Nonlinear System Theory
Homework 11

Due date: 6/4/2022

1. Consider the system

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2 - x_3 \\ \dot{x}_2 &= -x_1 x_3 - x_2 + u \\ \dot{x}_3 &= -x_1 + u \\ y &= x_3 \end{aligned}$$

   (a) Find the relative degree of the system.

   (b) Transform this nonlinear system into the normal form and specify the region over which the transformation is valid.

   (c) Is the system minimum phase?

2. Consider the 3rd-order system

$$\begin{aligned} \dot{x}_1 &= x_2 + \tan x_3 - 2u \\ \dot{x}_2 &= x_2^2 + x_3 \\ \dot{x}_3 &= -\sin x_3 - x_2 - x_1^3 + u \\ y &= x_2 \end{aligned}$$

   (a) Find the relative degree and the normal form of the system.

   (b) Write down the zero dynamics of the system and check whether the system is minimum phase or not.

   (c) Design an input-output feedback linearization controller such that the transfer function from an external command r to the output y is $\frac{100}{s^2+15s+100}$.