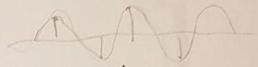
DSP First

Consider $x(t) = \cos(80\pi t + \frac{\pi}{3})$ 40Hz

① Plot spectrum of $x(t)$ (10%)

② Plot spectrum of $x(t+0.01)$ (10%)



③ Suppose $x[n]$ is obtained from sampling $x(t)$ at sampling rate f_s. Plot the spectrum of $x[n]$ between -3π and 3π for $f_s = 80$ Hz, 160 Hz and 800 Hz. (15%)

④ What are the three $\bar{x}(t)$ reconstructed from the three spectra in ③? (15%)

⑤ For $f_s = 32$ Hz, plot the spectrum of $x[n]$ in $(-3\pi, 3\pi)$. What is the $\bar{x}(t)$ reconstructed from spectrum of $x[n]$? (15%)

⑥ For $f_s = 50$ Hz, plot the spectrum of $x[n]$ in $(-3\pi, 3\pi)$. What is the $\bar{x}(t)$ reconstructed from spectrum of $x[n]$? (15%)

Consider $y[n] = 0.8\, y[n-1] + \frac{1}{3}\{x[n+1] + x[n] + x[n-1]\}$

⑦ Is this a causal system? Why? (10%)

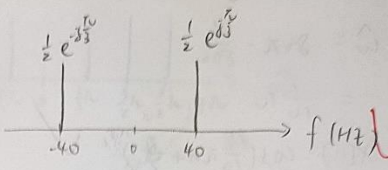
⑧ Is this system a FIR? Why? (10%)

$x(t) = \cos(80\pi t + \frac{\pi}{3})$

1.

$x(t) = \frac{e^{j(80\pi t+\frac{\pi}{3})} + e^{-j(80\pi t+\frac{\pi}{3})}}{2}$

10 $= \frac{1}{2} e^{j\frac{\pi}{3}} e^{j80\pi t} + \frac{1}{2} e^{-j\frac{\pi}{3}} e^{-j80\pi t}$



#

2. time shift

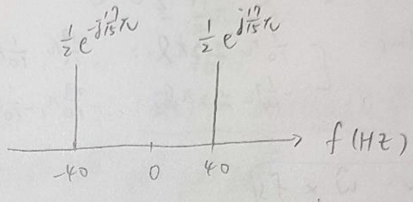
$\boxed{b_k = a_k \times e^{-j2\pi f_k 2d}}$

$x(t-2d)$

$2d = -0.01$

$\frac{1}{2} e^{j\frac{\pi}{3}} \times e^{-j2\pi 40\cdot(-0.01)} = \frac{1}{2} e^{j\frac{17}{15}\pi}$

~~5~~ 10 $\frac{1}{2} e^{-j\frac{\pi}{3}} \times e^{-j2\pi(-40)(-0.01)} = \frac{1}{2} e^{-j\frac{17}{15}\pi}$



#

x(t) function?

3.

(1) $f_s = 80$ Hz

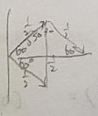
$\hat{w} = \omega \cdot \frac{1}{f_s}$

~~10~~ $= 80\pi \cdot \frac{1}{80}$

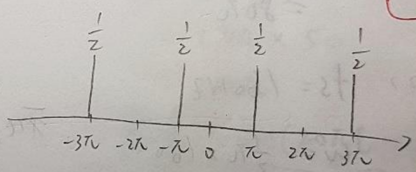
15 $= \pi$

$\frac{1}{2} e^{j\frac{\pi}{3}} + \frac{1}{2} e^{-j\frac{\pi}{3}}$

$= \frac{1}{2}$



x[n] function?



alias: $\pi \pm 2\pi l$: … $-\pi, \pi, 3\pi$ …
$-\pi \pm 2\pi l$: … $-3\pi, -\pi, \pi$ …

alias: $\pi \pm 2\pi l$
$-\pi \pm 2\pi l$

#

phase?

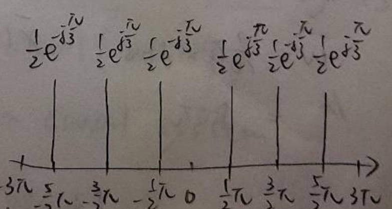
(2) $f_s = 160$ Hz

$\hat{w} = 80\pi \cdot \frac{1}{160}$

$= \frac{1}{2}\pi$

$\therefore x[n] = \cos(\frac{1}{2}\pi n + \frac{\pi}{3})$

$= \frac{1}{2} e^{j\frac{\pi}{3}} e^{j\frac{1}{2}\pi n} + \frac{1}{2} e^{-j\frac{\pi}{3}} e^{-j\frac{1}{2}\pi n}$



alias: $\frac{1}{2}\pi \pm 2\pi l$: … $-\frac{3}{2}\pi, \frac{1}{2}\pi, \frac{5}{2}\pi$ …
$-\frac{1}{2}\pi \pm 2\pi l$: … $-\frac{5}{2}\pi, -\frac{1}{2}\pi, \frac{3}{2}\pi$ …

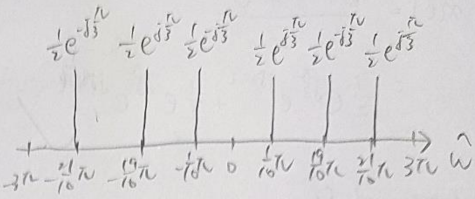
(3) $f_s = 800\,Hz$

$\hat{\omega} = 80\pi \cdot \frac{1}{800}$

$= \frac{1}{10}\pi$

$x(n) = \cos(\frac{1}{10}\pi n + \frac{\pi}{3})$

$= \frac{1}{2}e^{j\frac{\pi}{3}} e^{j\frac{1}{10}\pi n} + \frac{1}{2}e^{-j\frac{\pi}{3}} e^{-j\frac{1}{10}\pi n}$



#

alias: $\left[\begin{array}{l} \frac{1}{10}\pi \pm 2\pi l : \ldots -\frac{19}{10}\pi, \frac{1}{10}\pi, \frac{21}{10}\pi \ldots \\ -\frac{1}{10}\pi \pm 2\pi l : \ldots -\frac{21}{10}\pi, -\frac{1}{10}\pi, \frac{19}{10}\pi \ldots \end{array} \right.$

4.

15

$\boxed{\omega = \hat{\omega} \times f_s}$

(1) $f_s = 80\,Hz$

$\omega = \pi \times 80$

$= 80\pi$

$\therefore \bar{x}(t) = \cos(80\pi t)$ #

(2) $f_s = 160\,Hz$

$\omega = \frac{1}{2}\pi \times 160$

$= 80\pi$

$\therefore \bar{x}(t) = \cos(80\pi t + \frac{\pi}{3})$ #

(3) $f_s = 800\,Hz$

$\omega = \frac{1}{10}\pi \times 800$

$= 80\pi$

$\therefore \bar{x}(t) = \cos(80\pi t + \frac{\pi}{3})$ #

5. $f_s = 32\,Hz$

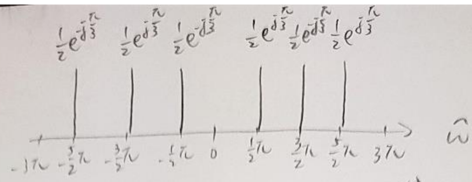
15

$\hat{\omega} = \omega \cdot \frac{1}{f_s}$

$= \overset{5}{\cancel{80}}\pi \times \frac{1}{32\times 2}$

$= \frac{5}{2}\pi$

alias: $\begin{cases} \frac{5}{2}\pi \pm 2\pi l : \ldots -\frac{3}{2}\pi, \frac{1}{2}\pi, \frac{5}{2}\pi \ldots \\ -\frac{5}{2}\pi \pm 2\pi l : \ldots -\frac{5}{2}\pi, -\frac{1}{2}\pi, \frac{3}{2}\pi \ldots \end{cases}$



#

$\omega = \hat{\omega} \times f_s$

$= \frac{1}{2}\pi \times 32$

$= 16\pi$

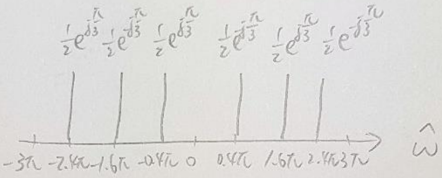
$\therefore \bar{x}(t) = \cos(16\pi t + \frac{\pi}{3})$ #

6. 15 $f_s = 50\,Hz$

$\hat{\omega} = 80\pi \times \frac{1}{50}$

$= 1.6\pi$

alias: $\begin{cases} 1.6\pi \pm 2\pi l : \ldots -2.4\pi, -0.4\pi, 1.6\pi \ldots \\ -1.6\pi \pm 2\pi l : \ldots -1.6\pi, 0.4\pi, 2.4\pi \ldots \end{cases}$



#

$\omega = \hat{\omega} \times f_s$

$= 0.4\pi \times 50$

$= 20\pi$

$\therefore \bar{x}(t) = \cos(20\pi t - \frac{\pi}{3})$ #

$y[n] = 0.8y[n-1] + \frac{1}{3}\{x[n+1] + x[n] + x[n-1]\}$

7. causal means to use only present or past signals, this system uses future signal ($x[n+1]$)

$\therefore$ No, this is not a causal system. #

8. FIR: $y[n] = \sum_{k=0}^{M} b_k x[n-k]$

該系統無法歸類成上述形式 #

$\therefore$ No, this system is not a FIR.