Nonlinear System Theory
Homework 8

Due date: 5/23/2023

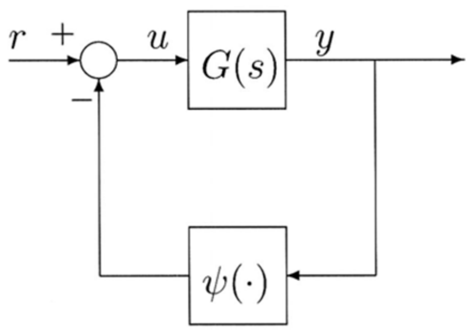

Figure 1: Feedback connection.

1. Consider the feedback connection in Figure 1. Use the circle criterion to find a sector $[\alpha, \beta]$ as large as possible such that for each of the following $G(s)$, if $\psi \in [\alpha, \beta]$, then the feedback system is absolutely stable. You can use Matlab to draw the Nyquist plot of $G(j\omega)$.

   (a) $G(s) = \frac{s}{s^2-s+1}$

   (b) $G(s) = \frac{1}{s^2+s+1}$

   (c) $G(s) = \frac{s+1}{(s+2)^2(s-1)}$

2. Repeat Problem 1 using the Popov criterion.
   Hint: For unstable $G(s)$, you need to apply the loop transformation to convert $G(s)$ to a stable system while maintaining an equivalent feedback loop.

   Remark: You can compare the results of Problem 1 with Problem 2, so the differences between the circle criterion and the Popov criterion will be clear.

3. Consider the following nonlinear system

$$\begin{aligned}
\dot{e}_1(t) &= e_2(t) \\
\dot{e}_2(t) &= -\tilde{a}(t)\phi(t) - k_1 e_1(t) - k_2 e_2(t) \\
\dot{\tilde{a}}(t) &= \gamma e_2(t)\phi(t)
\end{aligned}$$

   where k_1, k_2, γ are positive constants, and $\phi(t)$ is a bounded function of time.

   (a) Show that $(e_1, e_2, \tilde{a}) = (0, 0, 0)$ is a globally stable equilibrium point.

   (b) Show that $e_2 \to 0$ as $t \to \infty$.