Assignment 2

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Abstract—This document solves a problem based on the inclination of straight lines

1 Problem

If the lines

$$\begin{pmatrix} -3 & 1 \end{pmatrix} \mathbf{x} = 1 \tag{1.1}$$

$$\begin{pmatrix} -1 & 2 \end{pmatrix} \mathbf{x} = 3 \tag{1.2}$$

are equally inclined to the line

$$\begin{pmatrix} -m & 1 \end{pmatrix} \mathbf{x} = 4 \tag{1.3}$$

Find the value of m.

2 Solution

To find the angle between two lines we use-

$$\cos \theta = \frac{a^T b}{\|a\| \|b\|} \tag{2.1}$$

Assume line(1.1), (1.2) and (1.3) as vectors \mathbf{a} , \mathbf{b} and \mathbf{c} respectively

$$\mathbf{a} = \begin{pmatrix} -3\\1 \end{pmatrix} \tag{2.2}$$

$$\mathbf{b} = \begin{pmatrix} -1\\2 \end{pmatrix} \tag{2.3}$$

$$\mathbf{c} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \tag{2.4}$$

$$|\mathbf{a}| = \sqrt{10} \tag{2.5}$$

$$|\mathbf{b}| = \sqrt{5} \tag{2.6}$$

$$|\mathbf{c}| = \sqrt{m^2 + 1}$$
 (2.7)

$$\mathbf{a}^T \mathbf{c} = 3m + 1 \tag{2.8}$$

$$\mathbf{b}^T \mathbf{c} = m + 2 \tag{2.9}$$

Angle between a and c using (2.1)

$$\cos \theta = \frac{3m+1}{\sqrt{10}\sqrt{m^2+1}} \tag{2.10}$$

Angle between **b** and **c** using (2.1)

$$\cos \theta = \frac{m+2}{\sqrt{5}\sqrt{m^2+1}}$$
 (2.11)

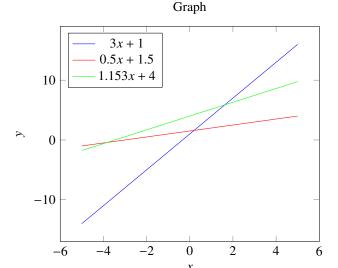


Fig. 1: This is a plot of lines (1.1),(1.2) and (1.3)

According to question \mathbf{a} and \mathbf{b} are equally inclined to \mathbf{c} Therefore, (2.10) and (2.11) are equal

$$\frac{3m+1}{\sqrt{10}\sqrt{m^2+1}} = \frac{m+2}{\sqrt{5}\sqrt{m^2+1}}$$
 (2.12)

$$\implies \frac{3m+1}{\sqrt{2}} = m+2 \tag{2.13}$$

$$\implies m = \frac{1 + 5\sqrt{2}}{7} \tag{2.14}$$