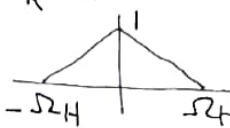


DSP Midterm 2023 Spring

1. Consider $g_a(t)$ with CTFT $G_a(j\Omega)$. If $g_a(t)$ is sampled at $t = nT$ to obtain $g[n]$ with DTFT $G(e^{j\omega})$

① Show that $G(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} G_a(j\Omega - jk\Omega_T)$ (*) (25%)

② Suppose $G_a(j\Omega) =$  and $\Omega_T = 3\Omega_H$

Plot $G(e^{j\omega})$ using (*)

2. ① If $g[n]$ has DTFT $G(e^{j\omega})$, explain why $g[n]$ and $g[n-na]$ have the same magnitude spectrum.

② In general, do $g[n]$ and $g[n-na]$ have the same phase spectrum? Why? (25%)

3. Use (*) to explain $G(e^{j\omega})$ is periodic and period is 2π . (25%)

4. ① Consider the system $h[n] = \begin{cases} 1/M, & 0 \leq n \leq M-1 \\ 0, & \text{otherwise} \end{cases}$

Show that

$$H(e^{j\omega}) = \frac{1}{M} \frac{\sin(M\omega/2)}{\sin(\omega/2)} e^{-j(M-1)\omega/2}$$

② If $M=3$, roughly plot its magnitude and phase spectrum. (25%)

③ Explain that the system has linear phase

④ Does the system α, β, α has linear phase? Why?