

The Application of Zero-Knowledge Proofs: Proof of Solvency

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Table of Contents

1. Introduction

1.1 What is ZKP

1.2 Commitment Scheme

2. Proof of Solvency

2.1 Proof of Liabilities

2.2 Proof of Assets

2.3 Proof of Solvency

2.4 Performance Evaluation

3. Outro

3.1 Conclusion

3.2 State of the Art in ZKP

What is ZKP

Goal: Prove a statement is true without revealing additional information, only the fact itself.

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- Compared to classic proofs
 - **Classic proofs** (pythagorean theorem, Euler's theorem, etc.):
verifier passively reads the proof
 - **ZKP**: two new ingredients, interaction and randomness

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 - **Classic proofs** (pythagorean theorem, Euler's theorem, etc.): verifier passively reads the proof
 - **ZKP:** two new ingredients, interaction and randomness

$\mathcal{P} \xrightarrow{m_1} \mathcal{V}$

$\xleftarrow{\$} c_1 \in \{0, 1\}^t$

$\xrightarrow{m_2}$

$\xleftarrow{\$} c_2 \in \{0, 1\}^t$

\leftarrow

...

interactive +
probabilistic proof
system

Zk-SNARKs

Succinctness: small proof size and short verifier time

Non-interactive: Fiat-Shamir transformation

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Building a zk-SNARK

commitment scheme + interactive oracle proof \Rightarrow zk-SNARK

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Building a zk-SNARK

commitment scheme + interactive oracle proof \Rightarrow zk-SNARK

- Commitment scheme: commit to a value and later reveal it
 - Binding: cannot open one commitment to two different values
 - Hiding: cannot learn the committed value from the commitment
- Random oracle: a black box that randomly uniformly outputs the same bits with the same input and different bits with different input

Important Commitment Schemes

- Polynomial commitments: univariate polynomial $f(X)$
- Multilinear commitments: multivariate polynomial $f(X_1, X_2, \dots, X_n)$
- Vector commitments: vector $\vec{v} = (v_1, v_2, \dots, v_n)$
- ...

Polynomial Commitment Scheme (PCS)

Given $\text{srs} = \{g, g^\tau, g^{\tau^2}, g^{\tau^3}, \dots\}$, the PCS allows \mathcal{P} to commit to a polynomial $f(X)$; denote the commitment to $f(X)$ by \mathcal{C}_f

$$\mathcal{C}_f = g^{f_0 + f_1\tau + f_2\tau^2 + f_3\tau^3 + f_4\tau^4 \dots}$$

where f_i is the coefficient of X^i in $f(X)$.

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Prove the evaluation of $f(a)$ is b

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Example

Prove the evaluation of $f(a)$ is b

A useful observation:

$$f(a) = b \iff q(X) \text{ exists s.t. } q(X) = \frac{f(X) - b}{X - a}$$

Application of PCS

\mathcal{P}

\mathcal{V}

$$q(X) = \frac{f(X) - b}{X - a}$$

commit(f, q)

$\mathcal{C}_f, \mathcal{C}_q$



check

$$e(\mathcal{C}_f/g^b, [1]_2) \stackrel{?}{=} e(\mathcal{C}_q, [X - a]_2)$$

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$$e(\mathcal{C}_f / g^b, [1]_2) \stackrel{?}{=} e(\mathcal{C}_q, [X - a]_2)$$

Schwartz-Zippel Lemma

Given a polynomial $f \in \mathbb{F}_{\leq d}[X]$. Let S be a finite subset of \mathbb{F} and $X \in S$ be chosen uniformly at random. Then

$$\Pr[f(X) = 0] \leq \frac{d}{|S|}$$

where d is the degree of f .

Polynomial Interactive Oracle Proof (Poly-IOP)

Polynomial Constraints: a set of polynomial relations among polynomials $f_1(X), f_2(X), \dots, f_n(X)$.

Polynomial Interactive Oracle Proof (Poly-IOP)

Example

Prove $f(X) = g(X)$ when $X = a$

Polynomial Interactive Oracle Proof (Poly-IOP)

Example

Prove $f(X) = g(X)$ when $X = a$

\mathcal{P}

\mathcal{V}

$$q(X) = (f(X) - g(X))/(X - a)$$

commit(f, g, q)

$\mathcal{C}_f, \mathcal{C}_g, \mathcal{C}_q$

$\gamma \xleftarrow{\$} \mathbb{F}_p$

$$\alpha_1 = f(\gamma), \alpha_2 = g(\gamma), \beta = q(\gamma)$$

check

(i) $\alpha_1 - \alpha_2 \stackrel{?}{=} \beta \cdot (\gamma - a)$

(ii) $\alpha_1 \stackrel{?}{=} f(\gamma), \alpha_2 \stackrel{?}{=} g(\gamma),$
 $\beta \stackrel{?}{=} q(\gamma)$

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What is PoS

Proof of Solvency

How can a centralized exchange prove that it has enough assets to cover all the liabilities of its customers?

- Reveal the assets directly?
- Trust third-party auditors?

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Numerous incidents

- Mt. Gox, 2014
- QuadrigaCX, 2019
- FTX, 2022
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Proof of Solvency (PoS)

- Proof of Liabilities (PoL)
- Proof of Assets (PoA)

PoS is not a silver bullet

- It may rely on a trusted setup (depending on the proof system)
- It requires honest verifier and user check
- It is not helpful to prevent hacks, scams, etc

Limitations

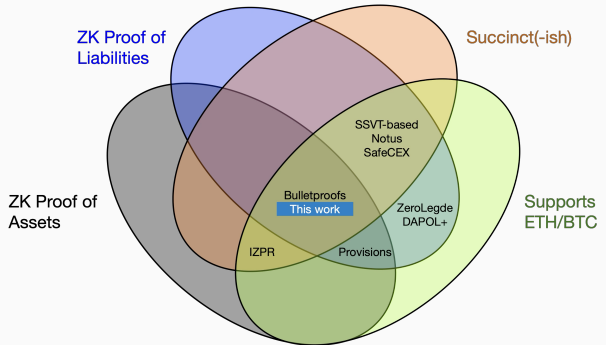
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It raises the bar for exchanges to cheat

Comparison with Previous Works

- Most of the previous works focus on only one side
- This work aims for a full end-to-end solution



Proof of Liabilities

Recall the problem

- Each user's balance is included in the proof
- The summation of all balances (total liability) is correct
- Each balance is in a specified range (range proof)

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$$p(X) : \quad \text{bal}_1 \quad \text{bal}_2 \quad \dots \quad \text{bal}_{n-1} \quad \text{bal}_n$$

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- Construct an accumulative polynomial

$$\begin{array}{llllll} f_{\text{liab}}(X) : & r_1 & r_2 & \dots & r_{n-1} & r_n \\ p(X) : & \text{bal}_1 & \text{bal}_2 & \dots & \text{bal}_{n-1} & \text{bal}_n \end{array}$$

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- Open the evaluation point for each user

Proof of Liabilities Example

User	Alice	Bob	Charlie	David
Balance	10	12	9	7

Table 1: User Balances

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<div>Poly \ X</div>	0	1	2	3
$f_{\text{liab}}(X)$	38	28	16	7
$p(X)$	10	12	9	7

Table 2: Polynomials

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- $f(X) = p(X), X = n - 1$
- $f(X) = p(X) + f(X + 1), \forall X \in [0, n - 1)$

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Table 2: Polynomials

- $f(X) = p(X), X = n - 1 \Rightarrow \frac{f(X) - p(X)}{X - (n - 1)} = q_1(X)$
- $f(X) = p(X) + f(X + 1), \forall X \in [0, n - 1]$
 $\Rightarrow \frac{f(X) - p(X) - f(X + 1)}{X(X - 1)(X - 2) \dots [X - (n - 2)]} = q_2(X)$

Range Proof

- Prove a value $x \in [0, 2^k)$
- Prevent attacks by overflowing the liabilities and inserting negative values
- Binary decomposition¹
 - $\bar{z} = \{z_1, z_2, \dots, z_k\}, x = 2^0 \cdot z_1 + 2^1 \cdot z_2 + \dots + 2^{k-1} \cdot z_k$
 - $x = z_1 + 2 \cdot (z_2 + 2 \cdot (z_3 + 2 \cdot (z_4 + \dots)))$

	2^0	2^1	2^2	2^3	...	2^{k-2}	2^{k-1}
\bar{z}	z_1	z_2	z_3	z_4	...	z_{k-1}	z_k
λ	$z_1 + 2(z_2 + \dots)$	$z_2 + 2(z_3 + \dots)$	$z_3 + 2(z_4 + \dots)$	$z_4 + 2(z_5 + \dots)$...	$z_{k-1} + 2z_k$	z_k

¹<https://hackmd.io/@dabo/B1U4kx8XI>

Range Proof

- Prove a value $x \in [0, 2^k)$
- Prevent attacks by overflowing the liabilities and inserting negative values
- Binary decomposition¹

Example

	2^0	2^1	2^2	2^3
$x = 13$	$\lambda_0 = 1$	$\lambda_1 = 0$	$\lambda_2 = 1$	$\lambda_3 = 1$
$f_{\text{range}}(X)$	13	6	3	1
	$f_{\text{range}}(0)$	$f_{\text{range}}(1)$	$f_{\text{range}}(2)$	$f_{\text{range}}(3)$

- $f_{\text{range}}(X) \in \{0, 1\}, X = k - 1$
- $f_{\text{range}}(X) - 2 \cdot f_{\text{range}}(X + 1) \in \{0, 1\}, X \in [0, k)$

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Proof of Liability + Range Proof

Poly \ X	0	1	2	3
$f_{\text{liab}}(X)$	38	28	16	7
$p(X)$	10	12	9	7
	5	6	4	3
	2	3	2	1
	1	1	1	0

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The degree of $p(X)$ is proportional to the size of the range proof

- The largest set of the roots we can use is 2^{32}
- One million $\approx 2^{20}$
- We can only prove values in $[0, 2^{12})$

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Solution

- Break the range polynomial

Proof of Liability + Range Proof

$\begin{array}{c} \text{Poly} \backslash X \\ \hline \end{array}$	0	1	2	3
$f_{\text{iab}}(X)$	38	28	16	7
$p_1(X)$	10	12	9	7
$p_2(X)$	5	6	4	3
$p_3(X)$	2	3	2	1
$p_4(X)$	1	1	1	0

- $f_{\text{iab}}(n-1) = p_1(n-1)$
- $f_{\text{iab}}(X) = f_{\text{iab}}(X+1) + p_1(X), X \in [0, n-1]$
- $p_k(X) \in \{0, 1\}$
- $p_i(X) - 2p_{i+1}(X) \in \{0, 1\}$

Proof of Liability + Range Proof

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Clash attack: count the same amount only once

Clash Attack

$\begin{array}{c} \diagdown \\ \text{Poly} \end{array} \quad X$	0	1	2	3
$f_{\text{liab}}(X)$	38	28	16	7
$p_1(X)$	10	12	9	7
$p_2(X)$	5	6	4	3
$p_3(X)$	2	3	2	1
$p_4(X)$	1	1	1	0
$u(X)$	A	B	C	D

- $p_k(X) \in \{0, 1\}$
- $p_i(X) - 2p_{i+1}(X) \in \{0, 1\}$
- $f_{\text{liab}}(n-1) = p_1(n-1)$
- $f_{\text{liab}}(X) = f_{\text{liab}}(X+1) + p_1(X), X \in [0, n-1]$
- For each user t , $p_1(t) = \text{bal}_t$, $u(t) = \text{uid}_t$

Proof of Assets

Recall the problem

- The exchange proves the ownership of some wallet accounts
- The total asset is the sum of these accounts

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Preliminaries

- Wallet address = Hash(pk)
- $pk = g^{sk}$, where g is a generator of an elliptic curve group (for Bitcoin and Ethereum, the curve is secp256k1; for the PCS, the curve is b1s12-381)
- Secured by the discrete logarithm assumption

Proof of Assets

Intuition

- Publish some public keys as an anonymity set $\{pk_i\}$

	pk_1	pk_2	pk_3	\dots	pk_{n-1}	pk_n
$b(X) :$	bal_1	bal_2	bal_3	\dots	bal_{n-1}	bal_n

Proof of Assets

Intuition

- Publish some public keys as an anonymity set $\{pk_i\}$
- Construct a selector polynomial $s(X)$ where each evaluation is in $\{0, 1\}$

	pk_1	pk_2	pk_3	\dots	pk_{n-1}	pk_n
$b(X) :$	bal_1	bal_2	bal_3	\dots	bal_{n-1}	bal_n
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	pk_1	pk_2	pk_3	\dots	pk_{n-1}	pk_n
$b(X) :$	bal_1	bal_2	bal_3	\dots	bal_{n-1}	bal_n
$s(X) :$	s_1	s_2	s_3	\dots	s_{n-1}	s_n
$f_{\text{assets}}(X) :$	r_1	r_2	r_3	\dots	r_{n-1}	r_n
	$s_1 \cdot \text{bal}_1 + r_2$	$s_2 \cdot \text{bal}_2 + r_3$	$s_3 \cdot \text{bal}_3 + r_4$	\dots	$s_{n-1} \cdot \text{bal}_{n-1} + r_n$	$s_n \cdot \text{bal}_n$

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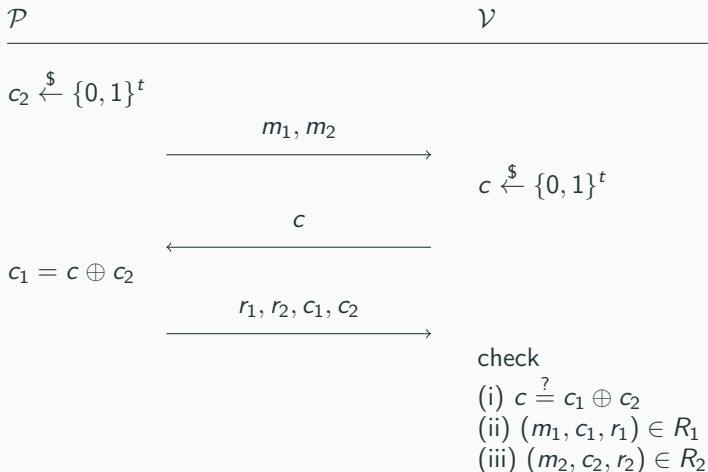
	pk_1	pk_2	pk_3	\dots	pk_{n-1}	pk_n
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$s(X) :$	s_1	s_2	s_3	\dots	s_{n-1}	s_n
$f_{\text{assets}}(X) :$	r_1	r_2	r_3	\dots	r_{n-1}	r_n
	$s_1 \cdot bal_1 + r_2$	$s_2 \cdot bal_2 + r_3$	$s_3 \cdot bal_3 + r_4$	\dots	$s_{n-1} \cdot bal_{n-1} + r_n$	$s_n \cdot bal_n$

How to construct such a selector polynomial in ZK?

Disjunctive Σ -Protocol (OR Proof)

OR Proof

Prove the claiming statement is R_1 or R_2 without telling which case it is.



Proof of Assets + OR Proof

A useful observation: the previous proof of assets has composite statements:

1. The selector s_i is 1 and prove the knowledge of the corresponding private key
 - $g_{\text{bls}}^{s_i} = g_{\text{bls}}^1$ (in bls12-381)
 - the knowledge of sk_i (in secp256k1)
2. the selector s_i is 0
 - $g_{\text{bls}}^{s_i} = g_{\text{bls}}^0$ (in bls12-381)

²See Section B of the paper.

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Can we open the commitment to the evaluation without revealing the evaluation itself?

²See Section B of the paper.

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PCS Extension

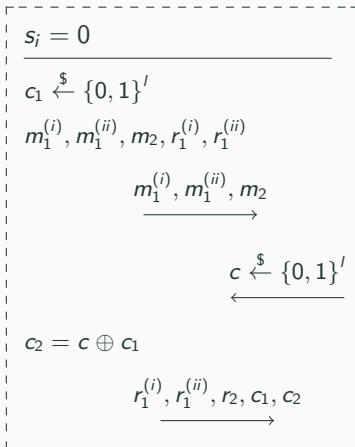
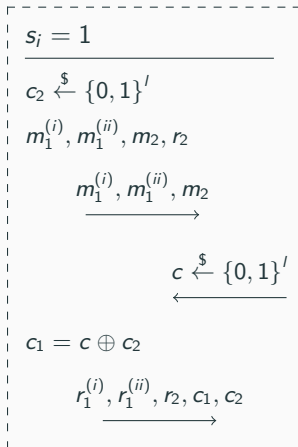
Open the committed evaluation g_{bls}^x ².

²See Section B of the paper.

Proof of Assets + OR Proof

\mathcal{P}

\mathcal{V}



check $(m_1^{(i)}, r_1^{(i)}, c_1), (m_1^{(ii)}, r_1^{(ii)}, c_1),$
 (m_2, r_2, c_2)

Full Protocol of Proof of Assets

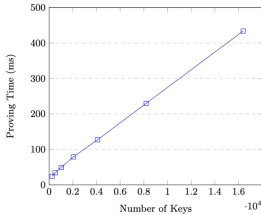
1. Bootstrapping (pre-computing)
 - 1.1 Interpolate selector polynomial $s(X)$
 - 1.2 Open the evaluations of $s(X)$
 - 1.3 Prove each evaluation is valid (the OR proof)
2. Interpolate the balance polynomial $b(X)$
3. Construct the accumulative polynomial $f_{\text{assets}}(X)$
4. Prove the polynomial constraints (accumulate assets) are correct

Proof of Solvency

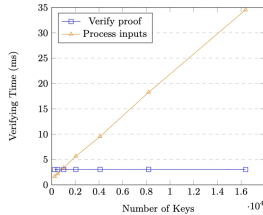
- Run the Proof of Liabilities to output $f_{\text{liab}}(X)$
- Run the Proof of Assets to output $f_{\text{assets}}(X)$
- Open the evaluations of $f_{\text{liab}}(0)$ and $f_{\text{assets}}(0)$ and compute $\Delta = f_{\text{assets}}(0) - f_{\text{liab}}(0)$
- Prove $\Delta \geq 0$ using the range proof

Benchmark Performance

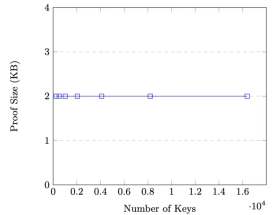
Proof of assets



(a)

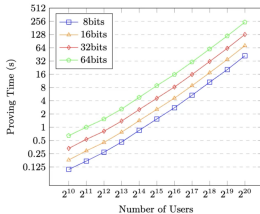


(b)

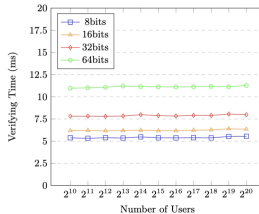


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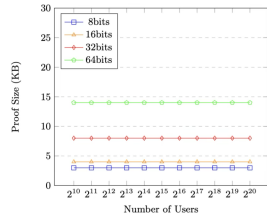
Proof of liability



(a)



(b)



(c)

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Conclusion

- This work is an almost succinct proof of solvency
- It is practical to be deployed for the real-world exchanges, implemented in Rust
(<https://github.com/Shvier/proof-of-solvency>)
- Future work
 - Multivariate polynomial for better performance
 - Incremental update (folding schemes)
 - Shorter range proof (lookup arguments)

State of the Art in ZKP

- Applications
 - In blockchain: zkRollup, zkBridge, zkEVM
 - In other fields: fight disinformation ³
- Limitations
 - Cost
 - Performance
 - Complexity
- Further reading
 - ZK MOOC
 - Vitalik's Blog
 - A Graduate Course in Applied Cryptography

³<https://rdi.berkeley.edu/zk-learning/assets/Lecture2-2023.pdf>