it shouldn't appear in the language

Using n-gram Language Models

If we have a sequence of words $w_1 \dots w_k$ then we can use the language model to predict the next word w_{k+1}:

$$\hat{w}_{k+1} = \underset{w_{k+1}}{\operatorname{argmax}} P(w_{k+1} | w_1 \dots w_k)$$

Being able to predict the next word is useful for applications that process input in real time (word-by-word).

Conditional language models

$p(yellow)$ $\quad$ $1 - p(yellow)$

$p(yellow)?$

$p(yellow)$?

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$$p(data) = p(yellow)^7 \times [1 - p(yellow)]^3$$

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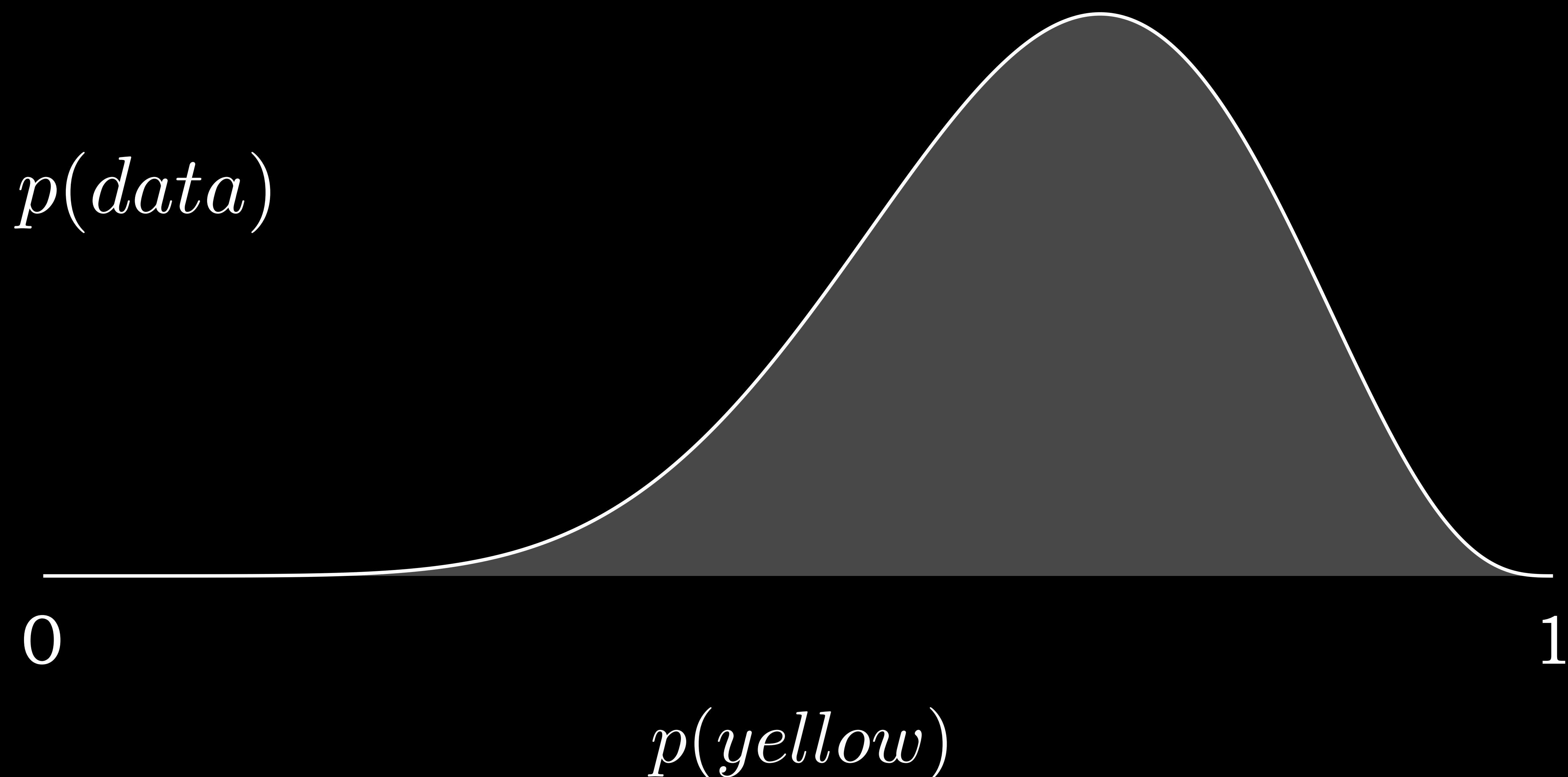
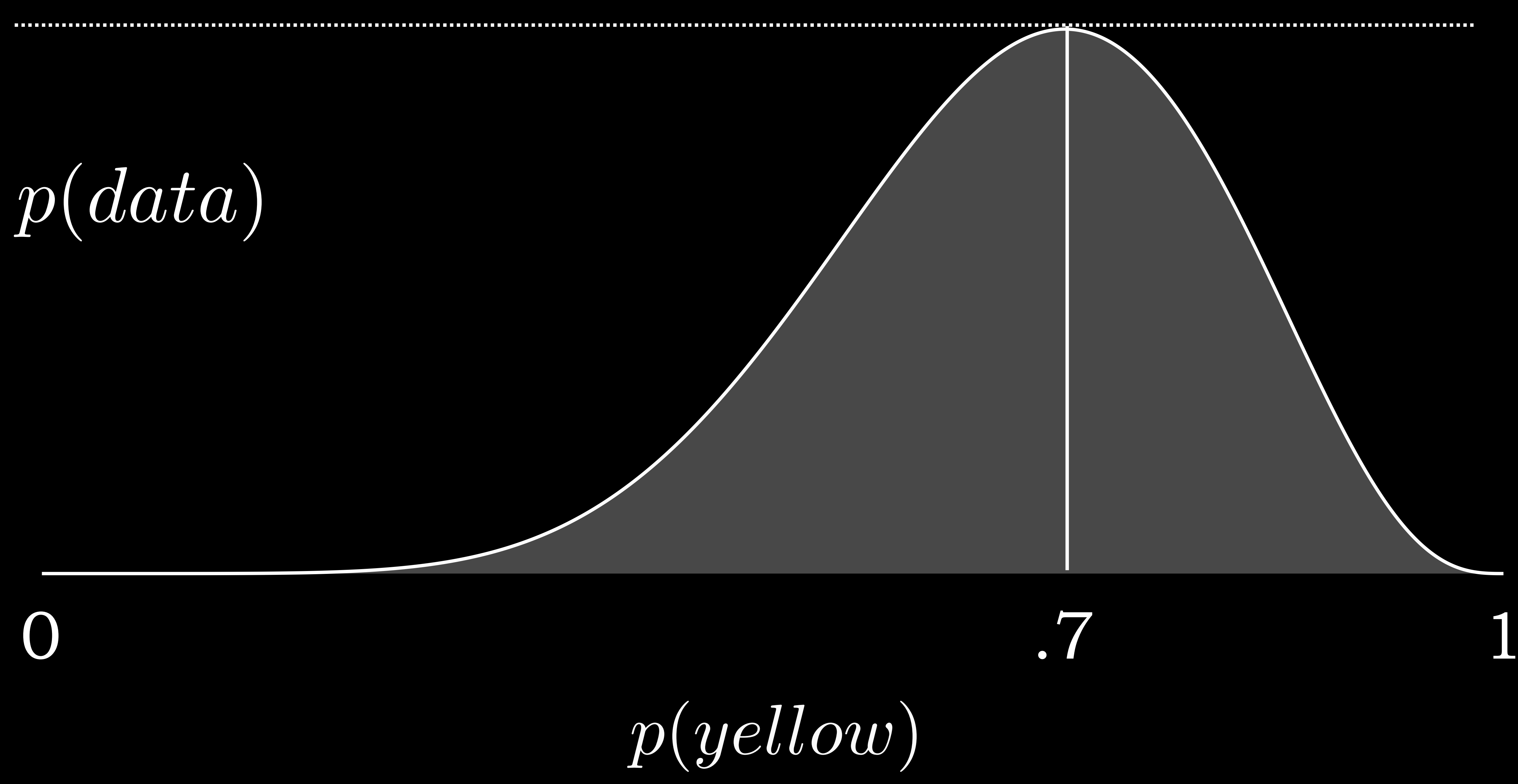
Maximum likelihood chooses parameters to maximize this function (called the likelihood).


$p(data)$
0
1
$p(yellow)$


$p(data)$
0
.7
1
$p(yellow)$

Machine Translation

This is just a **conditional** language model.
It generates Chinese, conditioned on English.

Question: Could we use *n*-gram models here?

Conditional LMs

Given Chinese word sequence $f = f_1 \ldots f_{|f|}$
predict English word sequence $e = e_1 \ldots e_{|f|}$

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Will this work?

Let $w = f_1 \dots f_{|f|} e_1 \dots e_{|e|}$

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Will this work?

Let $w = f_1 \ldots f_{|f|} e_1 \ldots e_{|e|}$

Problem: model forgets Chinese sentence after generating first n-1 words of English

Conditional LMs

Given Chinese word sequence $f = f_1 \ldots f_{|f|}$
predict English word sequence $e = e_1 \ldots e_{|f|}$

What about this?

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What about this?

Let $w = f_1 e_1 \ldots f_{|f|} e_{|f|} \ldots e_{|e|}$

Problem: sentences might not be in the same length or have the same word order.

Conditional LMs

General problem: *n*-grams condition on finite history.

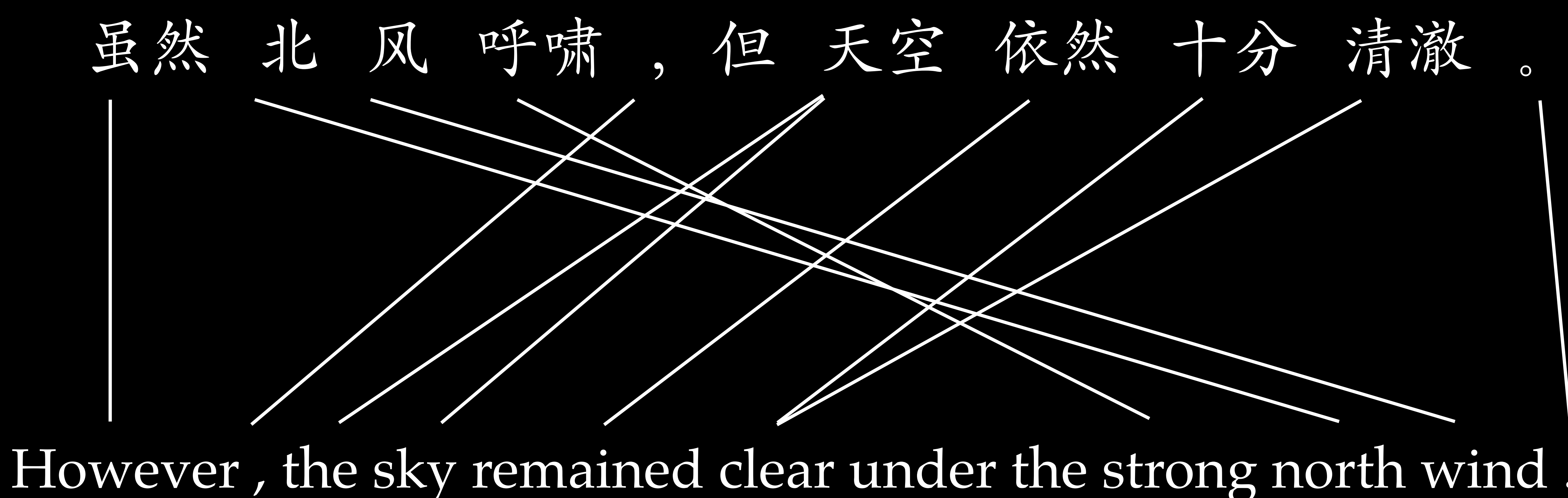
When we are generating an English word, how do we know which part of the history to condition on?

虽然 北 风 呼啸 ， 但 天空 依然 十分 清澈 。

However , the sky remained clear under the strong north wind .

Conditional LMs

Word alignments!



IBM Model 1

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Let’s write a simple model in terms of word-to-word alignments:

$$p(\mathbf{f}, \mathbf{a}|\mathbf{e})$$

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___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

predict English length given Chinese length

$$p(English\ length|Chinese\ length)$$

IBM Model 1

虽然 北 风 呼啸 ， 但 天空 依然 十分 清澈 。 ε

1. which Chinese word is in this position
2. on the condition the Chinese word I chose , which English word to generate
3. Concern about the unknown words

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

$$p(Chinese\ word\ position)$$

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___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

However

$$p(English\ word|Chinese\ word)$$

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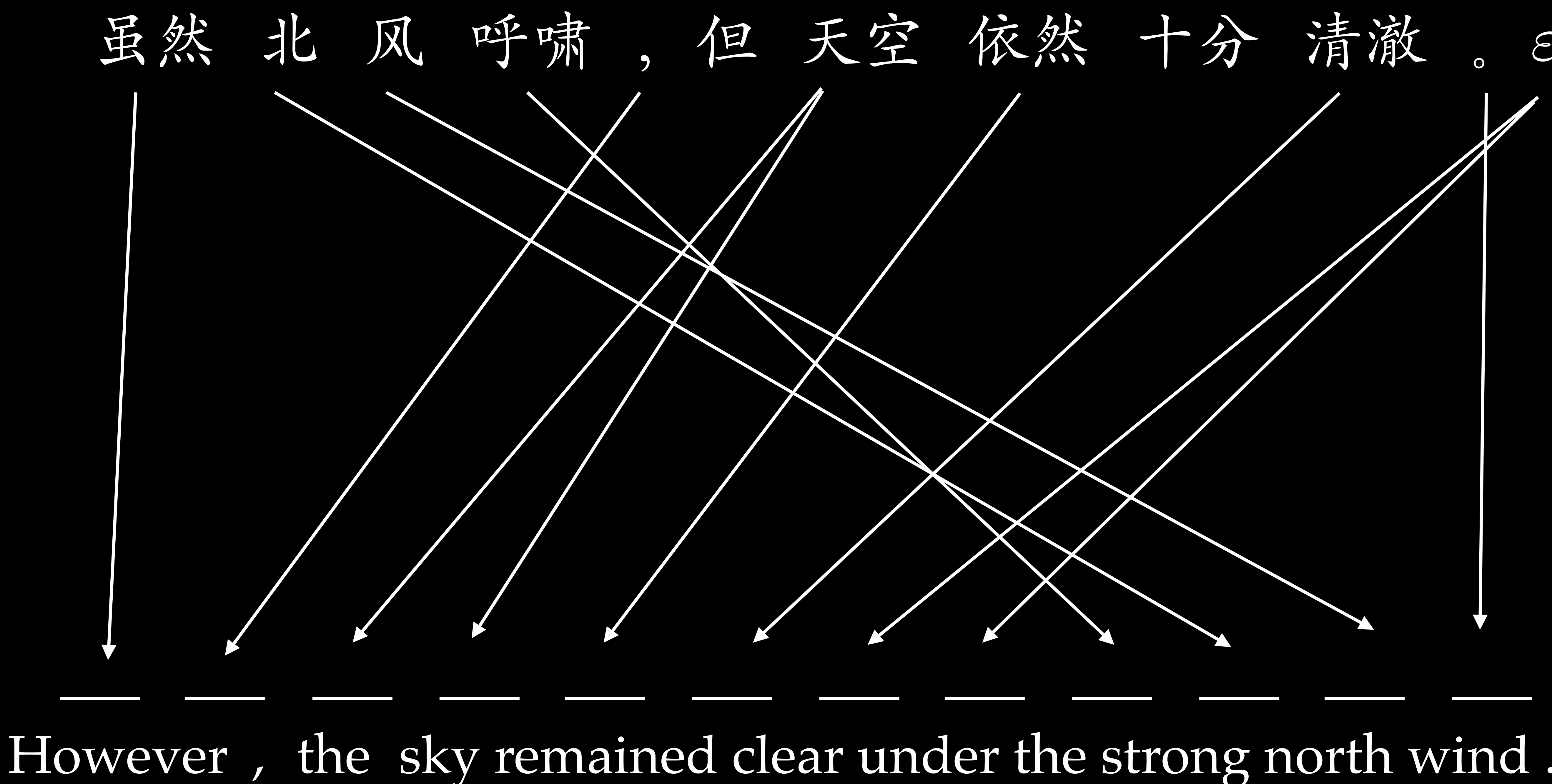
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IBM Model 1

alignment of French
word at position i

$$p(\mathbf{f}, \mathbf{a}|\mathbf{e}) = p(I|J) \prod_{i=1}^{I} p(a_i|J) \cdot p(f_i|e_{a_i})$$

French, English
sentence lengths

French, English
word pair

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The alignment is a **latent variable** whose value is a sequence over Chinese word positions: $\{1, \ldots, |f|\}^{|e|}$

IBM Model 1

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French, English
sentence lengths

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Does this equation look familiar?

IBM Model 1

transition to state at
position i

$$p(\mathbf{f}, \mathbf{a}|\mathbf{e}) = p(I|J) \prod_{i=1}^{I} p(a_i|J) \cdot p(f_i|e_{a_i})$$

emission at position i

Just a zero-order HMM!

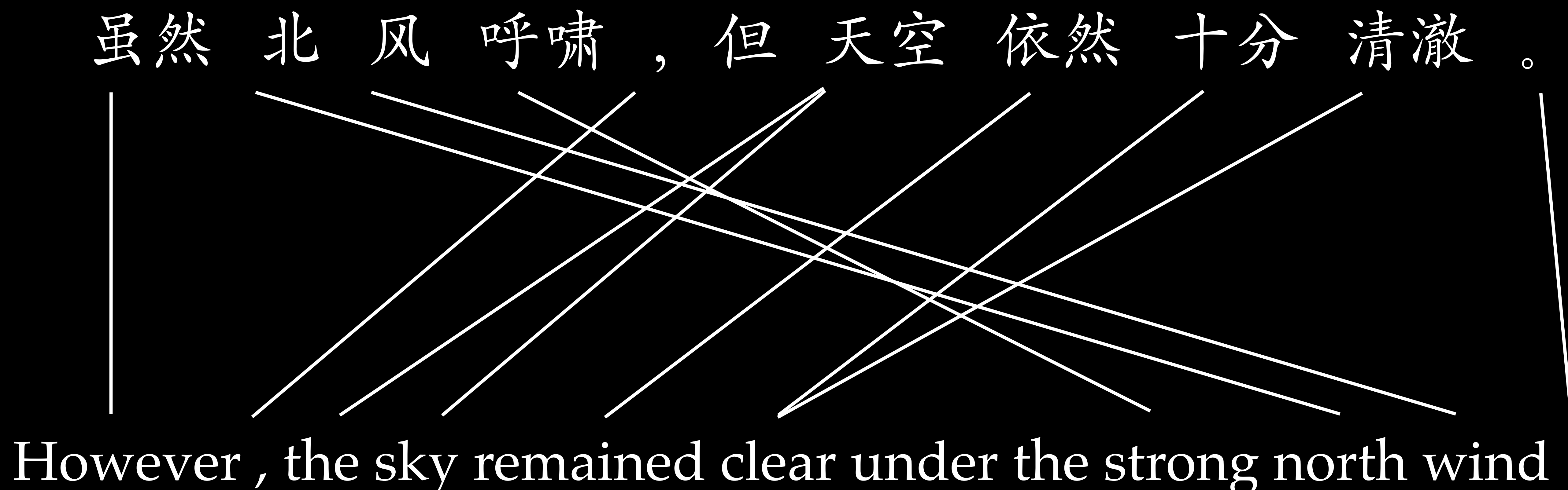
Only difference from standard HMM: set of tags (Chinese words) varies for each sentence.

IBM Model 1

$$\theta \begin{cases} p(despite|\text{虽然}) & ??? \\ p(however|\text{虽然}) & ??? \\ p(although|\text{虽然}) & ??? \\ \dots & \\ p(northern|\text{北}) & ??? \\ p(north|\text{北}) & ??? \end{cases}$$

Where do these parameters come from?

MLE for IBM Model 1



$$p(\text{north} \mid \text{北}) = \frac{\text{\# of times 北 aligns to “north”}}{\text{\# of times 北 aligns to any word}}$$

MLE for IBM Model 1

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$$p(\text{north} \mid \text{北}) = \frac{???}{???}$$

Problem: We do not get to observe the word alignments!

Expectation Maximization

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$$p(\text{north} \mid \text{北}) = \frac{\textit{Expected}\ \#\ \text{of times 北 aligns to "north"}}{\textit{Expected}\ \#\ \text{of times 北 aligns to any word}}$$

The same maths as for MLE leads to this.

What are expected counts?

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if we had observed the alignment, this line would either be here (count 1) or it wouldn't (count 0).

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But we need model parameters to compute this!

EM: the main idea

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Parameters and alignments are both unknown.

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$p(English\ word|Chinese\ word)$ unobserved!

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The Plan: Bootstrapping

- Arbitrarily select a set of parameters (say, uniform).
- Calculate ***expected counts*** of the unseen events.
- Choose new parameters to maximize likelihood, using expected counts as proxy for observed counts.
- Iterate.

Computing expected counts

- Main computational bottleneck.
- For this model: dynamic programming, specifically the forward-backward algorithm
  - This is a special case of backpropagation!
- For most models: sample for while, then compute a Monte Carlo estimate of the expected counts.

Why EM works

Observation 1: We are still solving a maximum likelihood estimation problem.

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MLE: choose parameters that maximize this expression.

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MLE: choose parameters that maximize this expression.

Minor problem: there is no analytic solution.

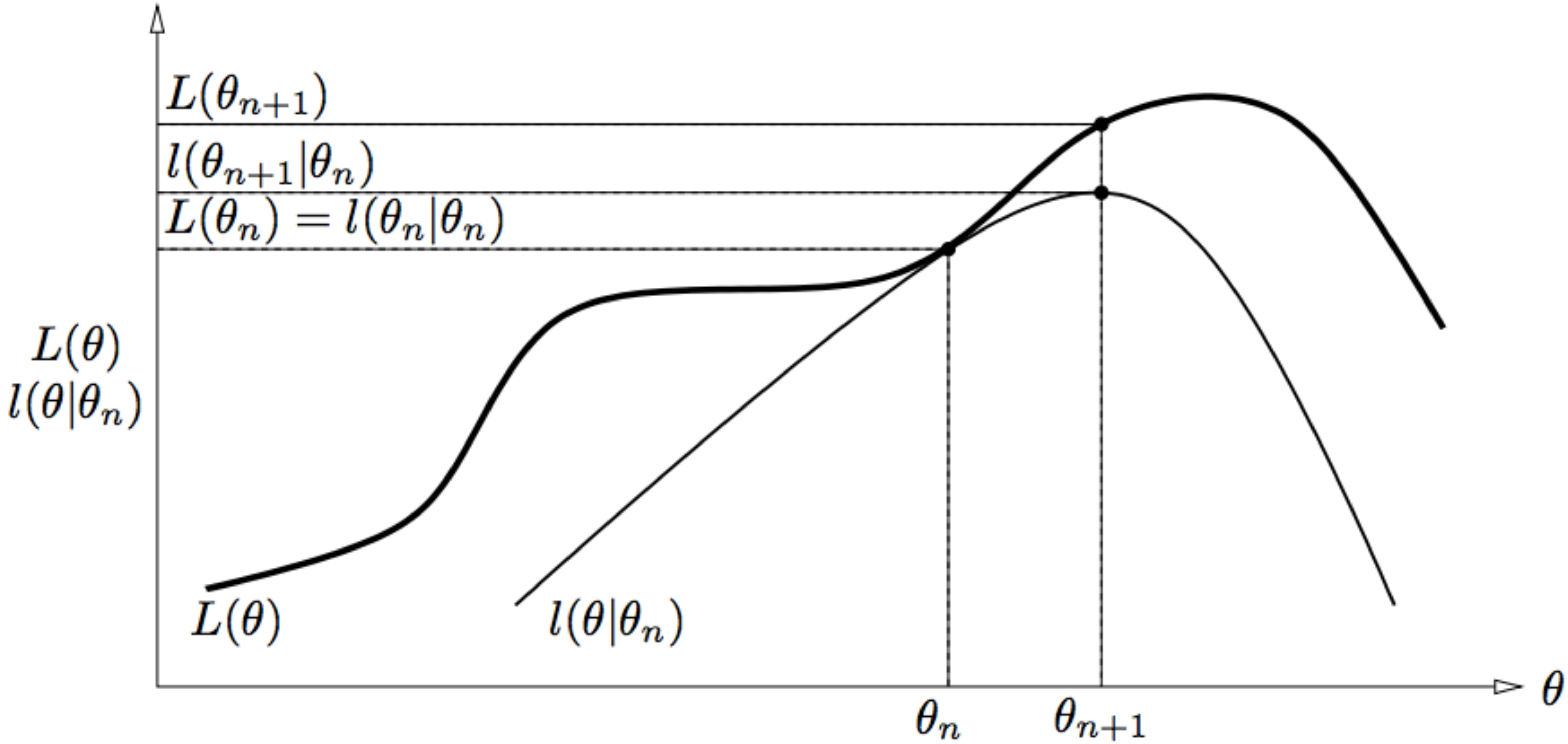

Figure 2: Graphical interpretation of a single iteration of the EM algorithm: The function $l(\theta|\theta_n)$ is bounded above by the likelihood function $L(\theta)$. The functions are equal at $\theta = \theta_n$. The EM algorithm chooses θ_{n+1} as the value of θ for which $l(\theta|\theta_n)$ is a maximum. Since $L(\theta) \geq l(\theta|\theta_n)$ increasing $l(\theta|\theta_n)$ ensures that the value of the likelihood function $L(\theta)$ is increased at each step.

(from Boorman '04)

Decoding

Once we have a model, we want to solve:

Given Chinese word sequence $f = f_1 \ldots f_{|f|}$
predict *most probable* English word sequence

- Doing this correctly involves Bayesian reasoning and NP-hard algorithms.
- Generally uses approximations (beam search).
- Will discuss similar approximations later in the course for neural MT.

Summary of key points (i.e. examinable content)

- Language models assign string probabilities
- Useful for word prediction in many NLP applications
- n-gram models use a Markov assumption to partition the infinite set of possible histories into a finite set of finite set of states, each with its own parameters.
- Machine translation is conditional language modeling.
- To model translation with n-grams, we need additional *latent variables* to model *word alignment*.
- One way to estimate the parameters of latent variable models is with a generalization of maximum likelihood estimation, called *expectation maximization*.