EE5609: Matrix Theory
Assignment-2

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***Abstract*—This document contains the solution to problem 75 from 3.9 Matrix Exercises**

Soln :

1 Problem

If

$$\mathbf{A} = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}, \ \mathbf{A}^2 = I \tag{1.0.1}$$

choose the correct relation
(a)$1+\alpha^2 + \beta\gamma = 0$ $(b) 1 - \alpha^2 + \beta\gamma = 0$
$(c) 1 - \alpha^2 - \beta\gamma = 0$ $(d) 1 + \alpha^2 - \beta\gamma = 0$

2 Solution

The characteristic equation is

$$|\mathbf{A} - \lambda\mathbf{I}| = 0 \implies \det\begin{pmatrix} \alpha - \lambda & \beta \\ \gamma & -\alpha - \lambda \end{pmatrix} = 0 \tag{2.0.1}$$

$$\implies (\alpha - \lambda)(-\alpha - \lambda) - \gamma\beta = 0 \tag{2.0.2}$$

$$\implies \lambda^2 - \alpha^2 - \gamma\beta = 0 \tag{2.0.3}$$

By the Cayley-Hamilton theorem, every square matrix satisfies its own characteristic equation.
Hence,on substituting from (1.0.1) in (2.0.3)
$\implies \mathbf{A^2} - \alpha^2 - \gamma\beta = 0.$
$\implies \mathbf{I} - \alpha^{\mathbf{2}} - \gamma\beta = 0$
Hence, (c) is the correct answer