312512011 李效賢 Machine Learning HW#4

RBF SVM 格子搜尋法輸出結果：

橫軸為 C 值，縱軸為 $\log_{1.05}$ (sigma)值，格子內為取到小數點第四位的分類正確率，而紅字者為同樣 C value 下，accuracy 最高的(C, sigma)組合；紅底白字的格子則為一組表格內分類率最高者(意味這對這組 dataset 而言的最佳參數)。

Fold No.1

| | 1 | 5 | 10 | 50 | 100 | 500 | 1000 |
|---|---|---|---|---|---|---|---|
| -100 | 48.00 | 62.67 | 36.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| -95 | 36.00 | 0.00 | 34.67 | 0.00 | 54.67 | 0.00 | 54.67 |
| -90 | 62.67 | 38.67 | 38.67 | 62.67 | 62.67 | 62.67 | 62.67 |
| -85 | 40.00 | 66.67 | 40.00 | 38.67 | 38.67 | 40.00 | 66.67 |
| -80 | 42.67 | 66.67 | 41.33 | 41.33 | 66.67 | 66.67 | 66.67 |
| -75 | 66.67 | 44.00 | 66.67 | 44.00 | 48.00 | 44.00 | 66.67 |
| -70 | 46.67 | 46.67 | 46.67 | 46.67 | 46.67 | 46.67 | 46.67 |
| -65 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 |
| -60 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 |
| -55 | 68.00 | 68.00 | 68.00 | 68.00 | 68.00 | 68.00 | 68.00 |
| -50 | 72.00 | 72.00 | 72.00 | 72.00 | 72.00 | 72.00 | 72.00 |
| -45 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 | 80.00 |
| -40 | 89.33 | 89.33 | 89.33 | 89.33 | 89.33 | 89.33 | 89.33 |
| -35 | 92.00 | 90.67 | 90.67 | 90.67 | 90.67 | 90.67 | 90.67 |
| -30 | 94.67 | 93.33 | 93.33 | 93.33 | 93.33 | 93.33 | 93.33 |
| -25 | 93.33 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 |
| -20 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 |
| -15 | 94.67 | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 |
| -10 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 |
| -5 | 96.00 | 97.33 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 |
| 0 | 96.00 | 94.67 | 93.33 | 97.33 | 97.33 | 97.33 | 97.33 |
| 5 | 96.00 | 94.67 | 97.33 | 94.67 | 97.33 | 97.33 | 97.33 |
| 10 | 94.67 | 94.67 | 94.67 | 93.33 | 97.33 | 97.33 | 97.33 |
| 15 | 94.67 | 94.67 | 93.33 | 93.33 | 94.67 | 97.33 | 97.33 |
| 20 | 93.33 | 94.67 | 93.33 | 93.33 | 93.33 | 97.33 | 97.33 |
| 25 | 93.33 | 94.67 | 94.67 | 93.33 | 93.33 | 97.33 | 97.33 |
| 30 | 94.67 | 94.67 | 94.67 | 93.33 | 93.33 | 94.67 | 97.33 |
| 35 | 90.67 | 94.67 | 94.67 | 94.67 | 93.33 | 93.33 | 96.00 |
| 40 | 90.67 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 |
| 45 | 90.67 | 92.00 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 50 | 89.33 | 92.00 | 93.33 | 94.67 | 94.67 | 94.67 | 94.67 |
| 55 | 89.33 | 94.67 | 92.00 | 94.67 | 94.67 | 94.67 | 94.67 |
| 60 | 90.67 | 94.67 | 93.33 | 94.67 | 94.67 | 94.67 | 94.67 |
| 65 | 77.33 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 |
| 70 | 69.33 | 94.67 | 94.67 | 92.00 | 94.67 | 94.67 | 94.67 |
| 75 | 69.33 | 94.67 | 94.67 | 93.33 | 92.00 | 94.67 | 94.67 |
| 80 | 70.67 | 94.67 | 94.67 | 94.67 | 93.33 | 94.67 | 94.67 |
| 85 | 66.67 | 94.67 | 94.67 | 94.67 | 93.33 | 94.67 | 94.67 |
| 90 | 66.67 | 94.67 | 94.67 | 94.67 | 94.67 | 93.33 | 94.67 |
| 95 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 | 92.00 | 94.67 |
| 100 | 94.67 | 94.67 | 94.67 | 94.67 | 94.67 | 93.33 | 92.00 |

最佳參數組合：

(C = 5, sigma = 1.05^-5)

(C = 10, sigma = 1.05^5)、

(C = 100, sigma = 1.05^0)、(C = 100, sigma = 1.05^5)

(C=500, sigma = 1.05^0)、(C=500, sigma = 1.05^5)、(C=500, sigma = 1.05^10)、

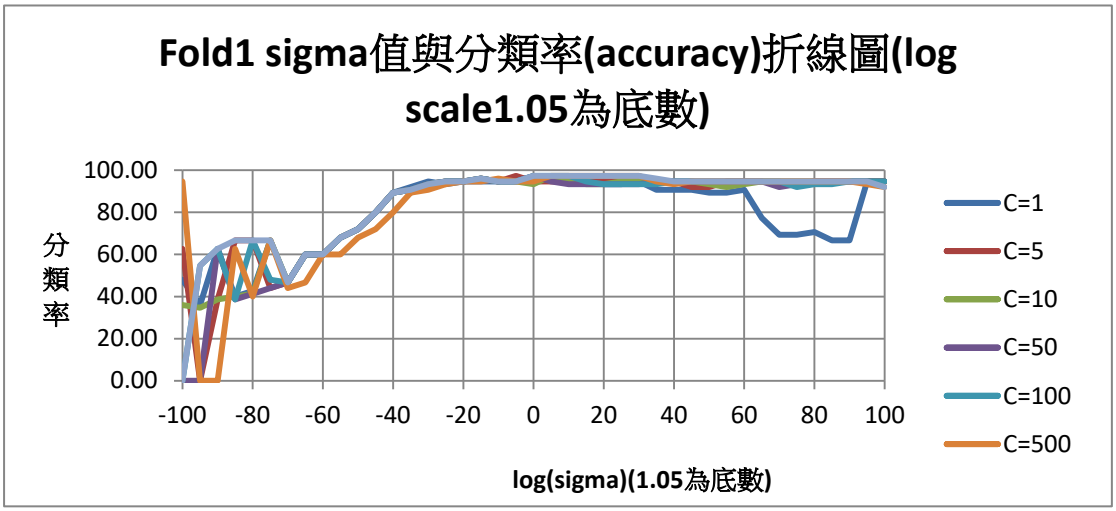
(C=500, sigma = 1.05^15)、(C=500, sigma = 1.05^20)、(C=500, sigma = 1.05^25)

(C = 1000, sigma = 1.05^0)、(C = 1000, sigma = 1.05^5) 、(C = 1000, sigma = 1.05^10)

(C = 1000, sigma = 1.05^15)、(C = 1000, sigma = 1.05^20)、(C = 1000, sigma = 1.05^25)

(C = 1000, sigma = 1.05^30)

(分類率 97.33%參數組合，共 17 組；另外分類率 0%者表示出現 Tied 情況)



Fold No.2

| | 1 | 5 | 10 | 50 | 100 | 500 | 1000 |
|---|---|---|---|---|---|---|---|
| -100 | 34.67 | 45.33 | 45.33 | 34.67 | 34.67 | 36.00 | 34.67 |
| -95 | 36.00 | 50.67 | 34.67 | 50.67 | 34.67 | 36.00 | 34.67 |
| -90 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 |
| -85 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 |
| -80 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 |
| -75 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 |
| -70 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 | 36.00 |
| -65 | 40.00 | 40.00 | 40.00 | 40.00 | 40.00 | 40.00 | 40.00 |
| -60 | 52.00 | 52.00 | 52.00 | 52.00 | 52.00 | 52.00 | 52.00 |
| -55 | 58.67 | 58.67 | 58.67 | 58.67 | 58.67 | 58.67 | 58.67 |
| -50 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 | 60.00 |
| -45 | 69.33 | 69.33 | 69.33 | 69.33 | 69.33 | 69.33 | 69.33 |
| -40 | 81.33 | 81.33 | 81.33 | 81.33 | 81.33 | 81.33 | 81.33 |
| -35 | 89.33 | 89.33 | 89.33 | 89.33 | 89.33 | 89.33 | 89.33 |
| -30 | 93.33 | 92.00 | 92.00 | 92.00 | 92.00 | 92.00 | 92.00 |
| -25 | 97.33 | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 |
| -20 | 96.00 | 94.67 | 97.33 | 97.33 | 97.33 | 97.33 | 97.33 |
| -15 | 96.00 | 94.67 | 94.67 | 97.33 | 97.33 | 97.33 | 97.33 |
| -10 | 96.00 | 96.00 | 94.67 | 96.00 | 96.00 | 96.00 | 96.00 |
| -5 | 96.00 | 96.00 | 96.00 | 94.67 | 96.00 | 96.00 | 96.00 |
| 0 | 96.00 | 96.00 | 96.00 | 96.00 | 94.67 | 96.00 | 96.00 |
| 5 | 97.33 | 97.33 | 96.00 | 96.00 | 97.33 | 96.00 | 96.00 |
| 10 | 97.33 | 96.00 | 97.33 | 96.00 | 96.00 | 94.67 | 96.00 |
| 15 | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 | 97.33 | 96.00 |
| 20 | 94.67 | 96.00 | 97.33 | 96.00 | 96.00 | 96.00 | 97.33 |
| 25 | 92.00 | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 | 97.33 |
| 30 | 94.67 | 97.33 | 97.33 | 97.33 | 96.00 | 97.33 | 96.00 |
| 35 | 93.33 | 96.00 | 97.33 | 97.33 | 98.67 | 98.67 | 97.33 |
| 40 | 93.33 | 96.00 | 97.33 | 97.33 | 98.67 | 98.67 | 98.67 |
| 45 | 90.67 | 94.67 | 94.67 | 97.33 | 97.33 | 98.67 | 98.67 |
| 50 | 89.33 | 93.33 | 96.00 | 96.00 | 97.33 | 98.67 | 98.67 |
| 55 | 90.67 | 93.33 | 94.67 | 97.33 | 96.00 | 98.67 | 98.67 |
| 60 | 89.33 | 93.33 | 93.33 | 94.67 | 97.33 | 97.33 | 98.67 |
| 65 | 85.33 | 92.00 | 93.33 | 96.00 | 96.00 | 97.33 | 98.67 |
| 70 | 82.67 | 92.00 | 92.00 | 94.67 | 94.67 | 97.33 | 97.33 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 75 | 77.33 | 92.00 | 92.00 | 93.33 | 96.00 | 97.33 | 97.33 |
| 80 | 73.33 | 92.00 | 92.00 | 93.33 | 94.67 | 97.33 | 96.00 |
| 85 | 92.00 | 92.00 | 92.00 | 92.00 | 93.33 | 94.67 | 97.33 |
| 90 | 92.00 | 92.00 | 92.00 | 92.00 | 93.33 | 96.00 | 96.00 |
| 95 | 92.00 | 92.00 | 92.00 | 92.00 | 92.00 | 94.67 | 96.00 |
| 100 | 92.00 | 92.00 | 92.00 | 92.00 | 92.00 | 93.33 | 94.67 |

以下為分類率 98.67%的參數組合

(C = 100, sigma = 1.05^35)、(C = 100, sigma = 1.05^40)

(C = 500, sigma = 1.05^35)、(C = 500, sigma = 1.05^40)、(C = 500, sigma = 1.05^45)、

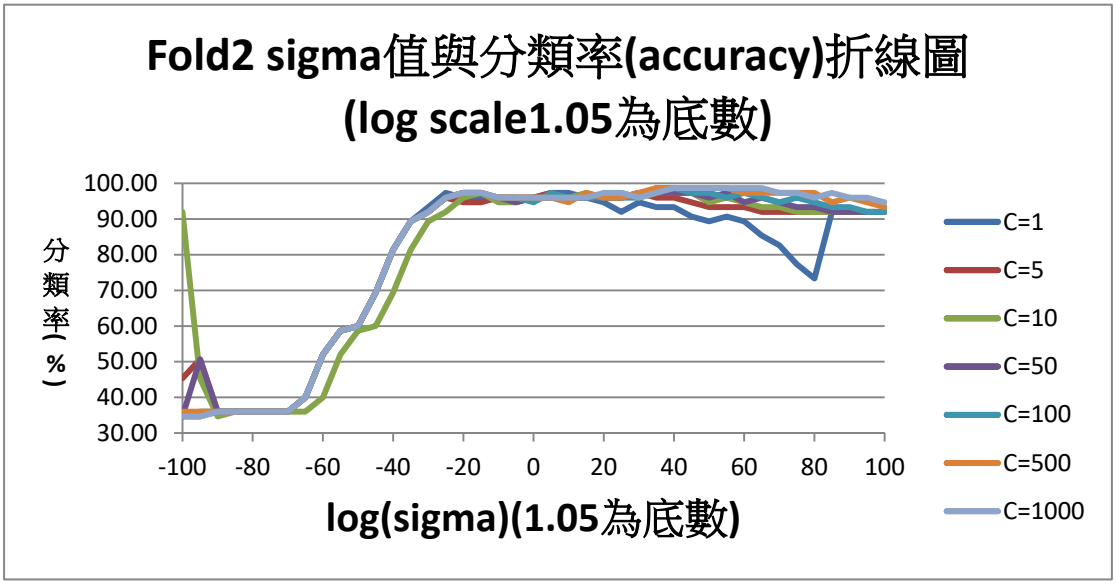
(C = 500, sigma = 1.05^50)、(C = 500, sigma = 1.05^55)

(C = 1000, sigma = 1.05^40)、(C = 1000, sigma = 1.05^45)、

(C = 1000, sigma = 1.05^50)、(C = 1000, sigma = 1.05^55)、(C = 1000, sigma = 1.05^60)

(C = 1000, sigma = 1.05^65)

(分類率 98.67%參數組合，共 13 組)



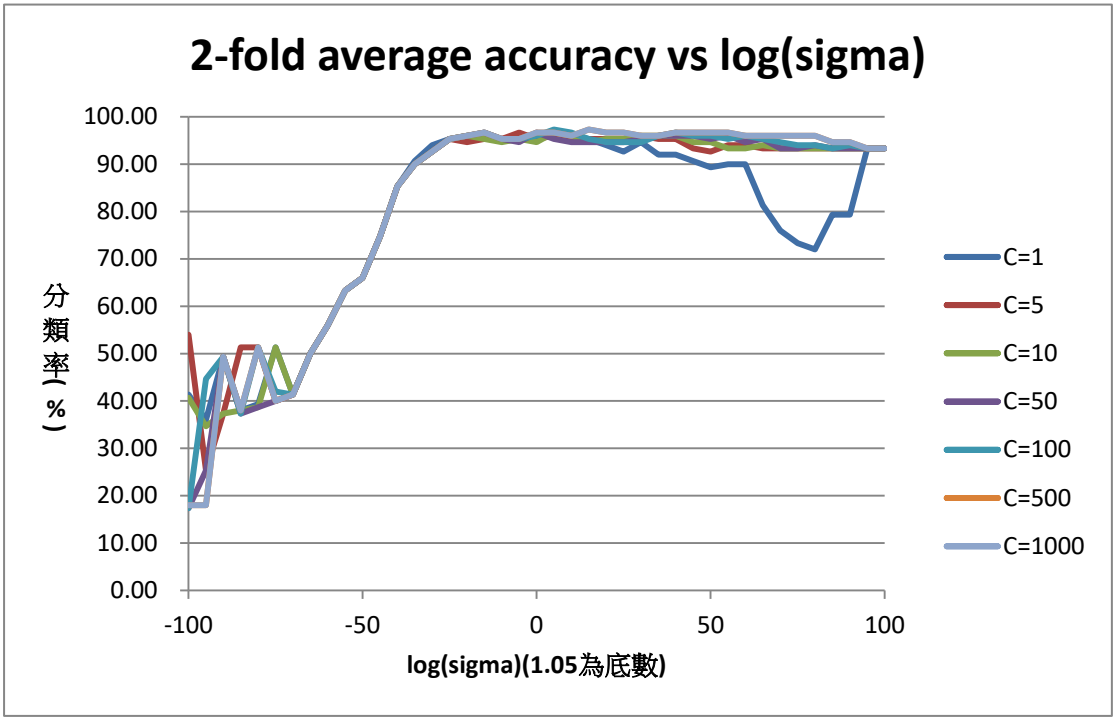
Average of 2 folds

| | 1 | 5 | 10 | 50 | 100 | 500 | 1000 |
|---|---|---|---|---|---|---|---|
| -100 | 41.33 | 54.00 | 40.67 | 17.33 | 17.33 | 18.00 | 17.33 |
| -95 | 36.00 | 25.33 | 34.67 | 25.33 | 44.67 | 18.00 | 44.67 |
| -90 | 49.33 | 37.33 | 37.33 | 49.33 | 49.33 | 49.33 | 49.33 |
| -85 | 38.00 | 51.33 | 38.00 | 37.33 | 37.33 | 38.00 | 51.33 |
| -80 | 39.33 | 51.33 | 38.67 | 38.67 | 51.33 | 51.33 | 51.33 |
| -75 | 51.33 | 40.00 | 51.33 | 40.00 | 42.00 | 40.00 | 51.33 |
| -70 | 41.33 | 41.33 | 41.33 | 41.33 | 41.33 | 41.33 | 41.33 |
| -65 | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 | 50.00 |
| -60 | 56.00 | 56.00 | 56.00 | 56.00 | 56.00 | 56.00 | 56.00 |
| -55 | 63.33 | 63.33 | 63.33 | 63.33 | 63.33 | 63.33 | 63.33 |
| -50 | 66.00 | 66.00 | 66.00 | 66.00 | 66.00 | 66.00 | 66.00 |
| -45 | 74.67 | 74.67 | 74.67 | 74.67 | 74.67 | 74.67 | 74.67 |
| -40 | 85.33 | 85.33 | 85.33 | 85.33 | 85.33 | 85.33 | 85.33 |
| -35 | 90.67 | 90.00 | 90.00 | 90.00 | 90.00 | 90.00 | 90.00 |
| -30 | 94.00 | 92.67 | 92.67 | 92.67 | 92.67 | 92.67 | 92.67 |
| -25 | 95.33 | 95.33 | 95.33 | 95.33 | 95.33 | 95.33 | 95.33 |
| -20 | 95.33 | 94.67 | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 |
| -15 | 95.33 | 95.33 | 95.33 | 96.67 | 96.67 | 96.67 | 96.67 |
| -10 | 95.33 | 95.33 | 94.67 | 95.33 | 95.33 | 95.33 | 95.33 |
| -5 | 96.00 | 96.67 | 95.33 | 94.67 | 95.33 | 95.33 | 95.33 |
| 0 | 96.00 | 95.33 | 94.67 | 96.67 | 96.00 | 96.67 | 96.67 |
| 5 | 96.67 | 96.00 | 96.67 | 95.33 | 97.33 | 96.67 | 96.67 |
| 10 | 96.00 | 95.33 | 96.00 | 94.67 | 96.67 | 96.00 | 96.67 |
| 15 | 95.33 | 95.33 | 94.67 | 94.67 | 95.33 | 97.33 | 96.67 |
| 20 | 94.00 | 95.33 | 95.33 | 94.67 | 94.67 | 96.67 | 97.33 |
| 25 | 92.67 | 95.33 | 95.33 | 94.67 | 94.67 | 96.67 | 97.33 |
| 30 | 94.67 | 96.00 | 96.00 | 95.33 | 94.67 | 96.00 | 96.67 |
| 35 | 92.00 | 95.33 | 96.00 | 96.00 | 96.00 | 96.00 | 96.67 |
| 40 | 92.00 | 95.33 | 96.00 | 96.00 | 96.67 | 96.67 | 96.67 |
| 45 | 90.67 | 93.33 | 94.67 | 96.00 | 96.00 | 96.67 | 96.67 |
| 50 | 89.33 | 92.67 | 94.67 | 95.33 | 96.00 | 96.67 | 96.67 |
| 55 | 90.00 | 94.00 | 93.33 | 96.00 | 95.33 | 96.67 | 96.67 |
| 60 | 90.00 | 94.00 | 93.33 | 94.67 | 96.00 | 96.00 | 96.67 |
| 65 | 81.33 | 93.33 | 94.00 | 95.33 | 95.33 | 96.00 | 96.67 |
| 70 | 76.00 | 93.33 | 93.33 | 93.33 | 94.67 | 96.00 | 96.00 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 75 | 73.33 | 93.33 | 93.33 | 93.33 | 94.00 | 96.00 | 96.00 |
| 80 | 72.00 | 93.33 | 93.33 | 94.00 | 94.00 | 96.00 | 95.33 |
| 85 | 79.33 | 93.33 | 93.33 | 93.33 | 93.33 | 94.67 | 96.00 |
| 90 | 79.33 | 93.33 | 93.33 | 93.33 | 94.00 | 94.67 | 95.33 |
| 95 | 93.33 | 93.33 | 93.33 | 93.33 | 93.33 | 93.33 | 95.33 |
| 100 | 93.33 | 93.33 | 93.33 | 93.33 | 93.33 | 93.33 | 93.33 |

最佳參數：

(C = 500, sigma = 1.05^50)、(C = 1000, sigma = 1.05^55)共兩組，平均分類率 98.67%



討論：

1. 請問在 grid search 的結果中，C 的大小與分類率高低有何關係?

Ans. C 在 SVM 演算法中代表 Punish weight(懲罰權重)，C 較小時訓練出的模型相對能夠容忍錯誤分類，一般會讓 bias 較大，但泛化程度會較佳；反過來 C 較大時，模型容忍的錯誤較少，會讓 bias 較小，但較有可能 overfitting。
而依照我所製作出的圖表觀察結果，在 $\sigma \in [1.05^{-40}, 1.05^{100}]$之間時，C 值對兩個 fold 以及全體平均的影響相當小，而以橫向 row 來看，相同 σ 值之下，C 值對整體分類正確率影響有限。

2. sigma 的大小的改變與分類率是否有關係? 若有，請探討 sigma 的差異與特徵的數值有什麼關聯性?

Ans. 是，在作業要求 $\sigma \in [1.05^{-100}, 1.05^{100}]$的集合中，當 σ 極小(越接近 $1.05^{-100}$)，分類率越低，特別是σ在 $1.05^{-35}$ 以下時，不論哪個 Fold 的分類正確率都低於 90%，依照 RBF kernel 定義，應為 overfitting 現象(整體 Kernel function 越尖，此時分類器對 dataset 中的 noise 越敏感)；另一方面，當 σ 在 $1.05^{80}$ 以上時，分類正確率會從 95%以上顯著下降，則是 underfitting (整體 Kernel function 較平滑，此時分類器可以說"不夠敏感"，但是泛化性(Generalization)較好)。當 $\sigma \in [1.05^{50}, 1.05^{70}]$ 時，通常會是最佳解發生的區域。
關於 sigma 差異與特徵數值的關聯性：如果特徵的數值範圍很大，那麼小的 sigma 可能會導致模型對特定數據的變化非常敏感，因為相似度的計算會更加依賴特徵值之間的距離。
在 sigma 值過小的情況下，在 Fold1 會出現平手情況(“Tied” in one-against-one strategy)，個人認為就是 overfitting 所造成的，其次 sigma 值相當大的時候，大多數的 C 值搭配下，模型分類率並沒有降到 90%以下，對比之下更能觀察出 sigma 過小時過度擬合(Overfitting)現象。

3. 若分析過程不採用 two-fold cross validation，則分類率是否會更高? 請探討之。

Ans. 以數據來說有，但是差異不明顯，甚至可以說在誤差範圍。
我在 excel 上有做出一個 Fold1 正確率減去平均正確率的表格，Fold1 整體正確率大約比平均的高 1.04%左右(也就是大約 1 組數據在 Fold1 會分對，在 Fold2 的模型會分錯)，個人認為這可能是由於數據組數量偏少(testset 75 組)造成的誤差，而非 cross-validation 所造成分類率下降。因為 cross-validation 本身就是在評估生

成模型的泛化性，確認是在排列組合都有不差的表現，還是只對某一組數據表現很好，其他排列組合分類率不佳。

在 sigma 極小的情況下會出現兩者相差很大，但在 sigma 較大時則無。

此圖為 Fold1 accuracy - average accuracy 對 log(sigma)的分布圖