

Assignment 2

Shweta Verma

Abstract—This document solves a problem based on the inclination of straight lines

1 PROBLEM

If the lines

$$(-3 \ 1)\mathbf{x} = 1 \quad (1.1)$$

$$(-1 \ 2)\mathbf{x} = 3 \quad (1.2)$$

are equally inclined to the line

$$(-m \ 1)\mathbf{x} = 4 \quad (1.3)$$

Find the value of m .

2 SOLUTION

To find the angle between two lines we use-

$$\cos \theta = \frac{\mathbf{a}^T \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} \quad (2.1)$$

Assume line(1.1), (1.2) and (1.3) as vectors \mathbf{a}, \mathbf{b} and \mathbf{c} respectively

$$\mathbf{a} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad (2.2)$$

$$\mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (2.3)$$

$$\mathbf{c} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \quad (2.4)$$

$$|\mathbf{a}| = \sqrt{10} \quad (2.5)$$

$$|\mathbf{b}| = \sqrt{5} \quad (2.6)$$

$$|\mathbf{c}| = \sqrt{m^2 + 1} \quad (2.7)$$

$$\mathbf{a}^T \mathbf{c} = 3m + 1 \quad (2.8)$$

$$\mathbf{b}^T \mathbf{c} = m + 2 \quad (2.9)$$

Angle between \mathbf{a} and \mathbf{c} using (2.1)

$$\cos \theta = \frac{3m + 1}{\sqrt{10} \sqrt{m^2 + 1}} \quad (2.10)$$

Angle between \mathbf{b} and \mathbf{c} using (2.1)

$$\cos \theta = \frac{m + 2}{\sqrt{5} \sqrt{m^2 + 1}} \quad (2.11)$$

Graph

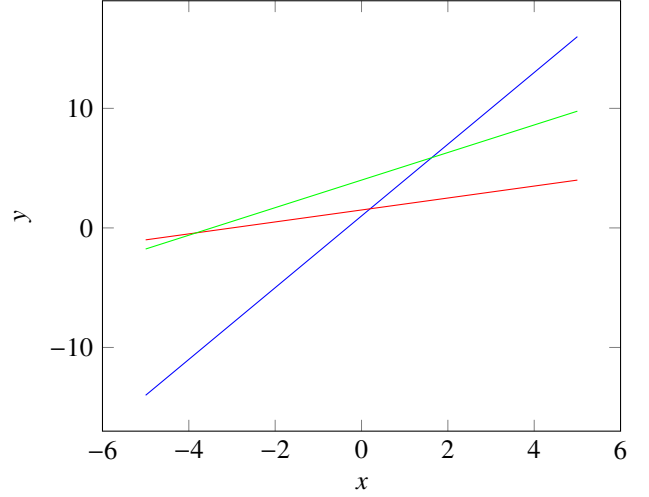


Fig. 1: This is a plot of lines (1.1),(1.2) and (1.3)

According to question \mathbf{a} and \mathbf{b} are equally inclined to \mathbf{c}
Therefore, (2.10) and (2.11) are equal

$$\frac{3m + 1}{\sqrt{10} \sqrt{m^2 + 1}} = \frac{m + 2}{\sqrt{5} \sqrt{m^2 + 1}} \quad (2.12)$$

$$\Rightarrow \frac{3m + 1}{\sqrt{2}} = m + 2 \quad (2.13)$$

$$\Rightarrow m = \frac{1 + 5\sqrt{2}}{7} \quad (2.14)$$