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Design and Analysis of Algorithms Lab

Assignment

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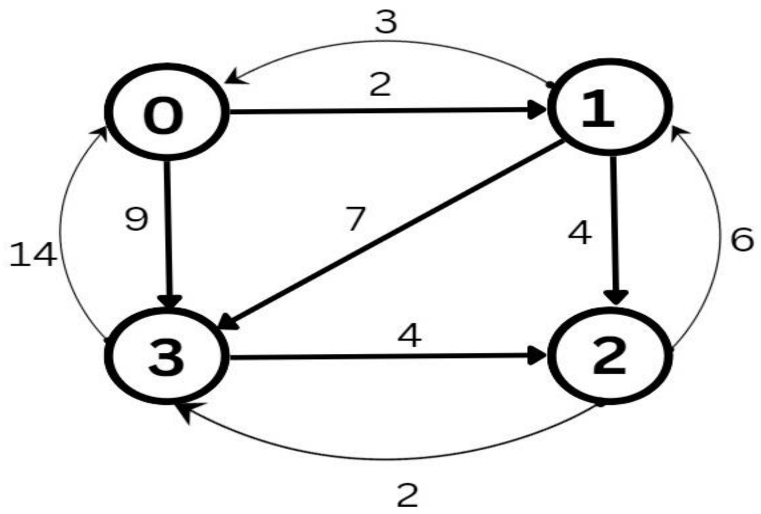
Trace the Floyd Warshall algorithm for the given graph .



Floyd – Warshall Algorithm :

This algorithm is an all pair shortest path problem , i.e. it finds the shortest path between all pairs of vertices in the weighted graph .

This algorithm follows the dynamic programming approach to find the shortest paths , i.e. it check every possible path going via every possible node in order to calculate shortest distance between every pair of nodes .

This algorithm works for both the directed and undirected weighted graph . And it is solved using the Adjacency Matrix representation of graph .

This algorithm is highly efficient with both positive and negative edge weights . But , it does not work for the negative cycles i.e. the sum of the edges in the graph is a negative number .

Procedure :

Step 1 : Initialize the solution matrix same as the input graph matrix .

If there is no path between two vertices mark the value as infinity (∞).

If there is no self loop , mark the value as zero (0) .

Step 2 : Then update the solution matrix by considering all vertices one - by - one as an intermediate vertex .

Step 3 : The idea is to pick all vertices one - by - one and update all shortest paths which include the picked vertex as an intermediate vertex in the shortest path .

Step 4 : The final adjacency matrix obtained is the final solution with all the shortest paths .

Complexity Analysis :

1 . Time Complexity :

O(V^3) , where V is the number of vertices in the graph .

In all the three cases , i.e. worst , average and best case the algorithm iterates through all vertices for each pair of vertices and checks if there is a shorter path through an intermediate vertex . This involves three nested loops , each iterating over all vertices , resulting in a cubic time complexity .

2 . Space Complexity :

O(V^2) , where V is the number of vertices in the graph .

This is because , it requires a matrix of size V × V (2D) to store the shortest path distance between all pairs of vertices .

Applications :

1 . Communication Networks : This algorithm helps in efficient data transmission by finding the shortest paths between nodes in communication networks .

2 . Traffic Management : This algorithm helps in optimizing traffic flow by determining the shortest routes between different locations in a transport network .

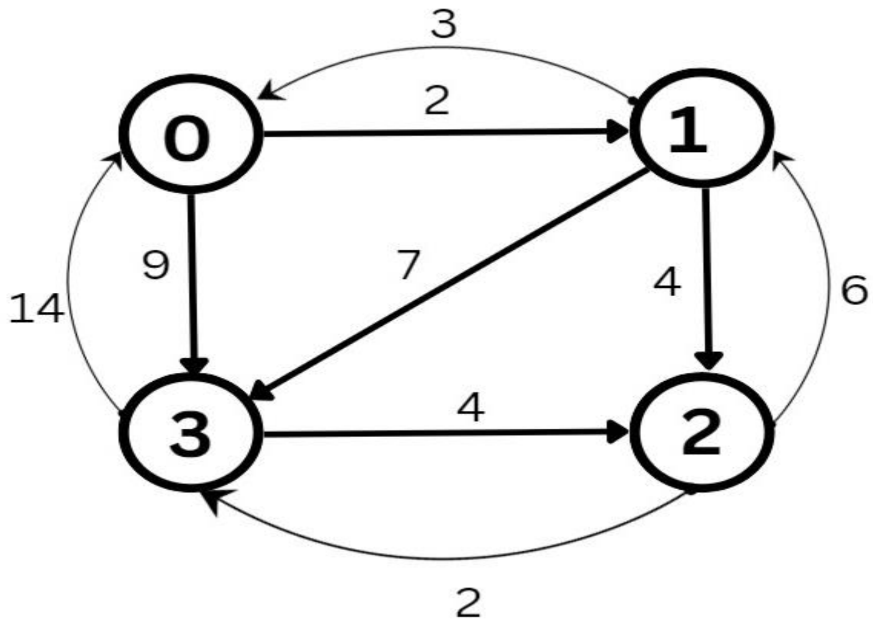
3 . Robotics : This algorithm is used in robotics for path planning , allowing robots to navigate through complex environments efficiently.

4 . Resource Allocation : This algorithm helps in determining the most efficient allocation of resources in distributed systems .

5 . Geographical Information Systems : This algorithm used to analyze spatial data , finding the shortest routes between locations on maps , which is useful in urban planning , logistics and navigation systems .

Example :

Consider the following directed - weighted graph .



Solution :

Adjacency matrix for given directed - weighted graph is :

$$\therefore\ A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 2 & \infty & 9 \\ 3 & 0 & 4 & 7 \\ \infty & 6 & 0 & 2 \\ 14 & \infty & 4 & 0 \end{bmatrix} \end{array}$$

Step 1 : To find A^0 by considering vertex 0 as an intermediate vertex with reference of A .

i) A [1 , 2] < A [1 , 0] + A [0 , 2]

= 4 < 3 + ∞

= 4

ii) A [1 , 3] > A [1 , 0] + A [0 , 3]

= 7 > 3 + 9

= 7

iii) A [2 , 1] < A [2 , 0] + A [0 , 1]

= 6 < ∞ + 2

= 6

iv) A [2 , 3] < A [2 , 0] + A [0 , 3]

= 2 < ∞ + 9

= 2

v) A [3 , 1] > A [3 , 0] + A [0 , 1]

= ∞ > 14 + 2

= 16

vi) A [3 , 2] < A [3 , 0] + A [0 , 2]

= 4 < 14 + 4

= 4

$$\therefore A^0 = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 2 & \infty & 9 \\ 3 & 0 & 4 & 7 \\ \infty & 6 & 0 & 2 \\ 14 & 16 & 4 & 0 \end{bmatrix} \end{array}$$

Step 2 : To find A^1 by considering vertex 1 as an intermediate vertex with reference of A^0 .

i) $A^0 [0 , 2] > A^0 [0 , 1] + A^0 [1 , 2]$

$= \infty > 2 + 4$

$= 6$

ii) $A^0 [0 , 3] = A^0 [0 , 1] + A^0 [1 , 3]$

$= 9 = 2 + 7$

$= 9$

iii) $A^0 [2 , 0] > A^0 [2 , 1] + A^0 [1 , 0]$

$= \infty > 6 + 3$

$= 9$

iv) $A^0 [2 , 3] < A^0 [2 , 1] + A^0 [1 , 3]$

$= 2 < 6 + 7$

$= 2$

v) $A^0 [3 , 0] < A^0 [3 , 1] + A^0 [1 , 0]$

$= 14 < 16 + 3$

$= 14$

vi) $A^0 [3 , 2] < A^0 [3 , 1] + A^0 [1 , 2]$

$= 4 < 16 + 4$

$= 4$

$$\therefore A^1 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 2 & 6 & 9 \\ 3 & 0 & 4 & 7 \\ 9 & 6 & 0 & 2 \\ 14 & 16 & 4 & 0 \end{bmatrix} \end{array}$$

Step 3 : To find A^2 by considering vertex 2 as an intermediate vertex with reference of A^1 .

i) $A^1 [0 , 1] < A^1 [0 , 2] + A^1 [2 , 1]$

$= 2 < 6 + 6$

$= 2$

ii) $A^1 [0 , 3] > A^1 [0 , 2] + A^1 [2 , 3]$

$= 9 > 6 + 2$

$= 8$

iii) $A^1 [1 , 0] < A^1 [1 , 2] + A^1 [2 , 0]$

$= 3 < 4 + 9$

$= 3$

iv) $A^1 [1 , 3] > A^1 [1 , 2] + A^1 [2 , 3]$

$= 7 > 4 + 2$

$= 6$

v) $A^1 [3 , 0] > A^1 [3 , 2] + A^1 [2 , 0]$

$= 14 > 4 + 9$

$= 13$

vi) $A^1 [3 , 1] > A^1 [3 , 2] + A^1 [2 , 1]$

$= 16 > 4 + 6$

$= 10$

$$\therefore A^2 = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 2 & 6 & 8 \\ 3 & 0 & 4 & 6 \\ 9 & 6 & 0 & 2 \\ 13 & 10 & 4 & 0 \end{bmatrix} \end{array}$$

Step 4 : To find A^3 by considering vertex 3 as an intermediate vertex with reference of A^2 .

i) $A^2 [0 , 1] < A^2 [0 , 3] + A^2 [3 , 1]$

$= 2 < 8 + 10$

$= 2$

ii) $A^2 [0 , 2] < A^2 [0 , 3] + A^2 [3 , 2]$

$= 6 < 8 + 4$

$= 6$

iii) $A^2 [1 , 0] < A^2 [1 , 3] + A^2 [3 , 0]$

$= 3 < 6 + 13$

$= 3$

iv) $A^2 [1 , 2] < A^2 [1 , 3] + A^2 [3 , 2]$

$= 4 < 6 + 4$

$= 4$

v) $A^2 [2 , 0] < A1 [2 , 3] + A^2 [3 , 0]$

$= 9 < 2 + 13$

$= 9$

vi) $A^2 [2 , 1] < A^2 [2 , 3] + A^2 [3 , 1]$

$= 6 < 2 + 10$

$= 6$

$$\therefore\ A^3 = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 2 & 6 & 8 \\ 3 & 0 & 4 & 6 \\ 9 & 6 & 0 & 2 \\ 13 & 10 & 4 & 0 \end{bmatrix} \end{array}$$