

# Assignment 4

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**Abstract**—This document determines the value of  $k$  for which the given equation represents two straight lines.

## 1 PROBLEM

For what value of  $k$  does the equation

$$\mathbf{x}^T \begin{pmatrix} 6 & k/2 \\ k/2 & -3 \end{pmatrix} \mathbf{x} + (4 \quad 5) \mathbf{x} - 2 = 0 \quad (1.1)$$

represent a pair of straight lines?

## 2 SOLUTION

The given equation can be represented as:

$$\mathbf{x}^T \begin{pmatrix} a & h \\ h & b \end{pmatrix} \mathbf{x} + (2g \quad 2f) \mathbf{x} + c = 0 \quad (2.1)$$

Comparing equation (1.1) and equation (2.1) we get  $a=6$ ,  $b=-3$ ,  $h=k/2$ ,  $g=2$ ,  $f=5/2$ ,  $c=-2$

Discriminant of the equation (2.1)

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \quad (2.2)$$

Substituting the values in equation (2.2)

$$\Delta = \begin{vmatrix} 6 & k/2 & 2 \\ k/2 & -3 & 5/2 \\ 2 & 5/2 & -2 \end{vmatrix} \quad (2.3)$$

If the equation (1.1) represents a pair of straight lines then Discriminant is zero

$$\Delta = 0 \quad (2.4)$$

$$\Rightarrow \begin{vmatrix} 6 & k/2 & 2 \\ k/2 & -3 & 5/2 \\ 2 & 5/2 & -2 \end{vmatrix} = 0 \quad (2.5)$$

$$k^2 + 10k + 21 = 0 \quad (2.6)$$

$$k = -3, -7 \quad (2.7)$$

Both the plots are same. Hence the value of  $k$  is verified.

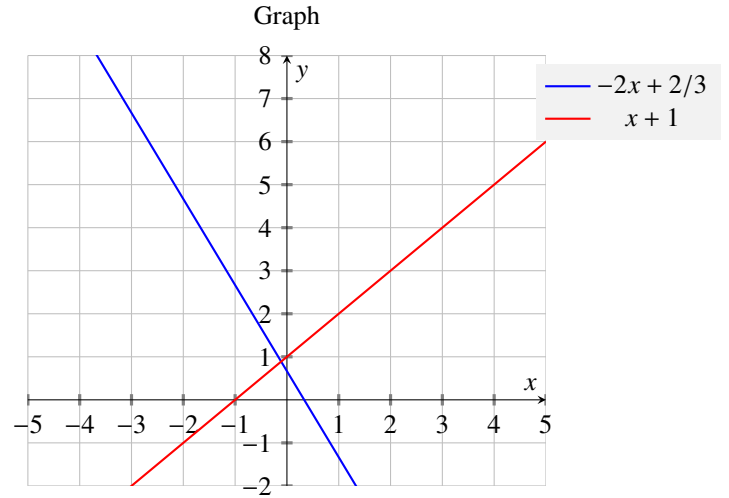


Fig. 1: This is a plot of pair of straight lines in (1.1) taking  $k=-3$

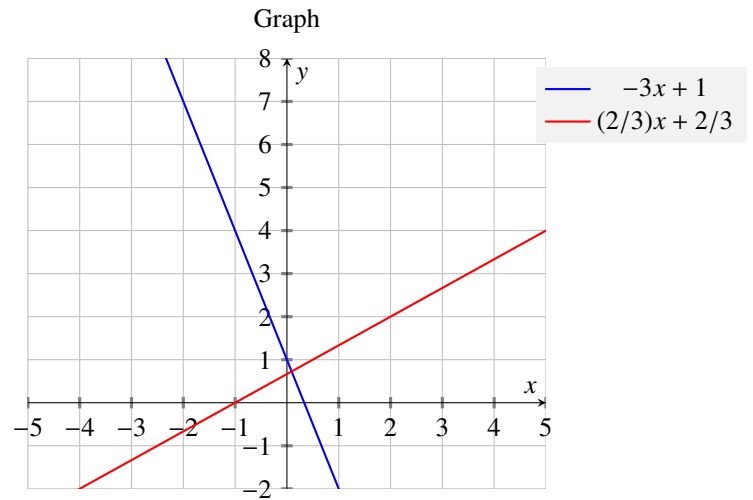


Fig. 2: This is a plot of pair of straight lines in (1.1) taking  $k=-7$