

EE5609: Matrix Theory

Assignment-2

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Abstract—This document contains the solution to problem 75 from 3.9 Matrix Exercises

Soln :

1 PROBLEM

If

$$\mathbf{A} = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}, \quad \mathbf{A}^2 = \mathbf{I} \quad (1.0.1)$$

choose the correct relation

- (a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 + \beta\gamma = 0$
 (c) $1 - \alpha^2 - \beta\gamma = 0$ (d) $1 - \alpha^2 - \beta\gamma = 0$

2 SOLUTION

The characteristic equation is

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \implies \det \begin{pmatrix} \alpha - \lambda & \beta \\ \gamma & -\alpha - \lambda \end{pmatrix} = 0 \quad (2.0.1)$$

$$\implies (\alpha - \lambda)(-\alpha - \lambda) - \gamma\beta = 0 \quad (2.0.2)$$

$$\implies \lambda^2 - \alpha^2 - \gamma\beta = 0 \quad (2.0.3)$$

By the Cayley-Hamilton theorem, every square matrix satisfies its own characteristic equation.

Hence, on substituting from (1.0.1) in (2.0.3)

$$\implies \mathbf{A}^2 - \alpha^2 - \gamma\beta = 0.$$

$$\implies \mathbf{I} - \alpha^2 - \gamma\beta = 0$$

Hence, (c) is the correct answer