Nonlinear System Theory
Homework 8

Due date: 5/17/2022

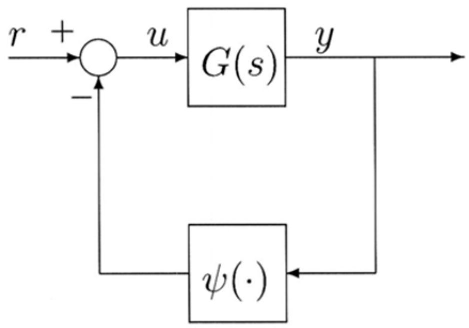

Figure 1: Feedback connection.

1. Consider the feedback connection in Figure 1. Use the circle criterion to find a sector $[\alpha,\ \beta]$ as large as possible such that for each of the following $G(s)$, if $\psi \in [\alpha,\ \beta]$, then the feedback system is absolutely stable. You can use Matlab to draw the Nyquist plot of $G(j\omega)$.

   (a) $G(s) = \frac{s}{s^2 - s + 1}$

   (b) $G(s) = \frac{1}{s^2 + s + 1}$

   (c) $G(s) = \frac{s+1}{(s+2)^2(s-1)}$

2. Consider the feedback system in Figure 1 and let $G(s) = \frac{2s}{s^2+s+1}$.

   (a) Show that the feedback system is absolutely stable for $\psi \in [0,\ 1]$.

   (b) Show that the system has no limit cycles with $\psi(y) = \mathrm{sat}(y)$, where $\mathrm{sat}(\cdot)$ denotes the saturation function.

3. Consider the feedback system in Figure 1. Let $G(s) = \frac{s+0.5}{s^2+2s+2}$.

   (a) Find all possible sectors of ψ such that the feedback system is absolutely stable.

   (b) Suppose that $\psi(y) = \sin(\omega_1 y)\cos(\omega_2 y)$, where $\omega_1 = 0.5$, $\omega_2 = 20$. Is the origin of the feedback system asymptotically stable?