DSP Final

1.1 For the ideal LP filter $h_{LP}[n] = \dfrac{\sin \omega_c n}{\pi n},\ -\infty < n < \infty$, design a length-9, causal, linear phase LP FIR $h_9[n]$. Also, provides $H_9(z)$. *(20%)*

(note: Express both $h_9[n]$ and $H_9(z)$ explicitly <u>in mathematical terms</u>.)

1.2 Consider the input to $H_9(z)$: $x[n] = \cos\omega_0 n,\ 0 \le n \le 1999$.

Segment $x[n]$ into 200 length-10 sequences: $x_0[n],\ x_1[n], \cdots,\ x_{199}[n]$

Express $x[n]$ in terms of $x_i[n],\ 0 \le i \le 199$. *(5%)*

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2. Suppose the DTFT, DFT and ZT of $x[n]$ are $X(e^{j\omega})$, $X[k]$ and $X(Z)$, respectively. It is known that $x[n-n_0] \Leftrightarrow e^{-j\omega n_0} X(e^{j\omega})$. Then, what is DFT of $x[<n-n_0>_N]$? And what is ZT of $x[n-n_0]$? *(15%)*

3. Use pole factors and zero factors to reason and explain the following:

3.1 What is the function of the filter:

$H(z) = \dfrac{1}{1-\alpha z^{-1}}$, $\le 0\alpha \le$, LP, HP or …, why? *(15%)*

3.2 What is the function of the filter:

$H(z) = \dfrac{1+z^{-1}}{1-\alpha z^{-1}}$, $0 \le \alpha \le 1$, LP, HP or …, why? *(15%)*

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If $h_9[n]$ is type I FIR, then $h_9[n]$ has linear phase

Let $h_9[n]=h_{LP}[n-4]$, $0\le n\le 8$ $\Rightarrow$ 9-length sequence

$\Rightarrow h_9[n]=\dfrac{\sin w_c(n-4)}{\pi(n-4)}$, $0\le n\le 8$ is type I FIR

(because $h_9[n]=h_9[8-n]$)

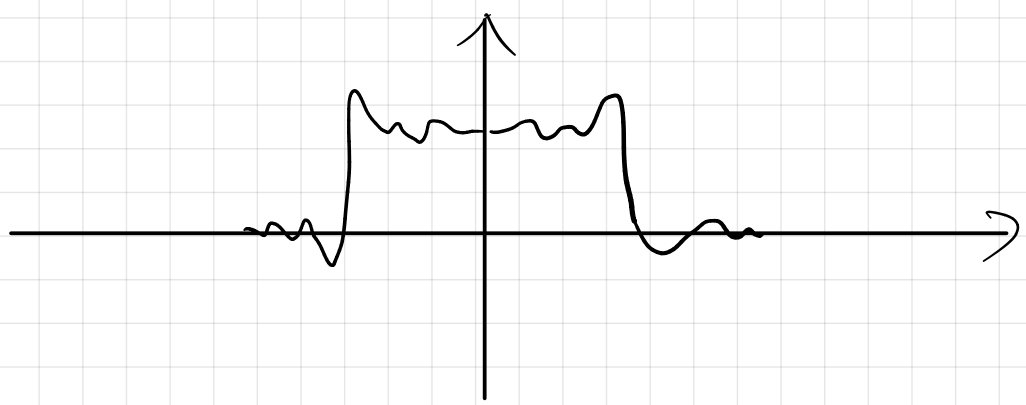
$$h_9[n]=\frac{\sin 4w_c}{4\pi}(\delta[n]+\delta[n-8])+\frac{\sin 3w_c}{3\pi}(\delta[n-1]+\delta[n-7])$$
$$+\frac{\sin 2w_c}{2\pi}(\delta[n-2]+\delta[n-6])+\frac{\sin w_c}{\pi}(\delta[n-3]+\delta[n-5])+\frac{w_c}{\pi}\delta[n-4]$$

the DTFT of $h_9[n]$

$$\Rightarrow H_9(e^{jw})=e^{-j4w}\Big(2\cos 4w\,\frac{\sin 4w_c}{4\pi}+2\cos 3w\,\frac{\sin 3w_c}{3\pi}$$
$$+2\cos 2w\,\frac{\sin 2w_c}{2\pi}+2\cos w\,\frac{\sin w_c}{\pi}+\frac{w_c}{\pi}\Big)$$

we can obtain the system $h_9[n]$ has linear phase by its DTFT

and the magnitude response shown in below



is a low pass filter

the z-transform of $h_9[n]$ is:

$$H_9(z) = \frac{\sin 4w_c}{4\pi}(1+z^{-8}) + \frac{\sin 3w_c}{3\pi}(z^{-1}+z^{-7}) + \frac{\sin 2w_c}{2\pi}(z^{-2}+z^{-6}) + \frac{\sin w_c}{\pi}(z^{-3}+z^{-5}) + \frac{w_c}{\pi}z^{-4}$$

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$\boxed{1.2}$ $x[n] = \cos w_0 n, \quad 0 \le n \le 1999$

let $x_i[n] = x[n+10i], \quad 0 \le n \le 9, \quad i = 0,1,2,\ldots,199$

$$x[n] = x_0[n] + x_1[n-10] + \cdots + x_{199}[n-1990] = \sum_{i=0}^{199} x_i[n-10i]$$

#

$\boxed{1.3}$ we know $y_i[n] = h_9[n] \circledast x_i[n]$

$\overset{ZT}{\Rightarrow} \quad Y_i(z) = H_9(z)X_i(z)$

So we can evaluate $y_i[n]$ by following step

step 1: calculate the z-transform of $h_9[n]$ and $x_i[n]$, name it as $H_9(z)$ and $X_i(z)$

step 2: calculate $H_9(z)X_i(z) \triangleq Y_i(z)$

step3: we can get $y_i[n]$ by the inverse z-transform of $Y_i(z)$

$\boxed{1.4}$ by $x[n] = \sum_{i=0}^{199} x_i[n-10i]$

$$\Rightarrow X(z) = \sum_{i=0}^{199} z^{-10i} X_i(z)$$

then $Y(z) = H_a(z) X(z)$

$$= \sum_{i=0}^{199} z^{-10i} H_a(z) X_i(z)$$

$$= \sum_{i=0}^{199} z^{-10i} Y_i(z)$$

$$\xrightarrow{IZT} y[n] = \sum_{i=0}^{199} y_i[n-10i]$$

$$= y_0[n] + y_1[n-10] + \dots + y_{199}[n-1990]$$

#

2. Suppose the DTFT, DFT and ZT of $x[n]$ are $X(e^{j\omega})$, $X[k]$ and $X(Z)$, respectively. It is known that $x[n-n_0] \Leftrightarrow e^{-j\omega n_0} X(e^{j\omega})$. Then, what is DFT of $x[<n-n_0>_N]$? And what is ZT of $x[n-n_0]$? *(15%)*

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$\boxed{2.}$

DFT is come from DTFT by sampling N point in $0 \sim 2\pi$

i.e. let $w = \dfrac{2\pi k}{N}$ substitute into $X(e^{jw})$

$$\Rightarrow X(e^{j\frac{2\pi k}{N}}) = X[k]$$

反正儿、代定义去証，可得

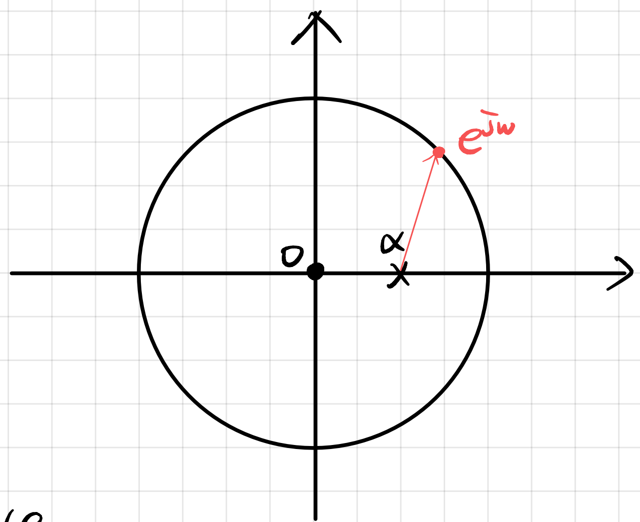
$$x[<n-n_0>_N] \xleftrightarrow{DFT} e^{-j\frac{2\pi <n_0>_N}{N}} X[k]$$

$$x[n-n_0] \xleftrightarrow{ZT} z^{-n_0} X(z)$$

$\boxed{3.}$

$$H(z) = \frac{1}{1-\alpha z^{-1}} = \frac{z}{z-\alpha}, \quad 0 \le \alpha \le 1$$

the pole, zero plot of $H(z)$ is :



the magnitude resp. of $H(z)$ is

$$|H(e^{jw})| = \frac{|e^{jw}|}{|e^{jw}-\alpha|} = \frac{1}{|e^{jw}-\alpha|}$$ → depends on the distance between e^{jw} and α

by the pole-zero plot of $H(z)$, we can obtain when w is small, $|e^{jw}-\alpha|$ is small → $|H(e^{jw})|$ 大

when w is bigger, $|e^{jw}-\alpha|$ become bigger → $|H(e^{jw})|$ 大

so the system is high pass filter

$\boxed{4}$ $\dfrac{1+z^{-1}}{1-\alpha z^{-1}}$, $0 \le \alpha \le 1$

$\Rightarrow \dfrac{z+1}{z-\alpha}$

$\Rightarrow$ low pass filter