

# Assignment 8

Shweta Verma

**Abstract**—This document explains the concept of self-adjoint matrix and properties of eigen values and vectors.

Download all python codes from

<https://github.com/Shweta-SV/Assignment-8>

Download latex-tikz codes from

<https://github.com/Shweta-SV/Assignment-8>

## 1 PROBLEM

Let  $\mathbf{A}$  be an  $n \times n$  complex matrix. Assume that  $\mathbf{A}$  is a self-adjoint matrix and let  $\mathbf{B}$  denotes the inverse of  $\mathbf{A} + i\mathbf{I}$ . Then all eigen values of  $(\mathbf{A} - i\mathbf{I})\mathbf{B}$  are:

- 1) purely imaginary
- 2) of modulus one
- 3) real
- 4) of modulus less than one

## 2 THEORY

- 1) If  $\mathbf{A}$  is a self-adjoint matrix, then it satisfies

$$\mathbf{A}^\theta = \mathbf{A} \quad (2.0.1)$$

where  $\mathbf{A}^\theta$  is the complex conjugate of  $\mathbf{A}$

- 2) Let  $\mathbf{A}$  be an  $n \times n$  matrix,  $\lambda_A$  be its eigen values and  $\mathbf{X}$  be its eigen vector.

$$\mathbf{A}\mathbf{X} = \lambda_A\mathbf{X} \quad (2.0.2)$$

- 3) If  $\lambda_A$  be the eigen value of  $\mathbf{A}$ , then
  - a) Eigen value of  $\mathbf{A} + k\mathbf{I}$  is  $\lambda_A + k$
  - b) Eigen value of  $\mathbf{A}^p$  is  $\lambda_A^p$
  - c) Eigen value of  $\mathbf{A}^{-1}$  is  $1/\lambda_A$

## 3 SOLUTION

Since  $\mathbf{A}$  is an  $n \times n$  complex matrix and a self-adjoint matrix. Hence, eigen values of  $\mathbf{A}$  are real. Let  $\lambda_A$  be the eigen value of  $\mathbf{A}$  and  $\mathbf{X}$  be its eigen vector.

$$\mathbf{A}\mathbf{X} = \lambda_A\mathbf{X} \quad (3.0.1)$$

The eigen value of  $\mathbf{B}$

$$\mathbf{B} = (\mathbf{A} + i\mathbf{I})^{-1}$$

Eigen value of  $\mathbf{A} + i\mathbf{I}$  is  $\lambda_A + i$

Eigen value of  $\mathbf{B}$  i.e.  $(\mathbf{A} + i\mathbf{I})^{-1}$  is  $\frac{1}{\lambda_A + i}$

Eigen value of  $\mathbf{A} - i\mathbf{I}$  is  $\lambda_A - i$

Now Using (3.0.1)

$$(\mathbf{A} + i\mathbf{I})^{-1}\mathbf{X} = \frac{1}{\lambda_A + i}\mathbf{X} \quad (3.0.2)$$

$$(\mathbf{A} - i\mathbf{I})\mathbf{X} = (\lambda_A - i)\mathbf{X} \quad (3.0.3)$$

Multiplying (3.0.2) by  $\mathbf{A} - i\mathbf{I}$

$$(\mathbf{A} - i\mathbf{I})(\mathbf{A} + i\mathbf{I})^{-1}\mathbf{X} = (\mathbf{A} - i\mathbf{I})\frac{1}{\lambda_A + i}\mathbf{X} \quad (3.0.4)$$

Using (3.0.3) in (3.0.4)

$$(\mathbf{A} - i\mathbf{I})(\mathbf{A} + i\mathbf{I})^{-1}\mathbf{X} = (\lambda_A - i)\frac{1}{\lambda_A + i}\mathbf{X}$$

$$(\mathbf{A} - i\mathbf{I})\mathbf{B}\mathbf{X} = \left(\frac{\lambda_A - i}{\lambda_A + i}\right)\mathbf{X} \quad (3.0.5)$$

From (3.0.5) the eigen values of  $(\mathbf{A} - i\mathbf{I})\mathbf{B}$  are:

- 1)  $\frac{\lambda_A - i}{\lambda_A + i}$
- 2) not real
- 3) Magnitude:

$$\left|\frac{\lambda_A - i}{\lambda_A + i}\right| = \frac{\sqrt{\lambda_A^2 + 1}}{\sqrt{\lambda_A^2 + 1}} = 1 \quad (3.0.6)$$

Therefore, option (2) is correct.