

Cluster analysis on fashion MNIST dataset using unsupervised learning

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In this project, we perform cluster analysis on fashion MNIST dataset using unsupervised learning. Cluster analysis is one of the unsupervised machine learning technique which doesn't require labeled data. The fashion MNIST data as described below comprises of 70000 instances of data with each image being a 28 x 28 grayscale reproduction of any of the 10 items described below. The purpose of this project dictates us to cluster the same using differing clustering algorithms, mainly K-Means and Gaussian Mixture Model, with and without the help of an autoencoder

We will be taking three different approaches to test and verify the accuracy of the algorithms on the dataset using the Scipy and sklearns libraries.

1. Cluster the training and test dataset using kmeans algorithm.
2. Build an Auto-Encoder based K-Means clustering model to cluster the condensed representation of the unlabeled fashion MNIST dataset using Keras and Sklearns library.
3. Build an Auto-Encoder based Gaussian Mixture Model clustering model to cluster the condensed representation of the unlabeled fashion MNIST dataset using Keras and Sklearns library.

Finally, we shall be reporting the accuracy of each of these clustering algorithms.

Problem Statement

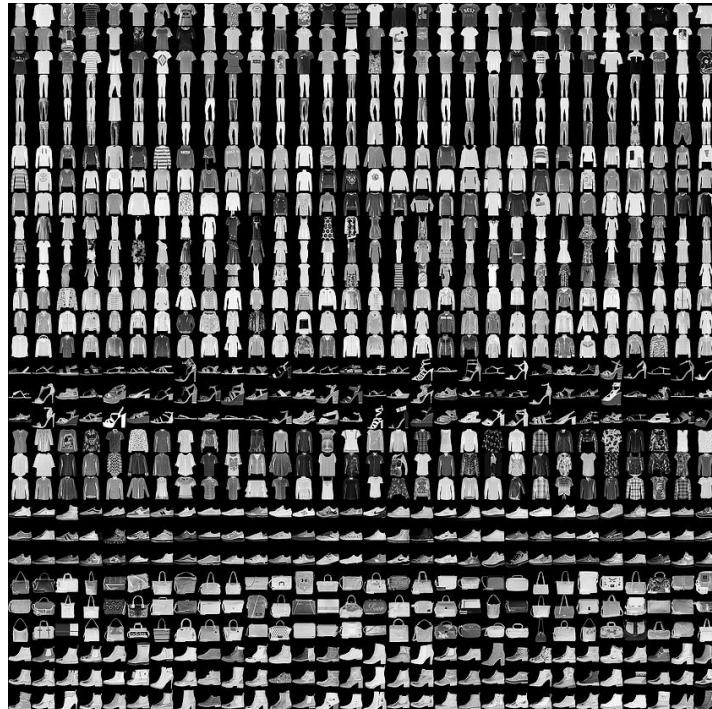
The fashion MNIST is a dataset comprising of 70000 data. Each image is a 28x28 grayscale reproduction, associated with a label from 10 classes. Labels so represented are noted in the next section. We aim to use kmeans and Gaussian Mixture Model- which are both clustering techniques in order to cluster the above dataset based on their likeliness. Initially, we simply use kmeans to cluster the dataset. However, we then parse the dataset through an autoencoder. Autoencoders are an unsupervised learning technique in which we leverage neural networks for the task of **representation learning**. Specifically, we'll design a neural network architecture such that we *impose a bottleneck in the network which forces a compressed knowledge representation of the original input*. After the compressed/ latent space representation of each image is formed, this is exploited to conduct kmeans clustering and GMM clustering on the same. This is done so as to gauge the performance of normal clustering on data v/s clustering on autoencoded/ compressed images of the same dataset.

1. Theory

1.1 Dataset Used

Fashion-MNIST is a dataset of Zalando's article images—consisting of a training set of 60,000 examples and a test set of 10,000 examples. Each example is a 28x28 grayscale image, associated with a label from 10 classes. We intend Fashion-MNIST to serve as a direct **drop-in replacement** for the original MNIST dataset for benchmarking machine learning algorithms. It shares the same image size and structure of training and testing splits.

Here's an example how the data looks (*each class takes three-rows*):



Label	Description
0	T-shirt/top
1	Trouser
2	Pullover
3	Dress
4	Coat
5	Sandal
6	Shirt
7	Sneaker
8	Bag
9	Ankle boot

1.2 What is clustering / cluster analysis?

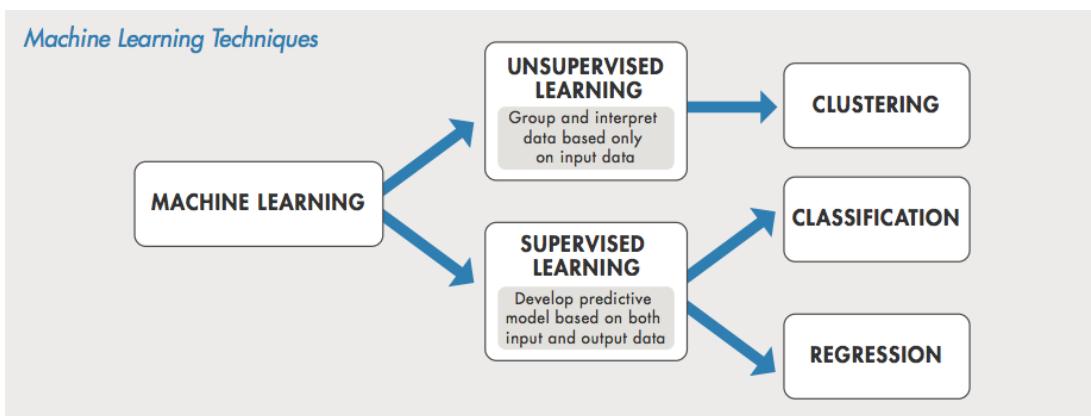
Cluster analysis divides data into groups (clusters) that are meaningful, useful, or both. If meaningful groups are the goal, then the clusters should capture the natural structure of the data. In some cases, however, cluster analysis is used for data summarization in order to reduce the size of the data.

Clusters are Classes, or conceptually meaningful groups of objects that share common characteristics, play an important role in how people analyse and describe the world. The greater the similarity within a group and the greater the difference between groups, the better or more distinct the clustering.

How is clustering a form of unsupervised learning?

The goal for unsupervised learning is to model the underlying structure or distribution in the data in order to learn more about the data. Clustering is unsupervised since with clustering we try to group some data-points into several groups. We call those groups as clusters. So usually clustering does not look at target/labels instead it groups the data considering the similarities in the features. Therefore, clustering employs a similarity function to measure the similarity between two data-points (e.g. k means clustering measures the euclidean distance).

Unlike supervised learning, there is no correct answers/labels and there is no teacher. Algorithms are left to their own to discover and present the interesting structure in the data.



Types of clustering

Clustering aims to find useful groups of objects (clusters), where usefulness is defined by the goals of the nature of the data. Some types of clusters are

- **Well-Separated:** A threshold is used to specify that all the objects in a cluster must be sufficiently close (or similar) to one another. This idealistic definition of a cluster is satisfied only when the data contains natural clusters that are quite far from each other.
- **Prototype-Based :** For data with continuous attributes, the prototype of a cluster is often a centroid, i.e., the average (mean) of all the points in the cluster.

- **Graph-Based:** If the data is represented as a graph, where the nodes are objects and the links represent connections among objects, then a cluster can be defined as a connected component; i.e., a group of objects that are connected to one another, but that have no connection to objects outside the group
- **Model-based:** In the model-based clustering approach, the data are viewed as coming from a mixture of probability distributions, each of which represents a different cluster. In other words, in model-based clustering, it is assumed that the data are generated by a mixture of probability distributions in which each component represents a different cluster. Thus a particular clustering method can be expected to work well when the data conform to the model.

For the purpose of our project, we shall be working with two very common and famous clustering algorithms. One of which is prototype based and the other is model based.

K-means Clustering

K-Means is one of the most popular clustering algorithms. It is an example of a prototype based clustering model. K-means stores “k” centroids that it uses to define clusters. A point is considered to be in a particular cluster if it is closer to that cluster's centroid than any other centroid.

It finds the best centroids by alternating between

1. Assigning data points to clusters based on the current centroids
2. Choosing centroids (points which are the center of a cluster) based on the current assignment of data points to clusters.

Algorithm:

1. Select K points as initial centroids
2. Repeat
 - a. Form K clusters by assigning each point to its closest centroid
 - b. Re-compute the centroid of each cluster
3. Until Centroids do not change

To assign a point to the closest centroid, we need a proximity measure that quantifies the notion of “closest” for the specific data under consideration. Euclidean (L2) distance is often used for data points in Euclidean space, while cosine similarity is more appropriate for documents.

For the purpose of our project, we have used a k-means++ approach (which is the default approach used in SKlearns library implementation)

Algorithm 7.2 K-means++ initialization algorithm.

- 1: For the first centroid, pick one of the points at random.
- 2: **for** $i = 1$ to *number of trials* **do**
- 3: Compute the distance, $d(x)$, of each point to its closest centroid.
- 4: Assign each point a probability proportional to each point's $d(x)^2$.
- 5: Pick new centroid from the remaining points using the weighted probabilities.
- 6: **end for**

Gaussian Mixture Model

Mixture models view the data as a set of observations from a mixture of differing probability distribution. More formally, if there are K distributions and m objects, then we find out the probability of the i -th object in the j -th distribution, which is given by a weight.

If the objects are generated in an independent manner, then the probability of the entire set of objects is just the product of the probabilities of individual x . By using statistical methods- MLE, we can estimate the parameters of these distributions from the data and thus describe these distributions as clusters.

Algorithm:

1. Select an initial set of model parameters (As with K means, this can be done randomly)
2. Repeat
 - a. **Expectation Step:** For each object calculate the probability that each object belongs to each distribution
 - b. **Maximization Step:** Given the probabilities from the estimation step, find the new estimates of the parameters that maximize the expected likelihood.
3. Until parameter stop changing or until convergence is met.

Given a Gaussian mixture model, the goal is to maximize the likelihood function with respect to the parameters comprising the means and covariances of the components and the mixing coefficients).

1. Initialize the means μ , covariances Σ and mixing coefficients π , and evaluate the initial value of the log likelihood.
2. E step: Evaluate the responsibilities using the current parameter values

$$\gamma_j(x) = \frac{\pi_k \mathcal{N}(x | \mu_k, \Sigma_k)}{\sum_{j=1}^K \pi_j \mathcal{N}(x | \mu_j, \Sigma_j)}$$

3. M step: Re-estimate the parameters using the current responsibilities

$$\boldsymbol{\mu}_j = \frac{\sum_{n=1}^N \gamma_j(x_n) \mathbf{x}_n}{\sum_{n=1}^N \gamma_j(x_n)}$$

$$\boldsymbol{\Sigma}_j = \frac{\sum_{n=1}^N \gamma_j(x_n) (\mathbf{x}_n - \boldsymbol{\mu}_j)(\mathbf{x}_n - \boldsymbol{\mu}_j)^T}{\sum_{n=1}^N \gamma_j(x_n)}$$

$$\pi_j = \frac{1}{N} \sum_{n=1}^N \gamma_j(x_n)$$

4. Evaluate log likelihood

$$\ln p(\mathbf{X} | \boldsymbol{\mu}, \boldsymbol{\Sigma}, \boldsymbol{\pi}) = \sum_{n=1}^N \ln \left\{ \sum_{k=1}^K \pi_k N(\mathbf{x}_n | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \right\}$$

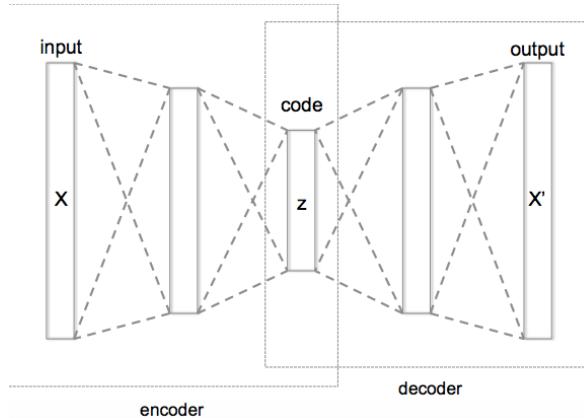
If there is no convergence, return to 2.

Autoencoders

An **autoencoder** neural network is an unsupervised learning algorithm that applies backpropagation, setting the target values to be equal to the inputs.

An autoencoder is a neural network that is trained to attempt to copy its input to its output. Internally, it has a hidden layer h that describes a code used to represent the input. The network may be viewed as consisting of two parts: an encoder function $h=f(x)$ and a decoder that produces a reconstruction $r=g(h)$.

If an autoencoder succeeds in simply learning to set $g(f(x))=x$ everywhere, then it is not especially useful. Instead, autoencoders are designed to be unable to learn to copy perfectly. Usually they are restricted in ways that allow them to copy only approximately, and to copy only input that resembles the training data. Because the model is forced to prioritize which aspects of the input should be copied, it often learns useful properties of the data



Autoencoders are similar to dimensionality reduction techniques like Principal Component Analysis (PCA). They project the data from a higher dimension to a lower dimension using linear transformation and try to preserve the important features of the data while removing the non-essential parts.

An auto encoder has two parts, namely the encoder and the decoder. While the encoder down-samples the image to a more reduced dimension, the decoder up-samples the image and reconstructs it to its former state. Auto encoders are useful for noise reduction and image reconstruction. For the purpose of this project, because we are dealing with 28 X 28 sized grayscale images which need to be reconstructed, we shall be using a convoluted autoencoder.

2. Implementation

In order to work on about any dataset, there are a few steps that we have to follow.

1. Importing datasets
2. Dataset normalization and preprocessing. Resizing 28 x 28 sized image into a 28 x 28 x 1 matrix to be fed into encoder.
3. Data partitioning into Training, Validation and Testing Sets
4. Run K-Means on Training Data. Subsequent calculation of accuracy using confusion matrix and comparison using NMI (Normalized Mutual Information Score)
5. Train convolutional auto encoder.
6. Run K-Means on latent space representation of encoded images. Subsequent calculation of accuracy using confusion matrix and comparison using NMI (Normalized Mutual Information Score)
7. Run GMM on latent space representation of encoded images. Subsequent calculation of accuracy using confusion matrix and comparison using NMI (Normalized Mutual Information Score)
8. Tuning of hyper-parameters.
9. Testing of each test-dataset against the same hyper-parameters to ascertain which tuning fits well.

2.1 Importing Datasets

The Fashion MNIST datasets can be imported using keras library by invoking `keras.datasets.fashion_mnist.load_data()`. It can also be directly downloaded from the official git page of fashion MNIST dataset as a .gz file and imported locally. We have used the latter approach. Four different datasets comprise of the fashion MNIST package. Train data, train label, test data and test label, i.e the label for the test and the train data are already bifurcated into separate datasets. Given below are the dataset sizes.

```
train_data shape: (60000, 28, 28) train_label shape: (60000, )
test_data shape: (10000, 28, 28) test_label shape: (10000, )
```

2.2 Dataset normalization and Preprocessing

The goal of normalization is to change the values of numeric columns in the dataset to a common scale, without distorting differences in the ranges of values, usually required for datasets with different kinds of values. This is required since if the features in the data set have values in different ranges, it takes the model a long time to converge and achieve the global/local minima. Hence, we do normalization to get the feature values in similar ranges and help the model to converge more quickly.

- For the purpose of this project, normalisation is done by dividing each image by 255 (range of pixels). The data is then concatenated together and shuffled using a common seed for the model to train well and learn on randomised data.
- The train data and test data are then reshaped into 28 x 28 x1 matrix in order to be fed into the auto encoder.

```
train_data = train_data.reshape(-1, 28, 28, 1)
test_data = test_data.reshape(-1, 28, 28, 1)
```

2.3 Data partitioning into Training, Validation and Testing

The data sets in this project are already separated into training and test data sets and labels.

Training Data: 48,000 rows, 784 labels | **Testing data:** 10000 rows, 784 labels | **Validation Data:** 12000 rows, 784 labels.

2.4 K- means implementation on Training Dataset

- We use the sklearns.cluster.Kmeans library to compute the K-means for the fashion mnist dataset, using all default values and specifying the no of clusters as 10. Method for initialization, defaults to ‘k-means++’: selects initial cluster centres for k-mean clustering in a smart way to speed up convergence

```
kmeans_data= train_data.reshape(len(train_data),784)
kmeans = KMeans(n_clusters=10, init='k-means++', max_iter=300, n_init=10, random_state=0)
kmeans_predicted_labels = kmeans.fit_predict(kmeans_data)
```

- Next we find the unique labels in each cluster. This is for us to understand that in the original dataset, we had 6000 images in each cluster, so as to draw a fair comparison (before and after the kmeans calculation).
- The scipy.stats.mode function calculates the mode of the array elements along the specified axis of the array. We simply find the indexes of the first cluster, for example, and then compare it with same indices of the predicted label from kmeans. The mode obtained from the original cluster becomes the final cluster label of the predicted list. This is reiteratively done for all clusters in the list
- The accuracy is then calculated using the confusion matrix.
- The normalized mutual information score is represented as follows

```

print(formatting.BOLD + "Normalized Mutual Information Score: " + formatting.END,
      NMI(train_label, predicted_kmean_train, average_method="arithmetic")*100)

```

Normalized Mutual Information (NMI) is a normalization of the Mutual Information (MI) score to scale the results between 0 (no mutual information) and 1 (perfect correlation). In this function, mutual information is normalized by some generalized mean of $H(\text{labels_true})$ and $H(\text{labels_pred})$, defined by the average method. It is also a means to generate clustering accuracy for unsupervised learning. The NMI is calculated in order to base our accuracies produced.

- We then apply PCA on the original dataset (train data and test data) to reduce the 10 feature dimension to a 2 component representation for better illustration.
- The scatter plots from PCA are plotted for the original labels and the clustered labels.

2.5 Train using Auto Encoder network

As previously mentioned, an Autoencoder can be broken in to three parts:

Encoder: this part of the network compresses or downsamples the input into a fewer number of bits.

The space represented by these fewer number of bits is often called the **latent-space** or bottleneck. The bottleneck is also called the "maximum point of compression" since at this point the input is compressed the maximum

Decoder: this part of the network tries to reconstruct the input using only the encoding of the input.

- For the purpose of our project, we constructed a deep convolutional autoencoder with 14 layers- inclusive of the encoder and decoder. The encoder layers are of the following sizes. The image (28 x28 x 1) is first made thicker (28 x 28 x 64) and then gradually downsampled into 7 x 7 x 7 which is the encoded image. Likewise, the encoded image with dimension 7 x 7 x 7 is upsampled in the decoder and reconstructed as a 28 x 28 x 1 image at the end of the model. The function getautoencoder() is constructed for this purpose. We are also taking help from the keras library to build the auto encoder network.
- The model is then trained with batch size and epochs and other parameters specified
- Model is compiled with mean squared error as loss type and adam optimizer.
- The performance (loss and accuracy is plotted for each epoch)
- Finally, after successful training, we evaluate the test data on the trained autoencoder using the autoencoder.evaluate and autoencoder.predict functions
- The reconstructed images from the test data are then plotted so as to visually analyse the differences between the original and the reconstructed set.
- In order to assess the performances of the clustering algorithms on the latent space representations or the encoded images, we extract the encoder function and use the encoded images so produced for the next parts of the project.

2.6 Using Kmeans on Latent Space

- we extract the encoder function and use the encoded images so produced in order to cluster them using K means.
- We follow the same steps as in section 2.4 with the only difference being the change in input and input dimension.
- Because we are sending in encoded images of the test set, the dimensions of the input label are different from the dataset label.

```
encoded_images = encoder_tr.predict(test_data)
encoded_images = encoded_images.reshape(-1,7*7*7)
print(encoded_images.shape)
```

(10000, 343)

2.6 Using GMM on Latent Space

- we extract the encoder function and use the encoded images so produced in order to cluster them using GMM
- Because we are sending in encoded images of the test set, the dimensions of the input label are different from the dataset label.

```
encoded_images = encoder_tr.predict(test_data)
encoded_images = encoded_images.reshape(-1,7*7*7)
print(encoded_images.shape)
```

(10000, 343)

- we use the sklearn.mixture.GaussianMixture library in order to use GMM on the encoded images.
- The parameters are mostly kept default, with the only exception being that of n_components where we mention the number of clusters we want and the max no of iterations to be 300.

```
gmm = GaussianMixture(n_components=10,max_iter=300,n_init=10).fit(encoded_images)
labels_gmm = gmm.predict(encoded_images)
```

- The confusion matrix and NMI are calculated in the same manner as mentioned above.
- Scatter plots representing the same are formulated and print.

2.5 Tuning of Hyper-parameter

Parameters are tuned to bring out the change in the accuracy, loss, clusters and to compare relative performances amongst each other

- Epochs and number of layers in deep autoencoder are tuned and analysed.

- Confusion matrix is calculated using the inbuilt sklearns metrics library and the accuracy, is calculated for the test set.

Thus

represented

as

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

$$Precision = \frac{TP}{TP + FP}$$

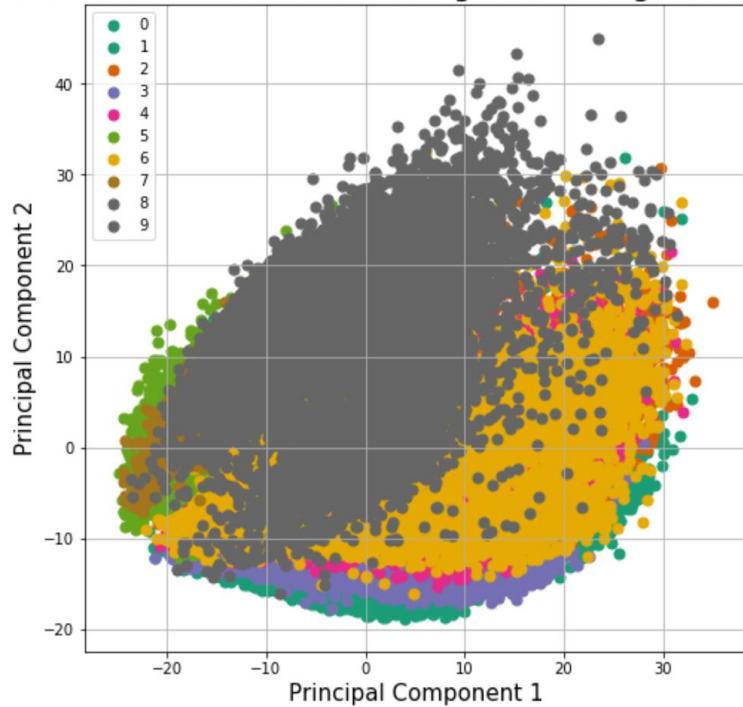
$$Recall = \frac{TP}{TP + FN}$$

Where TP = true positive, TN = true negative, FP = false positive and FN = false negative.

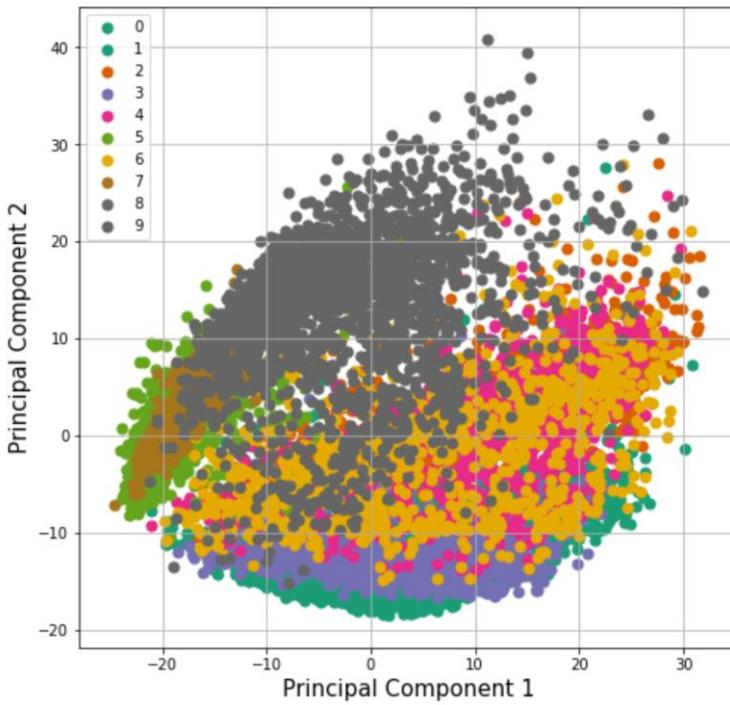
- NMI is also calculated, so as to double check our accuracies. NMI, as previously mentioned is also a good measure of unsupervised learning efficiency.
- The results are reported and analysed.

Training Data and Test Data representation with their original labels

2 component PCA for Fashion MNIST on original training data clusters with K=10



2 component PCA for Fashion MNIST on test data clusters with K=10



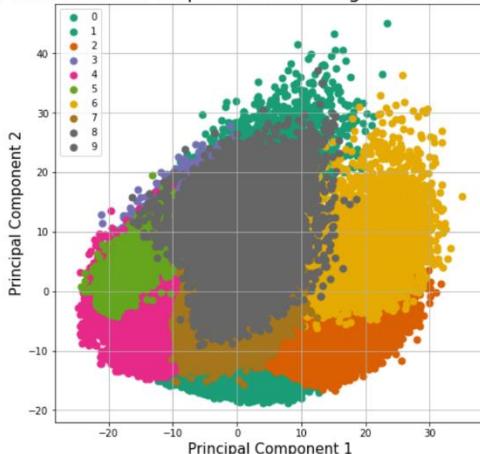
KMeans Clustering on Training Data

Accuracy: 55.33166666666667
Normalized Mutual Information Score: 51.16483920105648

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	3404	201	0	0	167	594	1582	2	50	0
1	236	5413	0	0	63	156	129	0	3	0
2	115	9	0	0	3519	515	1787	1	54	0
3	1684	3209	0	0	49	523	523	0	12	0
4	873	154	0	0	3596	252	1081	0	44	0
5	2	1	0	0	0	3766	29	1444	17	741
6	1053	62	0	0	1954	774	2071	6	79	1
7	0	0	0	0	0	508	0	4680	1	811
8	22	28	0	0	269	494	228	237	4650	72
9	2	2	0	0	1	170	38	164	4	5619

2 component PCA for Fashion MNIST on predicted training data clusters after K-means with K=10



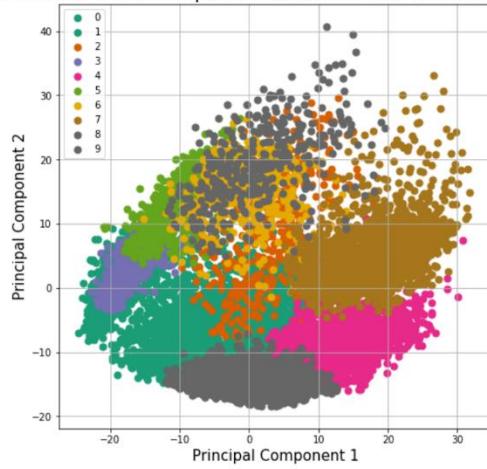
KMeans Clustering on Test Data

Accuracy: 56.03
Normalized Mutual Information Score: 51.45761149609541

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	594	29	0	0	35	80	251	1	10	0
1	52	890	0	0	9	20	29	0	0	0
2	24	4	0	0	568	49	347	0	8	0
3	276	506	0	0	9	84	120	0	5	0
4	148	27	0	0	619	39	158	0	9	0
5	0	0	0	0	0	663	6	221	0	110
6	190	12	0	0	317	108	359	0	14	0
7	0	0	0	0	0	66	0	774	0	160
8	3	6	0	0	63	78	44	35	762	9
9	0	0	0	0	0	29	4	23	2	942

2 component PCA for Fashion MNIST on predicted clusters for test data after K-means with K=10



AutoEncoder with KMeans and GMM

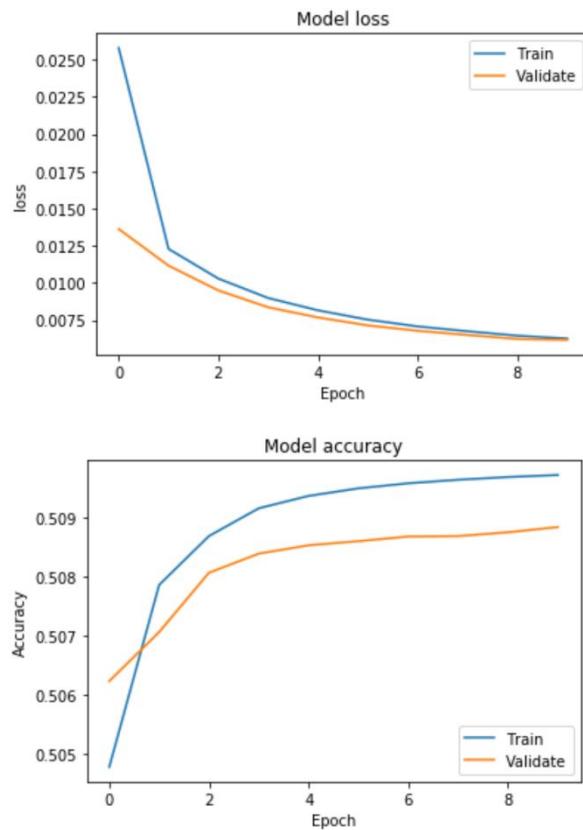
Case 1

Batch Size: 128

Epochs: 10

Encoder Layers:

Layer (type)	Output Shape	Param #
<hr/>		
input_28 (InputLayer)	(None, 28, 28, 1)	0
conv2d_164 (Conv2D)	(None, 28, 28, 64)	640
max_pooling2d_45 (MaxPooling)	(None, 14, 14, 64)	0
conv2d_165 (Conv2D)	(None, 14, 14, 32)	18464
max_pooling2d_46 (MaxPooling)	(None, 7, 7, 32)	0
conv2d_166 (Conv2D)	(None, 7, 7, 16)	4624
conv2d_167 (Conv2D)	(None, 7, 7, 7)	1015
<hr/>		

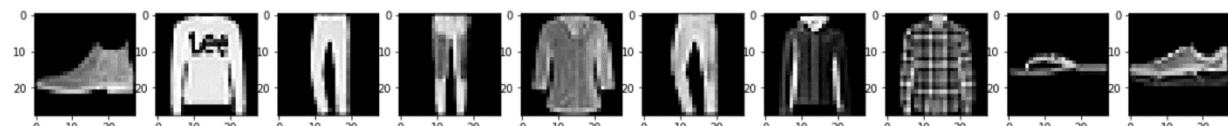


```

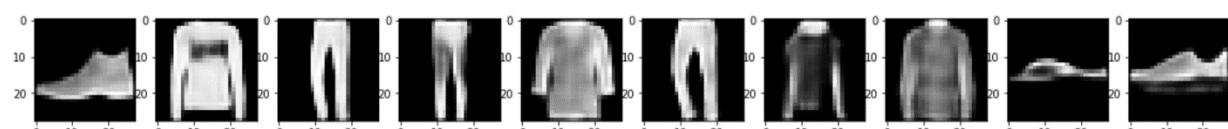
Autoencoder accuracy for training set: 50.97190702756246
Autoencoder loss for training set: 0.0062704156339168544
Autoencoder accuracy for validation set: 50.884002923965454
Autoencoder loss for validation set: 0.00618240815276901
Autoencoder accuracy for Test set: 50.73575259208679
Autoencoder loss for Test set: 0.006218375788629055

```

Test Images



Reconstruction of Test Images



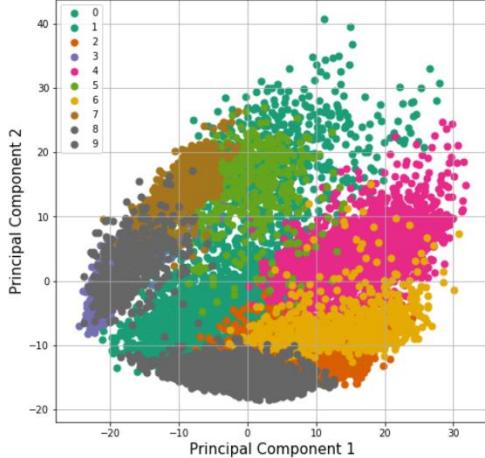
KMean on Encoded Images

Accuracy: 57.25
Normalized Mutual Information Score: 56.25728319297452

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	578	13	0	106	42	1	247	2	11	0
1	2	860	0	103	12	0	22	0	1	0
2	3	1	0	18	613	0	342	0	23	0
3	11	471	0	369	20	0	126	0	3	0
4	0	10	0	144	692	0	142	0	12	0
5	0	0	0	0	0	389	5	510	1	95
6	124	5	0	114	348	0	382	1	26	0
7	0	0	0	0	0	5	0	803	0	192
8	1	8	0	33	42	8	161	41	705	1
9	0	0	0	1	0	35	0	16	1	947

2 component PCA for Fashion MNIST on test data after KMeans with K=10



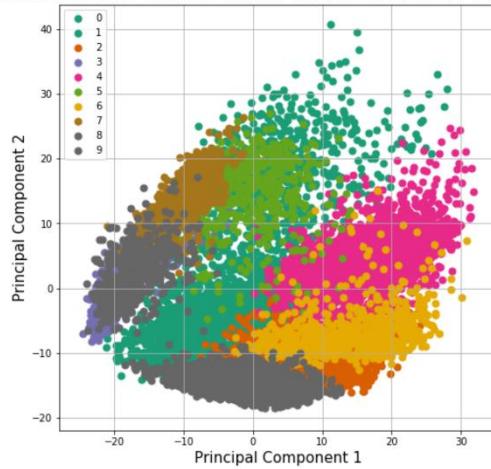
GMM on Encode Images

Accuracy: 61.02999999999994
Normalized Mutual Information Score: 59.38089706750023

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	673	2	38	96	32	2	145	1	11	0
1	0	808	9	129	33	0	20	0	1	0
2	6	1	816	3	57	0	105	0	12	0
3	13	238	11	471	172	0	92	0	3	0
4	0	1	670	36	227	0	59	0	7	0
5	0	0	0	1	0	402	3	535	2	57
6	156	4	517	50	84	0	166	1	22	0
7	0	0	0	0	0	6	0	892	0	102
8	1	1	13	21	63	6	133	31	730	1
9	0	0	0	3	0	26	2	49	2	918

2 component PCA for Fashion MNIST on test data after GMM with K=10



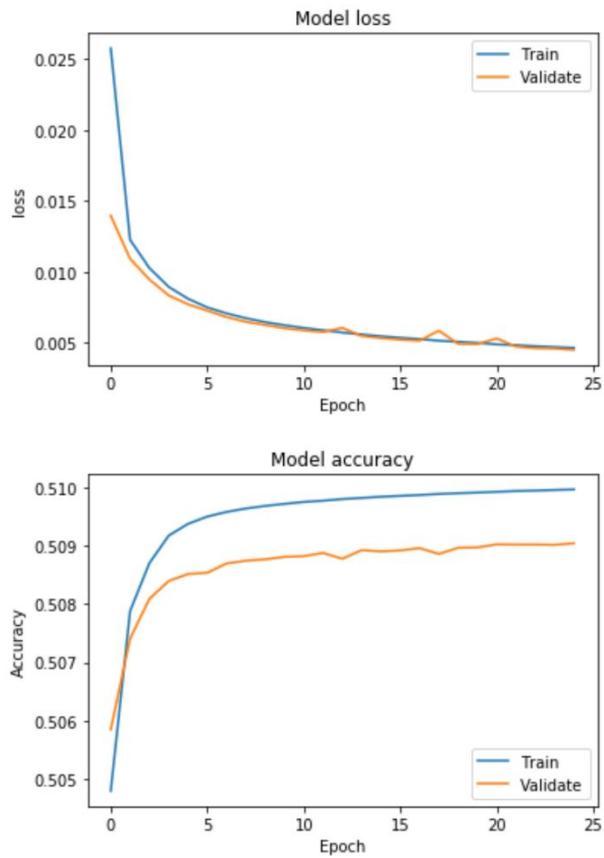
Case 2

Batch Size: 128

Epochs: 25

Encoder Layers:

Layer (type)	Output Shape	Param #
=====		
input_31 (InputLayer)	(None, 28, 28, 1)	0
conv2d_191 (Conv2D)	(None, 28, 28, 64)	640
max_pooling2d_51 (MaxPooling)	(None, 14, 14, 64)	0
conv2d_192 (Conv2D)	(None, 14, 14, 32)	18464
max_pooling2d_52 (MaxPooling)	(None, 7, 7, 32)	0
conv2d_193 (Conv2D)	(None, 7, 7, 16)	4624
conv2d_194 (Conv2D)	(None, 7, 7, 7)	1015
=====		



Autoencoder accuracy for training set: 50.99693876107534

Autoencoder loss for training set: 0.004620865081126491

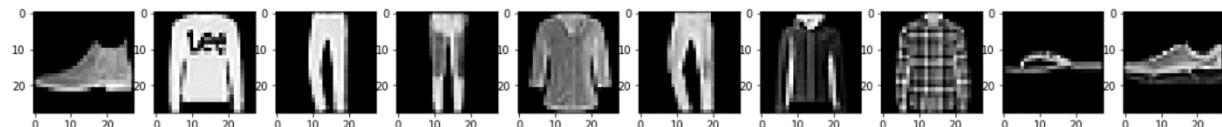
Autoencoder accuracy for validation set: 50.904559850692756

Autoencoder loss for validation set: 0.0044796687203149

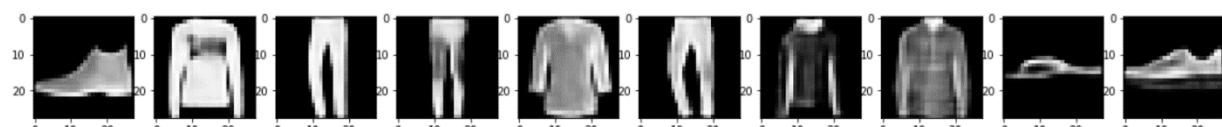
Autoencoder accuracy for Test set: 50.756249971389764

Autoencoder loss for Test set: 0.004517494411021471

Test Images



Reconstruction of Test Images



KMean on Encode Images

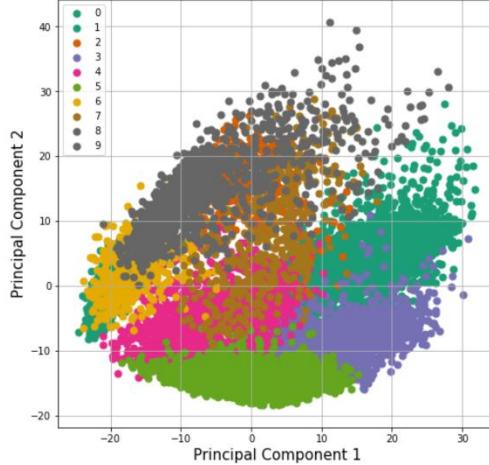
Accuracy: 56.08

Normalized Mutual Information Score: 56.48724338980167

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	625	47	0	0	45	2	268	1	12	0
1	42	919	0	0	12	0	26	0	1	0
2	15	3	0	0	628	0	347	0	7	0
3	270	581	0	0	16	0	130	0	3	0
4	121	23	0	0	700	0	147	0	9	0
5	0	0	0	0	0	391	6	509	1	93
6	211	18	0	0	350	0	399	1	21	0
7	0	0	0	0	0	4	0	784	0	212
8	3	14	0	0	42	10	54	32	843	2
9	1	0	0	0	0	35	0	16	1	947

2 component PCA for Fashion MNIST on test data after KMeans with K=10



GMM on Encode Images

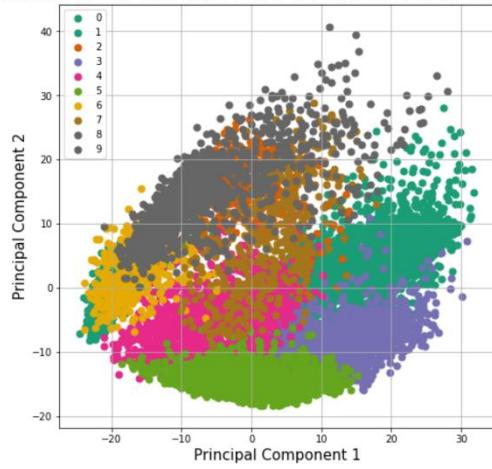
Accuracy: 59.36

Normalized Mutual Information Score: 60.088127806780975

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	692	0	43	126	0	0	125	2	11	1
1	0	866	9	106	0	0	18	0	1	0
2	7	0	835	46	0	0	102	0	10	0
3	15	4	17	886	0	0	75	0	3	0
4	0	1	714	217	0	0	61	0	7	0
5	0	0	0	0	0	0	5	527	0	468
6	169	1	534	130	0	0	144	1	21	0
7	0	0	0	0	0	0	0	791	0	209
8	1	1	14	67	0	0	142	30	744	1
9	0	0	0	2	0	0	0	18	2	978

2 component PCA for Fashion MNIST on test data after GMM with K=10



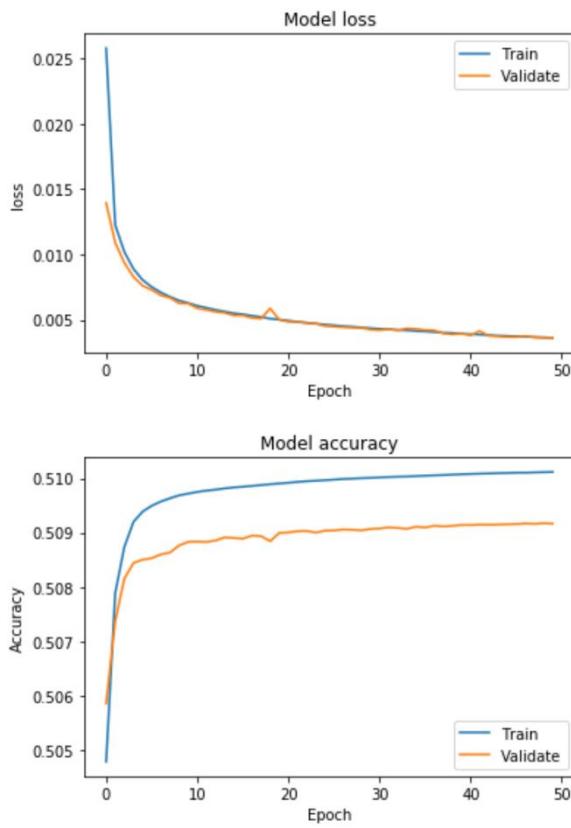
Case 3

Batch Size: 128

Epochs: 50

Encoder Layers:

Layer (type)	Output Shape	Param #
<hr/>		
input_29 (InputLayer)	(None, 28, 28, 1)	0
conv2d_173 (Conv2D)	(None, 28, 28, 64)	640
max_pooling2d_47 (MaxPooling)	(None, 14, 14, 64)	0
conv2d_174 (Conv2D)	(None, 14, 14, 32)	18464
max_pooling2d_48 (MaxPooling)	(None, 7, 7, 32)	0
conv2d_175 (Conv2D)	(None, 7, 7, 16)	4624
conv2d_176 (Conv2D)	(None, 7, 7, 7)	1015
<hr/>		

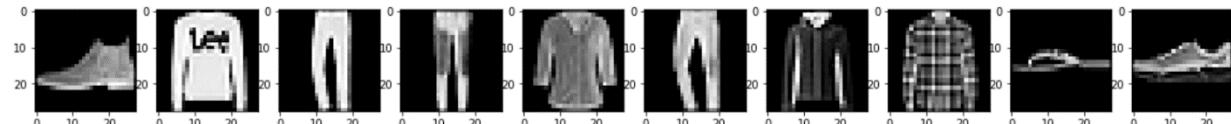


```

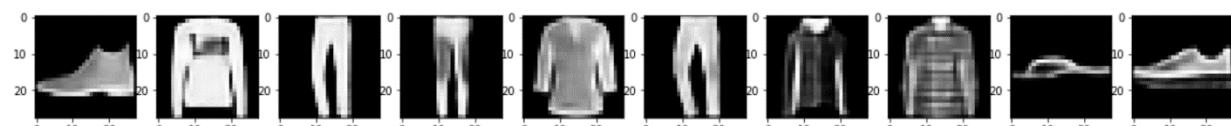
Autoencoder accuracy for training set: 51.01163368225098
Autoencoder loss for training set: 0.003611553032572071
Autoencoder accuracy for validation set: 50.91647534370423
Autoencoder loss for validation set: 0.0036327683261285224
Autoencoder accuracy for Test set: 50.77005103111267
Autoencoder loss for Test set: 0.0036662487968802454

```

Test Images



Reconstruction of Test Images



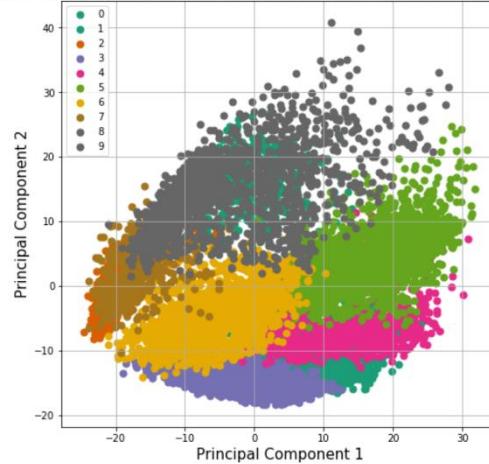
KMean on Encode Images

Accuracy: 57.89
Normalized Mutual Information Score: 56.55528651892057

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	589	19	0	107	45	1	227	1	11	0
1	4	884	0	78	13	0	20	0	1	0
2	5	1	0	15	620	0	339	0	20	0
3	11	466	0	408	21	0	91	0	3	0
4	0	10	0	134	702	0	144	0	10	0
5	0	0	0	0	0	405	4	510	0	81
6	135	7	0	107	344	0	375	1	31	0
7	0	0	0	0	0	7	0	792	0	201
8	1	11	0	29	40	13	169	47	687	3
9	0	0	0	2	0	31	0	19	1	947

2 component PCA for Fashion MNIST on test data after KMeans with K=10



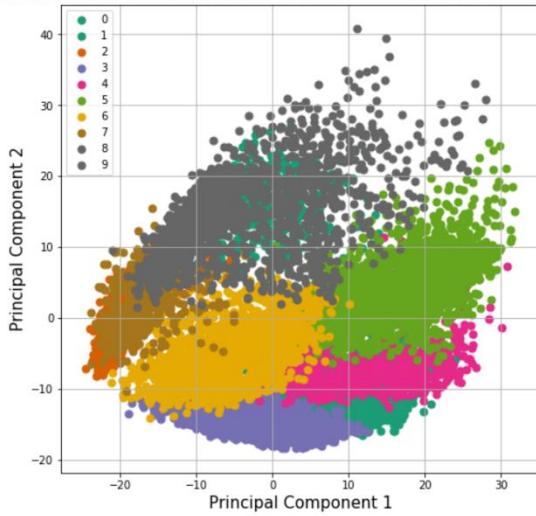
GMM on Encode Images

Accuracy: 59.64
Normalized Mutual Information Score: 60.2655374956955

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	698	1	42	131	0	0	113	2	13	0
1	2	866	8	106	0	0	17	0	1	0
2	6	0	843	43	0	0	97	0	11	0
3	12	11	15	899	0	0	60	0	3	0
4	0	1	715	219	0	0	58	0	7	0
5	0	0	0	0	0	0	3	577	0	420
6	168	2	528	127	0	0	143	1	31	0
7	0	0	0	0	0	0	0	808	0	192
8	1	1	12	71	0	0	142	35	735	3
9	0	0	0	2	0	0	0	24	2	972

2 component PCA for Fashion MNIST on test data after GMM with K=10



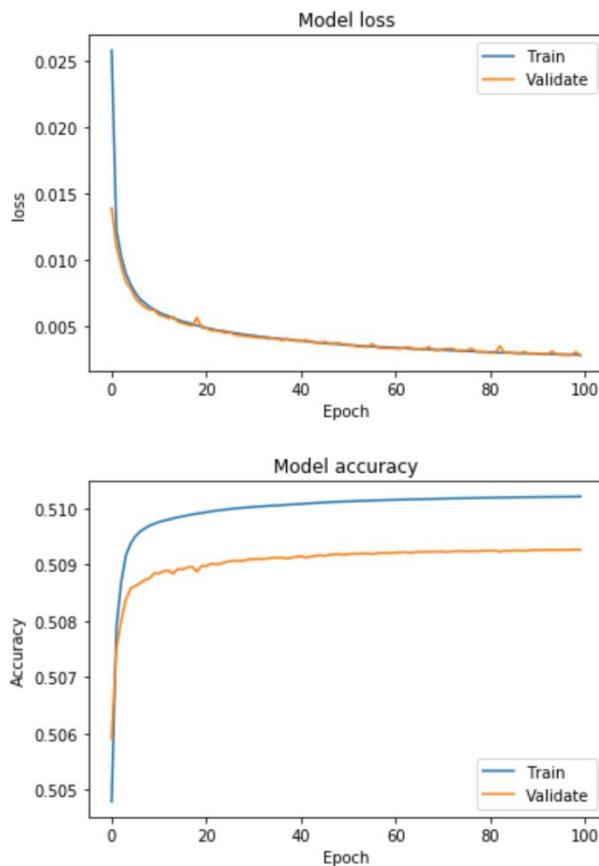
Case 4

Batch Size: 128

Epochs: 100

Encoder Layers:

Layer (type)	Output Shape	Param #
=====		
input_30 (InputLayer)	(None, 28, 28, 1)	0
conv2d_182 (Conv2D)	(None, 28, 28, 64)	640
max_pooling2d_49 (MaxPooling)	(None, 14, 14, 64)	0
conv2d_183 (Conv2D)	(None, 14, 14, 32)	18464
max_pooling2d_50 (MaxPooling)	(None, 7, 7, 32)	0
conv2d_184 (Conv2D)	(None, 7, 7, 16)	4624
conv2d_185 (Conv2D)	(None, 7, 7, 7)	1015
=====		



Autoencoder accuracy for training set: 51.02073504924775

Autoencoder loss for training set: 0.0028501253140469393

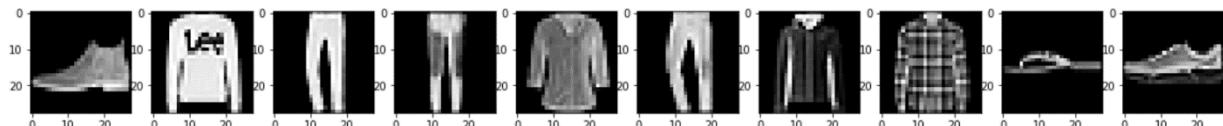
Autoencoder accuracy for validation set: 50.92626509666442

Autoencoder loss for validation set: 0.0028193462590376537

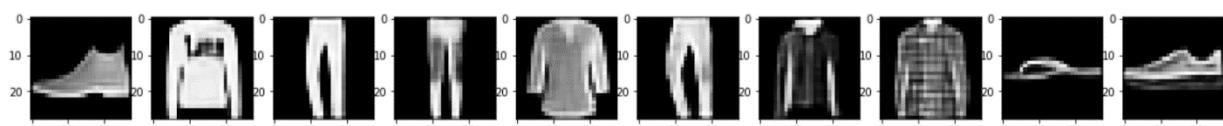
Autoencoder accuracy for Test set: 50.779451599121096

Autoencoder loss for Test set: 0.0028411265186965465

Test Images



Reconstruction of Test Images



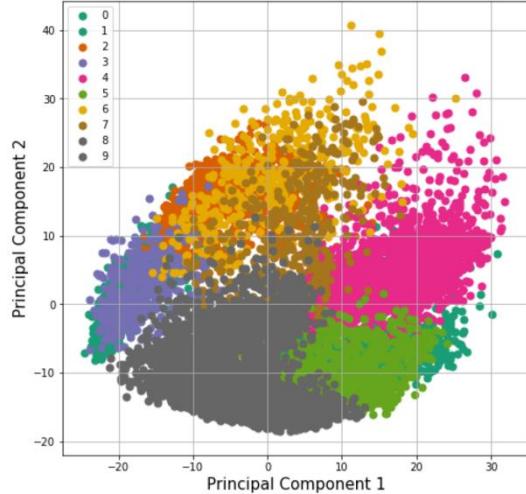
KMean on Encode Images

Accuracy: 58.60999999999999
Normalized Mutual Information Score: 57.74294012297651

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	574	30	0	101	47	1	236	1	10	0
1	4	892	0	69	10	0	24	0	1	0
2	4	1	0	23	610	0	357	0	5	0
3	12	536	0	343	17	0	91	0	1	0
4	0	14	0	166	659	0	153	0	8	0
5	0	0	0	0	0	401	5	542	2	50
6	126	11	0	111	349	0	387	2	14	0
7	0	0	0	0	0	6	0	908	0	86
8	2	19	0	11	62	8	58	37	801	2
9	0	0	0	5	0	27	4	67	1	896

2 component PCA for Fashion MNIST on test data after KMeans with K=10

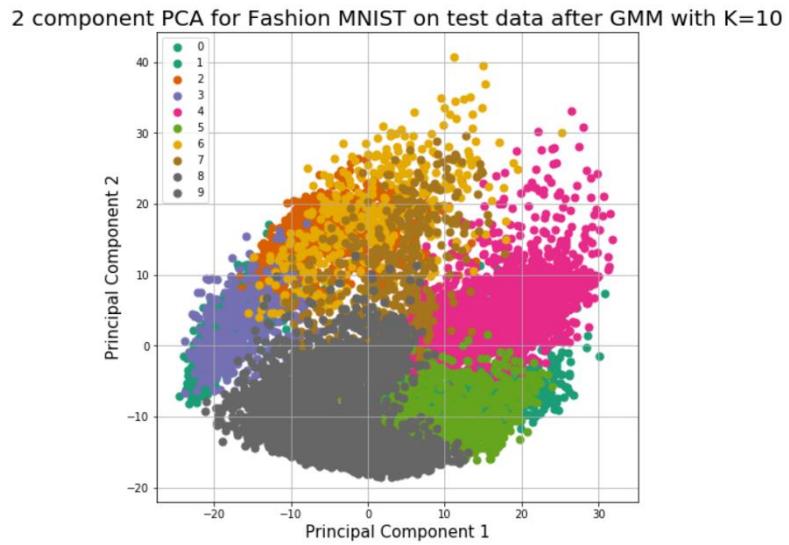


GMM on Encode Images

Accuracy: 58.98
Normalized Mutual Information Score: 59.57059036178043

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	699	37	48	98	0	1	0	1	116	0
1	4	902	8	63	0	0	0	0	23	0
2	5	1	878	22	0	0	0	0	94	0
3	12	524	21	371	0	0	0	0	72	0
4	0	11	756	164	0	0	0	0	69	0
5	0	0	0	0	0	398	0	516	4	82
6	169	14	543	116	0	0	0	2	156	0
7	0	0	0	0	0	4	0	790	0	206
8	2	14	16	21	0	9	0	31	903	4
9	0	0	0	2	0	21	0	18	2	957



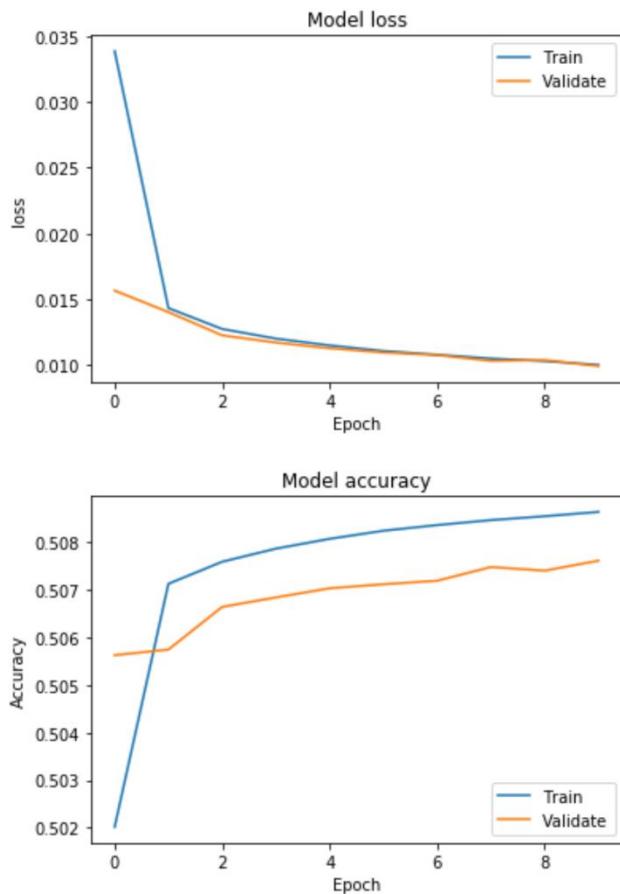
Case 5

Batch Size: 128

Epochs: 10

Encoder Layers:

Layer (type)	Output Shape	Param #
<hr/>		
input_33 (InputLayer)	(None, 28, 28, 1)	0
conv2d_209 (Conv2D)	(None, 28, 28, 64)	640
max_pooling2d_55 (MaxPooling)	(None, 14, 14, 64)	0
conv2d_210 (Conv2D)	(None, 14, 14, 32)	18464
max_pooling2d_56 (MaxPooling)	(None, 7, 7, 32)	0
conv2d_211 (Conv2D)	(None, 7, 7, 16)	4624
conv2d_212 (Conv2D)	(None, 7, 7, 7)	1015
conv2d_213 (Conv2D)	(None, 7, 7, 1)	64
<hr/>		

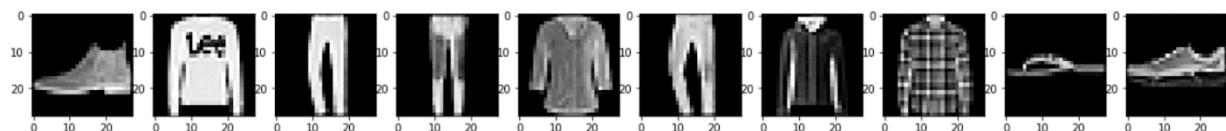


```

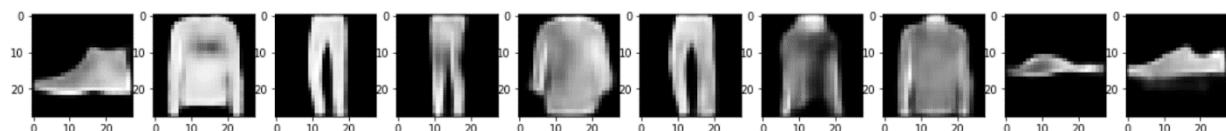
Autoencoder accuracy for training set: 50.86418205897013
Autoencoder loss for training set: 0.010009482945005099
Autoencoder accuracy for validation set: 50.761585744222
Autoencoder loss for validation set: 0.009918003735442956
Autoencoder accuracy for Test set: 50.616989879608155
Autoencoder loss for Test set: 0.009940810371935367

```

Test Images



Reconstruction of Test Images



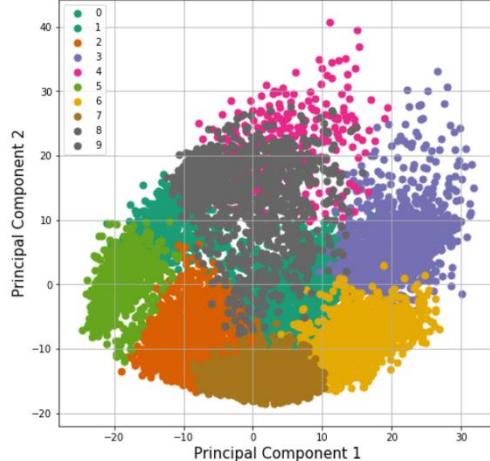
KMean on Encode Images

Accuracy: 50.46000000000001
Normalized Mutual Information Score: 48.202021408613035

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	446	37	215	0	49	12	233	2	6	0
1	45	794	21	0	5	0	134	0	1	0
2	21	4	358	0	421	4	184	0	8	0
3	260	441	64	0	8	2	222	0	3	0
4	128	29	261	0	475	6	92	0	9	0
5	0	0	1	0	0	760	16	189	0	34
6	138	24	301	0	238	25	261	2	11	0
7	0	0	0	0	0	403	0	581	0	16
8	2	6	15	0	53	27	88	110	699	0
9	3	0	3	0	2	27	2	288	3	672

2 component PCA for Fashion MNIST on test data after KMeans with K=10



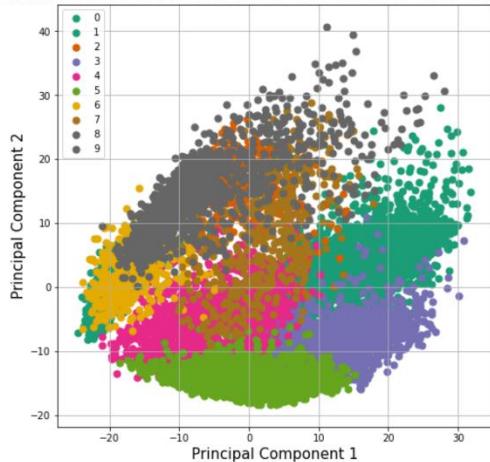
GMM on Encode Images

Accuracy: 64.03999999999999
Normalized Mutual Information Score: 61.252293036939356

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	689	6	5	100	41	0	0	0	159	0
1	0	878	0	94	9	0	0	0	19	0
2	9	0	575	8	312	0	0	0	96	0
3	40	182	2	616	109	0	0	0	51	0
4	1	0	450	42	452	0	0	0	55	0
5	0	0	0	0	0	514	0	474	6	6
6	195	4	381	88	200	0	0	0	132	0
7	0	0	0	0	0	45	0	837	1	117
8	0	1	0	5	2	2	0	2	987	1
9	0	0	0	0	0	103	0	39	2	856

2 component PCA for Fashion MNIST on test data after GMM with K=10



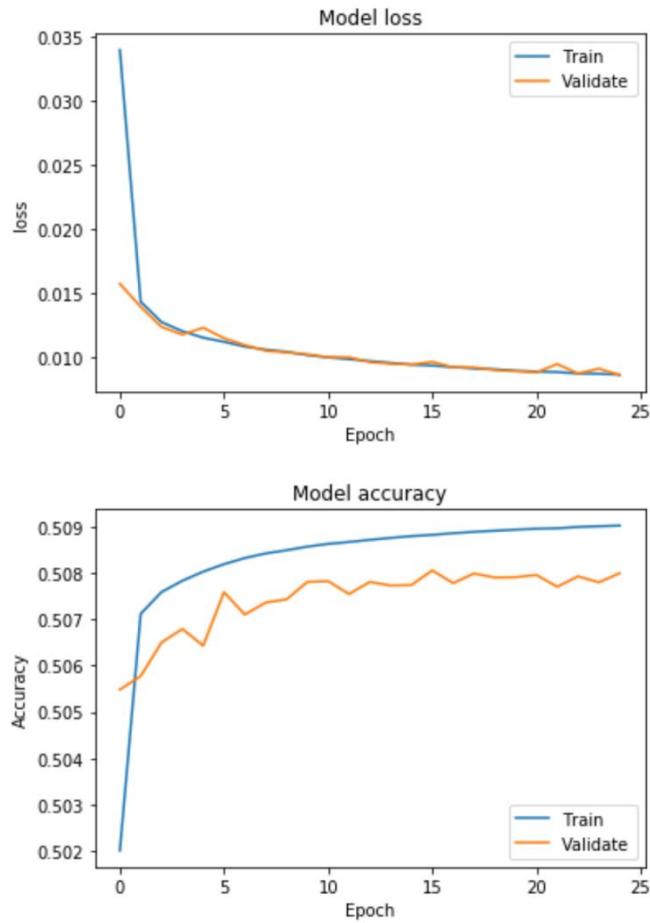
Case 6

Batch Size: 128

Epochs: 25

Encoder Layers:

Layer (type)	Output Shape	Param #
<hr/>		
input_34 (InputLayer)	(None, 28, 28, 1)	0
conv2d_220 (Conv2D)	(None, 28, 28, 64)	640
max_pooling2d_57 (MaxPooling)	(None, 14, 14, 64)	0
conv2d_221 (Conv2D)	(None, 14, 14, 32)	18464
max_pooling2d_58 (MaxPooling)	(None, 7, 7, 32)	0
conv2d_222 (Conv2D)	(None, 7, 7, 16)	4624
conv2d_223 (Conv2D)	(None, 7, 7, 7)	1015
conv2d_224 (Conv2D)	(None, 7, 7, 1)	64
<hr/>		

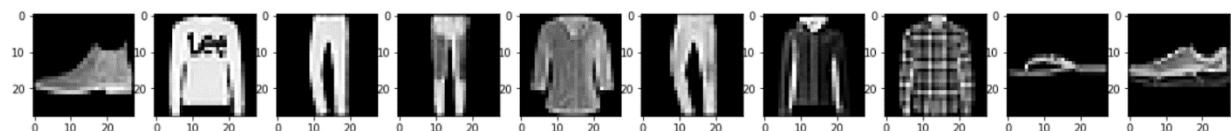


```

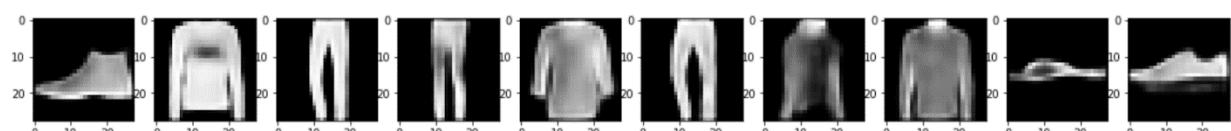
Autoencoder accuracy for training set: 50.902598937352494
Autoencoder loss for training set: 0.00861988844225804
Autoencoder accuracy for validation set: 50.79972364107768
Autoencoder loss for validation set: 0.008580441477398077
Autoencoder accuracy for Test set: 50.65727038383484
Autoencoder loss for Test set: 0.008601331052184105

```

Test Images



Reconstruction of Test Images



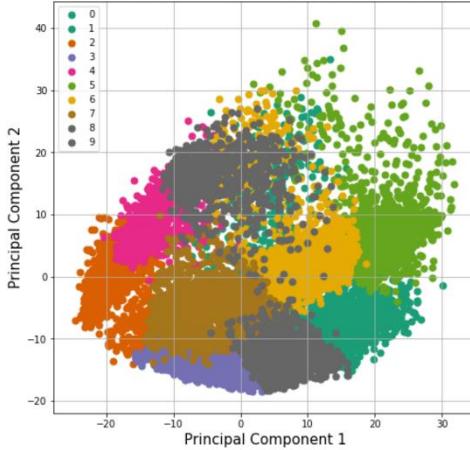
KMean on Encode Images

Accuracy: 50.05
Normalized Mutual Information Score: 46.33992717710239

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	488	20	41	95	30	37	285	2	2	0
1	24	722	12	221	3	3	15	0	0	0
2	28	2	315	7	347	18	281	0	2	0
3	146	278	13	428	2	26	106	0	1	0
4	136	11	305	53	357	16	116	0	6	0
5	0	0	0	0	0	769	18	178	1	34
6	137	7	225	54	186	52	337	2	0	0
7	0	0	0	0	0	437	0	550	0	13
8	2	6	194	9	114	30	94	159	390	2
9	0	0	0	2	2	28	4	314	1	649

2 component PCA for Fashion MNIST on test data after KMeans with K=10



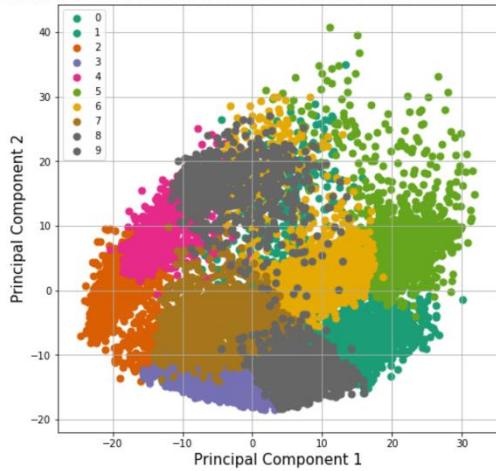
GMM on Encode Images

Accuracy: 58.96
Normalized Mutual Information Score: 60.482943802890965

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	789	1	2	84	0	0	116	0	8	0
1	67	915	0	14	0	0	4	0	0	0
2	136	0	746	13	0	0	105	0	0	0
3	149	86	2	685	0	0	77	0	1	0
4	105	0	628	105	0	0	162	0	0	0
5	4	0	0	0	0	0	0	423	5	568
6	247	1	449	69	0	0	229	0	5	0
7	0	0	0	0	0	0	0	714	1	285
8	100	1	0	1	0	0	66	1	828	3
9	2	0	0	0	0	0	0	8	0	990

2 component PCA for Fashion MNIST on test data after GMM with K=10



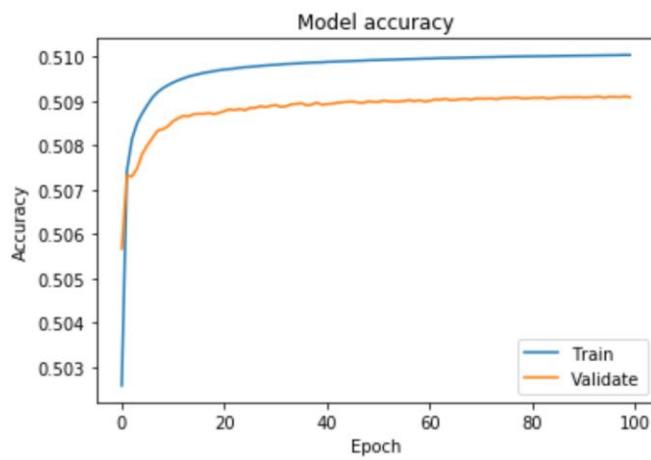
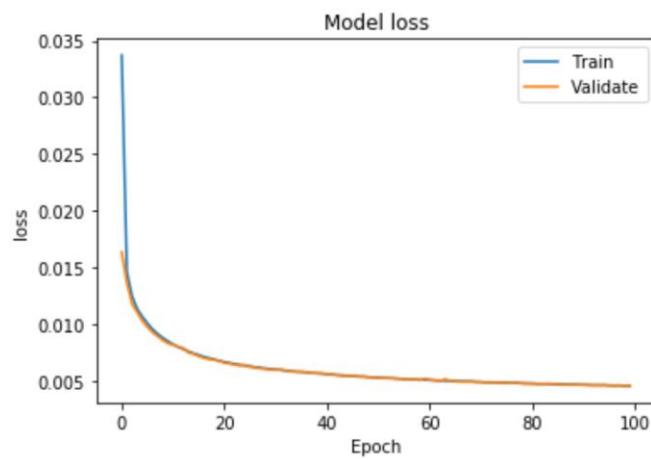
Case 7

Batch Size: 128

Epochs: 100

Encoder Layers:

Layer (type)	Output Shape	Param #
=====		
input_37 (InputLayer)	(None, 28, 28, 1)	0
conv2d_245 (Conv2D)	(None, 28, 28, 32)	320
max_pooling2d_63 (MaxPooling)	(None, 14, 14, 32)	0
conv2d_246 (Conv2D)	(None, 14, 14, 16)	4624
max_pooling2d_64 (MaxPooling)	(None, 7, 7, 16)	0
conv2d_247 (Conv2D)	(None, 7, 7, 7)	1015
=====		



Autoencoder accuracy for training set: 51.00404439767202

Autoencoder loss for training set: 0.004545479162906607

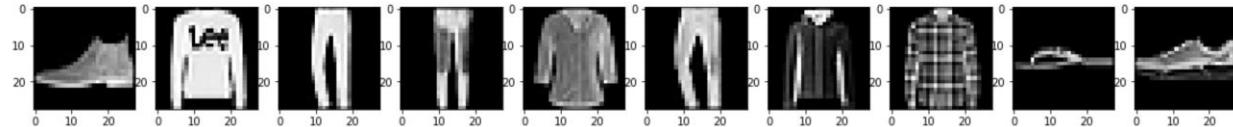
Autoencoder accuracy for validation set: 50.90880112648011

Autoencoder loss for validation set: 0.004527053128927946

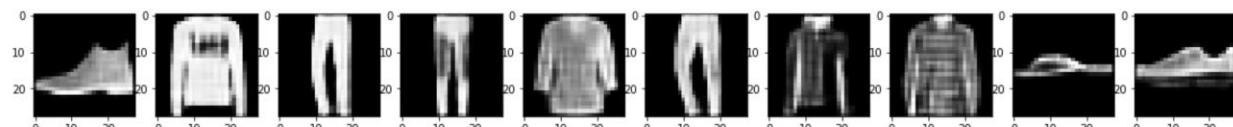
Autoencoder accuracy for Test set: 50.76146686553955

Autoencoder loss for Test set: 0.004563230994343757

Test Images



Reconstruction of Test Images



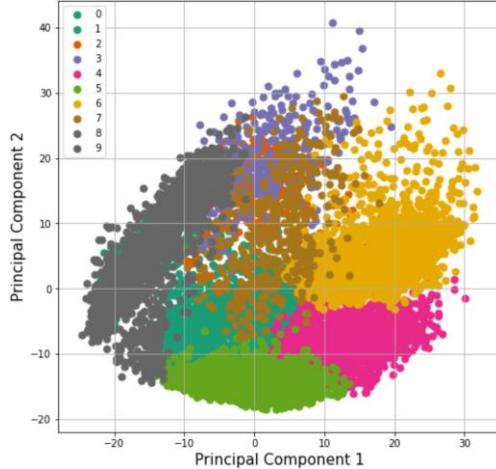
KMean on Encode Images

Accuracy: 55.45
Normalized Mutual Information Score: 52.46180545623302

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	596	36	0	0	49	45	263	1	10	0
1	59	899	0	0	10	7	24	0	1	0
2	36	3	0	0	581	16	356	0	8	0
3	275	557	0	0	8	40	115	0	5	0
4	172	25	0	0	617	15	163	0	8	0
5	0	0	0	0	0	546	6	322	1	125
6	195	17	0	0	342	52	381	1	12	0
7	0	0	0	0	0	35	0	808	1	156
8	3	11	0	0	74	38	73	42	758	1
9	0	2	0	0	0	29	0	28	1	940

2 component PCA for Fashion MNIST on test data after KMeans with K=10



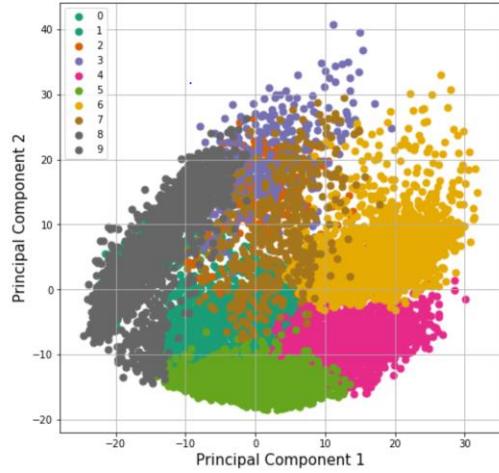
GMM on Encode Images

Accuracy: 59.830000000000005
Normalized Mutual Information Score: 60.24886018302886

Confusion Matrix:

	0	1	2	3	4	5	6	7	8	9
0	736	37	46	96	0	11	0	1	73	0
1	2	902	14	71	0	2	0	0	9	0
2	7	3	921	26	0	5	0	0	38	0
3	14	467	25	437	0	5	0	0	52	0
4	0	8	759	206	0	3	0	0	24	0
5	0	0	0	0	0	330	0	514	4	152
6	183	15	579	120	0	10	0	1	92	0
7	0	0	0	0	0	0	0	769	0	231
8	1	6	15	11	0	25	0	22	915	5
9	0	1	0	1	0	6	0	17	2	973

2 component PCA for Fashion MNIST on test data after GMM with K=10



Evaluations & Observations

1. GMM is a better clustering algorithm as compared to K-means for such type of datasets as we have used (datasets containing related data). It tends to perform better as it is a soft clustering approach compared to a hard one as Kmeans.
2. Kmeans on the original training dataset gives an accuracy of 55.33%, NMI: 51.16%
3. Kmeans on original test dataset gives an accuracy of 56.03%, NMI 51.46%
4. As observed, both Kmeans and GMM perform better after the dimensionality reduction using auto encoder.
5. As observed from the above results and test cases, GMM tends to perform better for Kmeans.
6. As number of epochs progresses, loss is reduced and more accurate images are produced from the auto encoder.
7. Also, as the number of epochs increases, accuracy for Kmeans increases. Also, the accuracy for GMM decreases slightly with the increase in epochs, but it still performs better than Kmeans.
8. From case 1-4 where we are reducing the dimensions to 7*7*7 representation, both the performances of Kmeans and GMM clustering have comparable performances and do not have a lot of gap in between their measures. But, GMM still performs better than KMeans Clustering with accuracy reaching to 60%.

With a 14 Layer network with dimensionality reduction to 7x7x7 following are the accuracies:

Epochs	K-Means	GMM
10	57.25	61.02
25	56.08	59.36
50	57.89	59.64
100	58.60	58.98

9. The best performances can be observed between epochs of 50-100, which shows a constant rise in accuracy of both the models and a constant decrease in loss.
10. On reducing the dimensionality of the encoded image to 7 * 7*1 instead of 7*7*7 by adding more convolutional layers, we get the following accuracies.

Epochs	Kmeans	GMM
10	50	64
25	50.05	58.96

11. Accuracy of GMM increases in the above case, but Kmeans decreases. Since, the accuracy of Kmeans is pretty low, this network is not used and the network with dimensionality reduction to $7 \times 7 \times 7$ is used.
12. In Case 7, where the number of total layers are 12 and the dimensionality reduction is still $7 \times 7 \times 7$ both KMeans and GMM perform well and give an accuracy of **55.45%** and **59.83%** respectively.
13. Also, from the PCA representations of the clustering algorithms and comparing it with original, we can see that the images are clustered better after applying the autoencoder network.

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