Thin Wire Antenna Analysis using Method of Moments (MOM)

[REPORT TO PROJECT III]

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Introduction

Method of Moments (MOM) is a numerical procedure to convert integral equations into a set of linear algebraic equations to obtain approximate solutions. MOM is particularly suited to open-region electromagnetic problems such as wave scattering and antenna radiation. Based on Jin's summary [1], even though MOM has similar numerical procedure as FEM such as expanding with respect to certain basis functions to form matrix problem, MOM surpasses FEM on the aspects of 1) more flexible basis functions such as zeroth order basis function due to the advantages of integral over differentiation, 2) discretization only over the surface domain instead of the entire volumetric domain, which reduces the problem dimension by 1, 3) free of truncating computation domain and absorbing boundary condition in open-region problem. On the other hand, MOM has certain disadvantages like the system matrix is full matrix, requiring an expensive solver.

In this project 3, I chose problem 27 and 28, the thin wire antenna, to apply the MOM method. By simulating the admittance and current distribution under delta-gap source, I found the MOM could give reasonable results.

Simplification for Thin Wire Antenna and Settings

The problem I simulated is a thin wire antenna as in Fig. 1. The wire has finite length L along the z-axis and has a circular cross-section of radius a. As required in the problem, $L/2a = 74.2$? 1. For such a thin wire, we can neglect the transverse component of the current on the wire and assume that the longitudinal component is uniformly distributed over the periphery of wire.

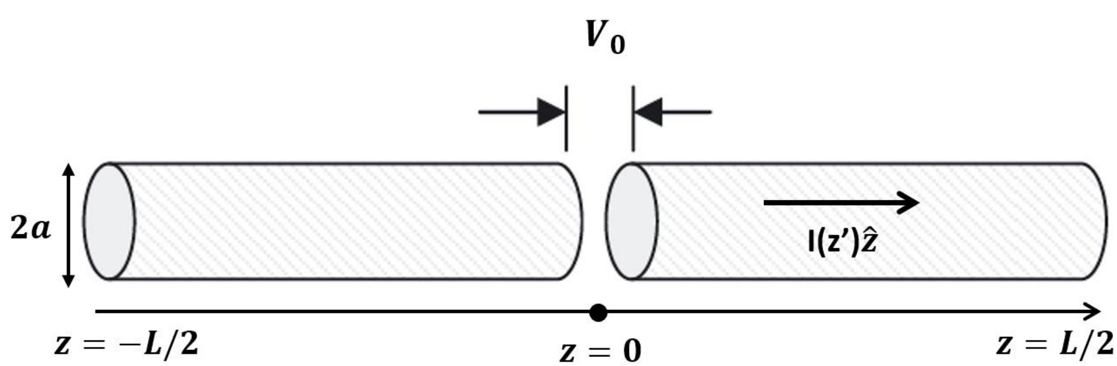


Figure 1 Thin wire antenna with center-fed delta-gap voltage source

General 3D MOM Formulation

Starting from 3D wave equations under certain incident current density distribution $\mathbf{J}_i$, the electromagnetic fields $(\mathbf{E},\mathbf{H})$ satisfy the vector wave **differential equations:**

$$\nabla\times\nabla\times\mathbf{E}(\mathbf{r})-k_0^2\mathbf{E}(\mathbf{r}) = -jk_0Z_0\mathbf{J}_i(\mathbf{r}) \qquad \mathbf{r}\in V_\infty$$

$$\nabla\times\nabla\times\mathbf{H}(\mathbf{r})-k_0^2\mathbf{H}(\mathbf{r}) = \nabla\times\mathbf{J}_i(\mathbf{r}) \qquad \mathbf{r}\in V_\infty$$

subjecting to Sommerfeld radiation **boundary conditions:**

$$r\left[\nabla\times\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix}+jk_0\hat{r}\times\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix}\right]=0 \qquad r\to\infty.$$

Following the procedures in Jin's book [1], page 416-420, based on above open-region wave equations, we can derive such **3D integral equations** for $(\mathbf{E},\mathbf{H})$

$$\mathbf{E}^{\text{inc}}(\mathbf{r})-\oiint_{S_o}\left[\frac{jZ_0}{k_0}\nabla'\cdot(\hat{n}'\times\mathbf{H})\nabla G_0+(\hat{n}'\times\mathbf{E})\times\nabla G_0+jk_0Z_0(\hat{n}'\times\mathbf{H})G_0\right]dS'$$
$$=\begin{cases}\mathbf{E}(\mathbf{r}) & \mathbf{r}\in V_\infty\\ 0 & \mathbf{r}\in V_o\end{cases}$$

$$\mathbf{H}^{\text{inc}}(\mathbf{r})-\oiint_{S_o}\left[\frac{Y_0}{jk_0}\nabla'\cdot(\hat{n}'\times\mathbf{E})\nabla G_0+(\hat{n}'\times\mathbf{H})\times\nabla G_0-jk_0Y_0(\hat{n}'\times\mathbf{E})G_0\right]dS'$$
$$=\begin{cases}\mathbf{H}(\mathbf{r}) & \mathbf{r}\in V_\infty\\ 0 & \mathbf{r}\in V_o.\end{cases}$$

Coding Analysis for Thin Wire Antenna

For such a conducting thin wire, only considering the electric field equation is enough because the $\hat{\mathbf{n}}\times\mathbf{E}=0$. Then, by writing $\hat{\mathbf{n}}'\times\mathbf{H}=\mathbf{j}_s$ and integrating over the side circle of this thin wire to replace surface current density $\mathbf{j}_s$ by current I, we end up with such a integral equation connecting incident electric field and current distribution I along the wire.

$$jk_0Z_0\int_C\left[\hat{l}\cdot\hat{l}'I(\mathbf{r}')G_0(\mathbf{r},\mathbf{r}')+\frac{1}{k_0^2}\frac{dI(\mathbf{r}')}{dl'}\frac{dG_0(\mathbf{r},\mathbf{r}')}{dl}\right]dl'=\hat{l}\cdot\mathbf{E}^{\text{inc}}(\mathbf{r}) \qquad \mathbf{r}\in C$$

Based on this integral equation, we can solve current I by expanding current by triangular basis function

$$I(\mathbf{r}') = \sum_{n=1}^{N-1} \Lambda_n(\mathbf{r}') I_n$$

Then, by choosing N-1 sample points, we can obtain a matrix equation

$$\sum_{n=1}^{N-1} Z_{mn} I_n = V_m \qquad m = 1, 2, \ldots, N-1$$

where

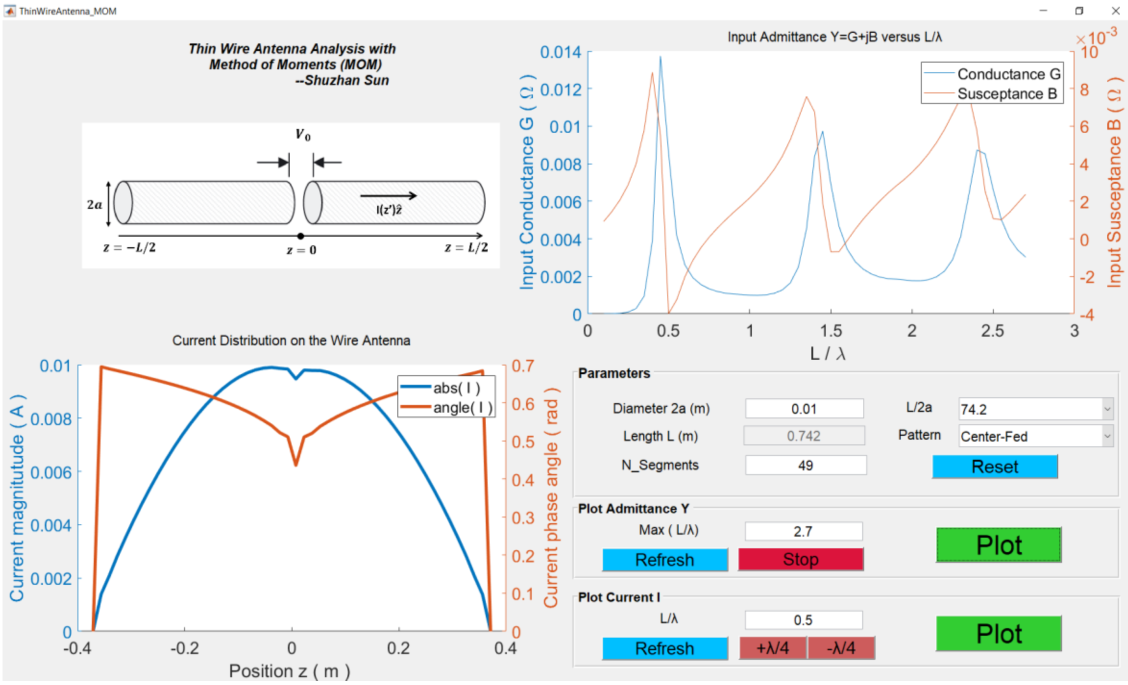
$$Z_{mn} = jk_0 Z_0 \int_C \Lambda_m(\mathbf{r}) \hat{l}_m \cdot \int_C \hat{l}'_n \Lambda_n(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}')\, dl' dl$$
$$- \frac{jZ_0}{k_0} \int_C \frac{d\Lambda_m(\mathbf{r})}{dl} \int_C \frac{d\Lambda_n(\mathbf{r}')}{dl'} G_0(\mathbf{r}, \mathbf{r}')\, dl' dl$$

$$V_m = \int_C \Lambda_m(\mathbf{r}) \hat{l}_m \cdot \mathbf{E}^{\text{inc}}(\mathbf{r})\, dl.$$

For calculating the admittance, we give a delta-gap source by assigning $V_m = V_0$ at the feed point and $V_m = 0$ elsewhere. Then, the input admittance of the antenna is given by $Y_{in} = \frac{I_m}{V_0} = G + jB$, where G is the conductance and B is the susceptance.

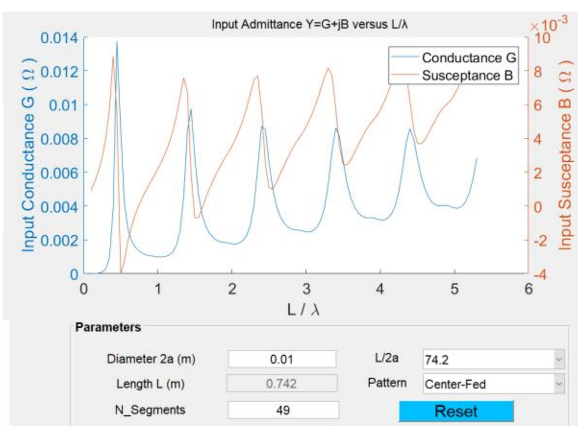
GUI and Simulation Results

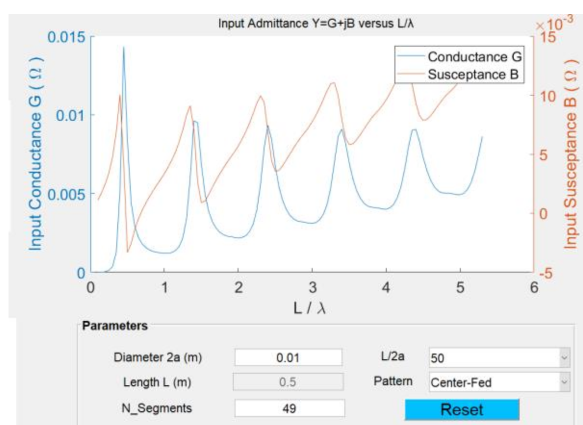
The GUI panel is designed as below. In this GUI, you can change the diameter 2a, $L/2a$ ratio, voltage source feed pattern, number of discretization segments in the parameter panel. A reset button is also provided to reset these parameters to default values. When plotting the admittance Y, you are free to change the maximum L/λ, meaning the maximum frequency. The code will calculate the Y from $L/\lambda = 0.1$ to that maximum L/λ, with a step 0.05 in unit of L/λ. This calculation for so many cases could be very long, so you may stop the Y calculation at any time by clicking the Stop button. The next panel is the current plotting for arbitrary L/λ ratio. Both magnitude and phase angle are plotted there. The user may be interested in the famous ¼ and ½ ratios, so, for convenience, I provide buttons to increase or decrease L by quarter wavelength.



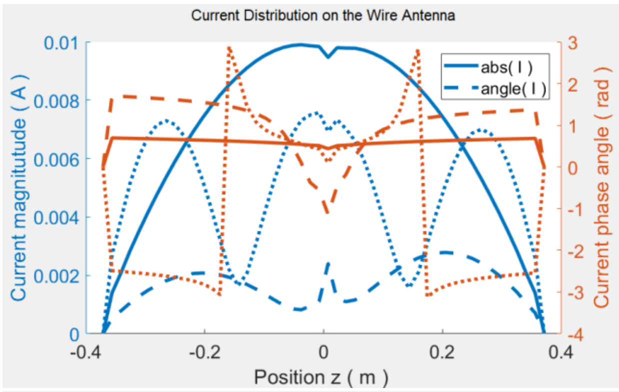
FOR THE CLASSIC CENTER-FED CASE:

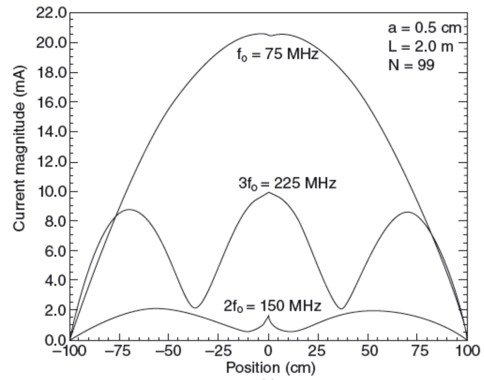
For the same cross section of diameter 0.01m, the Admittances for 2 different lengths are different. $L = 0.742m$ antenna and $L = 0.5m$ antenna have very similar conductance G, but the susceptances of $L = 0.5m$ antenna increases much faster than longer antenna, meaning the phase changes faster for short antenna.



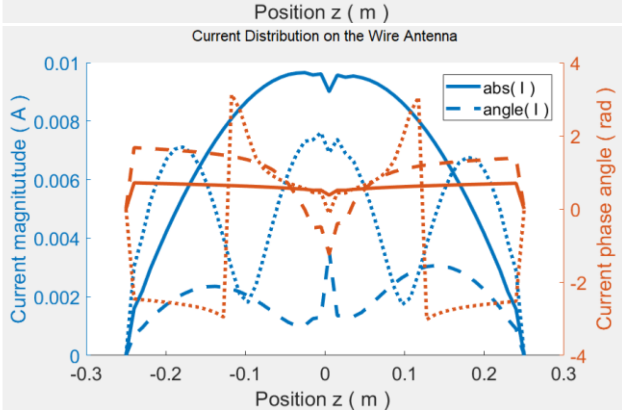


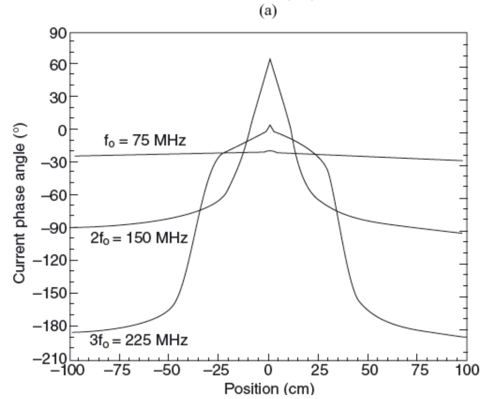
Then, the current distributions along the wire for $L/\lambda = 0.5, 1, 1.5$ are plotted together as below (blue lines are abs(I) whereas red lines are angle(I)). The first one is the $L = 0.742m$ where the second one is $L = 0.5m$ antenna. Both are similar.





(a)



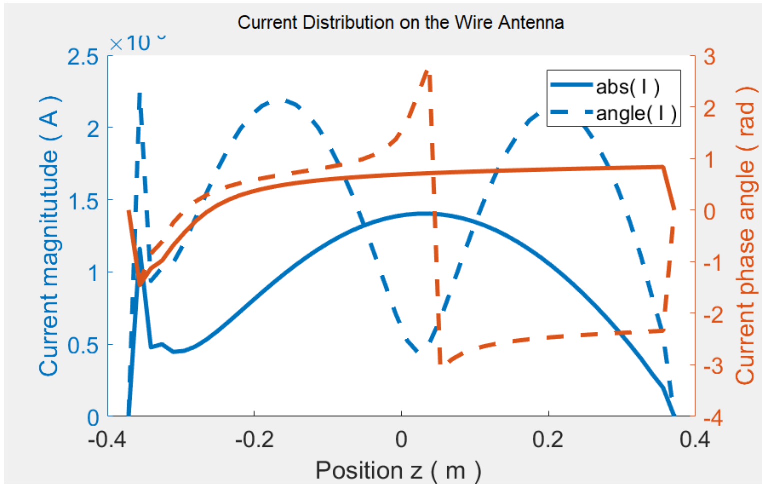


Also, the figures are similar to that of 90-degree bent antenna in the book. See the gray figure above. Though they are different antennas, the current magnitudes are similar, still, the phase differences are relatively large.

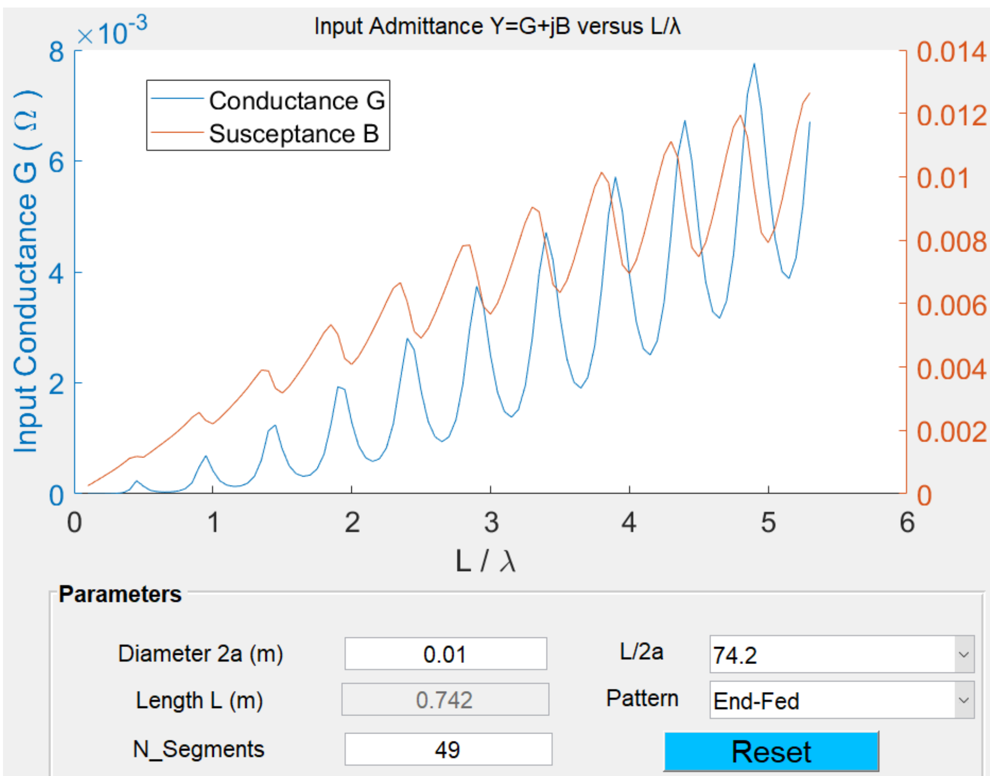
FOR THE END-FED CASE:

In this case, I only compare end-fed with center-fed using the longer antenna ratio. The source excitation was put on the left end in the end-fed case.

When considering the current distribution, the sharp difference between confined left end will give a sharp peak there, indicating a relatively large numerical error! The general current shape is still reasonable, though the magnitude is very different.



For the admittance vs. L/λ, peaks were observed at half integer peaks, which is twice the amount of center-fed antenna!



References

[1] J.-M. Jin, *Theory and computation of electromagnetic fields*: John Wiley & Sons, 2011.