Homework 0: CS 436/580L: Introduction to Machine learning

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1. Sample space when two standard six-sided dice rolled.:

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

let A be the event

~~Probability~~ of rolling doubles (both dice landing on the same numbers)

Probability of event A $= \frac{6}{36} = \frac{1}{6}$

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2. Given,

$P[X,Y] = 0.2$, $P[X] = 0.5$

By the definition of conditional probability

$$P[X|Y] = \frac{P[X,Y]}{P[Y]} \quad — \text{I}$$

Given that X and Y are two independent random variables.

Therefore, $P[X|Y] = P[X]$ — II

Hence I can be modified using II.

$$\frac{P[X,Y]}{P[Y]} = P[X]$$

$$\frac{0.2}{P[Y]} = 0.5$$

$$\frac{0.2}{0.5} = P[Y]$$

$$0.4 = P[Y]$$

3. P(stepping forward) = 0.6
P(stepping backward) = 0.4

To find probability that he is at his starting position after 10 steps, the ~~drunken~~ drunk person has to walk 5 steps forward and 5 steps backward.

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Hence,

Probability that he is at his starting position $= {}^{10}C_5 \times p(\text{moving forward})^5 \times p(\text{moving backward})^5$

$$= {}^{10}C_5 \times (0.6)^5 \times (0.4)^5$$

4. Given that $E[x] = 2$, $Var[X] = 1$, $E[Y] = 3$.

formula for.

Variance of random variable x.

$$Var[x] = E[x^2] - (E[x])^2$$

$$1 = E[x^2] - (2)^2$$

$$E[x^2] = 5$$

$E[Z] = E[X^2 Y]$ ... (Given that $Z = X^2 Y$)

$\therefore E[Z] = E[X^2] \times E[Y]$ .... (Since X and Y are independent random variables)

$E[Z] = 5 \times 3$

$\therefore E[Z] = 15$

5. Given series 1, 6, -1, 4, 10.

Mean $= \dfrac{1+6+(-1)+4+10}{5} = \dfrac{20}{5} = 4$

Arrange the series in ascending order to find median

-1, 1, 4, 6, 10

Median = 4.

$\bar{x}$ = mean, x = a value in the series

Variance $= \dfrac{\Sigma (x - \bar{x})^2}{n}$

$= \dfrac{(1-4)^2 + (6-4)^2 + (-1-4)^2 + (10-4)^2}{5}$

$$= \frac{(-3)^2 + (2)^2 + (-5)^2 + (6)^2}{5}$$

$$= \frac{9 + 4 + 25 + 36}{5}$$

$$= \frac{74}{5}$$

$$= 14.8$$

6. $P(\text{winning}) = 20\% = \frac{20}{100}$

$P(\text{losing}) = 80\% = \frac{80}{100}$

Expected gain $= P(\text{winning}) \times 10 - P(\text{losing}) \times 5$

$$= \frac{20}{100} \times 10 - \frac{80}{100} \times 5$$

$$= 2 - 4 = -2$$

$\therefore$ Hence, the gambler will lose \$2n after n bets. (Expected gain after n bets)

7. X be the event which denotes first card drawn from the deck

Y be the event which denotes second card drawn from the deck

Given that the first card is spade which was drawn from a deck consisting of the standard 52 cards.

Now, we need to find probability that the second card you draw is also spade given first card is spade from a deck consisting of 51 card. (Because the second card drawn without replacement).

By the rule of conditional probability

$$P(Y|X) = \frac{12}{51}$$

Given,

8. $P(\text{Getting a Head}) = \frac{1}{2}$, $P(\text{Getting a Tail}) = \frac{1}{2}$

$$P(\text{Draw a White ball from Urn 1} | \text{Head}) = \frac{2}{9} \quad \ldots \text{(I)}$$

$$P(\text{Draw a white ball from urn 2} | \text{Tail}) = \frac{5}{11} \quad \ldots \text{(II)}$$

We can find,

P(Draw a white ball) =

P(Draw a White ball from urn 1 ∩ Getting a Head) +

P(Draw a white ball from urn 2 ∩ Getting a tail)

= P(Draw a white ball from urn 1 | Given head is shown) ×

P(Getting a head) +

P(Draw a White ball from urn 2 | Given tail is shown)

∴ By the conditional probability rule

$$= \frac{2}{9} \times \frac{1}{2} + \frac{5}{11} \times \frac{1}{2} = \frac{1}{9} + \frac{5}{22} = \frac{67}{198}$$

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Now, we can find probability of getting a head given that a white ball was selected using Bayes formula:

$$P(\text{Getting a Head} \mid \text{Given white ball is } \cancel{\text{drawn}}\text{ selected}) = \frac{\text{I} \times P(\text{Getting a Head})}{\text{III}}$$

$$= \frac{2/9 \times 1/2}{\frac{67}{198}} = \frac{1}{9} \times \frac{198}{67} = \frac{22}{67}$$

9. Given,

$P(\text{Getting a Head}) = p$; n = # of total tosses

let X be the # of Heads in ~~one~~ 10 toss

$P(\text{Getting a Tail}) = 1 - p$

To find the probability that you get more than 6 heads in 10 toss, we will use binomial distribution theorm, as there are finite set of trials, trial outcome success or failure, trial results are independent and same ~~probility~~ probability on each trial.

So, By the binomial cumulative probability function

$$P(6 < X \leq 10) = \sum_{i=7}^{x} \binom{n}{i} p^i (1-p)^{(n-i)} =$$

$$P(7, p, 10) + P(8, p, 10) + P(9, p, 10) + P(10, p, 10) \ldots$$

$$\left\{ \begin{array}{l} \text{Probability mass function for} \\ \text{Binomial distribution} \\ P(x; p, n) = \binom{n}{x} p^x (1-p)^{n-x} \end{array} \right\}$$

$$= \binom{10}{7} p^7 (1-p)^3 + \binom{10}{8} p^8 (1-p)^2 + \binom{10}{9} p^9 (1-p)^1 +$$

$$\binom{10}{10} p^{10} (1-p)^0$$

10. $p(\text{winning}) = p$

Let X denotes # of bets until the man find his first success and X follows a geometric distribution".

The probability density function (P.d.f) for Geometric distribution :

$$P(X = x) = p(1-p)^{x-1}$$

Now find,

probability that he wins for the first time after n bets =

$$p(X = n+1) = p(1-p)^{n+1-1}$$

$$= p(1-p)^n$$

This is a geometric distribution as number of trials are not fixed / finite.