Assignment 6

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***Abstract*—This document explains the concept Positive Definite and Negative Definite matrices.**

Download all python codes from

https://github.com/Shweta-SV/Assignment-6/tree/code

Download latex-tikz codes from

https://github.com/Shweta-SV/Assignment-6/tree/Latex

1 Problem

The matrix

$$\mathbf{A} = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix} \tag{1.0.1}$$

is

1) positive definite.
2) non-negative definite but not positive definite.
3) negative definite.
4) neither negative definite nor positive definite. .

2 Theory

1) For a real symmetric matrix to be positive definite the eigen values of the matrix should be positive.
2) For a real symmetric matrix to be negative definite the eigen values of the matrix should be negative.

3 Solution

$$\mathbf{A} = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

The characteristic equation of the matrix **A**is given by

$$\begin{aligned} \left|V - \lambda\mathbf{I}\right| = \begin{vmatrix} 3-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{vmatrix} = 0 \\ \implies \lambda^3 - 8\lambda^2 + 19\lambda - 12 = 0 \end{aligned} \tag{3.0.1}$$

The Eigen values of **A** are:

$$\begin{aligned} \lambda_1 &= 5/2 \\ \lambda_2 &= 3/2 \\ \lambda_3 &= 4 \end{aligned} \tag{3.0.2}$$

Since all the eigen values of matrix **A** are positive, Therefore the matrix **A** is positive definite.