ARJUNA NEET BATCH

MOTION WITH CONSTANT ACCELERATION

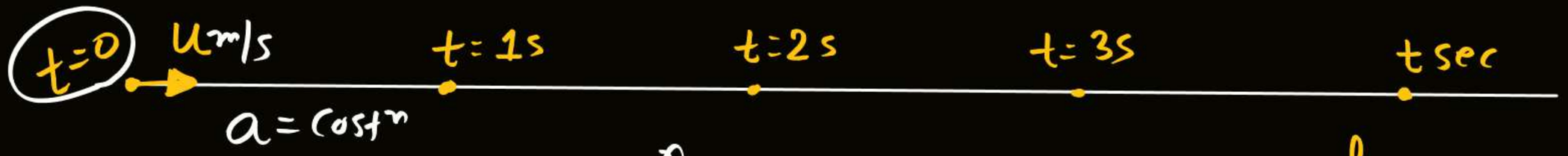
LECTURE - 07

Todays goal

Derivation of equation of motion using calculus

Question on Constant acceleration

object starts his motion with constantans acceleration and initial velocity u then Find displacement and velocity after time t (equation of motion)



"$a = \frac{dv}{dt}$"

$$\int a\,dt = \int dv$$

$a = \text{Cost}^n$

$$a\int_{t=0}^{t} dt = \int_{u}^{v} dv$$

$$at = [v]_u^v$$

$$v - u = at$$

$$\boxed{v = u + at} \quad (1)$$

$v = u + at$ → a = Const^n accⁿ.

↓ initial velocity

$$\Downarrow$$

$$\frac{d\vec{x}}{dt} = u + at$$

$$\int_{x_i}^{x_f} d\vec{x} = \int u\,dt + \int at\,dt$$

$$\vec{x}_f - \vec{x}_i = ut + a\frac{t^2}{2}$$

$$\boxed{s = ut + \frac{1}{2}at^2} \quad -(II)$$

3rd eqⁿ of motion

$$a = v\frac{dv}{dx}$$

$$\int_{x_i}^{x_f} a\,dx = \int_{u}^{v} v\,dv$$

$$a\left[x\right]_{x_i}^{x_f} = \left[\frac{v^2}{2}\right]_{u}^{v}$$

$$a\left[x_f - x_i\right] = \frac{v^2 - u^2}{2}$$

$$v^2 - u^2 = 2as$$

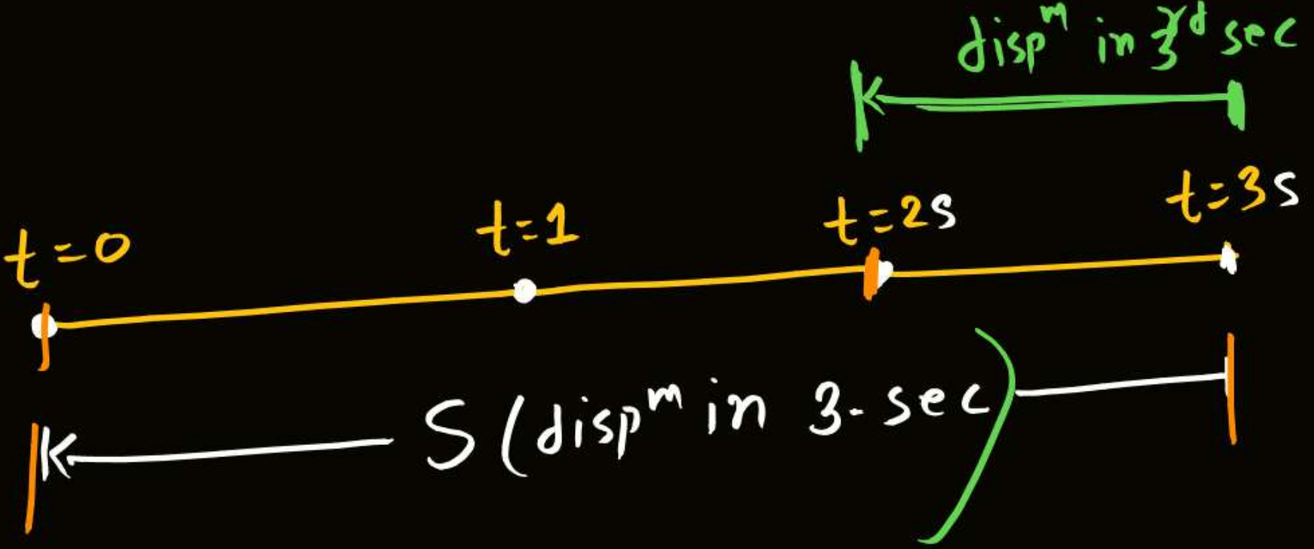
find velocity

$$\vec{V_f} = \vec{u_i} + \vec{a}t$$

$$\vec{S} = \vec{u_i}t + \frac{1}{2}\vec{a}t^2$$

$$V_f^2 - u_i^2 = 2\vec{a}\cdot\vec{S}$$

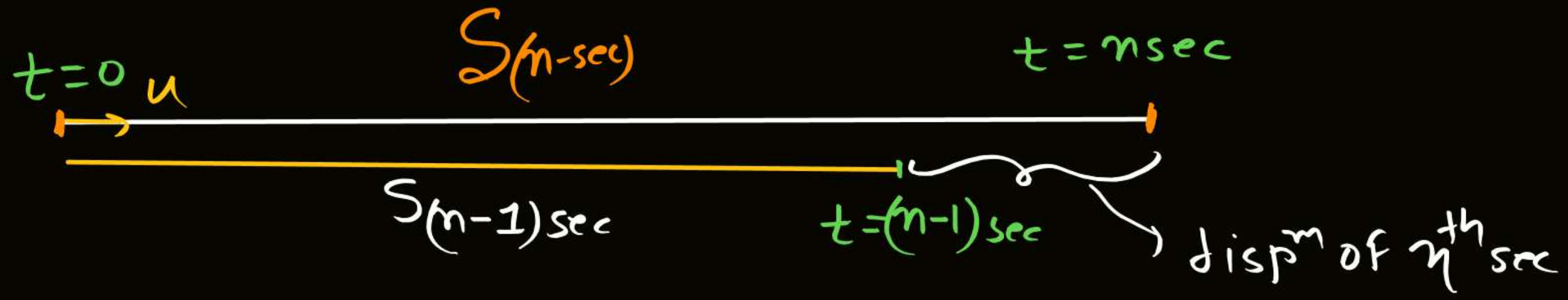
disp^m in t-sec



over	Run
1	4
2	2
3	6
4	3

Run in 3-over = 12

Run in 3rd over = 6



$$\text{disp}^{m} \text{ in } S_{n^{th}\,sec} = S_{n\,sec} - S_{(n-1)\,sec}$$

$$= \left(un + \frac{1}{2}a n^2\right) - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a\left[n^2 - 2n + 1\right]$$

$$= u + \frac{1}{2}an^2 - \frac{1}{2}an^2 + \frac{1}{2}a(2n) - \frac{1}{2}a$$

4^{th} eqn of motion

$$\boxed{S_{n^{th}} = u + \frac{a}{2}\left[2n-1\right]}$$

$$\text{Avg velocity} = \frac{\text{total disp}^m}{\text{total time}} = \frac{ut + \frac{1}{2}at^2}{t}$$

$$= u + \frac{1}{2}at$$

using 1st eqn of motion

$$v - u = at$$

$$V_{\text{Avg velocity}} = u + \frac{1}{2}(v-u)$$

$$= \frac{2u + v - u}{2} = \frac{\vec{u} + \vec{v}}{2}$$

$$\text{total disp}^m = V_{\text{Avg}} \times \text{time}$$

$$\vec{s} = \left(\frac{\vec{u} + \vec{v}}{2}\right) t$$

Motion with Constant Acceleration :

$$\vec{V} = \vec{u} + \vec{a}t \quad \text{—— (I)}$$

$$V^2 - u^2 = 2\vec{a}\cdot\vec{s} \quad \text{— (III)}$$

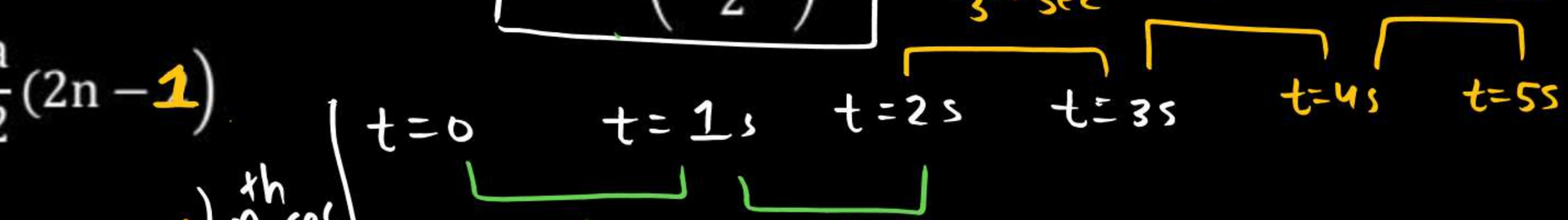
total disp^m in t-sec → $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$ — (II)

$S_{nth} = \vec{u} + \frac{a}{2}(2n-1)$

disp^m of (1-sec) n^{th} sec

$\vec{V}_{Arg} = \frac{\vec{u}+\vec{v}}{2}$ → Ratta always Valid for Cost^n acceleration

* $\vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t$



7^{th} sec = 6 sec to 7 sec = 1 sec

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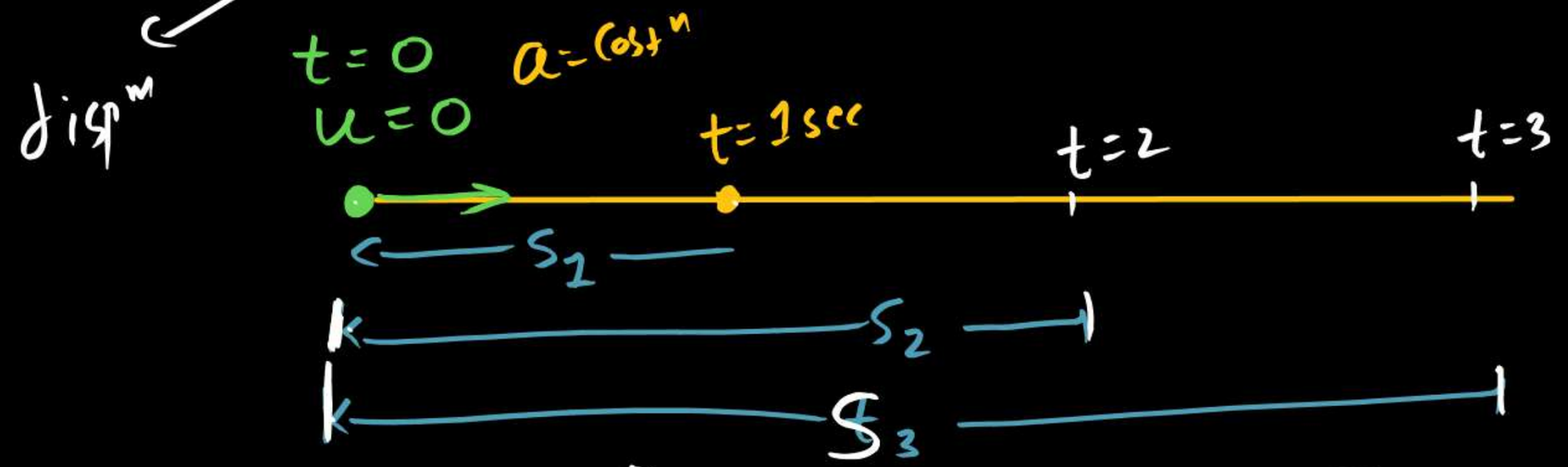
Object starts his motion from rest and constant acceleration then find Ratio of ~~distance~~ in 1-sec, 2-sec, 3-sec.



$$S_{1\text{-sec}} = \cancel{ut}^{0} + \frac{1}{2}at^2 = \frac{1}{2}a(1)^2 = \frac{a}{2}$$

$$S_{2\text{-sec}} = \frac{1}{2}a(2)^2 = 4 \times \left(\frac{a}{2}\right)$$

$$S_{3\text{-sec}} = \frac{1}{2}a(3)^2 = 9 \times \left(\frac{a}{2}\right)$$

$$S_{1\text{-sec}} : S_{2\text{-sec}} : S_{3\text{-sec}}$$

$$\frac{a}{2} : 4\frac{a}{2} : 9\left(\frac{a}{2}\right)$$

$$= x : 4x : 9x$$

$$= 1 : 4 : 9 : 16$$

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Object starts his motion from rest and constant acceleration then find ratio of displacement in 1st – 5 sec : 10 sec : 15 sec.

MR*

* $u = 0$ Cost^n accn

$$S_{(5\text{-sec})} : S_{10\text{sec}} : S_{15\text{sec}} = x : 4x : 9x = 25 : 100 : 225 = 1 : 4 : 9$$

$$S_{1\text{sec}} : S_{2\text{sec}} : S_{(3\text{t})\text{sec}} = x : 4x : 9x$$

$$S_{5\text{-sec}} = \frac{1}{2}a(5)^2 = 25\left(\frac{a}{2}\right)$$

$$S_{10\text{-sec}} = \frac{1}{2}a(10)^2 = 100\left(\frac{a}{2}\right)$$

$$S_{15\text{-sec}} = \frac{1}{2}a(15)^2 = 225\left(\frac{a}{2}\right)$$

object starts his motion from rest and constant acceleration it moves 80m in 10sec then disp^m is 20 sec. (NEET/AIPMT)

$x_t : x_{2t} = 1 : 4$

मजदूरी (School)

$u = 0 \quad a = \text{cost}^n$

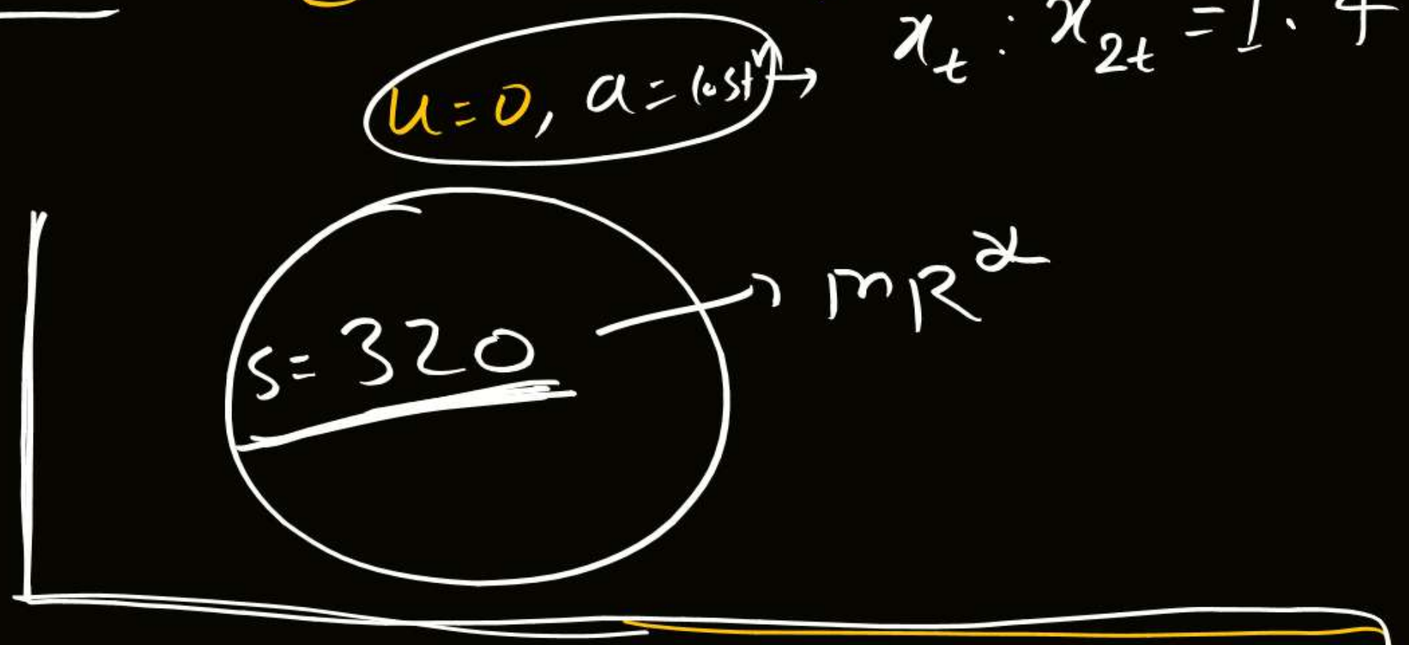
$S = 80m \quad t = 10sec$

$S = ut + \frac{1}{2}at^2$

$80 = \frac{1}{2}a(10)^2$

$160 = a \times 100$

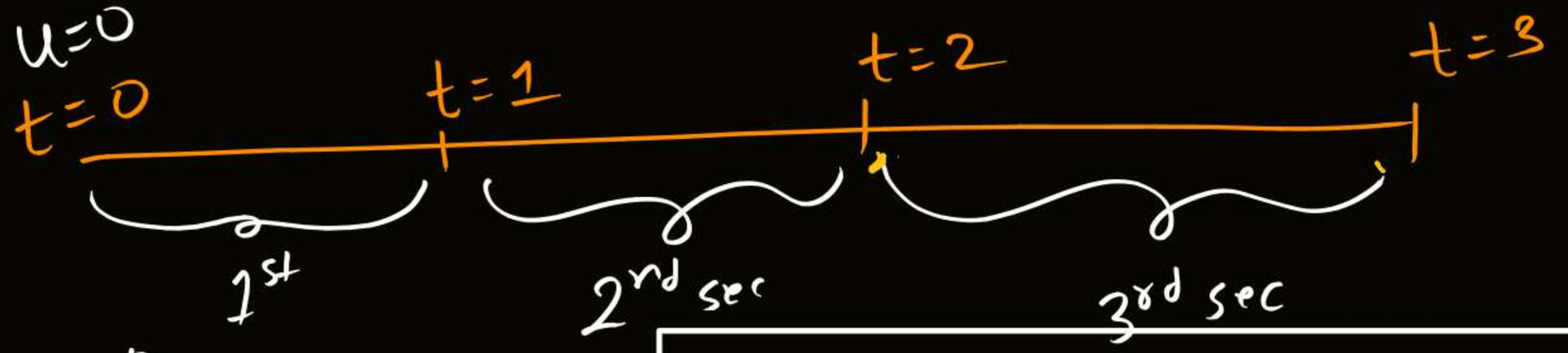
$\boxed{a = 1.6 m/s^2}$



$\Rightarrow S = ut + \frac{1}{2}at^2$

$S = 0 + \frac{1}{2}(1.6) \times 20 \times 20 = 20 \times 16 = 320 \ m$

Object starts his motion from rest and constant acceleration then find ratio of disp^m in 1sts : 2ndsec : 3rdsec



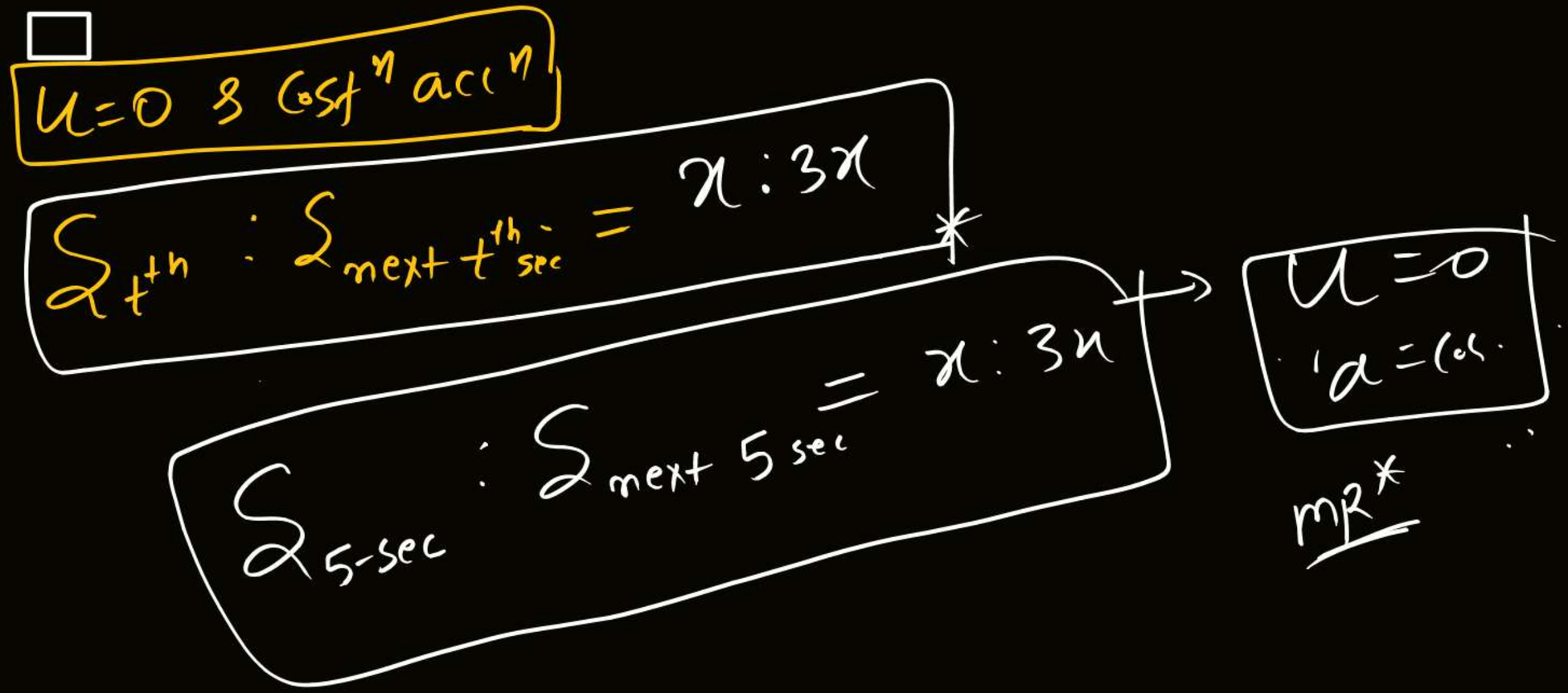
$$S_{n^{th}} = u + \frac{a}{2}(2n-1)$$

$$S_{1^{st}} = \frac{a}{2}[1] = x$$

$$S_{2nd} = \frac{a}{2}[3] = 3x$$

$$S_{3rd} = \frac{a}{2}[5] = 5x$$

$$S_{1^{st}} : S_{2^{nd}} : S_{3^{rd}} = x : 3x : 5x = 1 : 3 : 5$$


u=0 & Costⁿ accⁿ
S t^th : S next t^th sec = x : 3x
S 5-sec : S next 5 sec = x : 3x
u=0
a=(0
mR*

A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance S_1 in the first 10 sec and a distance S_2 in the next 10 sec, then :

(a) $S_1 = S_2$

(b) $S_1 = S_2/3$

(c) $S_1 = S_2/2$

(d) $S_1 = S_2/4$

$u = 0$
$a = \text{Constant}$

$S_1 : S_2 = 1 : 3$

$$\frac{S_1}{S_2} = \frac{1}{3}$$

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A body travels for 15 sec starting from rest with constant acceleration. If it travels distance S_1, S_2 and S_3 in the first five seconds, second five seconds and next five seconds respectively the relation between S_1, S_2 and S_3 is :

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(a) $S_1 = S_2 = S_3$ ✗

(b) $5S_1 = 3S_2 = S_3$

(c) $S_1 = \frac{1}{3}S_2 = \frac{1}{5}S_3$

(d) $S_1 = \frac{1}{5}S_2 = \frac{1}{3}S_3$

$S_1 : S_2 : S_3 = 1 : 3 : 5$

$\frac{S_1}{S_2} = \frac{1}{3}$

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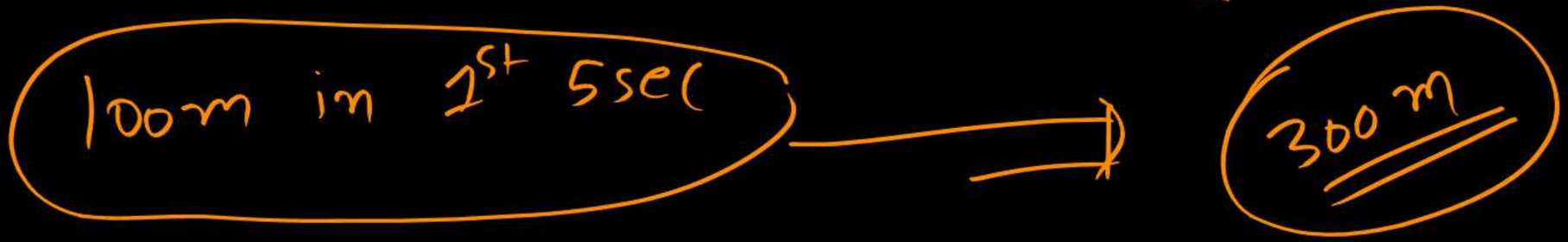
Object starts his motion from rest and constant acceleration and moves 100 m in 1st 5-sec then find displacement in next 5-sec.



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Velocity of object $v = \sqrt{25 - 8x}$ find initial velocity and acceleration.

$V = \sqrt{25 - 8x}$

$u_i\,(t=0) = ??$

$acc^n = Const^n$

$acc^n = ??$

$V^2 = 25 - 8x$

$V^2 = u^2 + 2ax$

$u = 5\,m/s$

$2ax = -8x$

$a = -4\,m/s^2$

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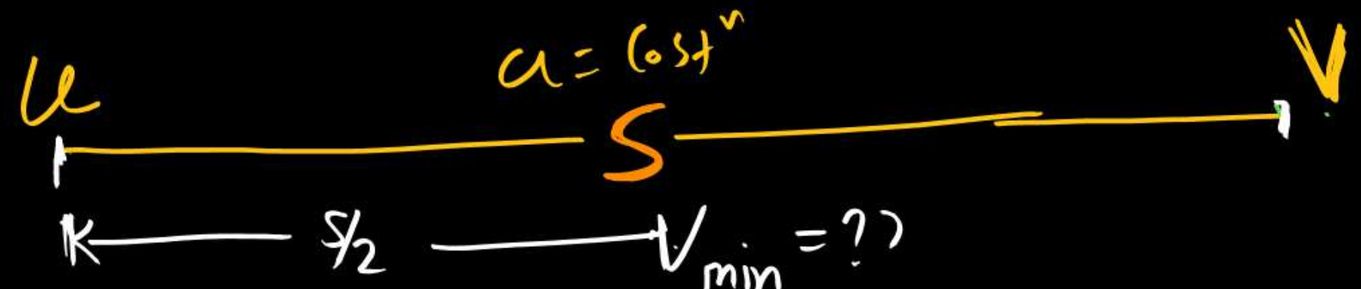
Object starts his motion from u and constant acceleration then find velocity at mid point if velocity at end point is V.



3rd eqn of motn

$$V^2 - u^2 = 2aS \quad \text{--- (1)}$$

3rd eqn of motn upto mid Point

$$V_{mid}^2 - u^2 = \frac{2aS}{2}$$

$$2V_{mid}^2 - 2u^2 = V^2 - u^2$$

$$2V_{mid}^2 = u^2 + V^2$$

$$V_{mid} = \sqrt{\frac{u^2+V^2}{2}}$$ Rat

$$v_{mid\ Point} = \sqrt{\frac{u^2 + v^2}{2}}$$

$\rightarrow a = \text{Const}^n$

Object starts his motion from rest and constant acceleration then find ratio of displacement in 6^{th} sec to 6-sec.

$$S_{6\text{-sec}} = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}a(6)^2 = \frac{a}{2}(36)$$

$$S_{6^{th}\,sec} = u + \frac{a}{2}(2n-1) = 0 + \frac{a}{2}(12-1) = \frac{a}{2}(11)$$

$$\frac{S_{6^{th}}}{S_{6\text{-sec}}} = \frac{11\,(a/2)}{36\,(a/2)} = \frac{11}{36}$$

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$u = 0 \therefore a = \text{constant}$

$$\frac{S_{n\,sec}}{S_{n^{th}\,sec}} = \frac{\frac{1}{2}a(n)^2}{\frac{a}{2}(2n-1)} = \boxed{\frac{n^2}{2n-1}}$$

NEET

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THANK YOU ☺