SHWETHAA·R
20MAI0043

# Mathematics for Machine Learning

## Digital Assignment - 1

(1) Given: $P(A|B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{32}$

'A' and 'B' are independent events

$\therefore \quad P(A \cap B) = P(A) \cdot P(B) \quad$ — (1)

To find: $P(A) = ?$ and $P(B) = ?$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad — (2)$$

using eq(1) in eq(2)

$$P(A|B) = \frac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

Substituting values in the above equation $\Rightarrow$ $P(A) = \frac{1}{4}$

Using eq(1) $\Rightarrow$ $P(A) \cdot P(B) = \frac{1}{32}$

$$\Rightarrow \quad P(B) = \frac{1/32}{1/4} = \frac{4}{32} = \frac{1}{8}$$

$$\therefore \quad \boxed{\begin{aligned} P(A) &= \frac{1}{4} \\ P(B) &= \frac{1}{8} \end{aligned}}$$

(2) Given: Bag M: 5 white and 7 Black balls

Bag N: 3 white and 8 Black balls

There are 3 possibilities when two balls are drawn from Bag M.

They are listed below along with their probabilities:

(i) 2 Black balls $= \frac{({}^7C_2)}{({}^{12}C_2)} = \frac{7 \times \cancel{6} \times \cancel{2}}{\cancel{2} \times \cancel{12}_2 \times 11} = \frac{7}{22}$

(ii) 1 Black ball and 1 white ball $= \frac{({}^5C_1)({}^7C_1)}{({}^{12}C_2)} = \frac{5 \times 7 \times \cancel{2}}{\cancel{12} \times 11_6} = \frac{35}{66}$

(iii) 2 white balls $= \frac{({}^5C_2)}{({}^{12}C_2)} = \frac{5 \times \cancel{4} \times \cancel{2}}{\cancel{2} \times \cancel{12} \times 11} = \frac{5}{33}$

The probabilities of picking a white ball from Bag N considering the above listed possibilities, are given below:

(i) 2 Black balls from Bag M, 1 white Ball from Bag N ⇒

$$\frac{7}{22} \times \frac{({}^3C_1)}{({}^{13}C_1)} = \frac{7}{22} \times \frac{3}{13} = \frac{21}{286} = \frac{63}{858}$$

(ii) 1 Black and 1 white ball from Bag M, 1 white Ball from Bag N ⇒

$$\frac{35}{66} \times \frac{({}^4C_1)}{({}^{13}C_1)} = \frac{35 \times \cancel{4}^{2}}{\cancel{66}_{33} \times 13} = \frac{70}{429} = \frac{140}{858}$$

(iii) 2 white balls from Bag M, 1 white Ball from Bag N ⇒

$$\frac{5}{33} \times \frac{({}^5C_1)}{({}^{13}C_1)} = \frac{5 \times 5}{33 \times 13} = \frac{25}{429} = \frac{50}{858}$$

Using Bayes Theorem, the probability of the balls transferred from Bag M being one white and one black, given that a white ball is chosen from Bag N is:

$$\Rightarrow \frac{\frac{140}{858}}{\frac{63 + 140 + 50}{858}} = \frac{140}{253} = 0.5534$$

Ans: 0.5534.

(3) Given: $P(A) = 0.25$; $P(B) = 0.35$; $P(C) = 0.4$

(i) With replacement, the two screws produced by Machine B will have probability ⇒ $P(B) \cdot P(B) = (0.35)(0.35)$

$= 0.1225$

(ii) With replacement, the probability of the two screws to be produced by Machines A and C ⇒ $P(A) \cdot P(C) = (0.25)(0.4)$

$= 0.1$

Ans: (i) 0.1225

(ii) 0.1

(4) The numbers on the faces of

(i) die B : $\{1, 2, 3, 4, 4, 6\}$

(ii) die A : $\{1, 2, 3, 3, 5, 6\}$

P(sum is an odd no.) = ?

odd no. + even no. = odd no.

odd no. + odd no. = even no.

even no. + even no. = even no.

Consider the numbers on die B :

(i) 1 - There are 2 ~~odd~~ even nos. in die A $(n_1 = 2)$

(ii) 2 - There are 4 odd nos. in die A $(n_2 = 4)$

(iii) 3 - There are 2 ~~odd~~ even nos. in die A $(n_3 = 2)$

(iv) 4 - There are 4 odd nos. in die A $(n_4 = 4)$

(v) 4 - There are 4 odd nos. in die A $(n_5 = 4)$

(vi) 6 - There are 4 odd nos. in die A $(n_6 = 4)$

$$\therefore \frac{n_1 + n_2 + n_3 + n_4 + n_5 + n_6}{\text{Total no. of possibilities}} = \frac{2+4+2+4+4+4}{6 \times 6} = \frac{20}{36}$$

$$= \frac{5}{9}$$

$\boxed{\text{Ans} : \frac{5}{9} = 0.\overline{55}}$

(5) Given : 3 Red and 4 Blue balls in a box.

To find : Probability that it will take more than four draws to remove all the red balls = ?

To find that, we can find the complement of that since that is easier to find.

Minimum no of draws to remove all the red balls is 3.

Ans : 1 - P(3 draws) - P(4 draws)

$$P(\cancel{3}\text{ draws}) = \frac{({}^3C_1)({}^2C_1)({}^1C_1)}{({}^7C_1)({}^6C_1)({}^5C_1)} = \frac{\cancel{3}\times\cancel{2}\times 1}{7\times\cancel{6}\times 5} = \frac{1}{35}$$

$$P(4\text{ draws}) = 3 \times \frac{({}^3C_1)({}^2C_1)({}^1C_1)({}^4C_1)}{({}^7C_1)({}^6C_1)({}^5C_1)({}^4C_1)}$$

$$= 3 \times \frac{\cancel{3}\times\cancel{2}\times 1\times\cancel{4}}{7\times\cancel{6}\times 5\times\cancel{4}} = \frac{3}{35}$$

(~~3~~ 4 draws means 3 Red and 1 Blue ball. There are 3 possibilities – BRRR, RBRR, RRBR, ~~RRRB~~)

$\therefore$ P(3 Red Balls in more than four draws) $= 1 - \frac{1}{35} - \frac{3}{35}$

$= 1 - \frac{4}{35} = \frac{31}{35}$

$$\boxed{\text{Ans: } \frac{31}{35} = 0.8857}$$