Nonlinear System Theory
Homework 10

Due date: 5/31/2022

1. A simplified model of the low-frequency motion of a ship is given by

$$\tau\ddot{\psi} + \dot{\psi} = k\delta$$

where ψ is the heading angle of the ship and δ is the rudder angle, viewed here as the control input. The time constant τ and the gain k depend on the forward speed of the ship v, according to the expression $\tau = \frac{\tau_0 v_0}{v}$ and $k = \frac{k_0 v}{v_0}$, where τ_0, k_0 and v_0 are constants.

   (a) Assuming a constant forward speed, design a state feedback integral controller so that ψ tracks a desired angle ψ_r. For step command ψ_r, the overshoot of the output response should be less than 25% and the settling time (1%) should be less than 10 sec.

   (b) Use gain scheduling to compensate for varying forward speed.

   Note: Use Matlab/Simulink for controller design and verification.

2. Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1^2 \\ \dot{x}_2 &= x_3 + u \\ \dot{x}_3 &= x_1 - x_3 \\ y &= x_1 \end{aligned}$$

   Design a state feedback control law such that the output y asymptotically tracks a twice differentiable reference signal $r(t)$.