Assignment 5

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***Abstract*—This document explains the the concept of finding two straight lines from given second degree equation**

Download all python codes from

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https://github.com/Shweta−SV/Assignment−5/blob/
    python−code/Assignment5.ipynb
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Download latex-tikz codes from

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https://github.com/Shweta−SV/Assignment−5/tree/
    latex
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1 Problem

Find the QR decomposition of $\mathbf{V}$.

$$\mathbf{V} = \begin{pmatrix} 6 & -3/2 \\ -3/2 & -3 \end{pmatrix} \tag{1.0.1}$$

2 Theory

A matrix $\mathbf{V}$ can be represented as

$$\mathbf{V} = \begin{pmatrix} \mathbf{v_1} & \mathbf{v_2} \end{pmatrix} \tag{2.0.1}$$

If $\mathbf{V} \in \mathbf{R}^{m \times n}$ has linearly independent columns then it can be decomposed as

$$\mathbf{V} = \mathbf{QR} \tag{2.0.2}$$

$\mathbf{Q}$ is $m \times n$ with orthogonal columns ($\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$) $\mathbf{R}$ is $n \times n$, upper triangular with non-zero diagonal elements

$$\begin{aligned} \mathbf{Q} &= \begin{pmatrix} \mathbf{q_1} & \mathbf{q_2} \end{pmatrix} \\ \mathbf{R} &= \begin{pmatrix} r_1 & r_2 \\ 0 & r_3 \end{pmatrix} \end{aligned} \tag{2.0.3}$$

$\mathbf{q_1}$, $\mathbf{q_2}$ and the values in $\mathbf{R}$ are given by,

$$r_1 = \|\mathbf{v_1}\| \tag{2.0.4}$$

$$\mathbf{q_1} = \frac{\mathbf{v_1}}{r_1} \tag{2.0.5}$$

$$r_2 = \frac{\mathbf{q_1^T v_2}}{\|\mathbf{q_1}\|^2} \tag{2.0.6}$$

$$\mathbf{q_2} = \frac{\mathbf{v_2} - r_2\mathbf{q_1}}{\|\mathbf{v_2} - r_2\mathbf{q_1}\|} \tag{2.0.7}$$

$$r_3 = \mathbf{q_2^T v_2} \tag{2.0.8}$$

3 Solution

$$\begin{aligned} \mathbf{v_1} &= \begin{pmatrix} 6 \\ -3/2 \end{pmatrix} \\ \mathbf{v_2} &= \begin{pmatrix} -3/2 \\ -3 \end{pmatrix} \end{aligned} \tag{3.0.1}$$

Using above equations we get

$$r_1 = \frac{\sqrt{153}}{2} = 6.1846 \tag{3.0.2}$$

$$\begin{aligned} \mathbf{q_1} &= \begin{pmatrix} \frac{12}{\sqrt{153}} \\ \frac{-3}{\sqrt{153}} \end{pmatrix} \\ &= \begin{pmatrix} 0.9701 \\ -0.2425 \end{pmatrix} \end{aligned} \tag{3.0.3}$$

$$r_2 = \frac{-9}{\sqrt{153}} = -0.7276 \tag{3.0.4}$$

$$\begin{aligned} \mathbf{q_2} &= \begin{pmatrix} -\frac{243}{\sqrt{1003833}} \\ -\frac{972}{\sqrt{1003833}} \end{pmatrix} \\ &= \begin{pmatrix} -0.2425 \\ -0.9701 \end{pmatrix} \end{aligned} \tag{3.0.5}$$

$$r_3 = \frac{6561}{2\sqrt{1003833}} = 3.2742 \tag{3.0.6}$$

Therefore

$$\mathbf{Q} = \begin{pmatrix} 0.9701 & -0.2425 \\ -0.2425 & -0.9701 \end{pmatrix} \tag{3.0.7}$$

$$\mathbf{R} = \begin{pmatrix} 6.1846 & -0.7276 \\ 0 & 3.2742 \end{pmatrix} \tag{3.0.8}$$

Using (3.0.7) and (3.0.8) in (2.0.2)

$$\begin{aligned} \mathbf{QR} &= \begin{pmatrix} 0.9701 & -0.2425 \\ -0.2425 & -0.9701 \end{pmatrix} \\ &\times \begin{pmatrix} 6.1846 & -0.7276 \\ 0 & 3.2742 \end{pmatrix} \\ &\approx \begin{pmatrix} 6 & -3/2 \\ -3/2 & -3 \end{pmatrix} \end{aligned} \tag{3.0.9}$$