Operations on Finite-Length Sequences

- Consider the length-N sequence $x[n]$ defined for $0 \leq n \leq N-1$
- Its sample values are equal to zero for $n < 0$ and $n \geq N$
- A time-reversal operation on $x[n]$ will result in a length-N sequence $x[-n]$ defined for $-(N-1) \leq n \leq 0$

Operations on Finite-Length Sequences

- Likewise, a linear time-shift of $x[n]$ by integer-valued M will result in a length-N sequence $x[n+M]$ no longer defined for $0 \le n \le N-1$
- Similarly, a convolution sum of two length-N sequences defined for $0 \le n \le N-1$ will result in a sequence of length $2M+1$ defined for $0 \le n \le 2N-2$

Operations on Finite-Length Sequences

- Thus we need to define new type of time-reversal and time-shifting operations, and also new type of convolution operation for length-N sequences defined for $0 \le n \le N-1$ so that the resultant length-N sequences are also are in the range $0 \le n \le N-1$

Modulo Operation

- The time-reversal operation on a finite-length sequence is obtained using the modulo operation
- Let $0,1,\ldots,N-1$ be a set of N positive integers and let m be any integer
- The integer r obtained by evaluating

  $$M \text{ modulo } N$$

  is called the residue

Modulo Operation

- The residue r is an integer with a value between 0 and $N-1$
- The modulo operation is denoted by the notation $\langle m \rangle_N = m$ modulo N
- If we let $r = \langle m \rangle_N$ then $r = m + \ell N$ where ℓ is a positive or negative integer chosen to make $m + \ell N$ an integer between 0 and $N-1$

Modulo Operation

- **Example** – For $N = 7$ and $m = 25$, we have

$$r = 25 + 7\ell = 25 - 7 \times 3 = 4$$

Thus, $\langle 25 \rangle_7 = 4$

- **Example** – For $N = 7$ and $m = -15$, we get

$$r = -15 + 7\ell = -15 + 7 \times 3 = 6$$

Thus, $\langle -15 \rangle_7 = 6$

Circular Time-Reversal Operation

- The circular time-reversal version $\{y[n]\}$ of a length-N sequence $\{x[n]\}$ defined for $0 \le n \le N-1$ is given by $\{y[n]\} = \{x[\langle -n \rangle_N]\}$
- **Example** – Consider

$$\{x[n]\} = \{x[0],\ x[1],\ x[2],\ x[3],\ x[4]\}$$

Its circular time-reversed version is given by $\{y[n]\} = \{x[\langle -n \rangle_5]\}$

$$= \{x[0],\ x[4],\ x[3],\ x[2],\ x[1]\}$$

Circular Shift of a Sequence

- The time shifting operation for a finite-length sequence, called circular shift operation, is defined using the modulo operation
- Let $x[n]$ be a length-N sequence defined for $0 \leq n \leq N-1$
- Its circularly shifted version $x_c[n]$, shifted n_o by samples, is given by

$$x_c[n] = x[\langle n - n_o \rangle_N]$$

Circular Shift of a Sequence

$$x_c[n] = x[\langle n - n_o \rangle_N]$$

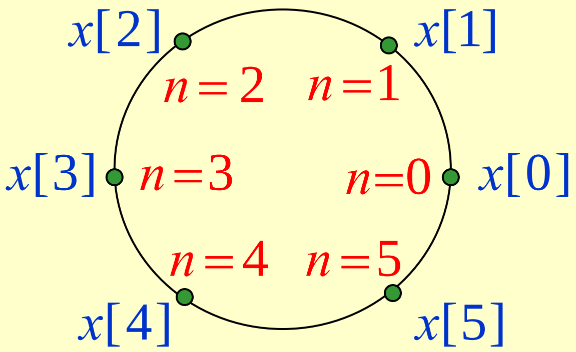
- $x_c[n]$ is also a length-N sequence defined for $0 \le n \le N-1$
- For $n_o > 0$ (right circular shift), the above equation implies

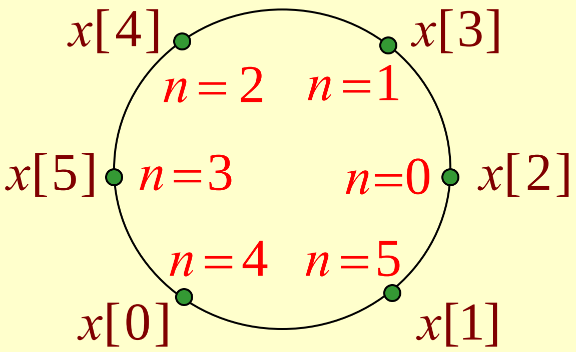
$$x_c[n] = \begin{cases} x[n - n_o], & \text{for } n_o \le n \le N-1 \\ x[N - n_o + n], & \text{for } 0 \le n < n_o \end{cases}$$

Circular Shift of a Sequence

- If the length-N sequence is displayed on a circle at N equally spaced points, then the circular shift operation can be viewed as a clockwise or anti-clockwise rotation of the sequence by n_o sample spacings as shown on the next slide

Circular Shift of a Sequence



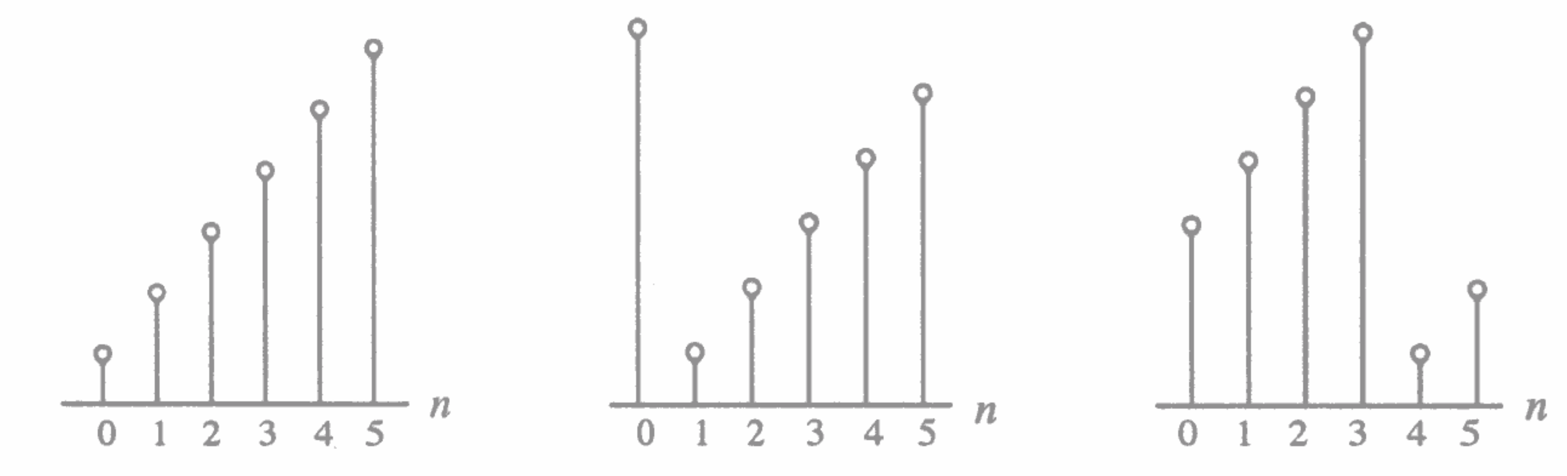


$x[n]$

$x[\langle n-4\rangle_6] = x[\langle n+2\rangle_6]$

Circular Shift of a Sequence

- Illustration of the concept of a circular shift



$x[n]$ $\quad$ $x[\langle n-1\rangle_6]$ $\quad$ $x[\langle n-4\rangle_6]$

$= x[\langle n+5\rangle_6]$ $\quad$ $= x[\langle n+2\rangle_6]$

Circular Shift of a Sequence

- As can be seen from the previous figure, a right circular shift by n_o is equivalent to a left circular shift by $N - n_o$ sample periods
- A circular shift by an integer number n_o greater than N is equivalent to a circular shift by $\langle n_o \rangle_N$

Classification of Sequences

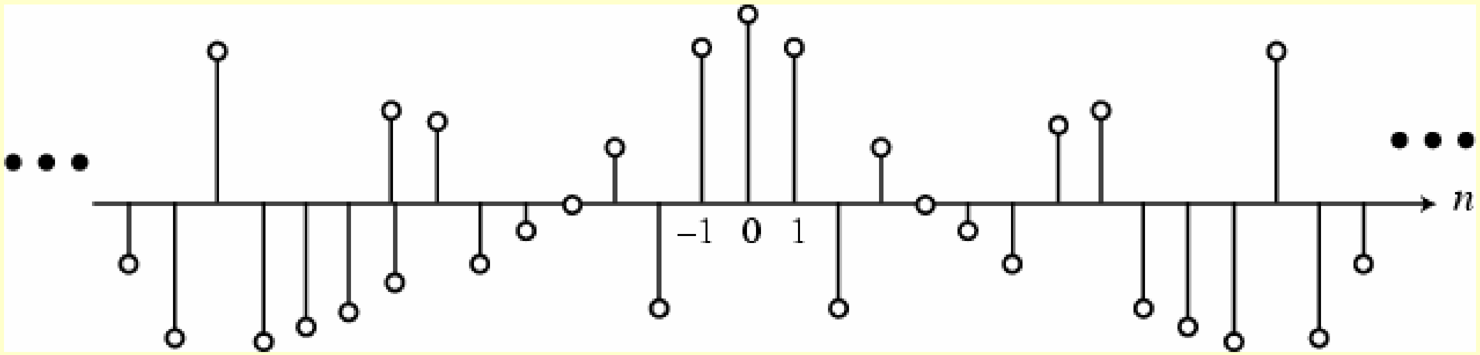
- There are several types of classification
- One classification is in terms of the number of samples defining the sequence
- Another classification is based on its symmetry with respect to time index $n = 0$
- Other classifications in terms of its other properties, such as periodicity, summability, energy and power

Classification of Sequences Based on Symmetry

- **Conjugate-symmetric sequence**:

$$x[n] = x^*[-n]$$

If $x[n]$ is real, then it is an **even sequence**



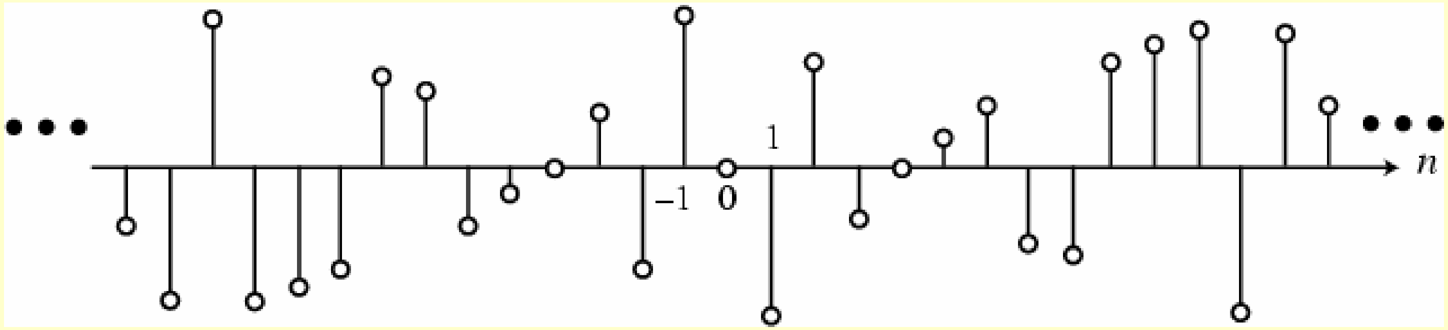
An even sequence

Classification of Sequences Based on Symmetry

- **Conjugate-antisymmetric sequence**:

$$x[n] = -x^*[-n]$$

If $x[n]$ is real, then it is an **odd sequence**



An odd sequence

Classification of Sequences Based on Symmetry

- It follows from the definition that for a conjugate-symmetric sequence $\{x[n]\}$, $x[0]$ must be a real number
- Likewise, it follows from the definition that for a conjugate anti-symmetric sequence $\{y[n]\}$, $y[0]$ must be an imaginary number
- From the above, it also follows that for an odd sequence $\{w[n]\}$, $w[0] = 0$

Classification of Sequences Based on Symmetry

- Any complex sequence can be expressed as a sum of its conjugate-symmetric part and its conjugate-antisymmetric part:

$$x[n] = x_{cs}[n] + x_{ca}[n]$$

where

$$x_{cs}[n] = \frac{1}{2}\left(x[n] + x^*[-n]\right)$$

$$x_{ca}[n] = \frac{1}{2}\left(x[n] - x^*[-n]\right)$$

Classification of Sequences Based on Symmetry

- As indicated in the previous slide, computation of conjugate-symmetric and conjugate anti-symmetric parts of a sequence involves conjugation, time-reversal, addition, and multiplication operations

Classification of Sequences Based on Symmetry

- The decomposition of a finite-length sequence into a sum of conjugate-symmetric and conjugate anti-symmetric sequences is possible if the parent sequence is an odd sequence defined for a symmetric interval, i.e.,

$$-M \leq n \leq M$$

Classification of Sequences Based on Symmetry

- Example - Consider the length-7 sequence defined for $-3 \le n \le 3$:

$$\{g[n]\} = \{0,\ 1+j4,\ -2+j3,\ \underset{\uparrow}{4-j2},\ -5-j6,\ -j2,\ 3\}$$

- Its conjugate sequence is then given by

$$\{g^*[n]\} = \{0,\ 1-j4,\ -2-j3,\ \underset{\uparrow}{4+j2},\ -5+j6,\ j2,\ 3\}$$

- The time-reversed version of the above is

$$\{g^*[-n]\} = \{3,\ j2,\ -5+j6,\ \underset{\uparrow}{4+j2},\ -2-j3,\ 1-j4,\ 0\}$$

Classification of Sequences Based on Symmetry

- Therefore $\{g_{cs}[n]\} = \frac{1}{2}\{g[n] + g^*[-n]\}$

$$= \{1.5,\ \ 0.5 + j3,\ \ -3.5 + j4.5,\ \ \underset{\uparrow}{4},\ \ -3.5 - j4.5,\ \ 0.5 - j3,\ \ 1.5\}$$

- Likewise $\{g_{ca}[n]\} = \frac{1}{2}\{g[n] - g^*[-n]\}$

$$= \{-1.5,\ \ 0.5 + j,\ \ 1.5 - j1.5,\ \ \underset{\uparrow}{-j2},\ \ -1.5 - j1.5,\ \ -0.5 - j,\ \ 1.5\}$$

- It can be easily verified that $g_{cs}[n] = g^*_{cs}[-n]$ and $g_{ca}[n] = -g^*_{ca}[-n]$

Classification of Sequences Based on Symmetry

- Any real sequence can be expressed as a sum of its even part and its odd part:

$$x[n] = x_{ev}[n] + x_{od}[n]$$

where

$$x_{ev}[n] = \frac{1}{2}\left(x[n] + x[-n]\right)$$

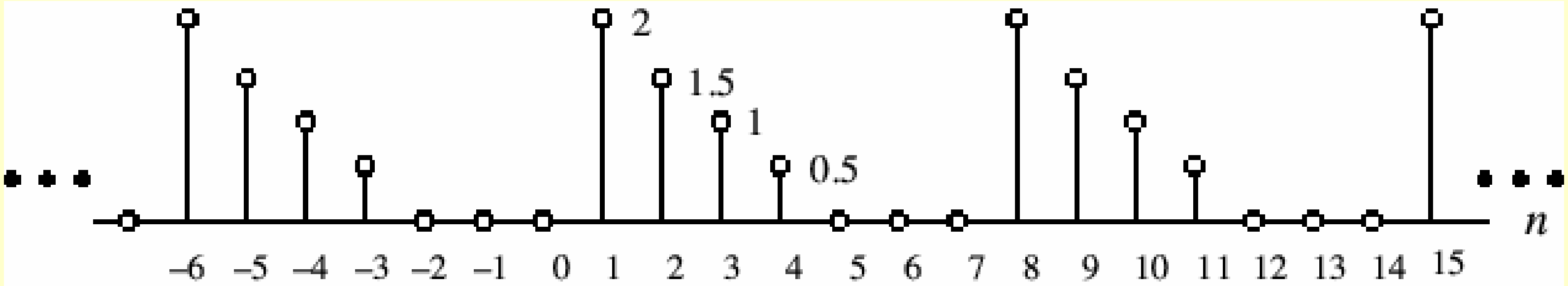
$$x_{od}[n] = \frac{1}{2}\left(x[n] - x[-n]\right)$$

Classification of Sequences Based on Periodicity

- A sequence $\tilde{x}[n]$ satisfying $\tilde{x}[n] = \tilde{x}[n + kN]$ is called a **periodic sequence** with a **period** N where N is a positive integer and k is any integer
- Smallest value of N satisfying $\tilde{x}[n] = \tilde{x}[n + kN]$ is called the **fundamental period**

Classification of Sequences Based on Periodicity

- Example -



- A sequence not satisfying the periodicity condition is called an **aperiodic sequence**

Classification of Sequences Based on Periodicity

- If $\tilde{x}_a[n]$ and $\tilde{x}_b[n]$ are two periodic sequences with fundamental periods N_a and N_b, respectively, then the sequence

$$\tilde{y}[n] = \tilde{x}_a[n] + \tilde{x}_b[n]$$

is a periodic sequence with a fundamental period N given by

$$N = \frac{N_a N_b}{GCD(N_a, N_b)}$$

Classification of Sequences Based on Periodicity

- If $\tilde{x}_a[n]$ and $\tilde{x}_b[n]$ are two periodic sequences with fundamental periods N_a and N_b, respectively, then the sequence

$$\tilde{y}[n] = \tilde{x}_a[n] \cdot \tilde{x}_b[n]$$

is a periodic sequence with a fundamental period N given by

$$N = \frac{N_a N_b}{GCD(N_a, N_b)}$$

Classification of Sequences: Energy and Power Signals

- Total **energy** of a sequence $x[n]$ is defined by

$$\mathcal{E}_{\mathrm{x}} = \sum_{n=-\infty}^{\infty} \left| x[n] \right|^2$$

- An infinite length sequence with finite sample values may or may not have finite energy
- A finite length sequence with finite sample values has finite energy

Classification of Sequences: Energy and Power Signals

- **Example** – The infinite-length sequence

$$x[n] = \begin{cases} 1/n, & n \geq 1, \\ 0, & n \leq 0, \end{cases}$$

has an energy equal to

$$\mathcal{E}_x = \sum_{n=1}^{\infty} (1/n)^2$$

which converges to $\pi^2/6$, indicating that $x[n]$ has finite energy

Classification of Sequences: Energy and Power Signals

- **Example** – The infinite-length sequence

$$y[n] = \begin{cases} 1/\sqrt{n}, & n \geq 1, \\ 0, & n \leq 0, \end{cases}$$

has an energy equal to

$$\mathcal{E}_y = \sum_{n=1}^{\infty} (1/n)$$

which does not converge indicating that $y[n]$ has infinite energy

Classification of Sequences: Energy and Power Signals

- The **average power** of an aperiodic sequence is defined by

$$P_x = \lim_{K \to \infty} \frac{1}{2K+1} \sum_{n=-K}^{K} \left| x[n] \right|^2$$

- Define the **energy** of a sequence $x[n]$ over a finite interval $-K \leq n \leq K$ as

$$\mathcal{E}_{x,K} = \sum_{n=-K}^{K} \left| x[n] \right|^2$$

Classification of Sequences: Energy and Power Signals

- Then

$$P_x = \lim_{K \to \infty} \frac{1}{2K+1} \mathcal{E}_{x.K}$$

- The **average power** of a periodic sequence $\tilde{x}[n]$ with a period N is given by

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} \left| \tilde{x}[n] \right|^2$$

- The average power of an infinite-length sequence may be finite or infinite

Classification of Sequences: Energy and Power Signals

- Example - Consider the causal sequence defined by

$$x[n] = \begin{cases} 3(-1)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

- Note: $x[n]$ has infinite energy
- Its average power is given by

$$P_x = \lim_{K\to\infty} \frac{1}{2K+1}\left(9\sum_{n=0}^{K} 1\right) = \lim_{K\to\infty} \frac{9(K+1)}{2K+1} = 4.5$$

Classification of Sequences: Energy and Power Signals

- An infinite energy signal with finite average power is called a **power signal**

  Example - A periodic sequence which has a finite average power but infinite energy

- A finite energy signal with zero average power is called an **energy signal**

  Example - A finite-length sequence which has finite energy but zero average power

Other Types of Classifications

- A sequence $x[n]$ is said to be **bounded** if

$$|x[n]| \leq B_x < \infty$$

- Example - The sequence $x[n] = \cos 0.3\pi n$ is a bounded sequence as

$$|x[n]| = |\cos 0.3\pi n| \leq 1$$

Other Types of Classifications

- A sequence $x[n]$ is said to be **absolutely summable** if

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

- Example - The sequence

$$y[n] = \begin{cases} 0.3^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

is an absolutely summable sequence as

$$\sum_{n=0}^{\infty} \left|0.3^n\right| = \frac{1}{1-0.3} = 1.42857 < \infty$$

Other Types of Classifications

- A sequence $x[n]$ is said to be **square-summable** if

$$\sum_{n=-\infty}^{\infty} \left| x[n] \right|^2 < \infty$$

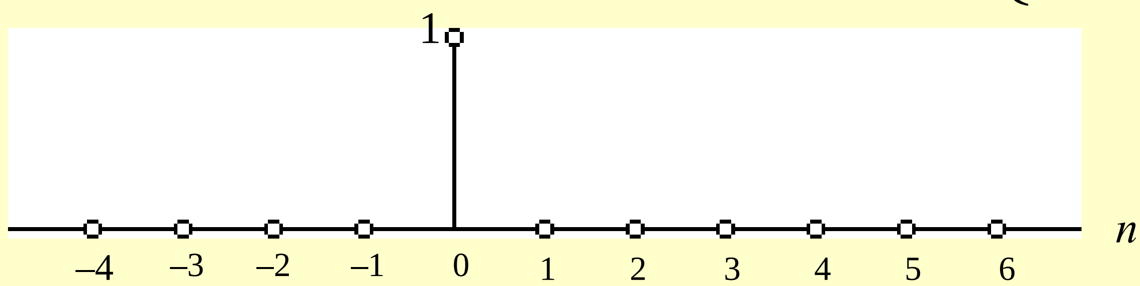
- Example - The sequence

$$h[n] = \frac{\sin 0.4n}{\pi n}$$

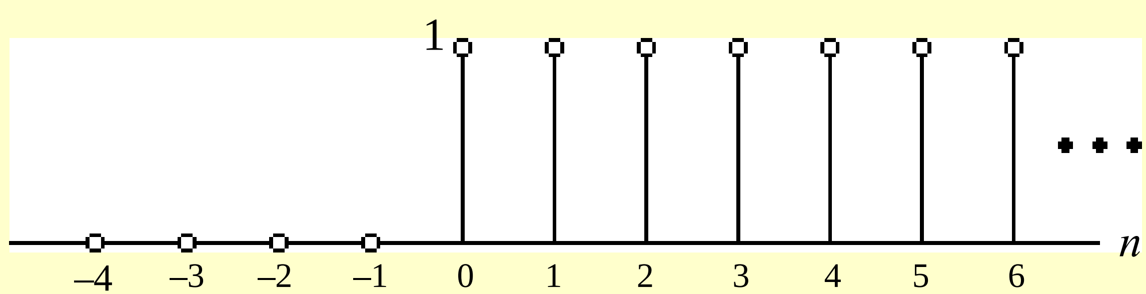
is square-summable but not absolutely summable

Basic Sequences

- **Unit sample sequence** - $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$



- **Unit step sequence** - $\mu[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$



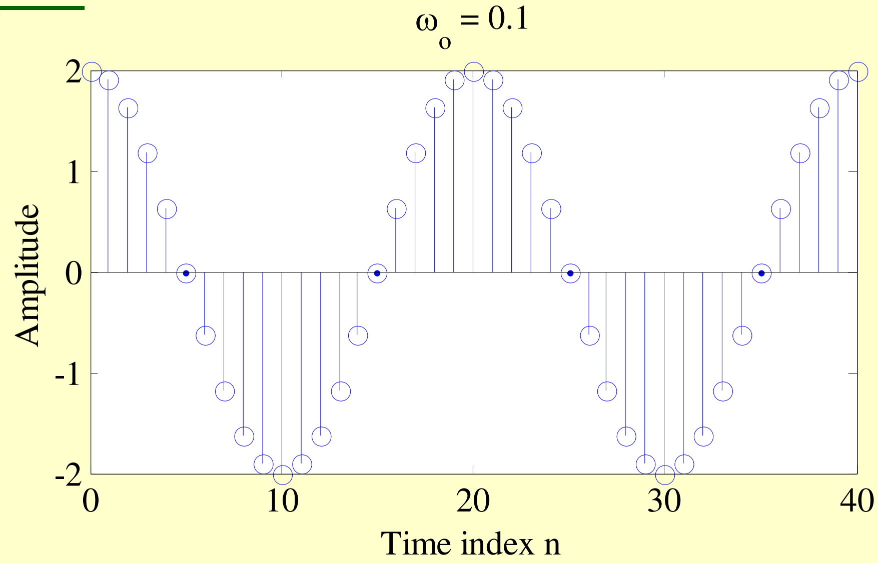
Basic Sequences

- **Real sinusoidal sequence** -

$$x[n] = A\cos(\omega_o n + \phi)$$

where A is the **amplitude**, ω_o is the **angular frequency**, and ϕ is the **phase** of $x[n]$

Example -



Basic Sequences

- **Exponential sequence -**

$$x[n] = A\alpha^n, \; -\infty < n < \infty$$

where A and α are real or complex numbers

- If we write $\alpha = e^{(\sigma_o + j\omega_o)}, A = |A|e^{j\phi},$

then we can express

$$x[n] = |A|e^{j\phi}e^{(\sigma_o + j\omega_o)n} = x_{re}[n] + j\,x_{im}[n],$$

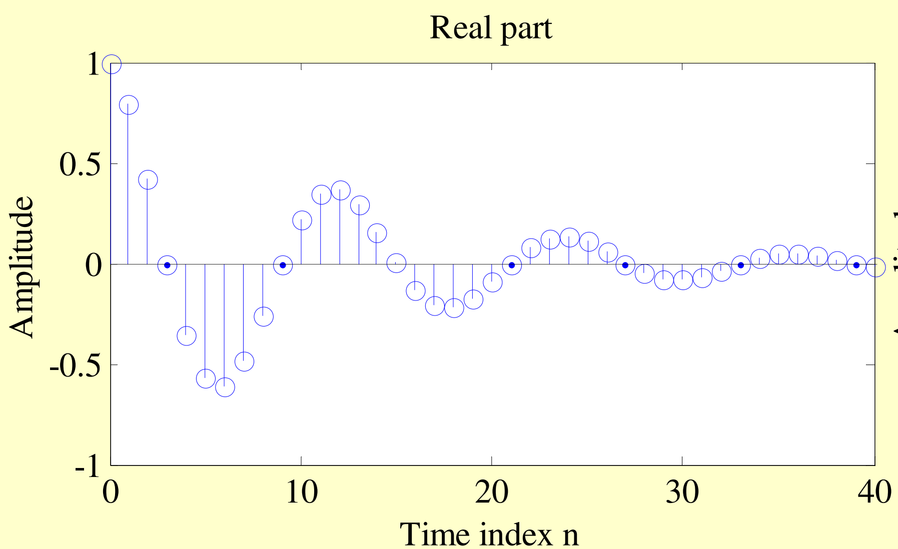
where

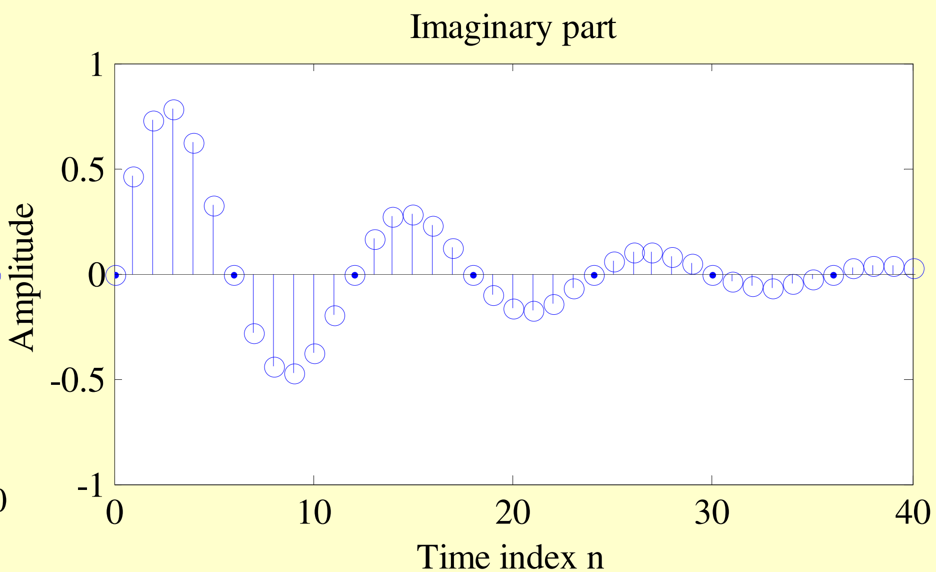
$$x_{re}[n] = |A|e^{\sigma_o n}\cos(\omega_o n + \phi),$$

$$x_{im}[n] = |A|e^{\sigma_o n}\sin(\omega_o n + \phi)$$

Basic Sequences

- $x_{re}[n]$ and $x_{im}[n]$ of a complex exponential sequence are real sinusoidal sequences with constant $(\sigma_o = 0)$, growing $(\sigma_o > 0)$, and decaying $(\sigma_o < 0)$ amplitudes for $n > 0$





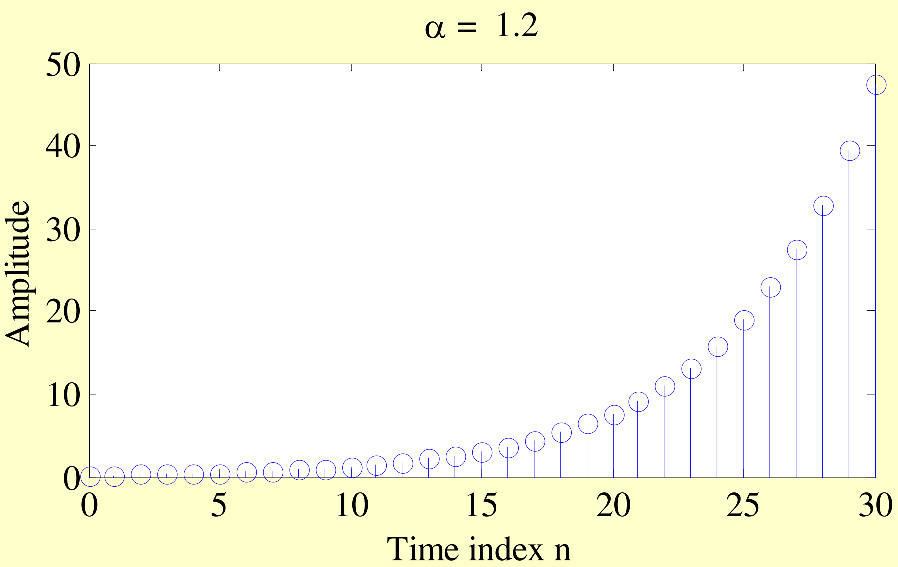
$$x[n] = \exp\left(-\frac{1}{12} + j\frac{\pi}{6}\right)n$$

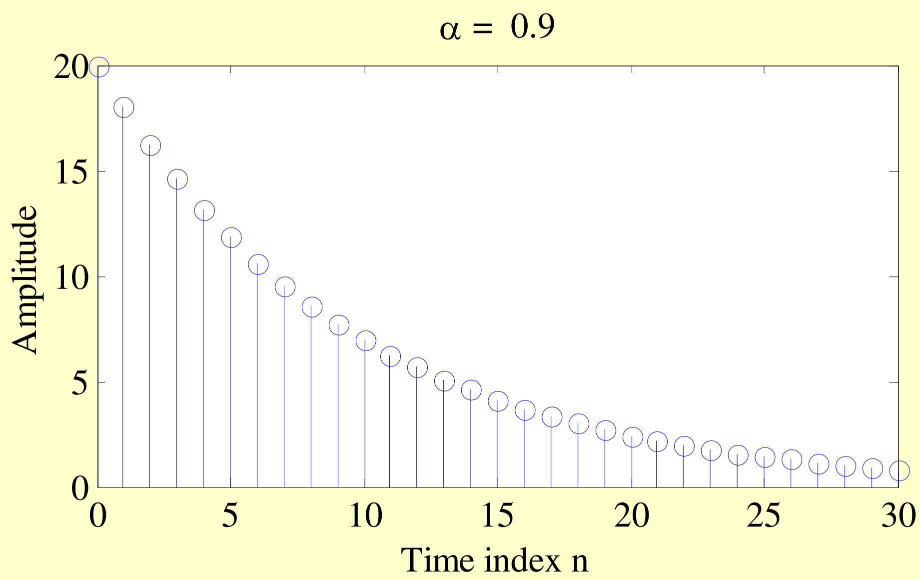
Basic Sequences

- **Real exponential sequence -**

$$x[n] = A\alpha^n, \ -\infty < n < \infty$$

where A and α are real numbers





Basic Sequences

- Sinusoidal sequence $A\cos(\omega_o n + \phi)$ and complex exponential sequence $B\exp(j\omega_o n)$ are periodic sequences of period N if $\omega_o N = 2\pi r$ where N and r are positive integers
- Smallest value of N satisfying $\omega_o N = 2\pi r$ is the **fundamental period** of the sequence
- To verify the above fact, consider

$$x_1[n] = \cos(\omega_o n + \phi)$$
$$x_2[n] = \cos(\omega_o (n + N) + \phi)$$

Basic Sequences

- Now $x_2[n] = \cos(\omega_o(n+N)+\phi)$

  $= \cos(\omega_o n+\phi)\cos\omega_o N - \sin(\omega_o n+\phi)\sin\omega_o N$

  which will be equal to $\cos(\omega_o n+\phi) = x_1[n]$ only if

  $$\sin\omega_o N = 0 \text{ and } \cos\omega_o N = 1$$

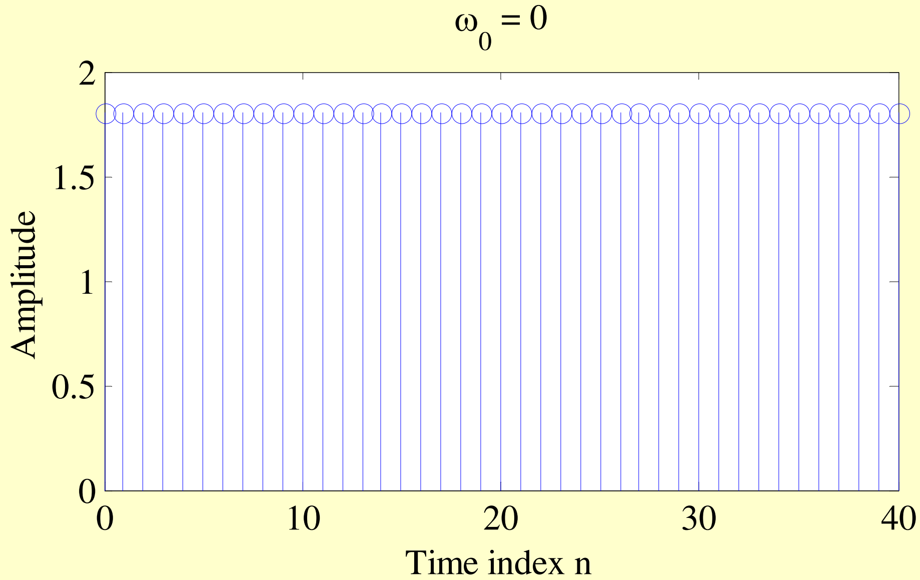
- These two conditions are met if and only if

  $$\omega_o N = 2\pi\, r \quad \text{or} \quad \frac{2\pi}{\omega_o} = \frac{N}{r}$$

Basic Sequences

- If $2\pi/\omega_o$ is a noninteger rational number, then the period will be a multiple of $2\pi/\omega_o$
- Otherwise, the sequence is **aperiodic**
- Example - $x[n] = \sin(\sqrt{3}n + \phi)$ is an aperiodic sequence

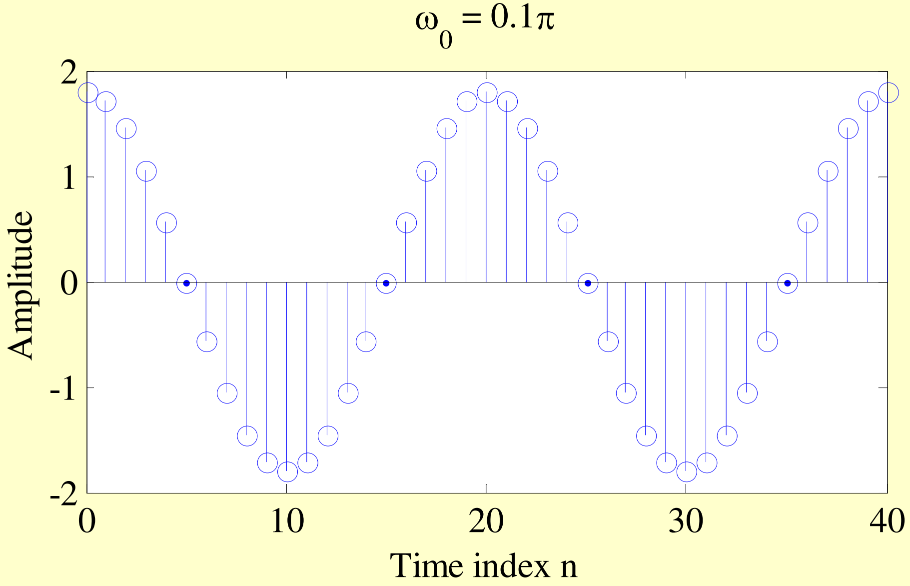
Basic Sequences



- Here $\omega_o = 0$

- Hence period $N = \dfrac{2\pi r}{0} = 1$ for $r = 0$

Basic Sequences



- Here $\omega_o = 0.1\pi$

- Hence $N = \dfrac{2\pi r}{0.1\pi} = 20$ for $r = 1$

Basic Sequences

- Property 1 - Consider $x[n] = \exp(j\omega_1 n)$ and $y[n] = \exp(j\omega_2 n)$ with $0 \leq \omega_1 < \pi$ and $2\pi k \leq \omega_2 < 2\pi(k+1)$ where k is any positive integer
- If $\omega_2 = \omega_1 + 2\pi k$, then $x[n] = y[n]$
- Thus, $x[n]$ and $y[n]$ are indistinguishable

Basic Sequences

- Property 2 - The frequency of oscillation of $A\cos(\omega_o n)$ increases as ω_o increases from 0 to π, and then decreases as ω_o increases from π to 2π
- Thus, frequencies in the neighborhood of $\omega = 0$ are called **low frequencies**, whereas, frequencies in the neighborhood of $\omega = \pi$ are called **high frequencies**

Basic Sequences

- Because of Property 1, a frequency ω_o in the neighborhood of $\omega = 2\pi\, k$ is indistinguishable from a frequency $\omega_o - 2\pi k$ in the neighborhood of $\omega = 0$

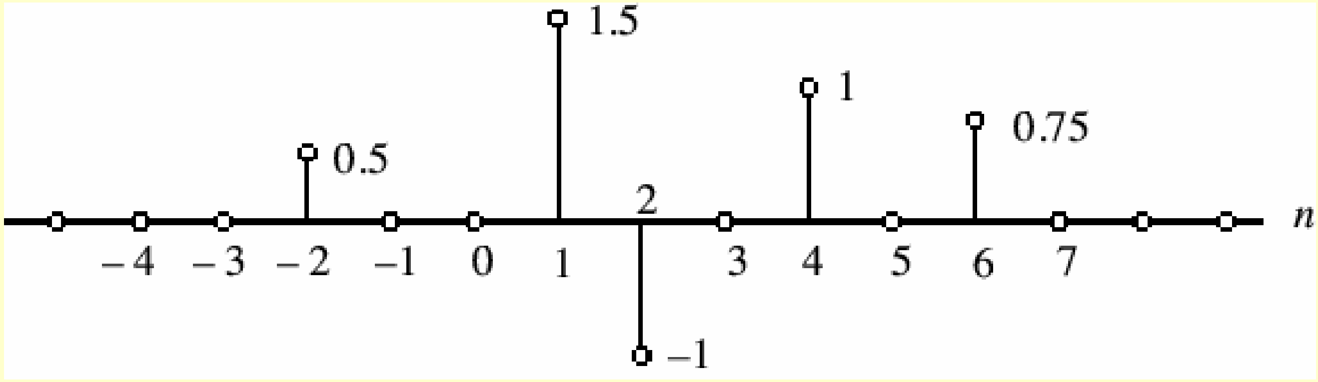
  and a frequency ω_o in the neighborhood of $\omega = \pi(2k+1)$ is indistinguishable from a frequency $\omega_o - \pi(2k+1)$ in the neighborhood of $\omega = \pi$

Basic Sequences

- Frequencies in the neighborhood of $\omega = 2\pi\, k$ are usually called **low frequencies**
- Frequencies in the neighborhood of $\omega = \pi\,(2k{+}1)$ are usually called **high frequencies**
- $v_1[n] = \cos(0.1\pi n) = \cos(1.9\pi n)$ is a low-frequency signal
- $v_2[n] = \cos(0.8\pi n) = \cos(1.2\pi n)$ is a high-frequency signal

Basic Sequences

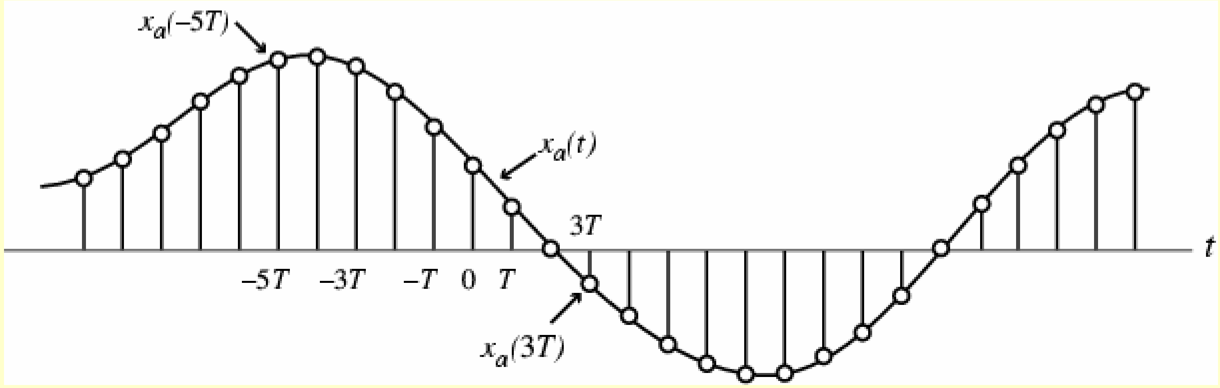
- An arbitrary sequence can be represented in the time-domain as a weighted sum of some basic sequence and its delayed (advanced) versions



$$x[n] = 0.5\delta[n+2] + 1.5\delta[n-1] - \delta[n-2] + \delta[n-4] + 0.75\delta[n-6]$$

The Sampling Process

- Often, a discrete-time sequence $x[n]$ is developed by uniformly sampling a continuous-time signal $x_a(t)$ as indicated below



- The relation between the two signals is

$$x[n] = x_a(t)\big|_{t=nT} = x_a(nT),\ n = \ldots, -2, -1, 0, 1, 2, \ldots$$

The Sampling Process

- Time variable t of $x_a(t)$ is related to the time variable n of $x[n]$ only at discrete-time instants t_n given by

$$t_n = nT = \frac{n}{F_T} = \frac{2\pi n}{\Omega_T}$$

with $F_T = 1/T$ denoting the sampling frequency and

$\Omega_T = 2\pi F_T$ denoting the sampling angular frequency

The Sampling Process

- Consider the continuous-time signal

$$x(t) = A\cos(2\pi f_o t + \phi) = A\cos(\Omega_o t + \phi)$$

- The corresponding discrete-time signal is

$$x[n] = A\cos(\Omega_o nT + \phi) = A\cos(\frac{2\pi\Omega_o}{\Omega_T} n + \phi)$$

$$= A\cos(\omega_o n + \phi)$$

where $\omega_o = 2\pi\Omega_o / \Omega_T = \Omega_o T$

is the normalized digital angular frequency of $x[n]$

The Sampling Process

- If the unit of sampling period T is in seconds
- The unit of normalized digital angular frequency ω_o is radians/sample
- The unit of normalized analog angular frequency Ω_o is radians/second
- The unit of analog frequency f_o is hertz (Hz)

The Sampling Process

- The three continuous-time signals

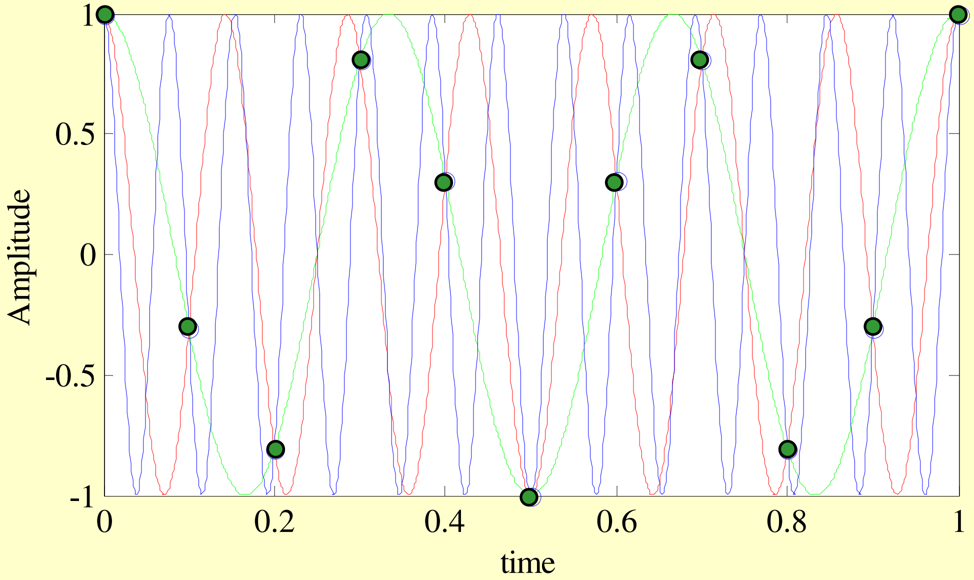
$$g_1(t) = \cos(6\pi t)$$
$$g_2(t) = \cos(14\pi t)$$
$$g_3(t) = \cos(26\pi t)$$

of frequencies 3 Hz, 7 Hz, and 13 Hz, are sampled at a sampling rate of 10 Hz, i.e. with $T = 0.1$ sec. generating the three sequences

$$g_1[n] = \cos(0.6\pi n) \qquad g_2[n] = \cos(1.4\pi n)$$
$$g_3[n] = \cos(2.6\pi n)$$

The Sampling Process

- Plots of these sequences (shown with circles) and their parent time functions are shown below:



- Note that each sequence has exactly the same sample value for any given n

The Sampling Process

- This fact can also be verified by observing that

$$g_2[n] = \cos(1.4\pi n) = \cos\big((2\pi - 0.6\pi)n\big) = \cos(0.6\pi n)$$

$$g_3[n] = \cos(2.6\pi n) = \cos\big((2\pi + 0.6\pi)n\big) = \cos(0.6\pi n)$$

- As a result, all three sequences are identical and it is difficult to associate a unique continuous-time function with each of these sequences

The Sampling Process

- The above phenomenon of a continuous-time signal of higher frequency acquiring the identity of a sinusoidal sequence of lower frequency after sampling is called **aliasing**

The Sampling Process

- Since there are an infinite number of continuous-time signals that can lead to the same sequence when sampled periodically, additional conditions need to imposed so that the sequence $\{x[n]\} = \{x_a(nT)\}$ can uniquely represent the parent continuous-time signal $x_a(t)$
- In this case, $x_a(t)$ can be fully recovered from $\{x[n]\}$

The Sampling Process

- Example - Determine the discrete-time signal $v[n]$ obtained by uniformly sampling at a sampling rate of 200 Hz the continuous-time signal

$$v_a(t) = 6\cos(60\pi t) + 3\sin(300\pi t) + 2\cos(340\pi t) + 4\cos(500\pi t) + 10\sin(660\pi t)$$

- Note: $v_a(t)$ is composed of 5 sinusoidal signals of frequencies 30 Hz, 150 Hz, 170 Hz, 250 Hz and 330 Hz

The Sampling Process

- The sampling period is $T = \dfrac{1}{200} = 0.005\ \text{sec}$
- The generated discrete-time signal $v[n]$ is thus given by

$$v[n] = 6\cos(0.3\pi n) + 3\sin(1.5\pi n) + 2\cos(1.7\pi n)$$
$$+\ 4\cos(2.5\pi n) + 10\sin(3.3\pi n)$$
$$= 6\cos(0.3\pi n) + 3\sin\big((2\pi - 0.5\pi)n\big) + 2\cos\big((2\pi - 0.3\pi)n\big)$$
$$+\ 4\cos\big((2\pi + 0.5\pi)n\big) + 10\sin\big((4\pi - 0.7\pi)n\big)$$

The Sampling Process

$$= 6\cos(0.3\pi n) - 3\sin(0.5\pi n) + 2\cos(0.3\pi n) + 4\cos(0.5\pi n)$$
$$- 10\sin(0.7\pi n)$$

$$= 8\cos(0.3\pi n) + 5\cos(0.5\pi n + 0.6435) - 10\sin(0.7\pi n)$$

- Note: $v[n]$ is composed of 3 discrete-time sinusoidal signals of normalized angular frequencies: 0.3π, 0.5π, and 0.7π

The Sampling Process

- Note: An identical discrete-time signal is also generated by uniformly sampling at a 200-Hz sampling rate the following continuous-time signals:

$$w_a(t) = 8\cos(60\pi t) + 5\cos(100\pi t + 0.6435) - 10\sin(140\pi t)$$

$$g_a(t) = 2\cos(60\pi t) + 4\cos(100\pi t) + 10\sin(260\pi t) \\ + 6\cos(460\pi t) + 3\sin(700\pi t)$$

The Sampling Process

- Recall $\omega_o = \dfrac{2\pi\Omega_o}{\Omega_T}$

- Thus if $\Omega_T > 2\Omega_o$, then the corresponding normalized digital angular frequency ω_o of the discrete-time signal obtained by sampling the parent continuous-time sinusoidal signal will be in the range $-\pi < \omega < \pi$

- $\Rightarrow$ No aliasing

The Sampling Process

- On the other hand, if $\Omega_T < 2\Omega_o$, the normalized digital angular frequency will foldover into a lower digital frequency $\omega_o = \langle 2\pi\Omega_o / \Omega_T \rangle_{2\pi}$ in the range $-\pi < \omega < \pi$ because of aliasing
- Hence, to prevent aliasing, the sampling frequency Ω_T should be greater than 2 times the frequency Ω_o of the sinusoidal signal being sampled

The Sampling Process

- Generalization: Consider an arbitrary continuous-time signal $x_a(t)$ composed of a weighted sum of a number of sinusoidal signals
- $x_a(t)$ can be represented uniquely by its sampled version $\{x[n]\}$ if the sampling frequency Ω_T is chosen to be greater than 2 times the highest frequency contained in $x_a(t)$

The Sampling Process

- The condition to be satisfied by the sampling frequency to prevent aliasing is called the **sampling theorem**
- A formal proof of this theorem will be presented later