Matrix Theory (EE5609) Assignment 3

Saurabh Joshi
MTech Artificial Intelligence
AI20MTECH13002

***Abstract*—This document shows that in a right angled triangle, the hypotenuse is the longest side.**

Download all latex-tikz codes from

https://github.com/saurabh13002/EE5609/tree/master

1 Problem

Show that in a right angled triangle, the hypotenuse is the longest side.

2 Solution

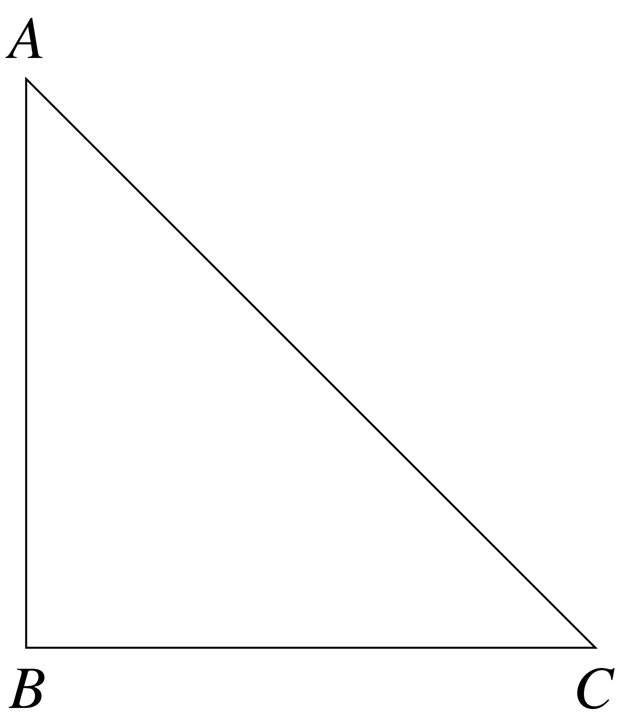

Fig. 1: $\triangle ABC$

Given conditions are : $\triangle ABC$ is a triangle right angled at $\mathbf{B}$,we have

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{B}) = \|\mathbf{A}-\mathbf{B}\|^2 \tag{2.0.1}$$

$$\implies (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}+\mathbf{C}-\mathbf{B}) = \|\mathbf{A}-\mathbf{B}\|^2 \tag{2.0.2}$$

$$\implies (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})+(\mathbf{A}-\mathbf{B})^T(\mathbf{C}-\mathbf{B}) = \|\mathbf{A}-\mathbf{B}\|^2 \tag{2.0.3}$$

$$\implies (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) + \|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{C}-\mathbf{B}\|\cos 90^\circ = \|\mathbf{A}-\mathbf{B}\|^2 \tag{2.0.4}$$

$$\implies (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) = \|\mathbf{A}-\mathbf{B}\|^2 \tag{2.0.5}$$

Similarly,

$$(\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{B}-\mathbf{C}\|^2 \tag{2.0.6}$$

$$\implies (\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{A}+\mathbf{A}-\mathbf{C}) = \|\mathbf{B}-\mathbf{C}\|^2 \tag{2.0.7}$$

$$\implies (\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{A}) + (\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{C}) = \|\mathbf{B}-\mathbf{C}\|^2 \tag{2.0.8}$$

$$\implies \|\mathbf{B}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{A}\|\cos 90^\circ + (\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{C}) = \|\mathbf{B}-\mathbf{C}\|^2 \tag{2.0.9}$$

$$\implies (\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{C}) = \|\mathbf{B}-\mathbf{C}\|^2 \tag{2.0.10}$$

Consider,

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{C}) = \|\mathbf{A}-\mathbf{C}\|^2 \tag{2.0.11}$$

$$\implies (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B}+\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{C}\|^2 \tag{2.0.12}$$

$$\implies (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})+(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{C}\|^2 \tag{2.0.13}$$

Substituting (2.0.5) and (2.0.10) in (2.0.13) above

$$\|\mathbf{A}-\mathbf{B}\|^2 + \|\mathbf{B}-\mathbf{C}\|^2 = \|\mathbf{A}-\mathbf{C}\|^2 \tag{2.0.14}$$

Therefore, in Right angled triangle, it follows that side

$$\|\mathbf{A} - \mathbf{C}\| > \|\mathbf{A} - \mathbf{B}\| \, and \, \|\mathbf{A} - \mathbf{C}\| > \|\mathbf{B} - \mathbf{C}\| \tag{2.0.15}$$

Hence, proved.