Nonlinear System Theory
Homework 4

Due date: 3/29/2022

1. Consider the system

$$\begin{aligned} \dot{x}_1 &= -x_1 \\ \dot{x}_2 &= (x_1x_2 - 1)x_2^3 + (x_1x_2 - 1 + x_1^2)x_2 \end{aligned}$$

   (a) Show that $x = 0$ is the unique equilibrium point.

   (b) Show, by using linearization, that $x = 0$ is asymptotically stable.

   (c) Show that $\Gamma = \{x \in \mathbb{R}^2 | x_1x_2 \geq 2\}$ is a positively invariant set.

   (d) Is $x = 0$ globally asymptotically stable?

2. Let α be a class $\mathcal{K}$ function on $[0,\, a)$. Show that

$$\alpha(r_1 + r_2) \leq \alpha(2r_1) + \alpha(2r_2), \quad \forall r_1, r_2 \in \big[0,\, a/2\big)$$

3. Consider the system

$$\dot{x} = -a\big(I + S(x) + xx^T\big)x$$

   where $x \in \mathbf{R}^n$, $a > 0$ is a constant, I is the $n \times n$ identity matrix, and $S(x)$ is a state-dependent skew symmetric matrix, i.e. $v^T S(x)v = 0$ for all $v, x \in \mathbf{R}^n$. Show that $x = 0$ is globally asymptotically stable.