Assignment 3

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***Abstract*—This document examines the consistency of the system of equations.**

1 Problem

Examine the consistency of the system of given equations:

$$3x - y - 2z = 2 \tag{1.1}$$
$$2y - z = -1 \tag{1.2}$$
$$3x - 5y = 3 \tag{1.3}$$

2 Solution

The given system of equations can be represented as:

$$\mathbf{Ax} = \mathbf{B} \tag{2.1}$$

Coefficient Matrix **A**

$$\mathbf{A} = \begin{pmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{pmatrix} \tag{2.2}$$

Constant Vector **B**

$$\mathbf{B} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \tag{2.3}$$

$$\det A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix} \tag{2.4}$$
$$\det A = 0 \tag{2.5}$$

The given system of equations are either inconsistent or has infinitely many solutions. If

$$Adj(\mathbf{A}).\mathbf{B} = O \tag{2.6}$$

then infinitely many solutions otherwise system of equations are inconsistent.

$$Adj(\mathbf{A}) = \begin{pmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{pmatrix} \tag{2.7}$$
$$Adj(\mathbf{A}).\mathbf{B} = \begin{pmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \tag{2.8}$$
$$Adj(\mathbf{A}).\mathbf{B} = \begin{pmatrix} -5 \\ -3 \\ -6 \end{pmatrix} \tag{2.9}$$
$$\because Adj(\mathbf{A}).\mathbf{B} \neq O \tag{2.10}$$

Therefore the given system of equations are inconsistent.

OR

Augumented Matrix-

$$\mathbf{A}|\mathbf{B} = \left(\begin{array}{ccc|c} 3 & -1 & -2 & 2 \\ 0 & 2 & -1 & -1 \\ 3 & -5 & 0 & 3 \end{array}\right) \tag{2.11}$$

Applying Row operations on (2.11)

$$\left(\begin{array}{ccc|c} 3 & -1 & -2 & 2 \\ 0 & 2 & -1 & -1 \\ 3 & -5 & 0 & 3 \end{array}\right) \xleftrightarrow{R3 \to R1 - R3} \left(\begin{array}{ccc|c} 3 & -1 & -2 & 2 \\ 0 & 2 & -1 & -1 \\ 0 & 4 & -2 & -1 \end{array}\right) \tag{2.12}$$

$$\left(\begin{array}{ccc|c} 3 & -1 & -2 & 2 \\ 0 & 2 & -1 & -1 \\ 0 & 4 & -2 & -1 \end{array}\right) \xleftrightarrow{R3 \to R3 - R2} \left(\begin{array}{ccc|c} 3 & -1 & -2 & 2 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right) \tag{2.13}$$

$$\mathbf{A}|\mathbf{B} = \left(\begin{array}{ccc|c} 3 & -1 & -2 & 2 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right) \mathbf{A} = \begin{pmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.14}$$

$$R(\mathbf{A}|\mathbf{B}) = 3 \tag{2.15}$$
$$R(\mathbf{A} = 2 \tag{2.16}$$
$$R(\mathbf{A}|\mathbf{B}) \neq R(\mathbf{A}) \tag{2.17}$$

The given system of equations are inconsistent.