EE5609: Matrix Theory Assignment-2

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***Abstract*—This document contains the solution to problem 75 from 3.9 Matrix Exercises**

Soln :

1 Problem

If

$$\mathbf{A} = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}, \ \mathbf{A}^2 = I \tag{1.0.1}$$

choose the correct relation
(a)$1+\alpha^2 + \beta\gamma = 0$ $(b)1 - \alpha^2 + \beta\gamma = 0$
$(c)1 - \alpha^2 - \beta\gamma = 0$ $(d)1 - \alpha^2 - \beta\gamma = 0$

2 Solution

$$\mathbf{A}^2 = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix} = \begin{pmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \gamma\beta + \alpha^2 \end{pmatrix} \tag{2.0.1}$$

from (1.0.1)

$$\implies \begin{pmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \gamma\beta + \alpha^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.2}$$

Therefore,on comparing (2.0.2)
$\alpha^2 + \beta\gamma = 1$
$\implies 1 - \alpha^2 - \beta\gamma = 0$
Hence, (c) is the correct answer