Surprise

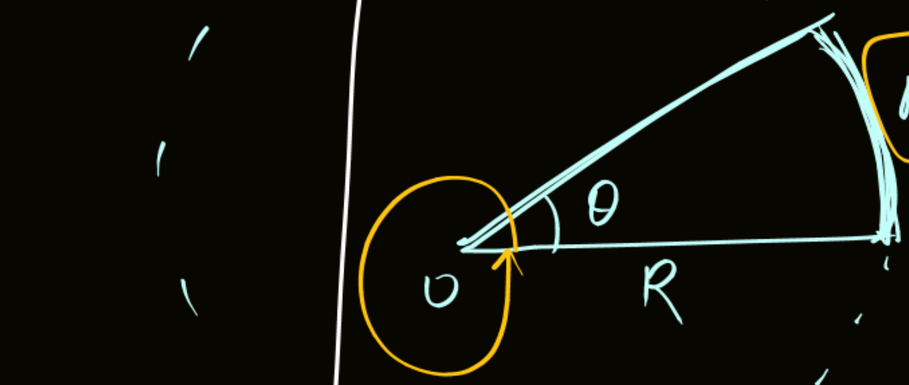
Revision For Basic math

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

	0°	30	45	60	90	120	135°	150	180
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos\theta$	$+1$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

* $\sin(270^\circ) = -1$

$\infty \hookrightarrow$ not defined



$\text{Arc} = R\theta$

↳ Must be in rad.

$r_{atom} \ll 1\,m$

$2\pi\,\text{rad} = 360^\circ$

$\pi\,\text{rad} = 180^\circ$

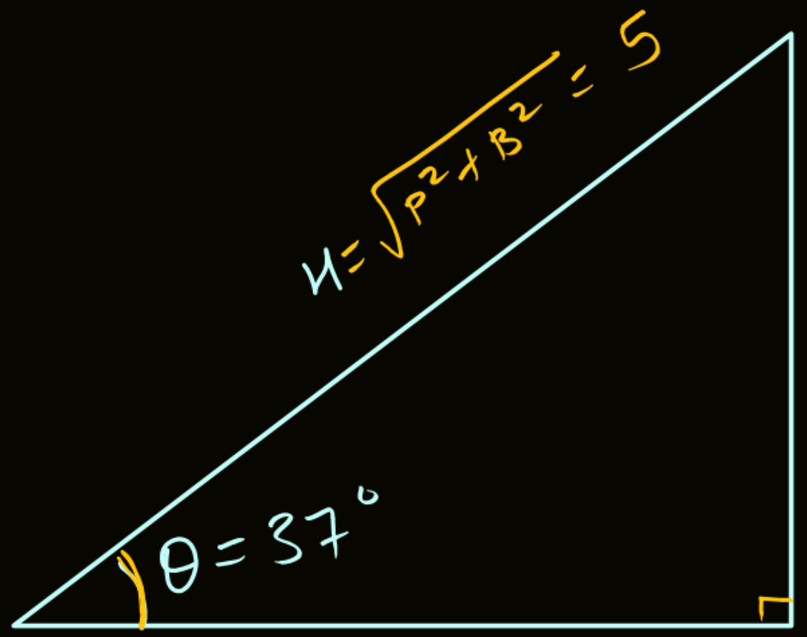
↳ Ratio

$\sin(\theta)$

rad / degree

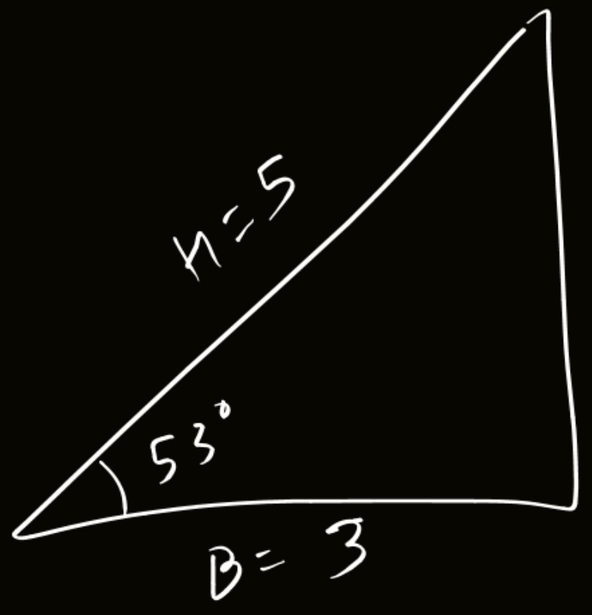
$\frac{\pi}{6}\,\text{rad} = \frac{180^\circ}{6} = 30^\circ$

$\frac{2\pi}{3}\,\text{rad} = \frac{2}{3} \times 180^\circ = \underline{120^\circ}$



$\sin 37^\circ = \frac{3}{5}$ $\quad \cos 37^\circ = \frac{4}{5}$

$\tan 37^\circ = \frac{3}{4}$



$\sin 53^\circ = \frac{4}{5}$

$\tan 53^\circ = \frac{4}{3}$

mR*

$$-1 \le \sin\theta = \frac{P}{H} \le 1$$

$$-1 \le \cos\theta = \frac{B}{H} \le 1$$

$$-\infty \le \tan\theta \le +\infty$$

$$P^2 + B^2 = H^2$$

divided by H^2

$$\left(\frac{P^2}{H^2}\right) + \left(\frac{B^2}{H^2}\right) = 1$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

(a) If $\tan\theta = 4$ then find $\sin\theta$

Soln $\tan\theta = \frac{4}{1} = \frac{P}{B}$

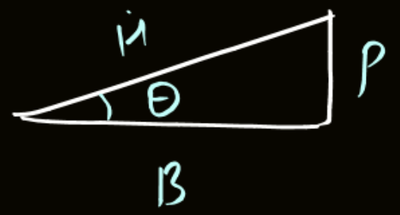
If $P = 4$ then $B = 1$

$$H = \sqrt{P^2 + (B)^2} = \sqrt{17}$$

$$\sin\theta = \frac{P}{H} = \frac{4}{\sqrt{17}}$$ R

Small angle approximation

$$\sin\theta = \frac{P}{H}$$



$$\sin\theta = \theta = \tan\theta$$

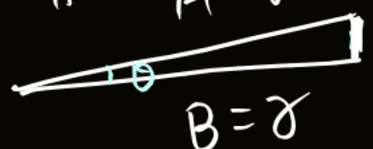
If θ is very small

$$\sin\theta = \frac{P}{H} = \frac{\gamma\theta}{\gamma}$$

$$\# \quad \sin\theta = \theta$$

$$\tan\theta = \frac{P}{B} = \frac{\gamma\theta}{\gamma} = \theta$$

$\#$ $H = \gamma$



$P = \text{Small} = \gamma\theta$ (Arc)

$B = \gamma$

$$H = \sqrt{P^2 + (B)^2}$$

$$H = B = \gamma \text{ (Let)}$$

$$\# \cos\theta = \frac{B}{H} = \frac{\gamma}{\gamma} 1$$

$\cos 3^\circ = 1$

$\cos(2^\circ) = 1$

$\cos(1^\circ) = 1$

$\cos(4^\circ) = 1$

$\cos(0^\circ) = 1$

$$\sin(2^\circ) = 2^\circ \times \frac{\pi \text{ rad}}{180} = \frac{\pi \text{ rad}}{90}$$

$\pi \text{ rad} = 180^\circ$

Maxm value

$$Y = a\sin\theta + b\cos\theta$$

→ Ratta

$$Y_{max} = \sqrt{a^2 + b^2}$$

Ex- $Y = 3\sin\theta + 2\cos\theta$

$$Y_{max} = \sqrt{(3)^2 + (2)^2}$$

$$= \sqrt{9+4} = \sqrt{13}$$

$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \sin\beta\cdot\cos\alpha$$

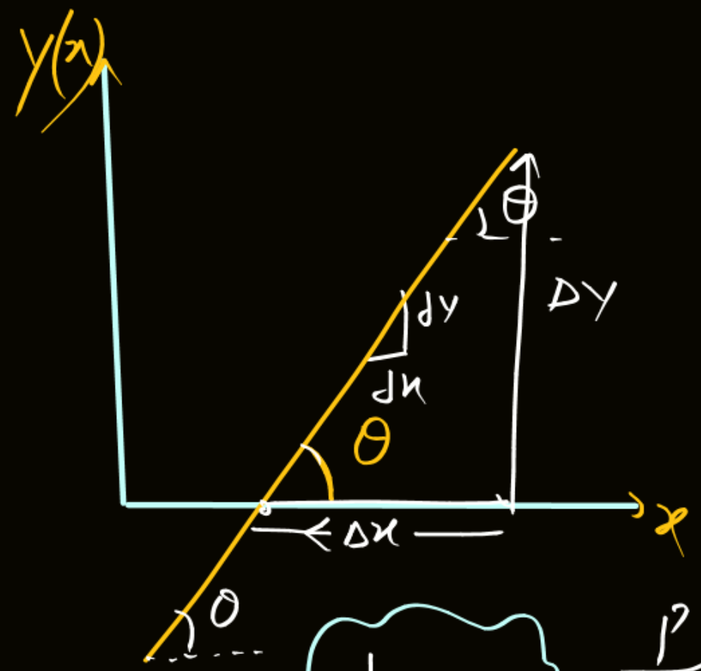
$$\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \sin\beta\cdot\cos\alpha$$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$

$$\# \quad \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\cdot\tan B}$$

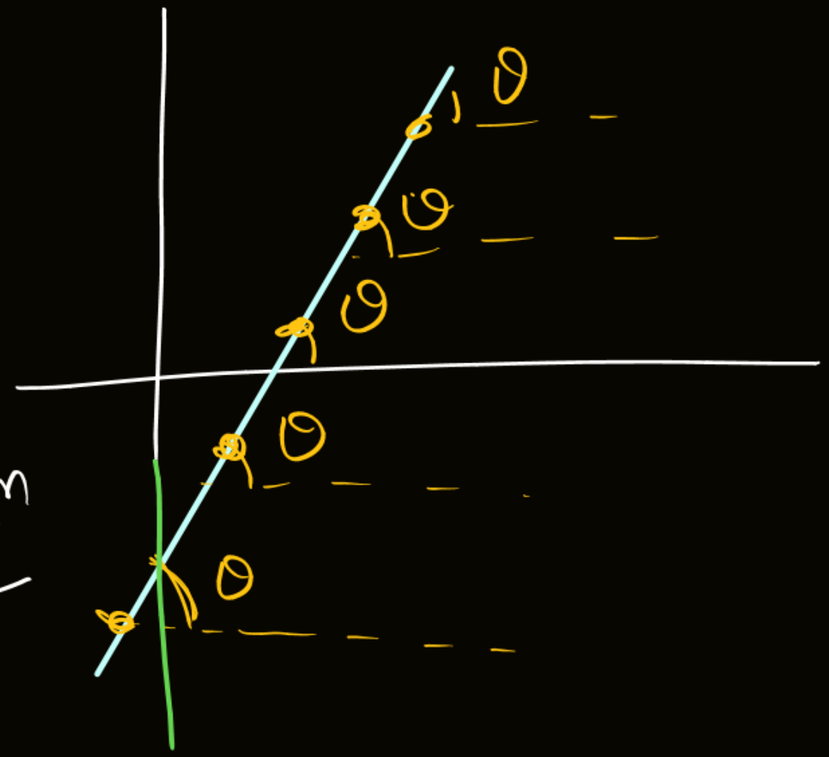
Graph : Straight line → C = Intercept

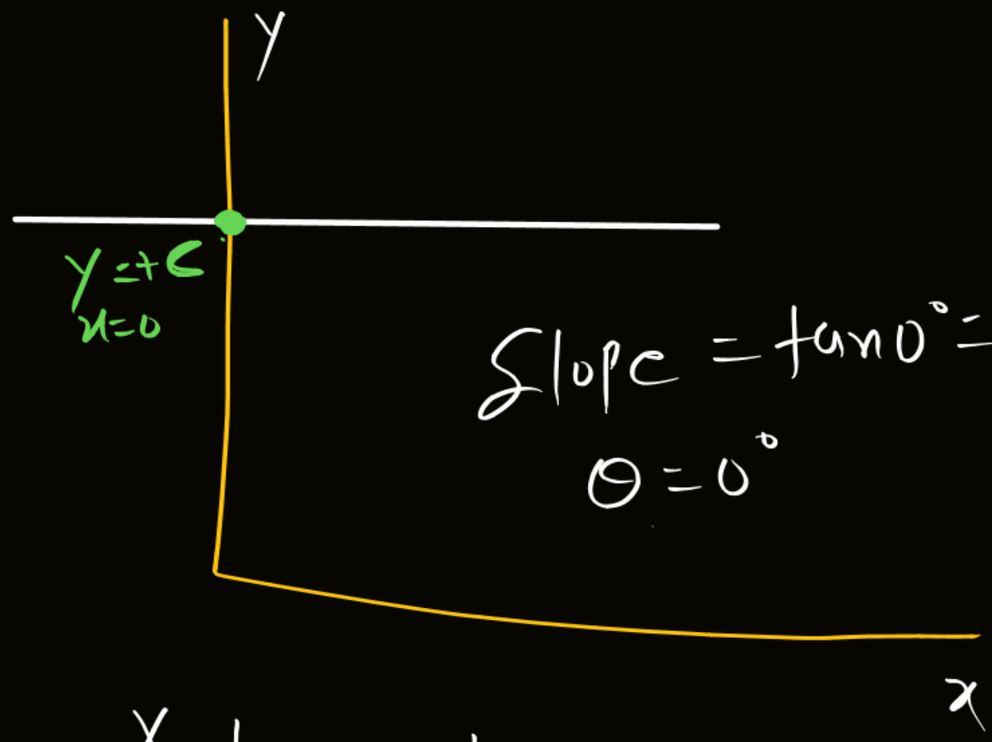
→ Slope (remains same at all the Point on straight line)

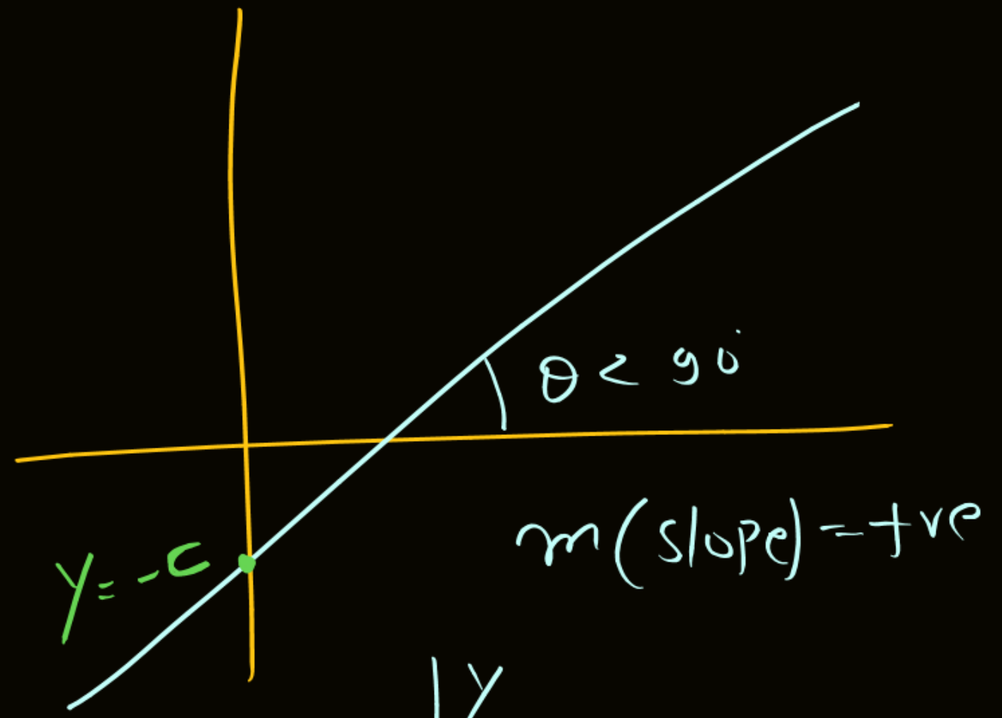


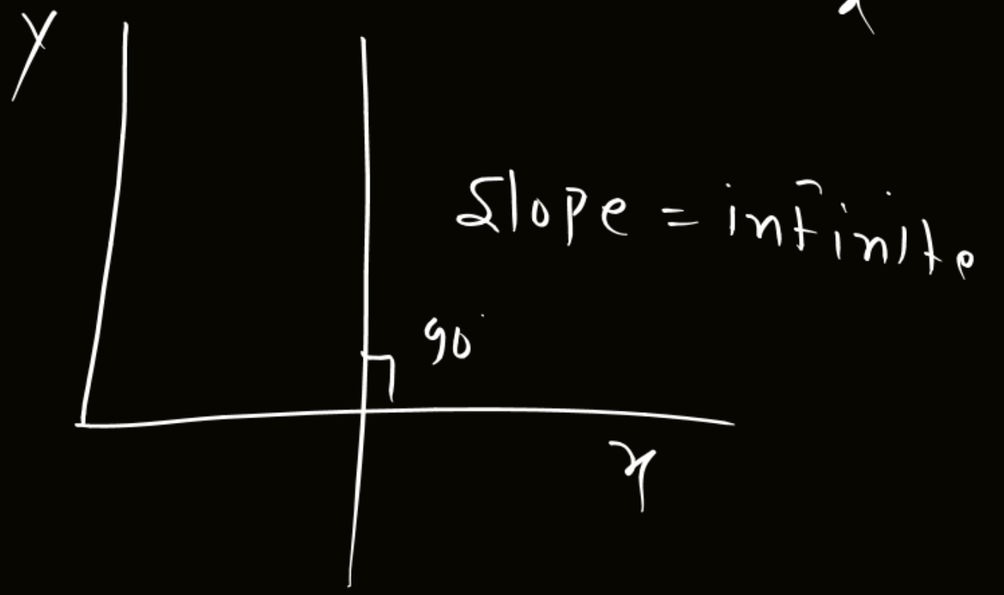
$$\tan\theta = \frac{P}{B} = \frac{\Delta Y}{\Delta x} = \frac{dy}{dx}$$

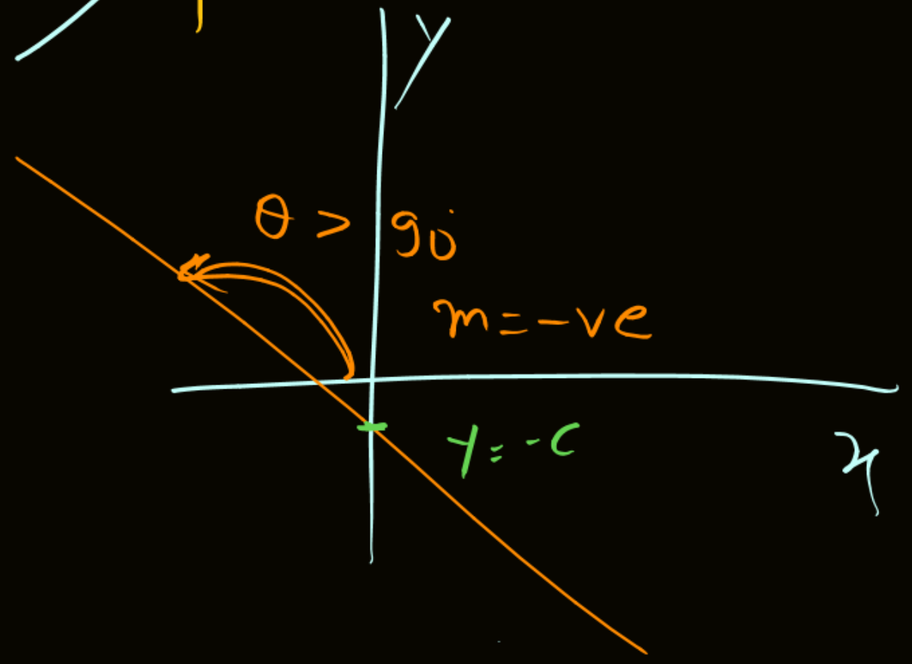
$$\text{Slope} = \frac{\Delta Y}{\Delta x} = \frac{dy}{dx} \rightarrow \text{diff}^{n}$$

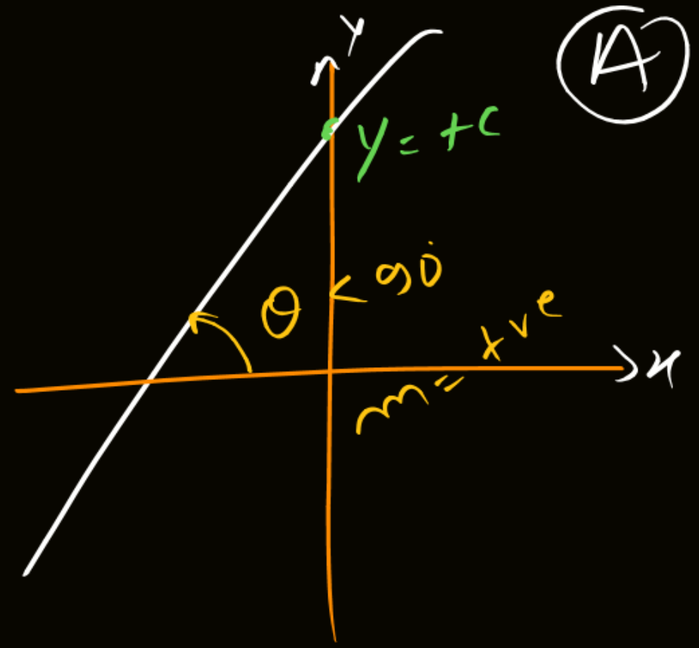



Y
y = +c
x = 0
Slope = tan 0° = 0
θ = 0°
x


θ < 90°
y = -c
m (slope) = +ve

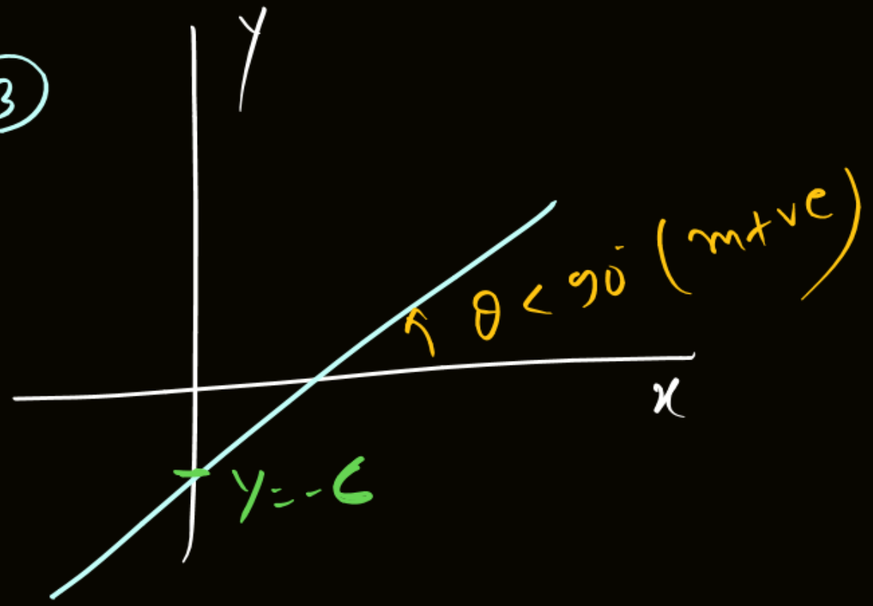

Y
Slope = infinite
90°
x

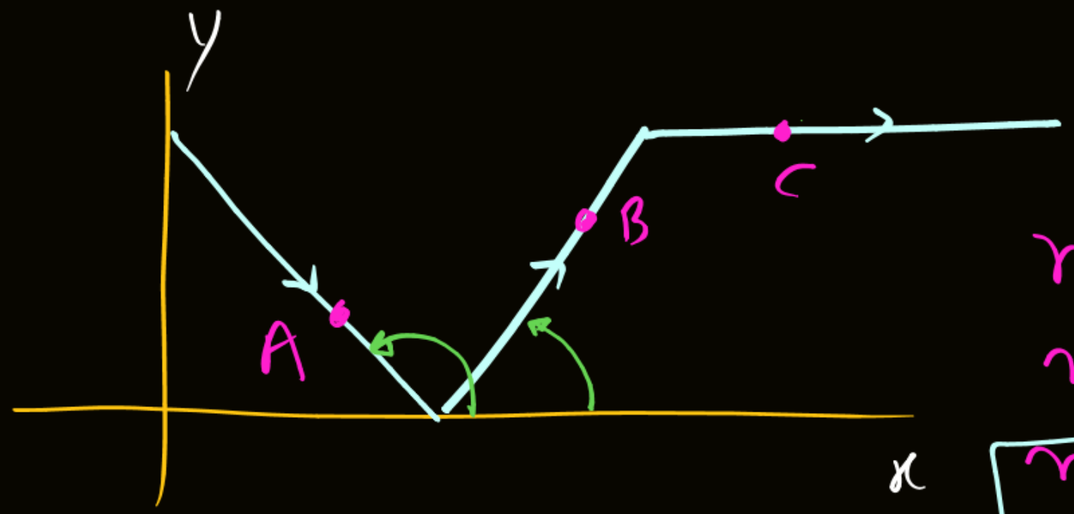

Y
θ > 90°
m = -ve
y = -c
x



(A)

(B)





m_A = -ve

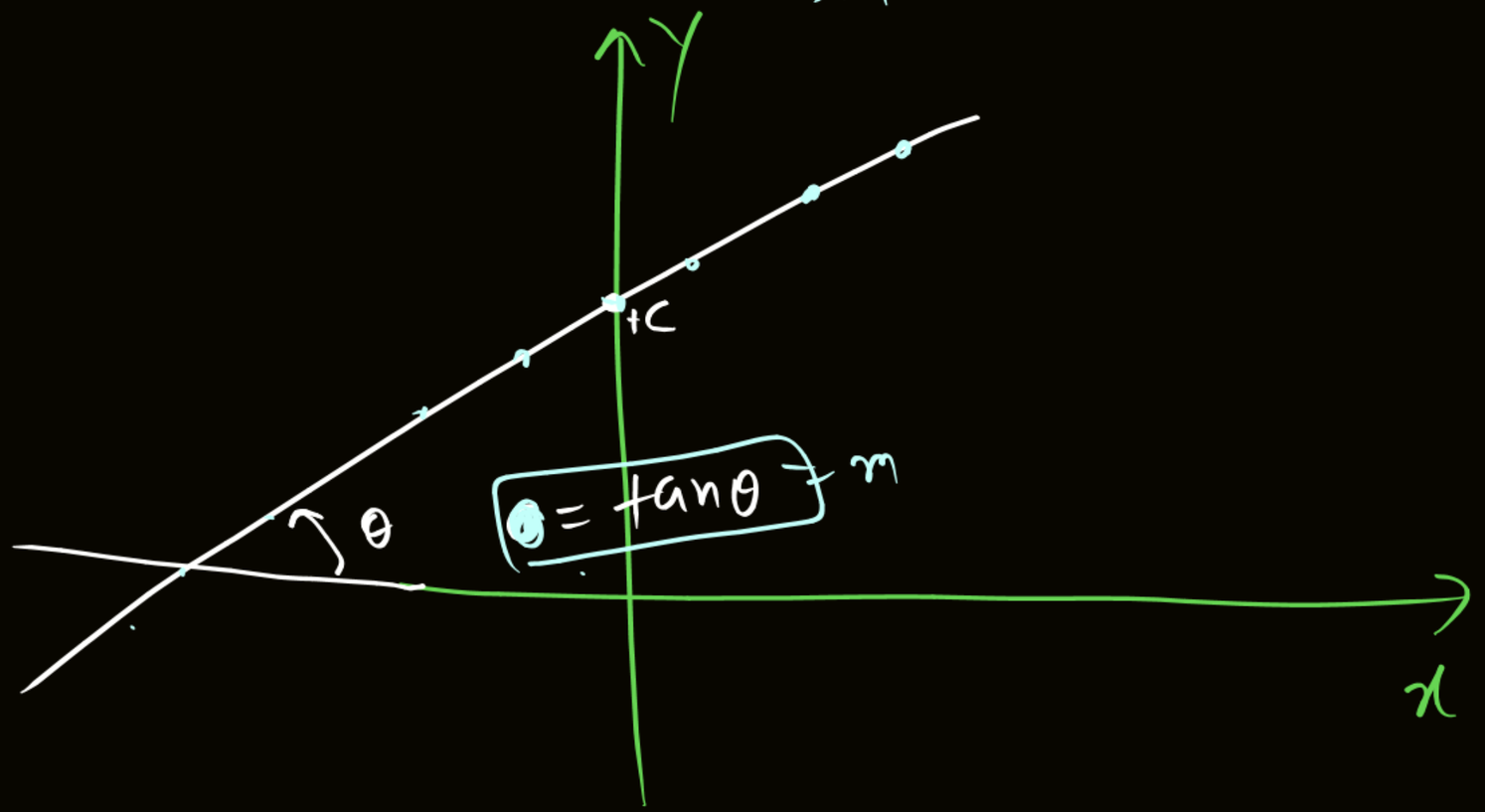
m_B = +ve

$m_C = 0$

$$y = mx + c$$

m → slope

c → Intercept

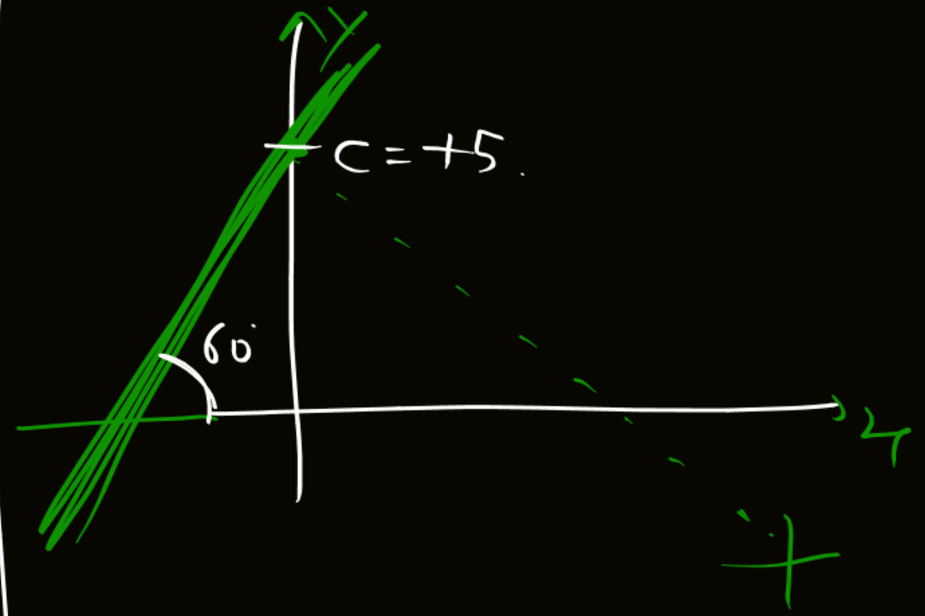


$$y = \sqrt{3}x + 5$$

$c = 5$

$\tan\theta = \sqrt{3}$

$\theta = 60$

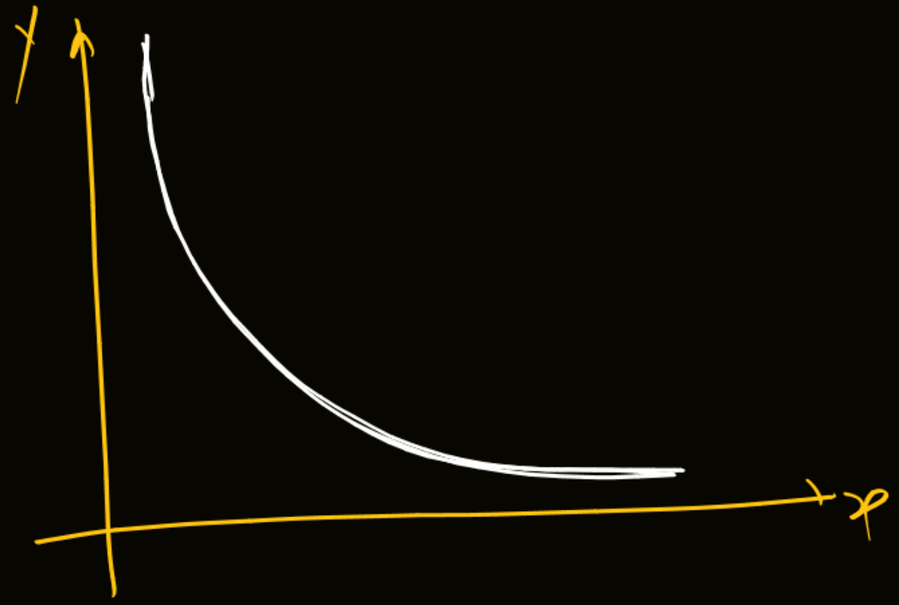


Rectangular hyperbola (Inverse relation)

$$y^{-1} = \frac{K}{x} = K x^{-1}$$

$$yx = \text{Const}^n$$

$$\boxed{y = \frac{\text{Const}^n}{x}} = \text{Const} \cdot x^{-1}$$

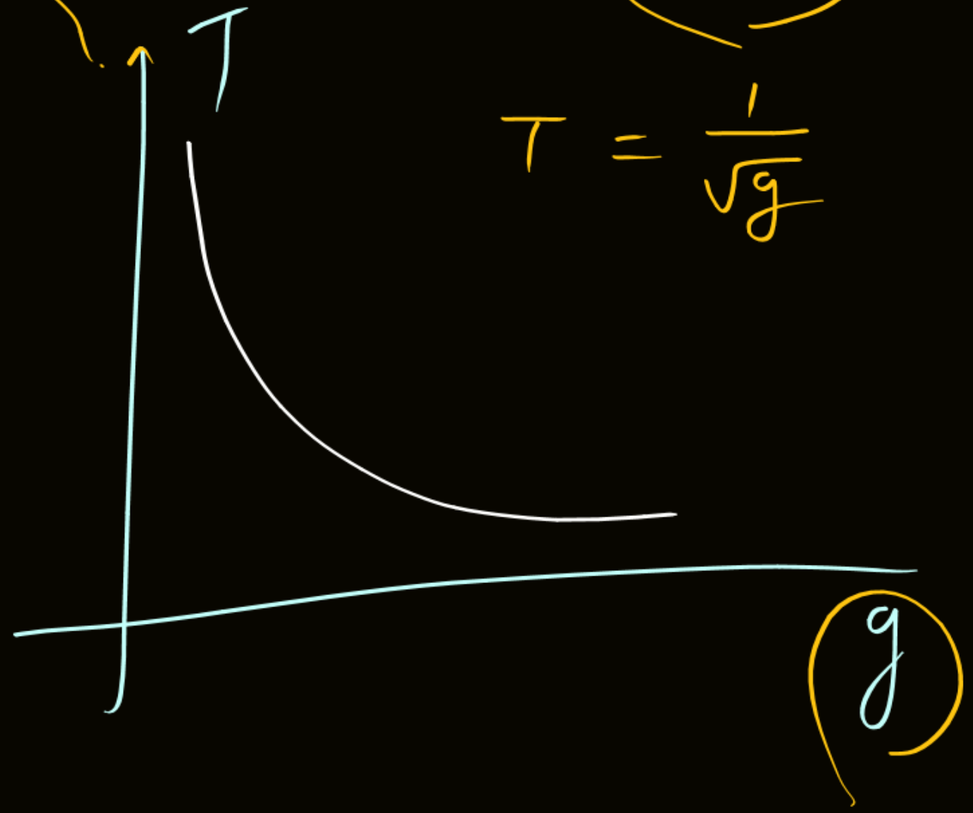


$$y = \frac{1}{x^2} = x^{-2}$$

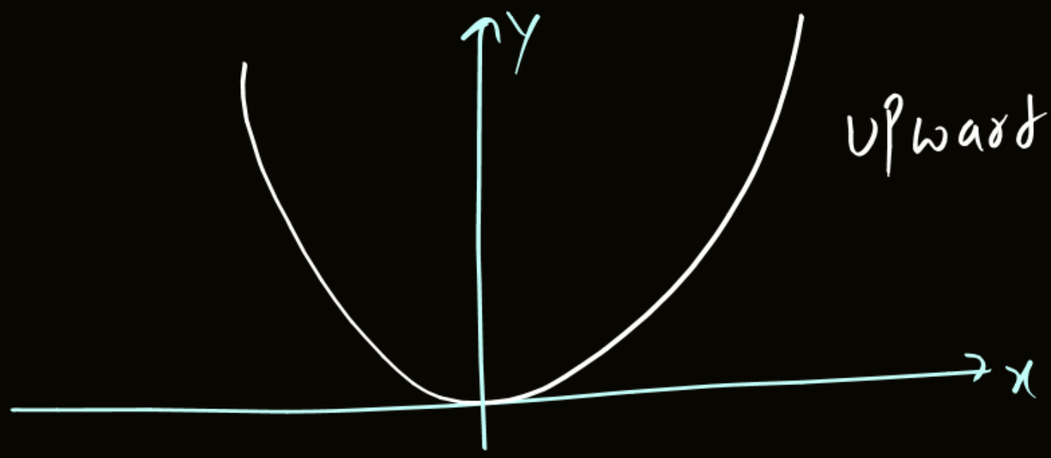
$$T = 2\pi\sqrt{\frac{l}{g}}$$

$l = 1\,\text{m}$

$$T = \frac{1}{\sqrt{g}}$$

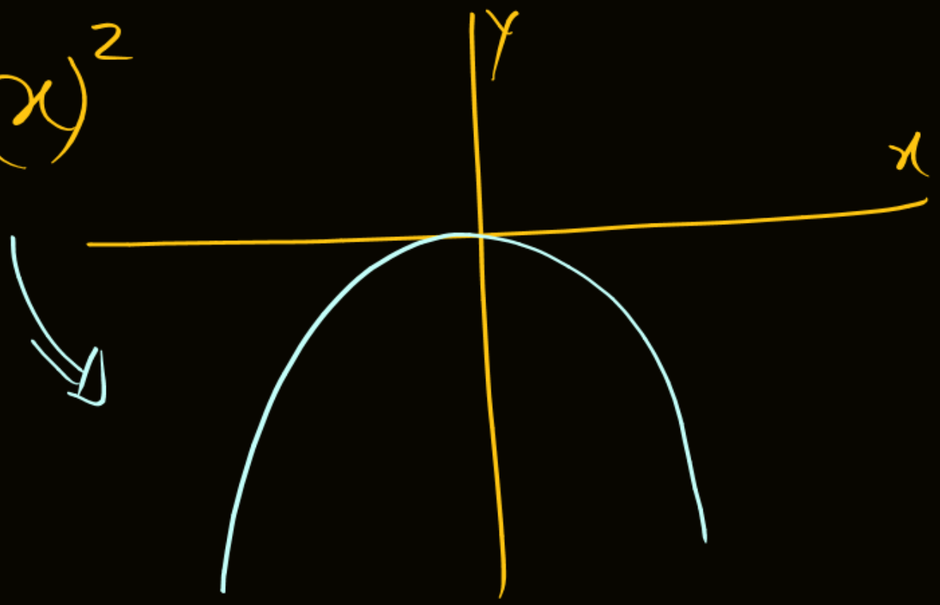


$y = x^2$ (Parabola)

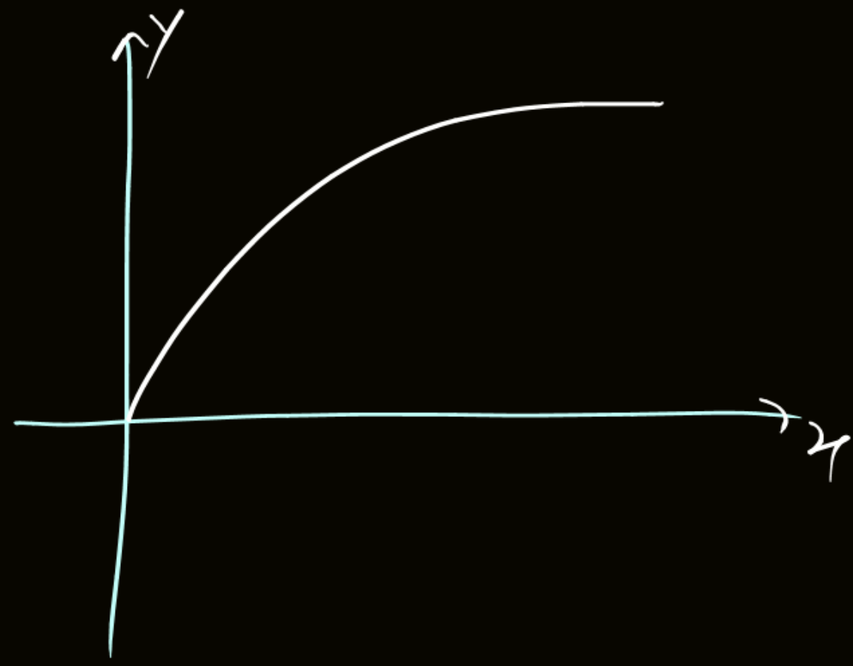


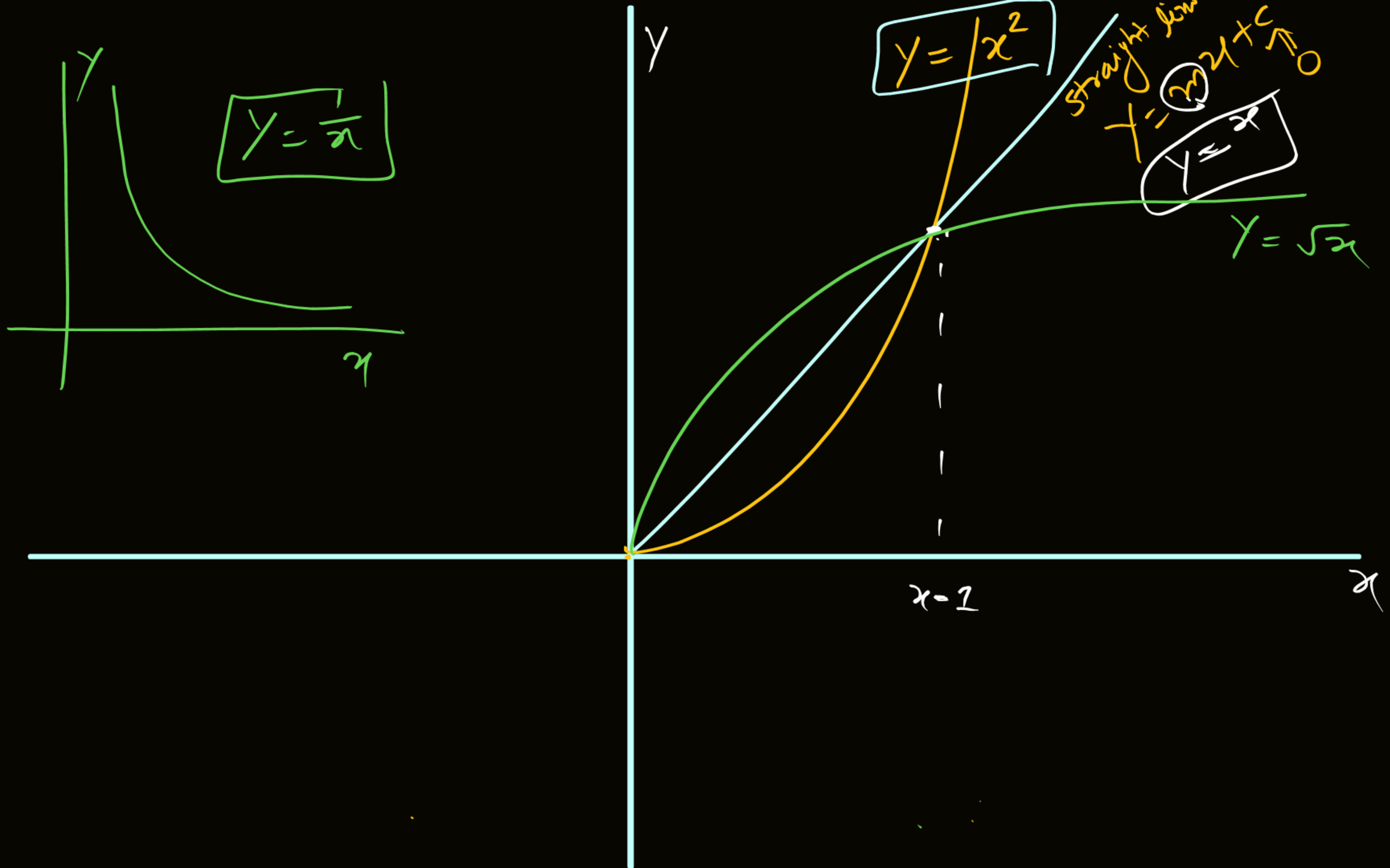
Upward opning Parabola

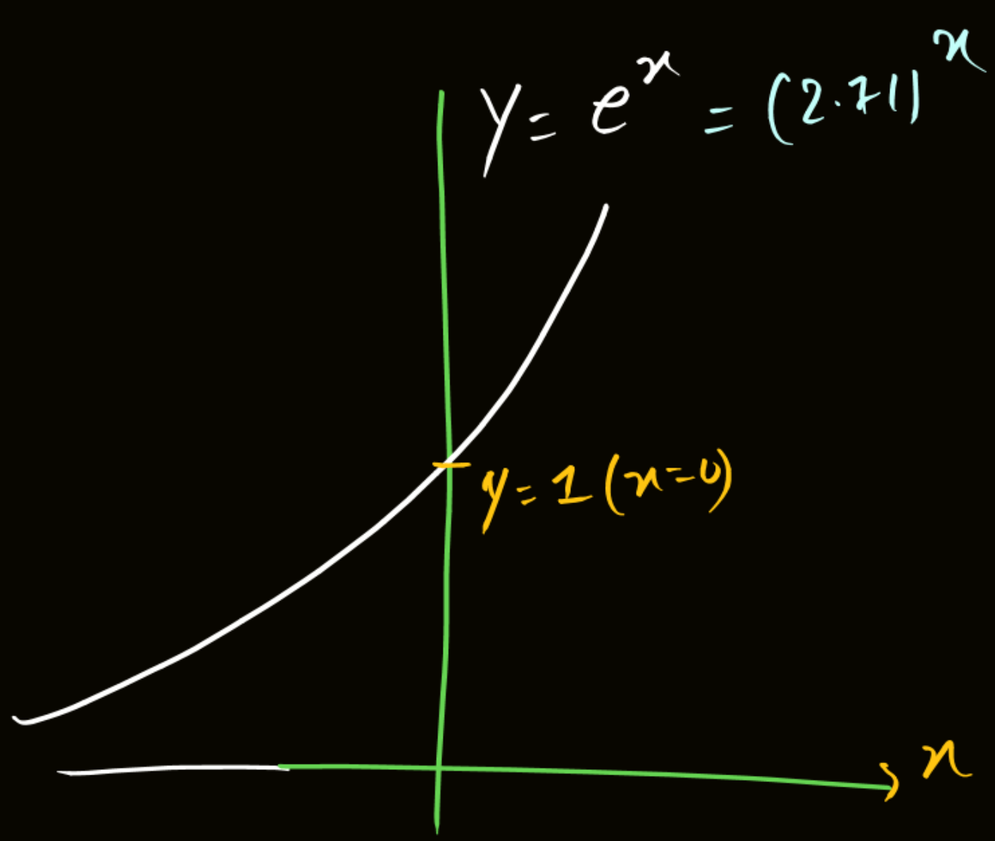
$y = -(x)^2$

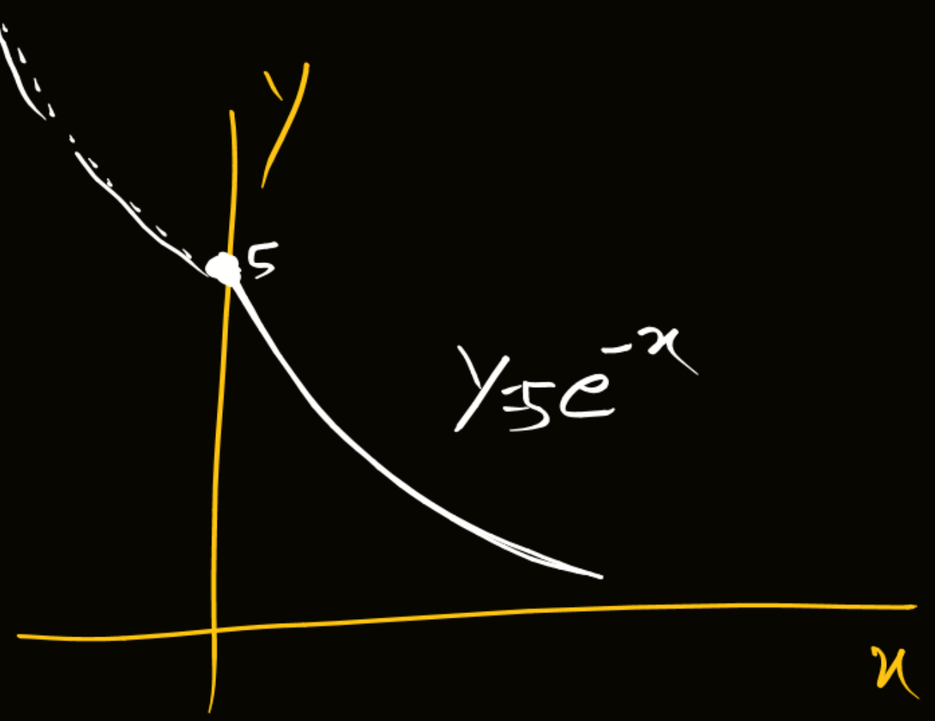


$y = \sqrt{x} = x^{1/2}$



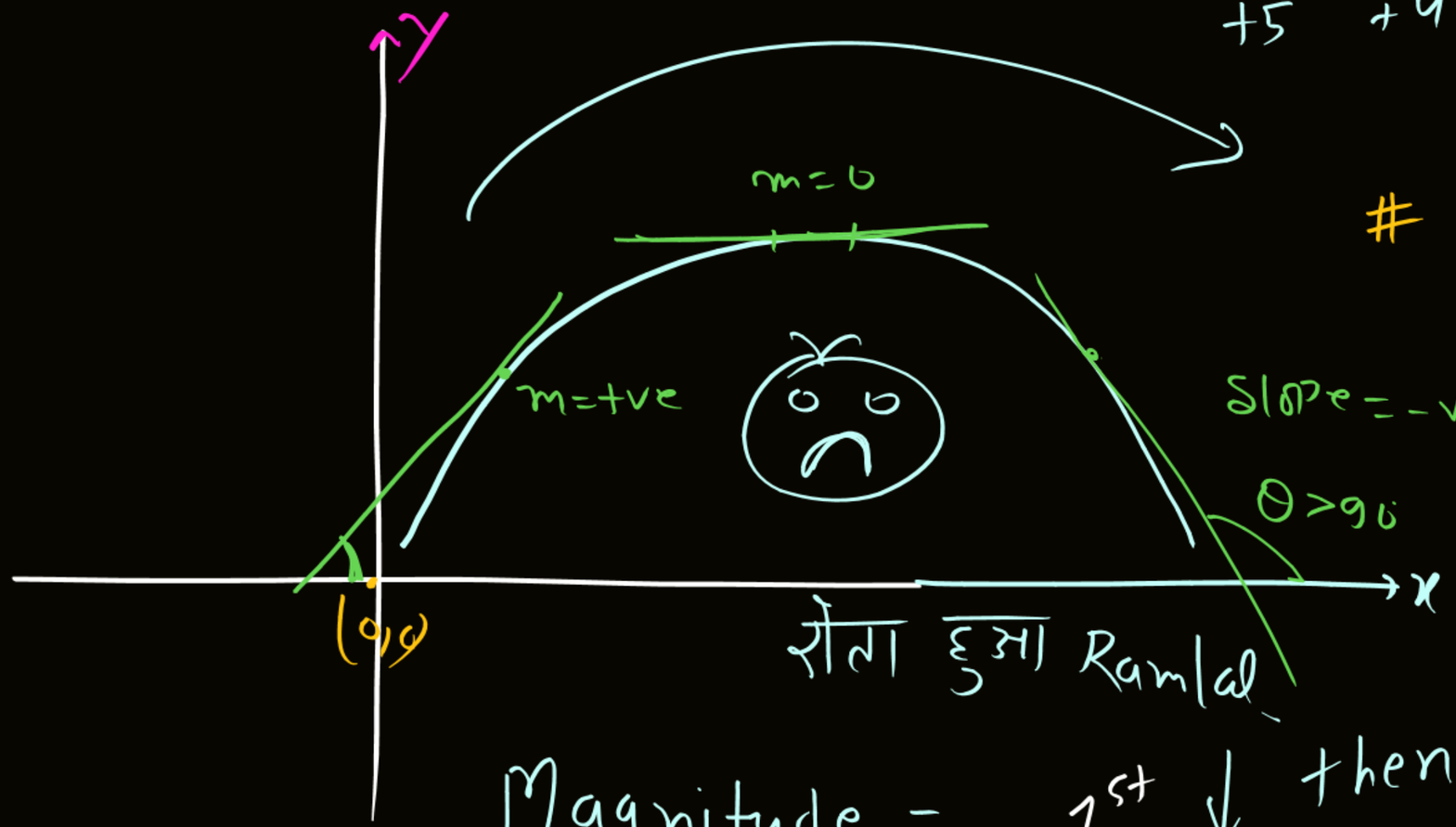

y
x
$y = \frac{1}{x}$
y
$y = x^2$
Straight line
$y = mx + c$
$y = x$
$y = \sqrt{x}$
x = 1
x


$y = e^x = (2.71)^x$
$y = 1 \ (x = 0)$
x


y
5
$y = 5e^{-x}$
x

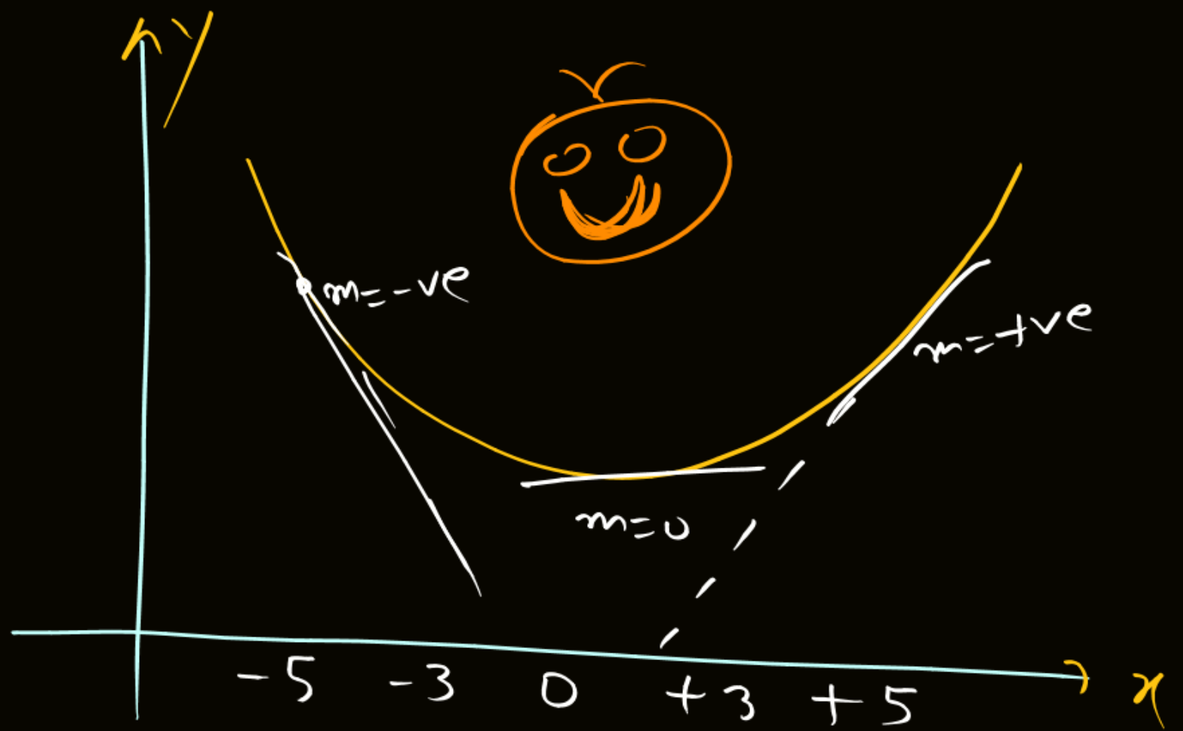
Slope

Slope ⇒
+5 +4 +2 0 −2 −3 −5



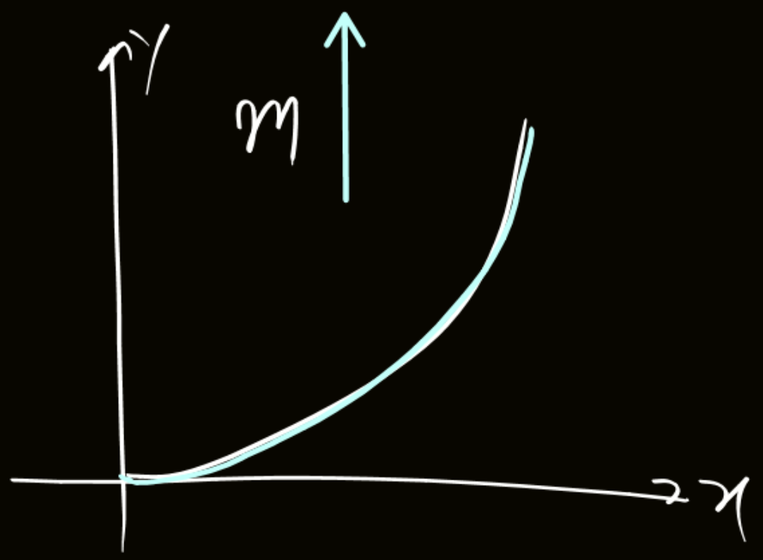
Slope is Continously decreasing

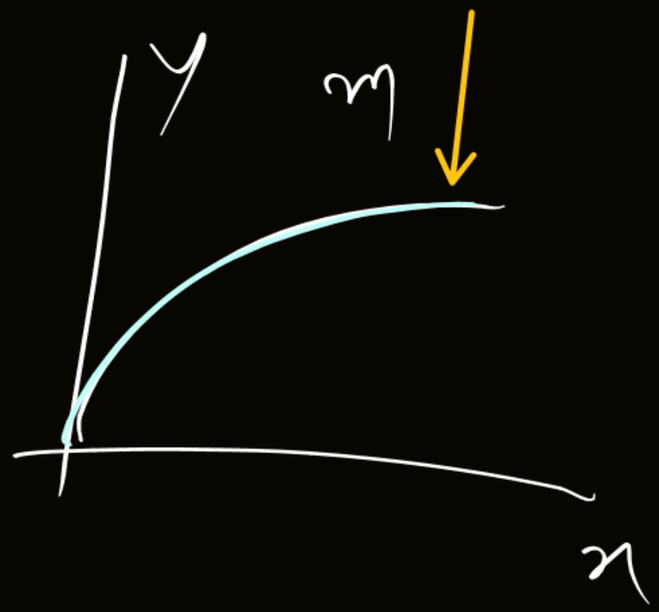
Magnitude = 1st ↓ then ↑
(+ve or -ve नहीं देखना)

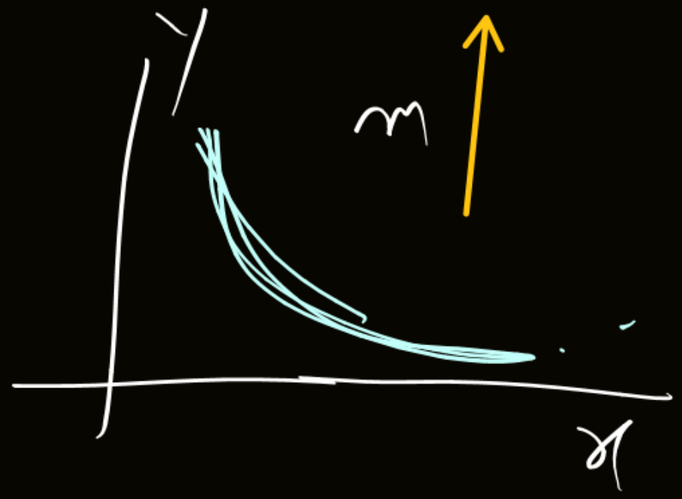


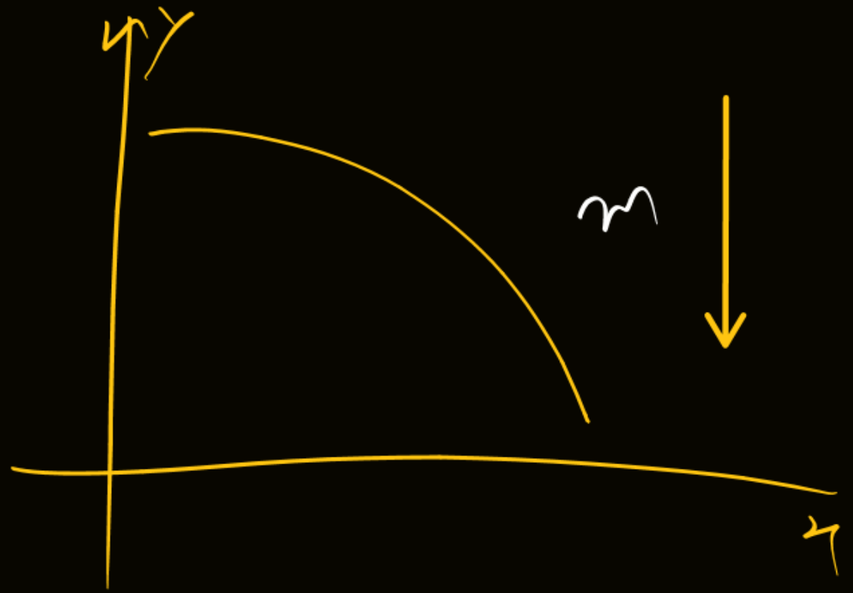
Slope => always Increasing

Magnitude = 1st ↓ then ↑


y
m
x


y
m
x


y
m
x


y
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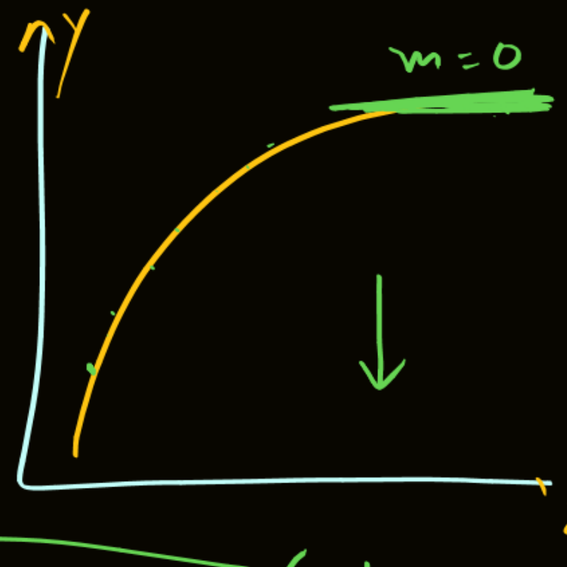
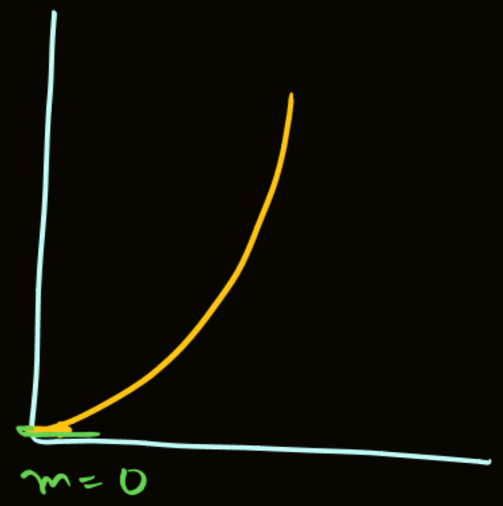
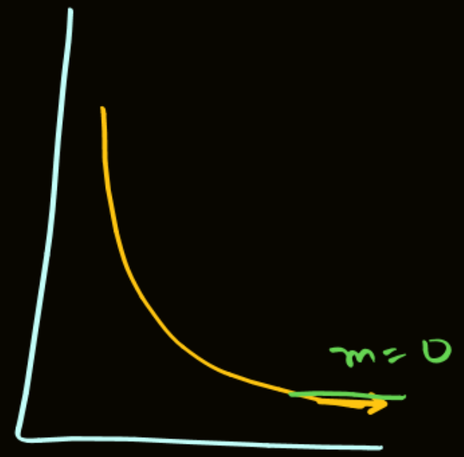
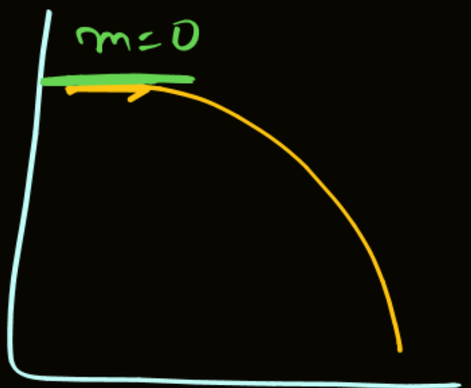


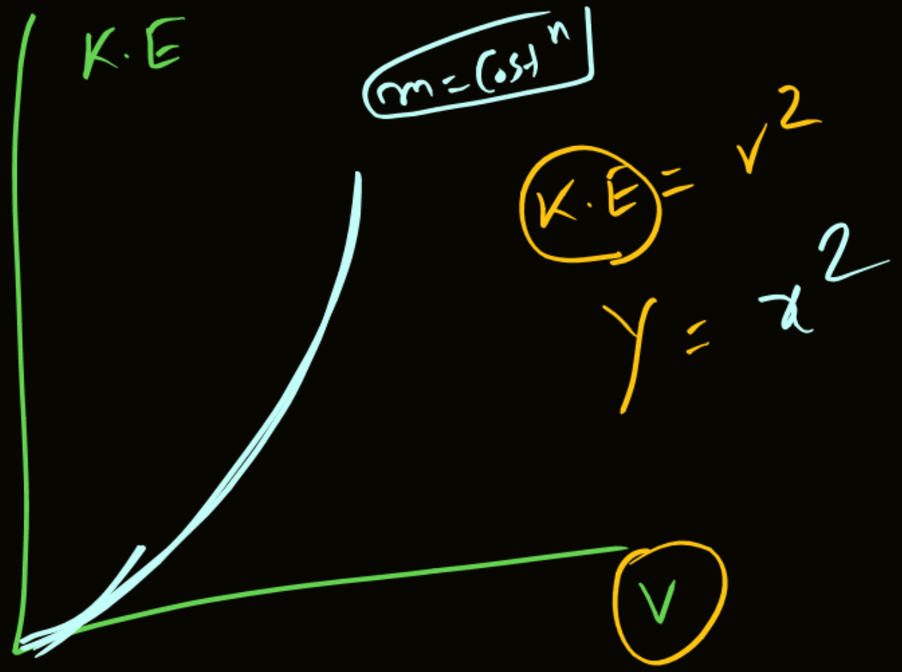






magnitude of slope.

$$K.E = \frac{1}{2} m v^2$$



Differentiation

$$\frac{d(\sin\theta)}{d\theta} = \cos\theta \qquad \frac{d\cos\theta}{d\theta} = -\sin\theta$$

$$\frac{d\tan\theta}{d\theta} = \sec^2\theta$$

$$\frac{d\sec\theta}{d\theta} = \sec\theta \cdot \tan\theta$$

$$\frac{de^x}{dx} = e^x$$

$$\frac{d\log x}{dx} = \frac{1}{x}$$

$$\frac{d\ln x}{dx} = \frac{1}{x}$$

$\frac{d\square}{dx}$ = The rate of change in $\square$ w.r.t. 'x'

$\frac{dy}{dx}$ = The rate of change in y w.r.t x

$v = \frac{d(x)}{dt}$ = The rate of chang in x (disp^{m}) w.r.t. time

$$\frac{dx^{n}}{dx} = n\,x^{n-1}$$

$$\frac{dt^{3}}{dt} = 3\,t^{3-1} = 3t^{2}$$

diff^{n} of constant

$$\frac{dG}{dt} = 0$$

$$\frac{d\pi}{dx} = 0$$

$$y = 5x^3$$

$$\frac{dy}{dx} = 5\frac{dx^3}{dx} = 5 \times 3\, x^{3-1}$$

$$= 15\, x^2$$

$$y = \frac{1}{x} = x^{-1}$$

$$\frac{dy}{dx} = \frac{dx^{-1}}{dx} = -1\, x^{-1-1}$$

$$= \frac{-1}{x^2}$$

$$\# \quad y = \frac{2}{x^2} = 2x^{-2}$$

$$\# \quad \frac{dy}{dx} = 2\frac{dx^{-2}}{dx}$$

$$= 2(-2)\, x^{-2-1}$$

$$= \frac{-4}{x^3}$$

Addition

$$Y = A + B$$

$$\frac{dY}{dx} = \left(\frac{dA}{dx}\right) + \left(\frac{dB}{dx}\right)$$

Subⁿ

$$Y = A - B$$

$$\frac{dY}{dx} = \frac{dA}{dx} - \frac{dB}{dx}$$

multiplica

$$Y = (A \cdot B)$$

$$\frac{dY}{dx} = \frac{dA}{dx}(B) + A\frac{dB}{dx}$$

Division.

$$Y = \frac{A}{B}$$

$$\frac{dY}{dx} = \frac{\left(\frac{dA}{dx}\right)B - A\frac{dB}{dx}}{(B)^2}$$

$$y = x^2 e^x$$

$$\frac{dy}{dx} = 2x(e^x) + x^2(e^x)$$

Chain Rule

MR^{V+} (Outside - Inside Rule)

(diff^n of outer function
Keep Inside as it is) $\times$

diff^n of Inner function.

outer ↓ Inner ←

$$Y = \sin(2x)$$

Derivative (differentiation)

$$\frac{dy}{dx} = \cos(2x) \times \frac{d(2x)}{dx}$$

$$= 2\cos(2x)$$

$$Y = \sin(x^2)$$

$$\frac{dy}{dx} = \cos(x^2)\,\frac{dx^2}{dx} = \cos(x^2)\,2x$$

$$\frac{dx^2}{dx} = 2x$$

\# $Y = \sin^2 x = (\sin x)^2$

Inner ↓ Outer

$$\frac{dy}{dx} = 2(\sin x)^{2-1} \times \frac{d\sin x}{dx}$$

$$= 2\sin x \cos x$$

$$\frac{dy}{dx} = \sin(2x)$$

$$y = e^{(2x)}$$

$$\frac{dy}{dx} = e^{(2x)} \times \frac{d(2x)}{dx}$$

$$= 2e^{(2x)}$$

$$y = e^{(x^2)}$$

$$\frac{dy}{dx} = e^{(x^2)} \times 2x$$

$$y = (x^2+1) \cdot \left(x^2 + 5 + \frac{1}{x}\right)$$

$$\frac{dy}{dx} = (2x)\left(x^2+5+\frac{1}{x}\right) + (x^2+1)\left(2x - \frac{1}{x^2}\right)$$

$$y = A \sin(Kx)$$

find double derivative w.r.t x

K and A are Costant.

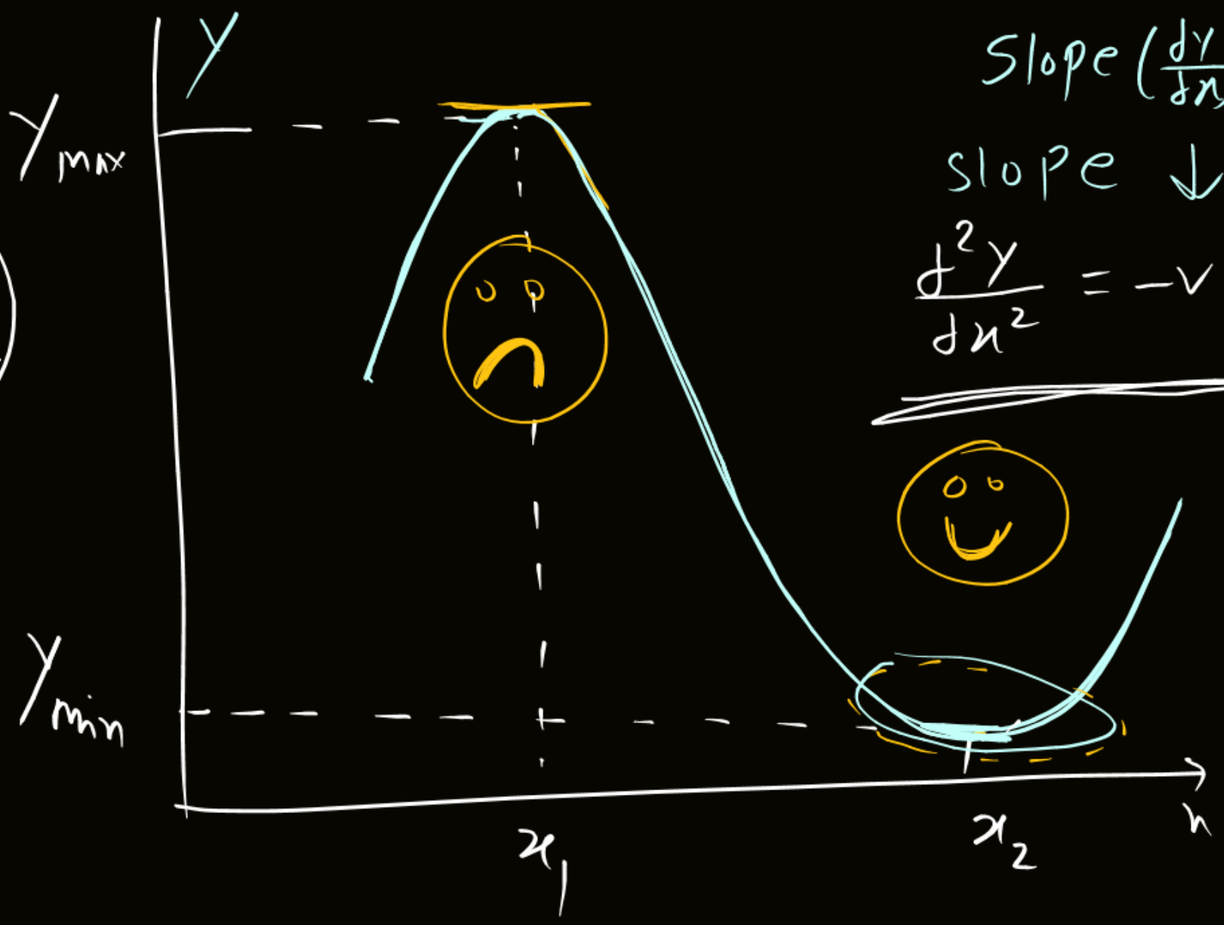
$$\frac{dy}{dx} = A \frac{d \sin(Kx)}{dx} = A \cos(Kx) \times K = AK \cos(Kx)$$

$$\frac{d^2y}{dx^2} = AK\left[-\sin(Kx)\right] \times K$$

$$\therefore \frac{d^2y}{dx^2} = -AK^2 \sin(Kx)$$

max^m & min^m.

diff^n (slope of graph)



Maxima

Slope $\left(\frac{dy}{dx}\right) = 0$

slope ↓

$\frac{d^2y}{dx^2} = -ve$

minima

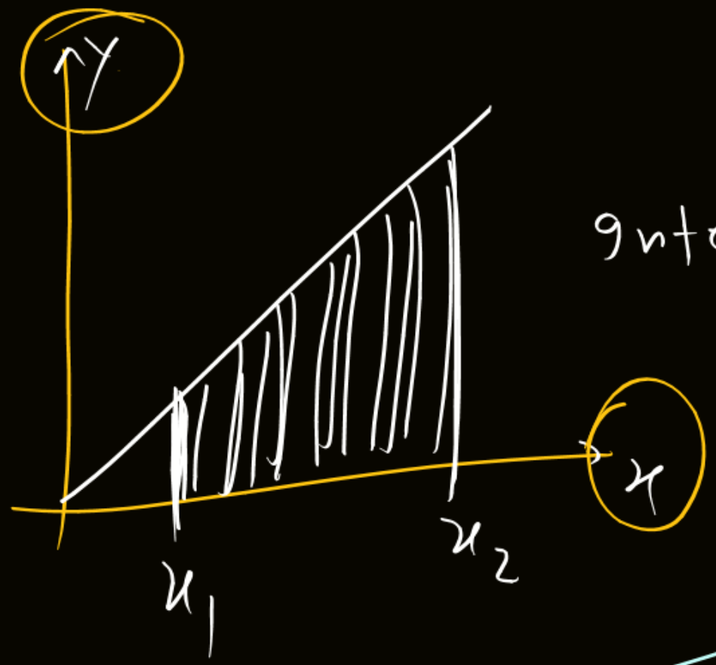
Slope $\left(\frac{dy}{dx}\right) = 0$

Slope ↑

$\frac{d^2y}{dx^2} = +ve$

Integration

→ Area under the Cure



Integral

$$\int_{x_1}^{x_2} Y\,dx = \text{Area}$$

$\frac{dy}{dx}$ = slope

$$\int \sin\theta \, d\theta = -\cos\theta$$

$$\int \cos\theta \, d\theta = \sin\theta$$

$$\# \int e^x \, dx = e^x$$

$$\int x^n \, dx = \frac{x^{n+1}}{(n+1)}$$

$$y = 3x^2 + 4$$

$$\int y \, dx = \int (3x^2 + 4) \, dx$$

$$= 3\int x^2 \, dx + 4\int dx$$

$$= x^3 + 4x$$

A-2

$S = 4\pi r^2$

then find relation b/w $\frac{ds}{dt}$ & $\frac{dr}{dt}$

Diff^n w.r.t (r)

$$\Rightarrow \frac{ds}{dr} = 4\pi \frac{d(r^2)}{dr}$$

$$\frac{ds}{dt} = 4\pi 2r \frac{dr}{dt} = 8\pi r \frac{dr}{dt}$$

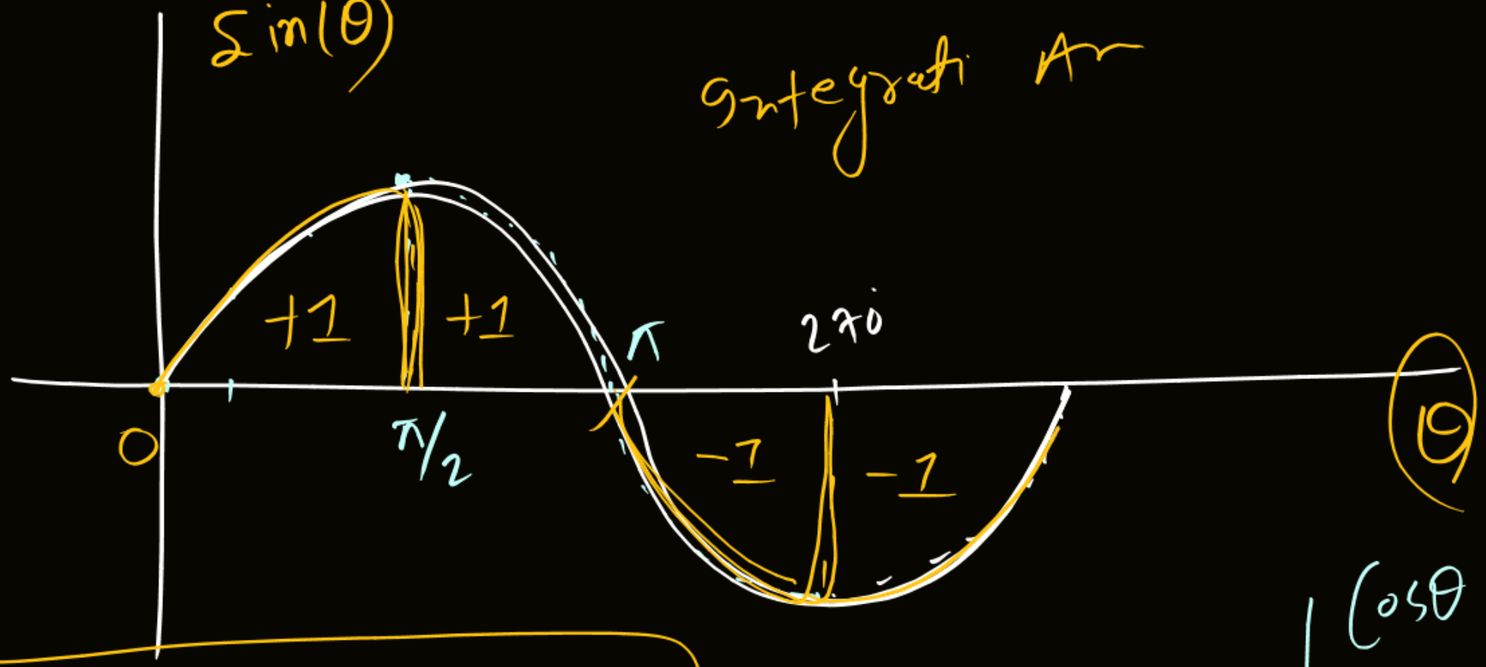
$$\int_0^{\pi/2} \sin\theta \, d\theta = \left[-\cos\theta \right]_0^{\pi/2}$$

mR^2

$$= \cos(0^\circ) - \cos(\pi/2)$$

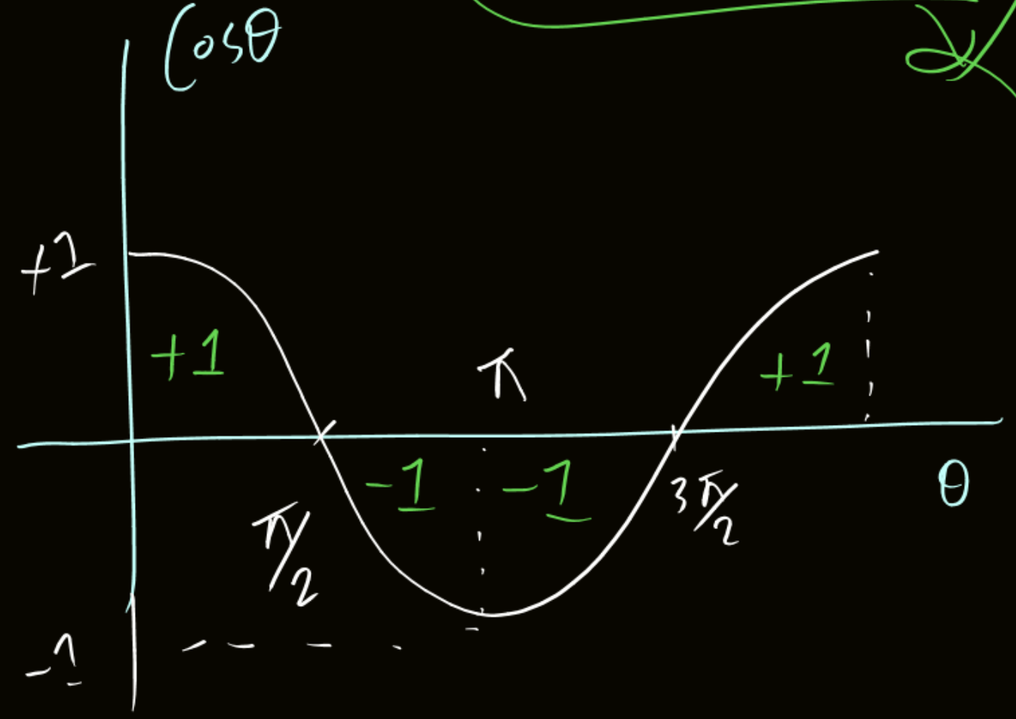
$$= 1$$ Ans

$$\int_0^{\pi} \cos\theta \, d\theta = 0$$



$$\int_0^{\pi} \sin\theta \, d\theta = +2$$

$$\int_{\pi/2}^{\pi} (\cos\theta) \, d\theta = -1$$



thanks
for watching

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