ARJUNA NEET BATCH

MOTION IN A PLANE

LECTURE - 08

Todays Goal

question on circular motion

Moksha

U.C.M

- $\boxed{\text{Speed} = \text{const}^n}$ $\rightarrow a_T = 0$

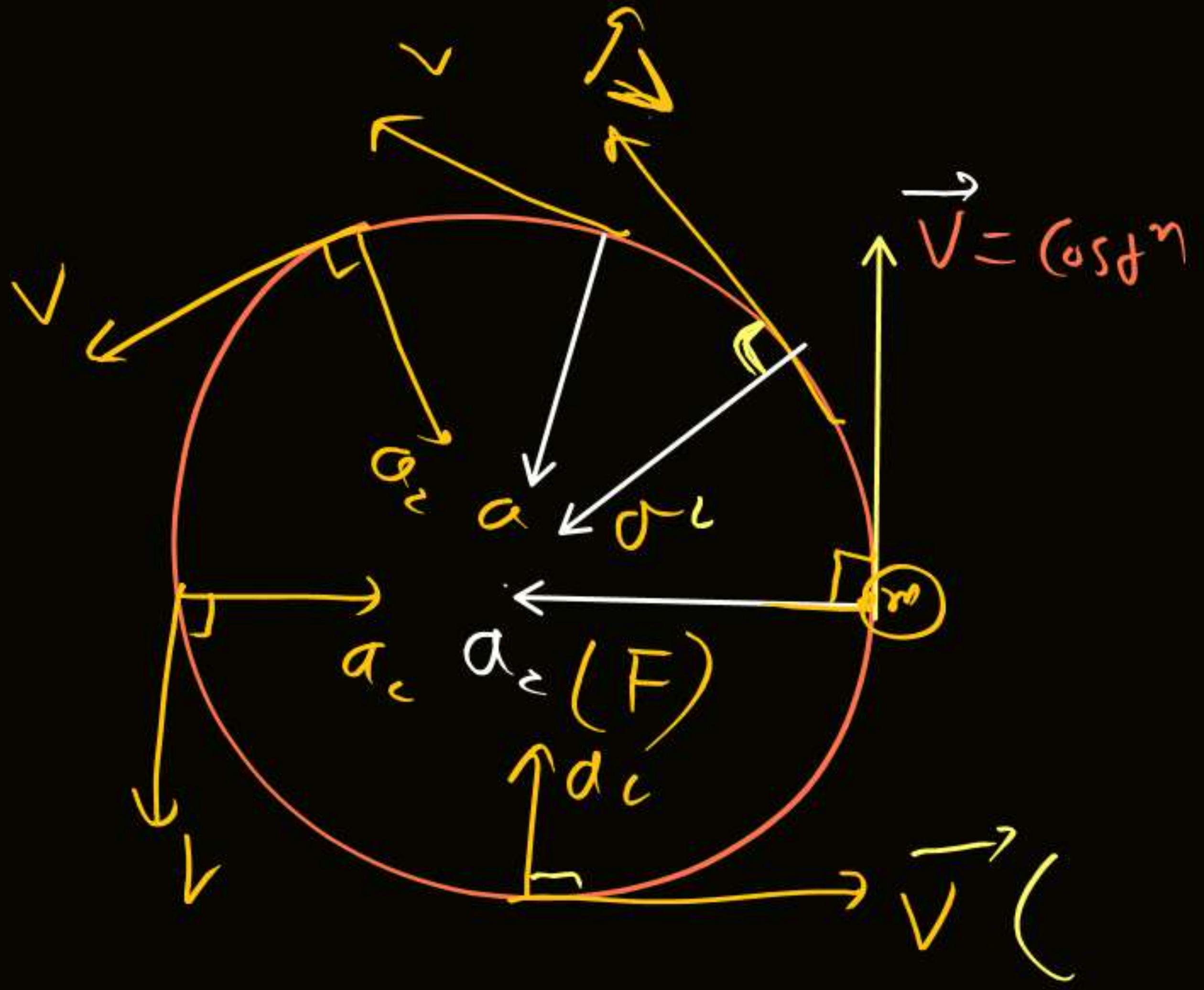
$\vec{a} \neq 0$

$acc^n = a_c$

$\boxed{\omega = \text{const}^n}$ $\rightarrow \alpha = 0$

$v = r\omega$

$$\boxed{\vec{a} = \vec{a}_c = \frac{v^2}{R} = \omega^2 R}$$

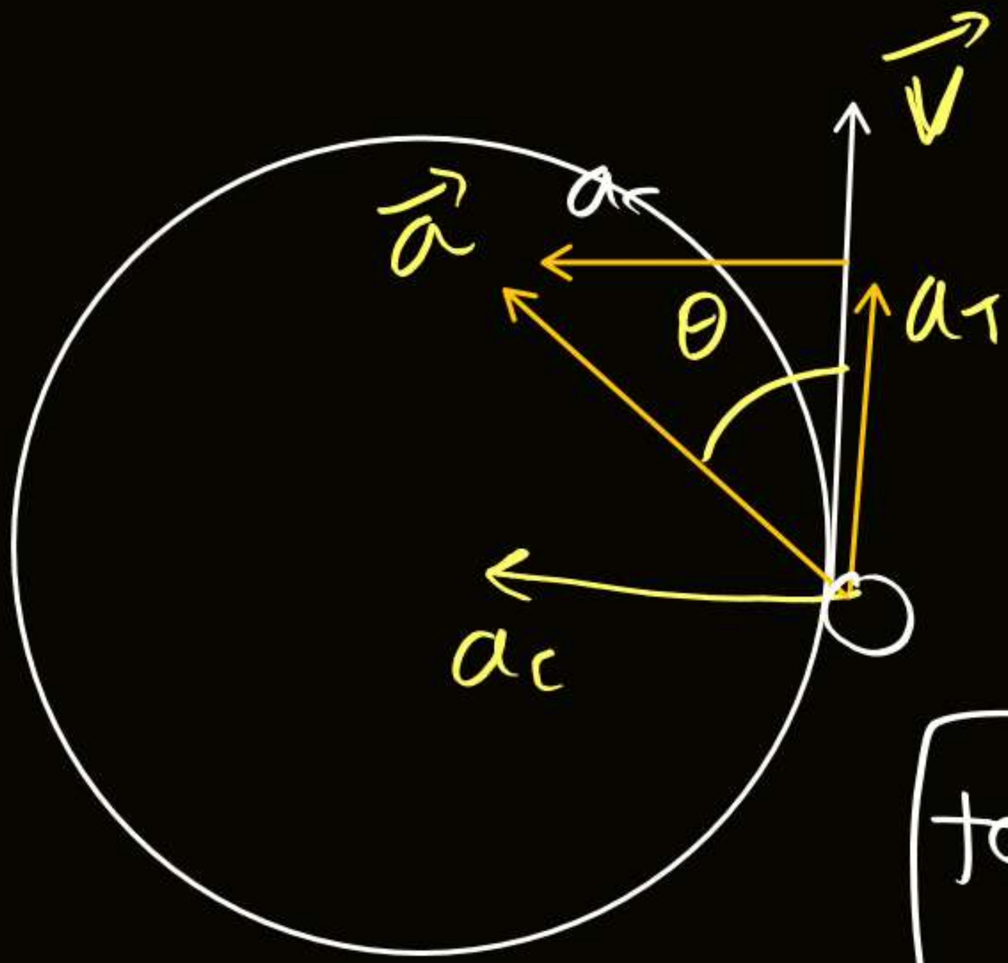


N.U.C.M

$$\boxed{0^\circ < \theta < 90^\circ}$$

Speed ↑

dir^n = change

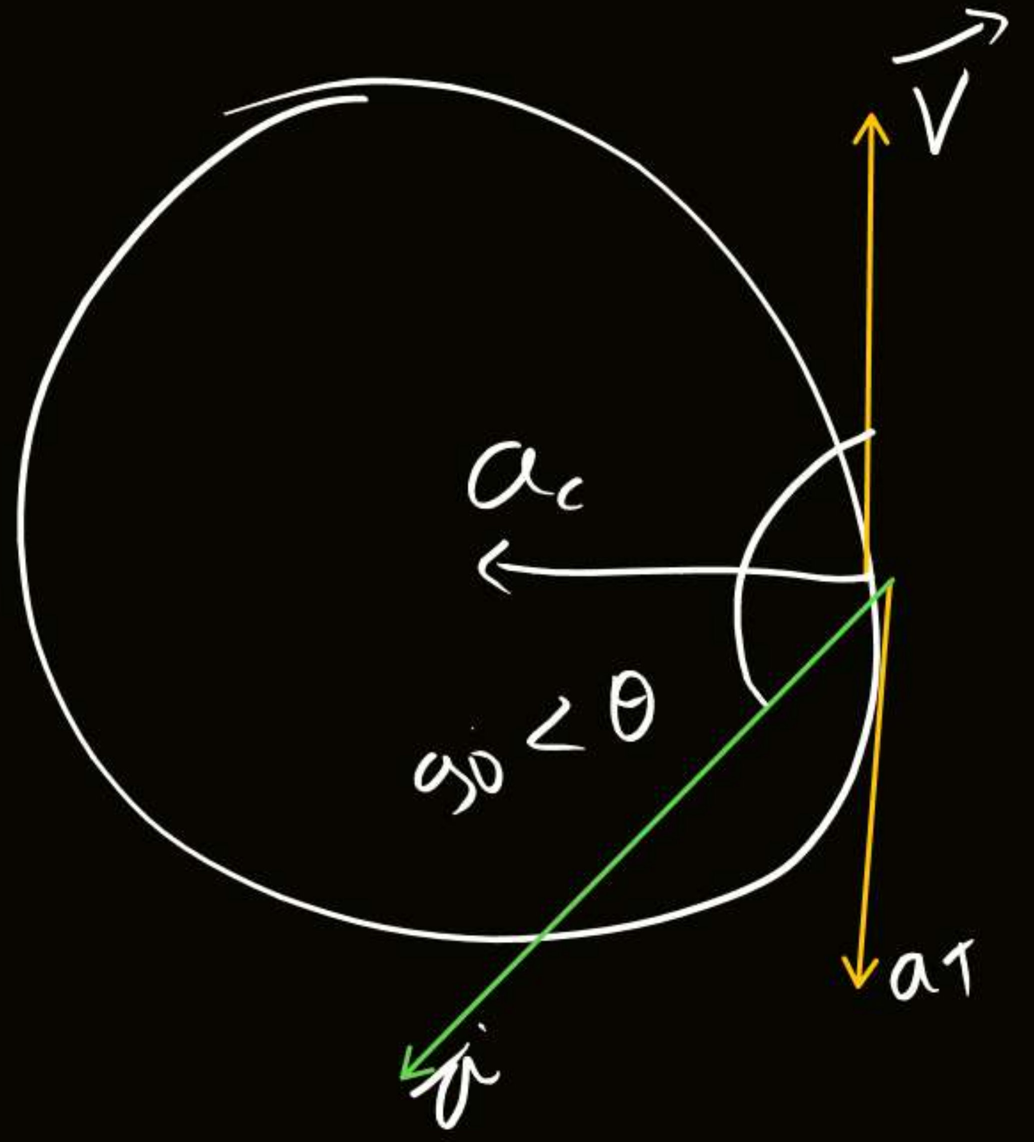


$$\vec{a} = \vec{a}_T + \vec{a}_c$$

$$\boxed{\tan\theta = \frac{a_c}{a_T}}$$

$$|\vec{a}| = \sqrt{a_T^2 + a_c^2}$$

N.U.C.M speed $\downarrow$



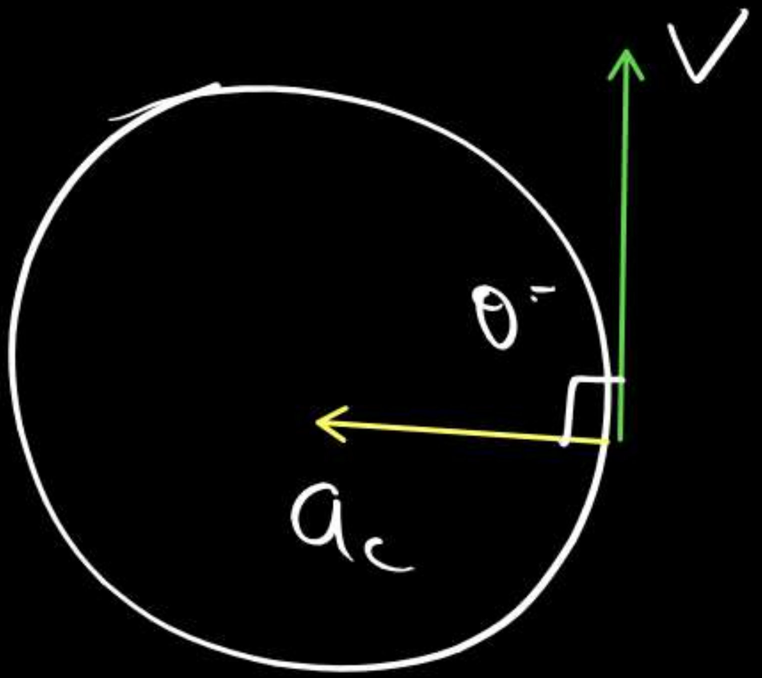
The angle between velocity vector and acceleration vector in uniform circular motion is

(a) 0°

(b) 180°

(c) 90°

(d) 45°



Two cyclists cycle along circular tracks of radii R_1 and R_2 at uniform rates. If both of them take same time to complete one revolution, then their angular speeds are in the ratio

$T = \text{same}$

(a) $R_1 : R_2$

(b) $R_2 : R$

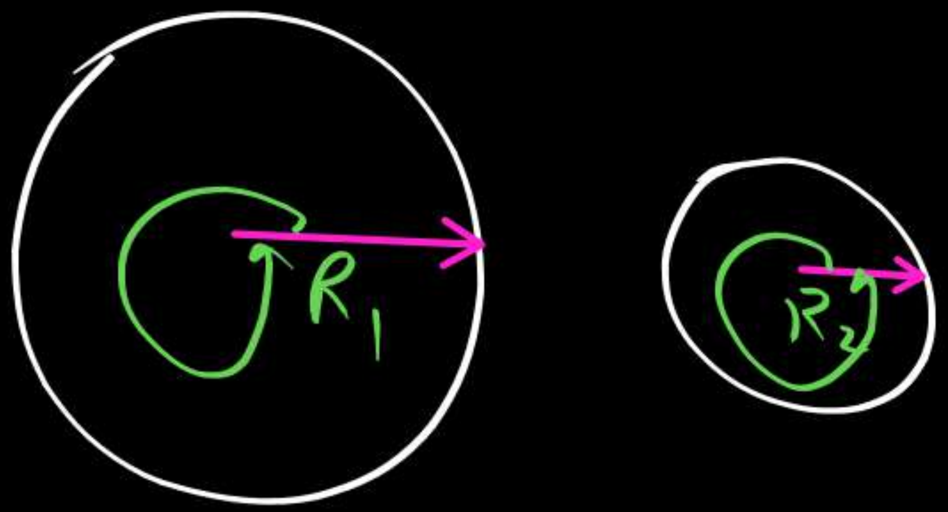
(c) $1 : 1$

(d) $R_1R_2 : 1$

$$\frac{\omega_1}{\omega_2} = ??$$

$$\omega = \frac{2\pi}{T} = \text{const}^n$$

$= \text{sam.}$



Angular speed of a uniformly circulating body with time period T is

(a) $2\pi T$

(b) $\frac{2\pi}{T}$

(c) πT

(d) $\frac{\pi}{T}$

$$\omega = \frac{d\theta}{dt} = \frac{2\pi}{T} = 2\pi f$$

$$\alpha = \left(\frac{d\omega}{dt}\right)$$

An object moving in a circular path at constant speed has constant

(a) Energy

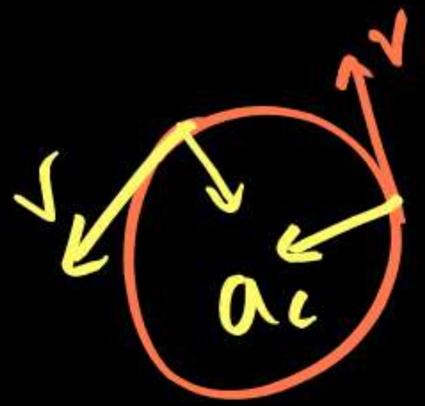
(b) ~~Velocity~~

(c) Acceleration (Variable due to dir^n)

(d) ~~Displacement~~

$\boxed{\text{U.C.M}}$

→ Uniform circular motion is a non Uniform Motion ($\vec{V}$ = variable) with non-uniform acceleration $\left[a_c = v^2/R \text{ (dir}^n = \text{varible)}\right]$



$K.E = \frac{1}{2} m (\text{speed})^2$

↳ scalar.

Speed of an object moving in circular path of radius 10 m with angular speed 2 rad/s is

(a) 10 m/s $\rightarrow \omega = 2\ rad/s$

(b) 5 m/s

(c) 20 m/s

(d) 30 m/s

$R = 10m \quad \omega = 2\ rad/sec$

$\boxed{V = r\omega}$

$V = (10 \times 2)$

MR* (सोच)

⟹ circular motion के question को solve करने से पहले decide कर लो $\boxed{U.C.M}$ है या $\boxed{N.U.C.M}$*

A body performing uniform circular motion completed 140 revolution in a second. Its angular speed is

(a) 880 rad/s (b) ~~440~~ rad/s

(c) 220 rad/s (d) ~~240~~ rad/s

Speed = Cost^n

$n = 140$ rev/sec

$f = 140$ Hz

$\omega = 2\pi f$

$= 2\pi \times 140$

$= 2 \times \frac{22}{7} \times \cancel{140}^{20}$

$= (40 \times 22)$

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Centripetal acceleration of a cyclist completing 7 rounds in a minute along a circular track of radius 5 m with a constant speed, is

(a) $2.7\ m/s^2$ (b) $4\ m/s^2$

(c) $3.78\ m/s^2$ (d) $6\ m/s^2$

V.C.M

1 mint = 60 sec = 7 round

$$\omega = \frac{\theta}{t} = \frac{14\pi}{60\,sec} = \frac{7\pi}{30}$$

$$a_c = \frac{v^2}{R} = \omega^2 R$$

$$= \frac{7\times 7\,\pi^2}{30\times 30}\times 5 = \frac{\cancel{7\times 7}}{\cancel{30\times 30}}\times\frac{\overset{11}{\cancel{22}}\times\overset{11}{\cancel{22}}}{\cancel{7\times 7}}\times\cancel{5} = \frac{121}{45}$$

$15\ \cancel{15}\ 3$

If the frequency of an object in uniform circular motion is doubled, its acceleration becomes

(a) Two times
(b) Four times
(c) Half
(d) One fourth

f = double

U.C.M

$a = \frac{v^2}{R} = \omega^2 R = (2\pi f)^2 R$

$= 4\pi^2 f^2 R$

$\vec{a_c} = \vec{a}$

$a_c \propto \boxed{f^2}$

ARJUNA

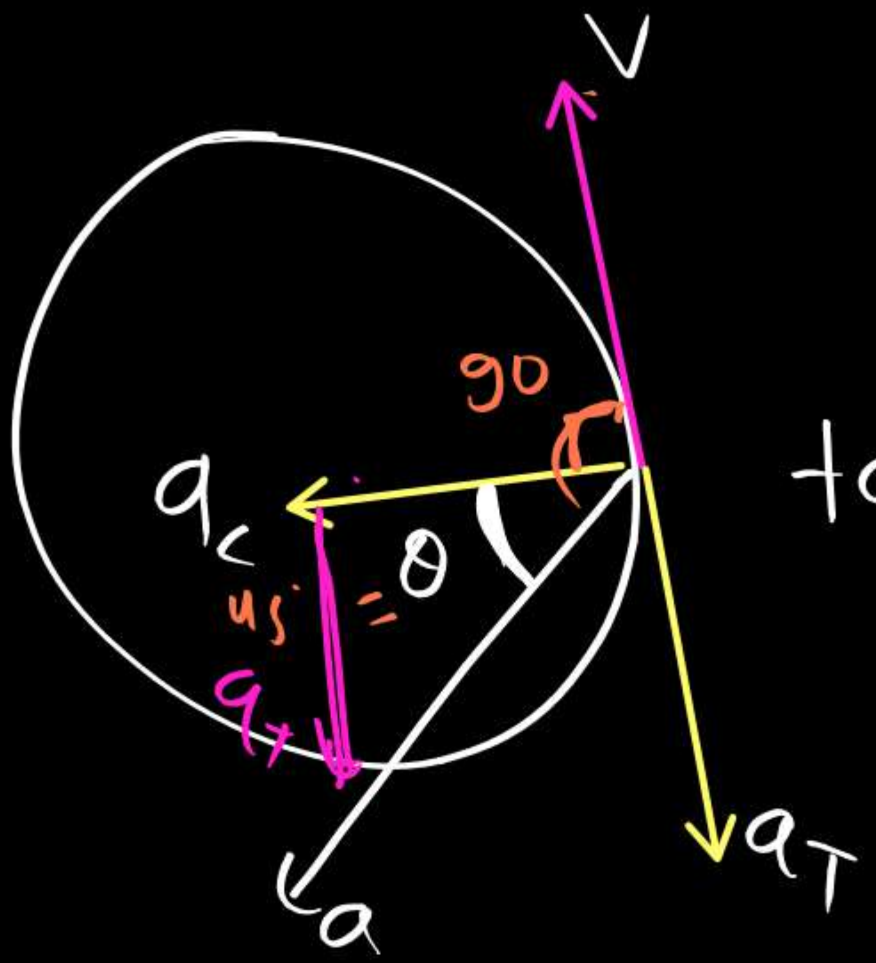
A body is moving on a circle of radius 80 m with a speed 20 m/s which is decreasing at the rate 5 m/s^2 at an instant. The angle made by its acceleration with its velocity is

(a) 45°
(b) 90°
(c) 135°
(d) 0°

1mR* सोच

Speed ↓

90 < θ



$R = 80\,m$

$V = 20\,m/s$

$a_t = 5\,m/s^2$

$\boxed{N.U.C.M}$

$\tan\theta = \frac{a_T}{a_c} = \frac{5}{v^2/R} = \frac{5}{\frac{20\times 20}{80}} = \frac{80\times 5}{20\times 20} = 1$

$\theta = 45°$

A particles is moving in a circle of radius r having centre at O, with a constant speed v. The magnitude of change in velocity in moving from A to B is

(a) $2v$ (b) 0

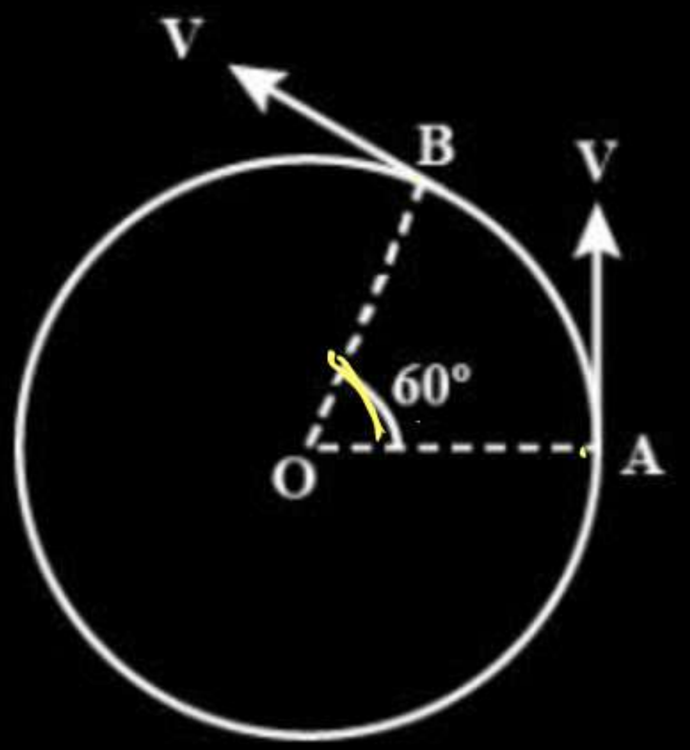
(c) $\sqrt{3}\,v$ (d) v

MR* (Bade आराम से)

$$\vec{\Delta v} = \vec{v_f} - \vec{v_i}$$

Change in velocity

$\boxed{\text{Change in magnitude of velocity} = 0}$



V.C.M

A car is moving at a speed of 40 m/s on a circular track of radius 400 m. This speed is increasing at the rate of 3 m/s^2. The acceleration of car is

(a) 4 m/s^2 (b) 7 m/s^2

(c) 5 m/s^2 (d) 3 m/s^2

N.U.C.M

* $a_T = 3\ m/s^2$

$\vec{a} = \vec{a}_T + \vec{a}_c$

$\vec{a}_c = \frac{v^2}{R} = \frac{(40)^2}{400}$

$= \frac{40 \times 40}{400} = 4$

$\vec{a} = \sqrt{(a_T)^2 + (a_c)^2} = \sqrt{(4)^2 + (3)^2}$

$\boxed{\begin{array}{l} a_T = \text{The rate of change in Magnitude of velocity} \\ \Downarrow \\ a_T = \text{The rate of change in speed.} \end{array}}$

ARJUNA

A car is going round a circle of radius R_1 with constant speed. Another car is going round a circle of radius R_2 with constant speed. If both of them take same time to complete the circles, the ratio of their angular speeds and linear speeds will be

(a) $\sqrt{\frac{R_1}{R_2}}, \frac{R_1}{R_2}$

(b) 1, 1

(c) $1, \frac{R_1}{R_2}$

(d) $\frac{R_1}{R_2}, 1$

$\frac{\omega_1}{\omega_2} = \frac{1}{1} \Rightarrow \omega = \frac{2\pi}{T}$

$V = R\omega$

$\boxed{\frac{V_1}{V_2} = \frac{R_1}{R_2}}$

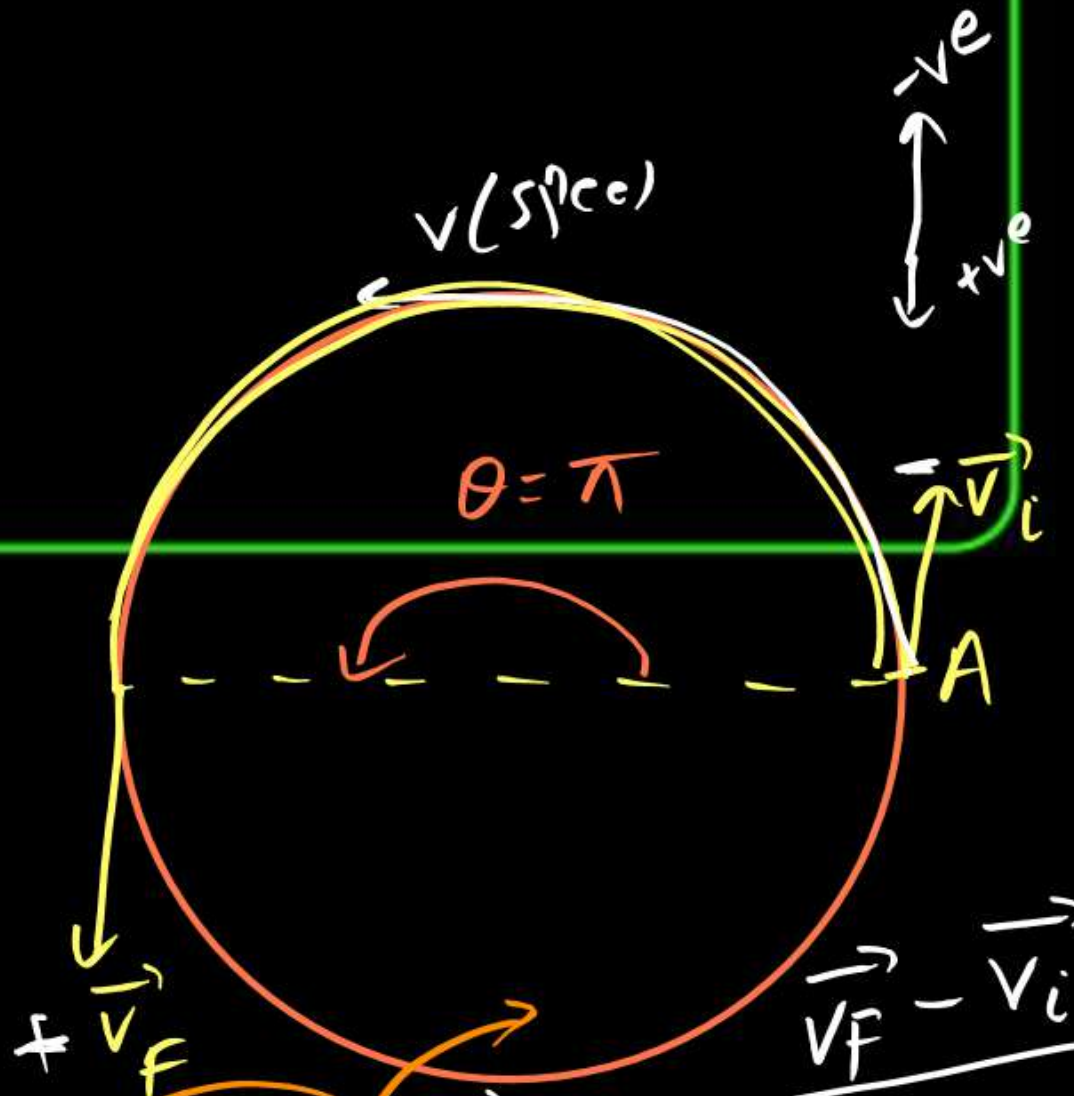
ARJUNA

A body revolves with constant speed v in a circular path of radius r. The magnitude of its average acceleration during motion between two points in diametrically opposite direction is

(a) ~~Zero~~ (b) $\cancel{\frac{v^2}{r}}$

$\boxed{\text{(c) } \frac{2v^2}{\pi r}}$ (d) $\frac{v^2}{2r}$

→ V.C.M



$$\vec{a}_{Avg} = \frac{V^2}{R}\,\frac{\sin\theta/2}{\theta/2}$$

$$= \frac{V^2}{R}\,\frac{\sin\pi/2}{\pi/2} = \frac{2V^2}{\pi R}$$

MOKSHA

$$(\vec{a})_{Avg} = \frac{\vec{V}_F - \vec{V}_i}{\Delta t} = \frac{+\vec{V}-(-V)}{\text{dist}^n/\text{speed}}$$

$$= \frac{2V}{\pi R/V} = \frac{2V^2}{\pi R}$$

If θ is angle between the velocity and acceleration of a particle moving on a circular path with decreasing speed, then

(a) $\theta = 90°$

(b) $0° < \theta < 90°$

(c) $90° < \theta < 180°$

(d) $0° \le \theta \le 180°$

If speed of an object revolving in a circular path is doubled angular speed is reduced to half of original value, then centripetal acceleration will become/remain

(a) Same (b) Double

(c) Half (d) Quadruple

$$\vec{a}_c = \frac{v^2}{R} = \frac{v^2}{\left(\frac{v}{\omega}\right)} = v\omega$$

$$V = \boxed{R}\,\omega$$

$$a_c = 2v\left(\frac{\omega}{2}\right) = \text{same}$$

$$V = R\omega$$

$R \rightarrow$ const^n

#

The position vector of a particle $\vec{R}$ as a function of time is given by $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$, where R is in meters, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x-and y-directions, respectively. Which one of the following statements is wrong for the motion of particle?

NEET-

(a) Path of the particle is a circle of radius 4 m

(b) Acceleration vector of along $-\vec{R}$

$a \leftrightarrows R$

(c) Magnitude of acceleration vector is v^2/R, where v is the velocity of particle

(d) Magnitude of the velocity of particle is 8 meter/second

wrong state

$\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$

Radius

$\theta = 2\pi t$

$\left(\frac{d\theta}{dt}\right) = \omega = 2\pi \frac{dt}{dt} = 2\pi = \text{const}$

U.C.M

$V = R\omega$
$= 4(2\pi)$
$= 8\pi$ m/s

ARJUNA

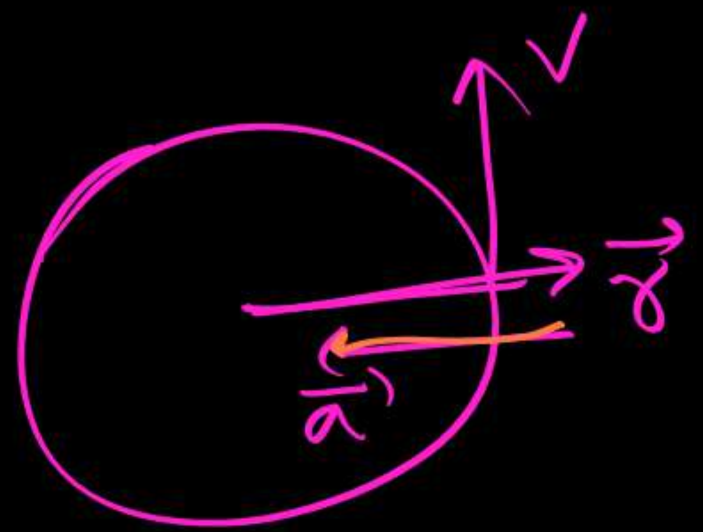
A particle moves so that its position vector is given by $\vec{r} = \cos \omega t \, \hat{x} + \sin \omega t \, \hat{y}$, where ω is a constant. Which of the following is true?

(a) Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin.

(b) Velocity and acceleration both the perpendicular to $\vec{r}$.

(c) Velocity and acceleration both are parallel to $\vec{r}$.

(d) Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin.



$\vec{r} = \cos(\omega t)\, \hat{x} + \sin(\omega t)\, \hat{y}$

→ circula Motⁿ

→ v.c.m

A motor car is travelling at 30 m/sec on a circular road of radius 500 m. It is increasing its speed at the rate of 2.0 ms^{-2}. The total acceleration is:

(a) 1.8 ms^{-2}
(b) 2 ms^{-2}
(c) 3.8 ms^{-2}
(d) 2.7 ms^{-2}

N.U.C.M

$$\vec{a} = \vec{a}_c + \vec{a}_T$$

$$a_T = 2 m/s^2$$

$$a_c = \frac{v^2}{R} = \frac{900}{500} = 9/5$$

$$a = \sqrt{(2)^2 + \left(\frac{9}{5}\right)^2} = 2\sqrt{2}$$

$$\boxed{= 2 \times 1.41}$$

$$= 2.82$$

IMP*

ARJUNA

The distance of a particle moving on a circle of radius 12 m measured from a fixed point on the circle and measured along the circle is given by $s = 2t^3$ (in meters). The ratio of its tangential to centripetal accelerationat $t = 2s$ is

(a) 1 : 1 (b) 1 : 2

(c) 2 : 1 (d) 3 : 1

$$\frac{a_T}{a_c} = ?? = \frac{\cancel{24}^{\,6} \times \cancel{3}}{\cancel{36}_{\,12} \times \cancel{4}} = \frac{1}{2}$$

distans R = 12 m

$s = 2t^3$

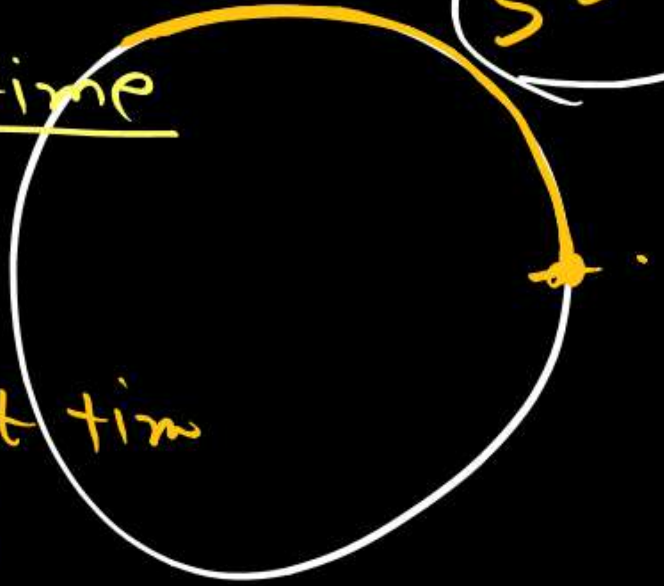
$S = 2t^3$

diff^n w.r.t. time

speed $= 6t^2$

diff^n w.r.t time

$\boxed{a_t = 12t}$



$|a_t| = 12 \times 2 = 24$ m/s^2

$$|a_c| = \frac{v^2}{R} = \frac{36t^4}{12} = \frac{36 \times 16}{\cancel{12}\,3}$$

A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that:

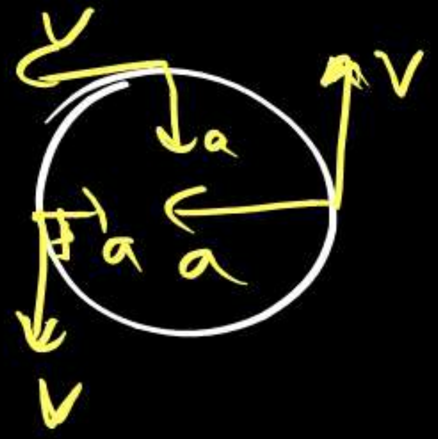
(a) Its velocity is constant

(b) Its acceleration is constant

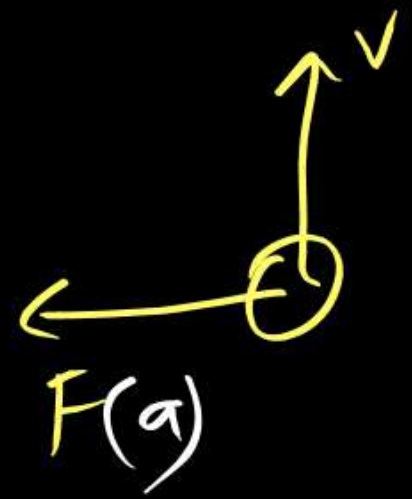
(c) Its kinetic energy is constant

(d) It moves in a straight line

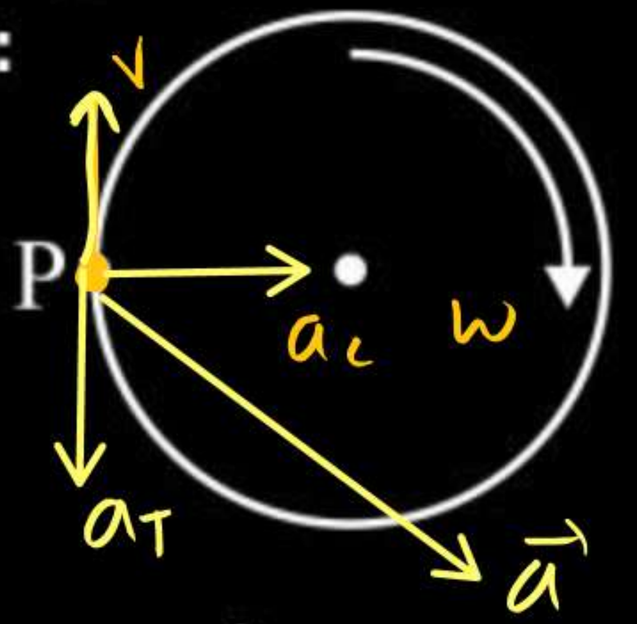
U.C.M



Speed = C.



A music CD of ‘Bajiro Mastani’ is rotating clockwise (as shown). After turing it off, the CD slows down. Assuming it has not come to a stop yet, the direction of acceleration at point P is:



(MR* Prob^m)

Speed ↓

(a) ↑

(b) →

(c) ↗

(d) ↘

a_r and a_t represent radial and tangential acceleration. The motion of a particle will be uniform circular motion if:-

(a) $a_r = 0$ and $a_t = 0$

(b) $a_r = 0$ but $a_t \neq 0$

(c) $a_r \neq 0$ but $a_t = 0$

(d) $a_r \neq 0$ and $a_t \neq 0$

centripetal

U.C.M

$\vec{a}_c = \vec{a} \neq 0$

$a_t = 0$

Angular velocity of minute hand of a clock is:-

(a) $\frac{\pi}{30}$ rad / s

(b) 8π rad/s

(c) $\frac{2\pi}{1800}$ rad / s

(d) $\frac{\pi}{1800}$ rad / s

$$(\omega)_{U.C.M} = \frac{2\pi}{T} = \frac{2\pi}{1 hr} = \frac{2\pi}{60 \times 60} =$$

ARJUNA

If the equation for the displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after 2s from its start is:-

(a) 8 rad/s
(b) 12 rad/s
(c) 24 rad/s
(d) 36 rad/s

$\theta = 2t^3 + 0.5$ (Angular displ)

$$\omega = \frac{d\theta}{dt} = 2(3t^2)$$

$$= 6(t^2)$$

$$= 6(2)^2 = 6 \times 4$$

A particle of mass 'm' describes a circle of radius (r). The centripetal acceleration of the particle is $4/r^2$. The momentum of the particle:-

(a) $\frac{2m}{r}$

(b) $\frac{2m}{\sqrt{r}}$

(c) $\frac{4m}{r}$

(d) $\frac{4m}{\sqrt{r}}$

$$a_c = \frac{4}{r\,\cancel{r}} = \frac{v^2}{\cancel{r}}$$

$$v^2 = \frac{4}{r}$$

$$v = \sqrt{\frac{4}{r}} = \frac{2}{\sqrt{r}}$$

$$p = mv = \frac{m\,2}{\sqrt{r}}$$

A particle is moving around a circular path with uniform angular speed (ω). The radius of the circular path is r. The acceleration of the particle is:-

(a) $\frac{\omega^2}{r}$

(b) $\frac{\omega}{r}$

(c) $v\omega$

(d) vr

$$a_c = \frac{v^2}{R} = \omega^2 R = \omega v$$

ARJUNA

Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their time period of rotation is the same. The ratio of angular speed of A to that of B will be

NEET-2019

(a) $1:1$

(b) $r_A : r_B$

(c) $v_A : v_B$

(d) $r_B : r_A$

$\omega = \frac{2\pi}{T} = \text{same}$

A particle starting from rest, moves in a circle of radius 'r'. It attains a velocity of V_0 m/s in the n^{th} round. Its angular acceleration will be

NEET-19

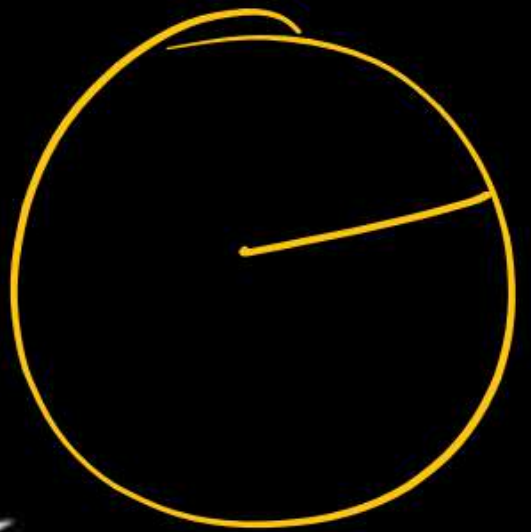
$\alpha = ??$

(a) $\dfrac{V_0}{n}\text{rad/s}^2$

(b) $\dfrac{V_0}{2\pi n r^2}\text{rad/s}^2$

(c) $\dfrac{V_0^2}{4\pi n r^2}\text{rad/s}^2$ (mR^2 unit)

(d) $\dfrac{V_0^2}{4\pi n r}\text{rad/s}^2$



$V_i = 0$

$\omega_i = 0$

$\Rightarrow \boxed{V_f = V_0 \text{ m/s}}$

n^{th} round $\Rightarrow$

$\theta = (2\pi)n$

$\text{dist}^n = 2n\pi R$

$\boxed{\text{N.U.C.M}}$

$\omega_f^2 - \cancel{\omega_i^2}^{\,0} = 2\alpha\theta$

$\dfrac{V_0^2}{R^2} = 2\alpha(2n\pi)$

$\alpha = \dfrac{V_0^2}{4n\pi R^2}$

In the given figure, $a = 15\ \text{m s}^{-2}$ represents the total acceleration of a particle moving in the clockwise direction in a circle of radius $R = 2.5$ m at a given instant of time. The speed of the particle is

(a) $4.5\ \text{m s}^{-1}$ (b) $5.0\ \text{m s}^{-1}$

(c) $5.7\ \text{m s}^{-1}$ (d) $6.2\ \text{m s}^{-1}$

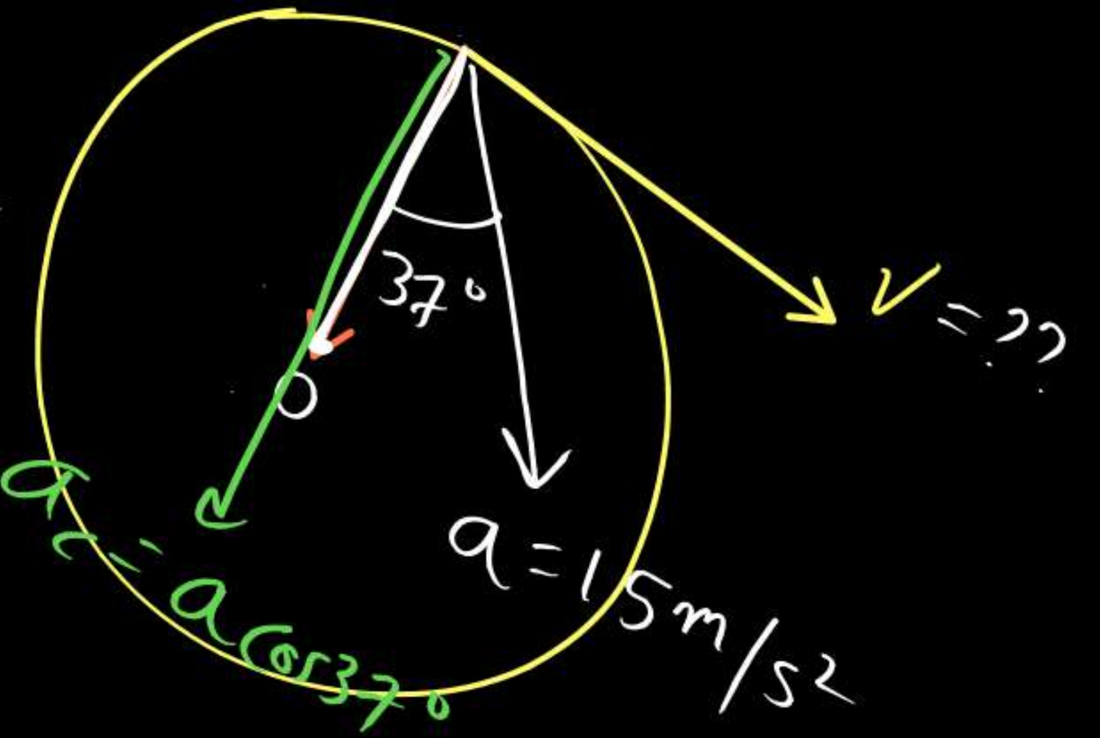
N.U.C.M

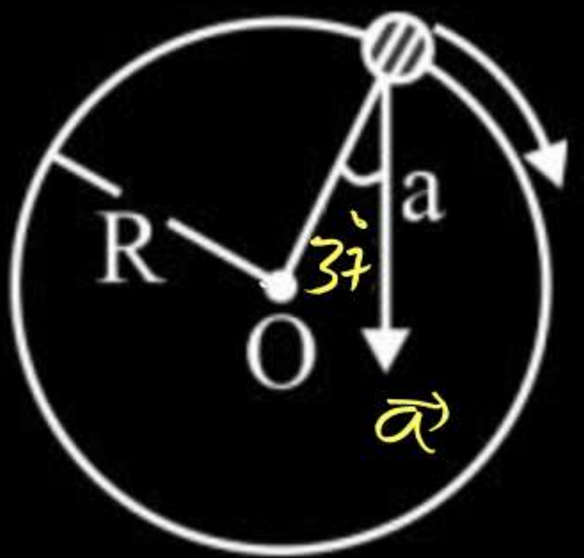
#) $a_c = \frac{v^2}{R} = a\cos 37°$

$v^2 = 15 \times \frac{4}{\cancel{5}} \times \frac{\cancel{5}}{2}$

$V^2 = \frac{6\cancel{0}}{\cancel{2}} = 30$

$V = \sqrt{30}$





A particle moves in a circle of radius 5 cm with constant speed and time period 0.2π s. The acceleration of the particle is

(a) $15\ m/s^2$

(b) $25\ m/s^2$

(c) $36\ m/s^2$

(d) $5\ m/s^2$

U.C.M

$T = 0.2\pi$

$\omega = \frac{2\pi}{T} = \frac{2\cancel{\pi}}{0.2\cancel{\pi}} = 10$

$a_c = \omega^2 R = 1\cancel{00} \times \frac{5}{1\cancel{00}} = 5\ m/s^2$

ARJUNA

A stone tied to the end of a string of 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes $\boxed{\text{22 revolutions in 44 seconds}}$, what is the magnitude and direction of acceleration of the stone?

(a) π^2 m s^{-2} and direction along the radius towards the centre

(b) π^2 m s^{-2} and direction along the radius away from the centre

(c) π^2 m s^{-2} and direction along the tangent to the circle

(d) $\pi^2/4$ m s^{-2} and direction along the radius towards the centre.

U.C.M

$$a_c = \omega^2 R = (2\pi f) R$$

$$= \left(\cancel{2} \times \pi \times \frac{1}{\cancel{2}}\right)^2 R$$

$=$

A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is

(a) $40\ m/s^2$ (b) $640\pi\ m/s^2$

(c) $160\pi\ m/s^2$ (d) $40\pi\ m/s^2$

$\boxed{\omega_f^2 - \omega_i^2 = 2\theta\alpha}$

$\boxed{a_T = R\alpha}$

$R = \frac{20}{\pi}$ m

N.U.C.M

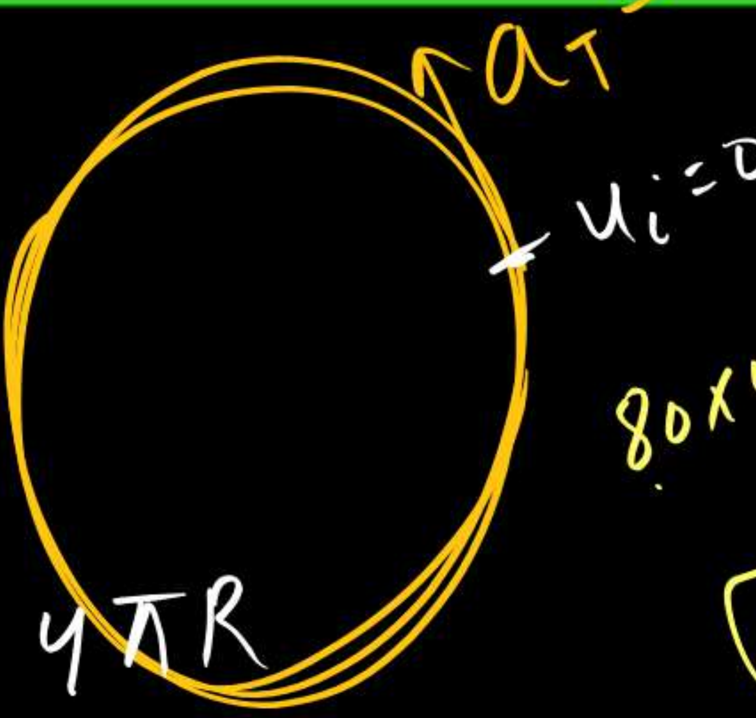
2nd rev

$v_f = 80$ m/s

$a_T = ??$

$v_f^2 - \cancel{v_i^2}^{\,0} = 2 a_T S$

$80 \times \cancel{80} = \cancel{2} a_T 4\pi R$



$80 \times 40 = a_T 4 \cancel{\pi} \frac{20}{\cancel{\pi}}$

$\boxed{a_T = 40\ m/s^2}$

Two particles having mass M and m are moving in a circular path having radius R and r. If their time period are same then the ratio of angular velocity will be

ω = sa

(a) $\frac{r}{R}$

(b) $\frac{R}{r}$

(c) 1

(d) $\sqrt{\frac{R}{r}}$

Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time t. The ratio of the angular speeds of the first to the second car is

AIPMT

(a) $r_1 : r_2$

(b) $m_1 : m_2$

(c) $1 : 1$

(d) $m_1 m_2 : r_1 r_2$

$$\omega = \frac{2\pi}{t}$$

t = same

ARJUNA

A body is whirled in a horizontal circle of radius 20 cm. It has an angular velocity of 10 rad/s. What is its linear velocity at any point on circular path?

(a) 20 m/s

(b) $\sqrt{2}$ m/s

(c) 10 m/s

(d) 2 m/s

$V = r\omega$

ARJUNA

The angular speed of a flywheel making 120 revolutions/minute is

(a) 4π rad/s (b) $4\pi^2$ rad/s

(c) π rad/s (d) 2π rad/s

$$f = \frac{12\cancel{0}}{6\cancel{0}} = 2 \text{ rvs}$$

$$\omega = 2\pi f$$

$$= 2\pi (2)$$

$$= 4\pi \text{ rad/sec}$$

$\cancel{\omega}$ / $\cancel{f}$

ω (rad/sec) ✓

f (Hz) (rev/s) ✓

ARJUNA

An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rpm, the acceleration of a point on the tip of the blade is

(a) 1600 m s^{-2} (b) 47.4 m s^{-2}

(c) 23.7 m s^{-2} (d) 50.55 m s^{-2}

$f = \frac{120}{60} = 2 \text{ Hz}$

$\boxed{\omega = 2\pi f}$

V.C.M

$a_c = \omega^2 R$

circular motion

$\boxed{\text{Kinematics of circular motion}}$

THANK YOU ☺

Dynamics of circular motion (Laws of motion) के last में

vertical circular motion → (work energy Pu

Rotational Mo