DSP

$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

1. $h[n]$ is LTI. Show that $y[n] = x[n] * h[n] = \sum_{\ell=-\infty}^{\infty} x[\ell]\, h[n-\ell]$

2. It is known that $x[n] * h[n] = h[n] * x[n]$. Show that if $h[n]$ is a causal FIR, then $y[n] = h[n] * x[n] = \sum_{\ell=0}^{M} h[\ell]\, x[n-\ell]$

3. $h[n]$ is a causal FIR. Use the input $x[n] = A e^{j\phi} e^{j\hat{\omega} n}$ to show that the frequency-response of the system is

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k]\, e^{-j\hat{\omega} k}$$

4. Consider $h[n] = \underset{\uparrow}{1}, 2, 1$
   ① Show that $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$ $-\pi \sim \pi$
   ② Plot the magnitude and phase of $H(e^{j\hat{\omega}})$

5. Let $h[n] = \underset{\uparrow}{\overset{0}{1}}, \overset{1}{2}, \overset{2}{1}, 0, 0, 0, 0, 0, 0$
   ① Give an expression of $H[k]$ $\overset{0 \sim N-1}{\boxed{\text{DFT}}}$
   ② Plot the magnitude and phase of $H[k]$

6. Consider FIR $h[n] = b_0, b_1, \ldots, b_{M-1}$
   It is known that $h[n-n_0] \rightleftharpoons e^{-j\hat{\omega} n_0} H(e^{j\hat{\omega}})$
   What is the $f(k)$ in $\underbrace{h[((n-n_0))_M]}_{\text{circular shift}} \rightleftharpoons f(k)\, H[k]$?

7. Consider $h[n] = b_0, b_1, b_2$ and $x[n] = x_0, x_1$
   Provide the algorithm 步驟 to use DFT and IDFT to do the linear convolution $h[n] * x[n]$.

Note: Problem 2 is 10%. All other problems are 15%

1.

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$h[n]$ is LTI : time invariant and linearity

①

$\delta[n] \longmapsto h[n]$

$\delta[n-1] \longmapsto h[n-1]$

$\vdots$

$\delta[n-l] \longmapsto h[n-l]$

time invariant

② $x[n]$ input $\quad$ output $y[n]$

$x[0]\,\delta[n] \longmapsto x[0]\,h[n]$

$x[1]\,\delta[n-1] \longmapsto x[1]\,h[n-1]$

$\vdots$

$x[l]\,\delta[n-l] \longmapsto x[l]\,h[n-l]$

linearity

∴將 ② 的各 input sum 起來 : $x[n] = \sum_{l=-\infty}^{\infty} x[l]\,\delta[n-l]$

∴ $y[n]$ = ② 的各 output sum 起來

$$= \sum_{l=-\infty}^{\infty} x[l]\,h[n-l] = x[n] * h[n]$$ verified.

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known : $x[n] * h[n] = h[n] * x[n]$ $\qquad h[n] = \sum_{k=0}^{M} h[k]\,\delta[n-k]$

$\begin{cases} h[n] \text{ is causal } \therefore h[n]=0 \text{ for } n<0 \\ h[n] \text{ is FIR } \therefore h[n]=0 \text{ for } n>M \end{cases}$

$$\therefore y[n] = h[n] * x[n]$$

$$= x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

$n-k=l$

$k=n-l$

$\begin{cases} k=-\infty \sim \infty \\ l=\infty \sim -\infty \end{cases}$

$$= \sum_{l=-\infty}^{\infty} h[l]\,x[n-l]$$

$$= \sum_{l=0}^{M} h[l]\,x[n-l]$$ verified.

3.

$$y[n] = h[n] * x[n]$$

15.

$$= \sum_{k=0}^{M} h[k]\, x[n-k]$$

$$= \sum_{k=0}^{M} h[k]\, A e^{j\phi} e^{j\hat{\omega}(n-k)}$$

$$= (A e^{j\phi} e^{j\hat{\omega} n}) \sum_{k=0}^{M} h[k]\, e^{-j\hat{\omega} k}$$

$$= x[n]\, H(e^{j\hat{\omega}})$$

$\therefore H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k]\, e^{-j\hat{\omega} k}$ verified.

4.

① $H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k]\, e^{-j\hat{\omega} k}$

$$= h[0] + h[1] e^{-j\hat{\omega}} + h[2] e^{-j\hat{\omega} 2}$$

$$= 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega} 2}$$

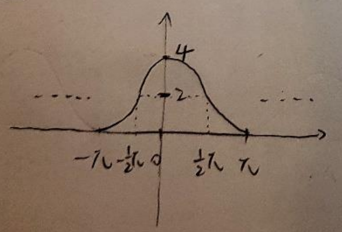
$$= 2e^{-j\hat{\omega}}\left(\tfrac{1}{2} e^{j\hat{\omega}} + 1 + \tfrac{1}{2} e^{-j\hat{\omega}}\right)$$

$$= 2e^{-j\hat{\omega}}(1 + \cos\hat{\omega})$$

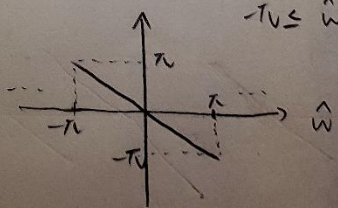
$$= e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$$ verified.

② 畫 $-\pi \le \hat{\omega} \le \pi$

magnitude: $|H(e^{j\hat{\omega}})| = |2 + 2\cos\hat{\omega}|$



phase: $\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$ for $-\pi \le \hat{\omega} \le \pi$



5. DFT:

$$\boxed{X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}n}}$$

now $N = 9$

① $H[k] = \sum_{n=0}^{8} h[n] e^{-j\frac{2\pi k}{9}n}$

$= 1 + 2e^{-j\frac{2\pi k}{9}\cdot 1} + e^{-j\frac{2\pi k}{9}\cdot 2}$

$= \underline{1 + 2e^{-j\frac{2\pi k}{9}} + e^{-j\frac{4\pi k}{9}}}$ #

②

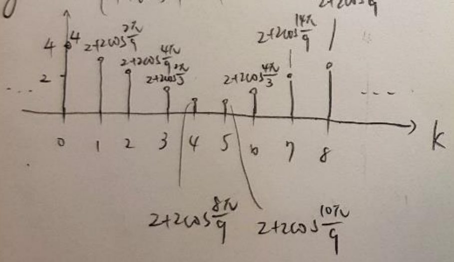
畫 $[0, N-1]$ $H[k] = 2e^{-j\frac{2\pi k}{9}}\left(\frac{1}{2}e^{j\frac{2\pi k}{9}} + 1 + \frac{1}{2}e^{-j\frac{2\pi k}{9}}\right)$

$= 2e^{-j\frac{2\pi k}{9}}\left(1 + \cos\left(\frac{2\pi k}{9}\right)\right)$

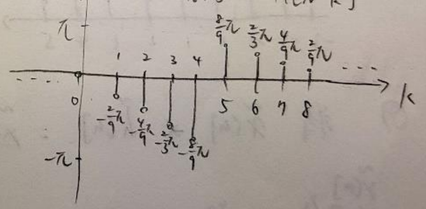
$= e^{-j\frac{2\pi k}{9}}\left(2 + 2\cos\frac{2\pi k}{9}\right)$

mag: $|H[k]| = \left|2 + 2\cos\frac{2\pi k}{9}\right|$

phase: $\angle H[k] = -\frac{2\pi k}{9}$, $-\frac{N}{2} \le k \le \frac{N}{2}$



$\angle H[-k] = \angle H[N-k]$



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6.

DFT: $X[k] = X(e^{j\hat{\omega}})\big|_{\hat{\omega} = \frac{2\pi k}{N}}$

now $h[n - n_0] \rightleftharpoons e^{-j\hat{\omega}n_0}H(e^{j\hat{\omega}})$

$\therefore \underline{f[k] = e^{-j\frac{2\pi k}{M}n_0}}$ #

7.

10, DFT : $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}n}$

15 IDFT : $\hat{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi k}{N}n}$

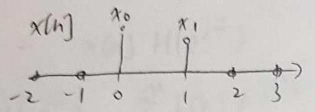
① zero-padding, Lang 4

② DFT

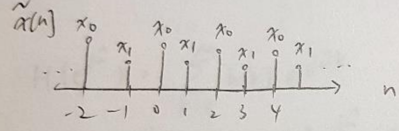
③ Convolution

④ IDFT,

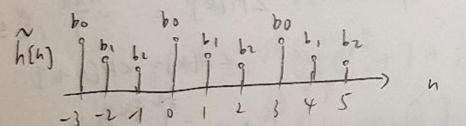
① $x[n]$ 的長度為 2，設 $x[n]$ 如下圖



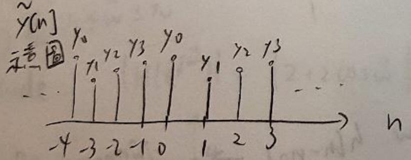
② 將 $x[n]$ 做 DFT，再用 IDFT 得 $\tilde{x}[n]$



③ 同①②，得 $\tilde{h}[n]$

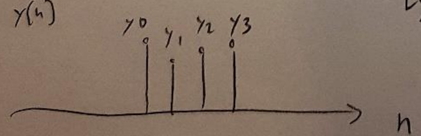


④ 將 $\tilde{x}[n] * \tilde{h}[n] = \tilde{y}[n]$



⑤ $\tilde{y}[n]$ 是 $y[n]$ 的 circular shift，

∴ $y[n]$ 為 $\tilde{y}[n]$ 的其中一週期



$L_y = L_x + M$

$= 2+2$

$= 4$