ARJUNA NEET BATCH

LAWS OF MOTION

LECTURE - 4

To Days Goal

questio'n's on newtons 2^{nd} Law

Newton's 3^{rd} Law ✓

Conservation of momentum ✓ (Basic)

Gun bullet system ✓

Rocket Prob^{m} ✓

Connected body system ✓

Newton's 2nd Law

Rate of change in momentum w.r.t. time is called force

$$\boxed{\vec{F}_{Avg} = \frac{\vec{\Delta p}}{\Delta t}}$$

#

$$\vec{F}_{inst} = \frac{d\vec{P}}{dt}$$ (slope of momentum-time graph is called force)

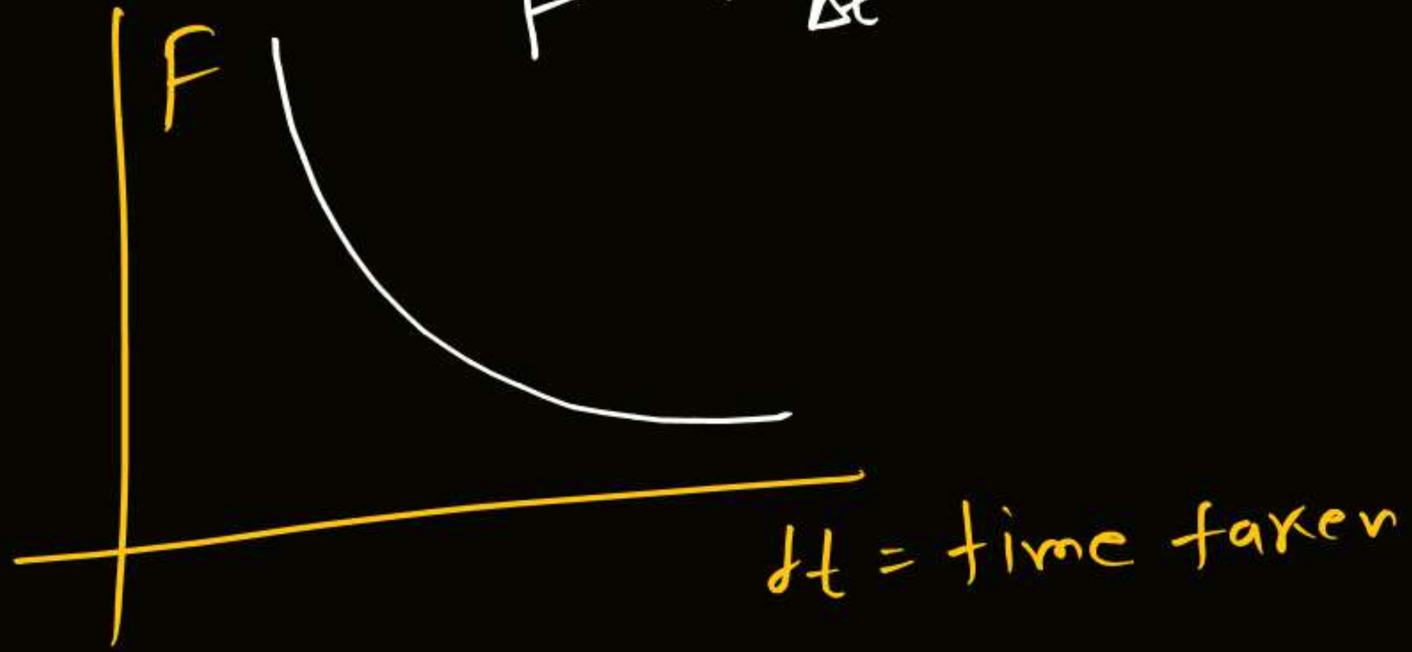
↳ vector (direction force is along change in momentum)

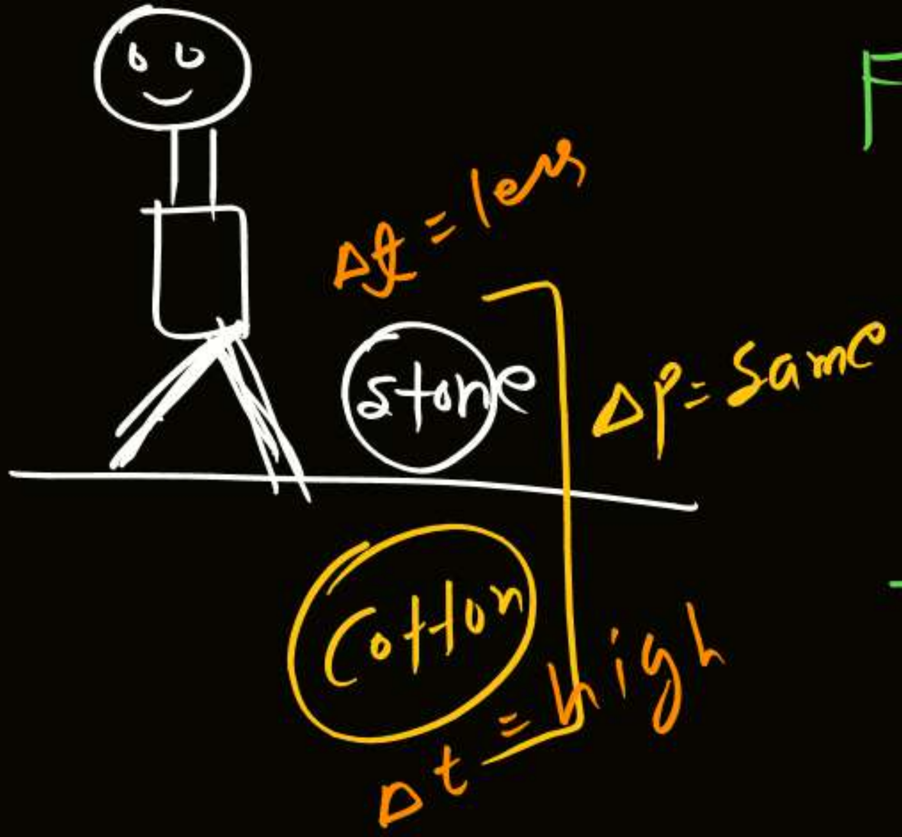
$\int F\,dt = \int dP$ = change in momentum = Area of force time graph

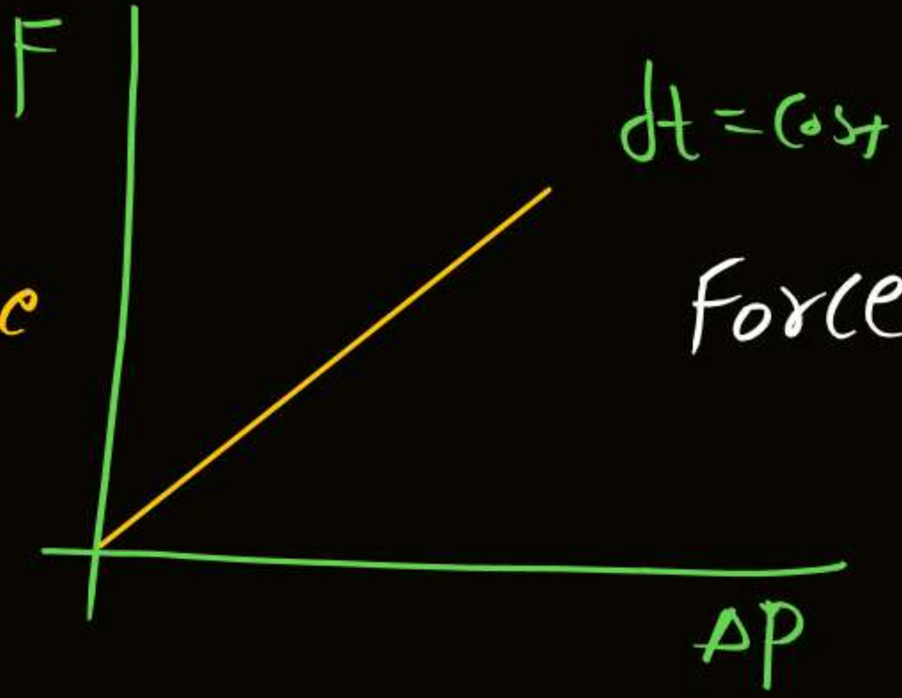
$$\vec{F} = \frac{d\vec{P}}{dt}$$

$dP = \text{Cost}^n$

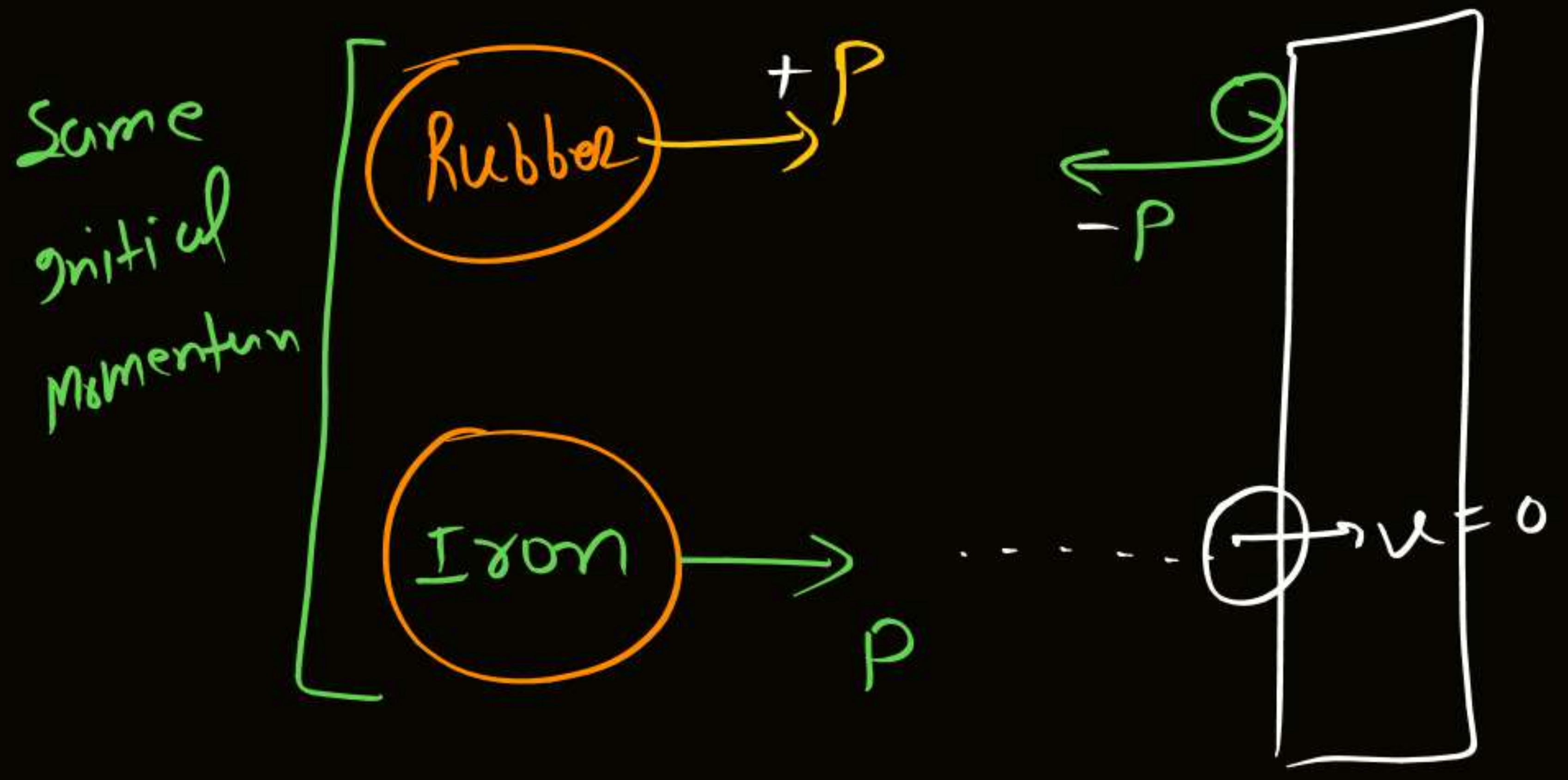
$F \propto \frac{1}{\Delta t}$







Force $\propto dp$



Force on Surface is more by

In case of Iron

$$\vec{F}_{Iron} = \frac{\Delta p}{\Delta t} = \frac{0 - p}{\Delta t} = \frac{-p}{\Delta t}$$

In Case of Rubber

$$\vec{F}_{Rub^r} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{-p - (p)}{\Delta t} = -\frac{2p}{\Delta t}$$

$$\boxed{\vec{F} = \frac{d\vec{P}}{dt}}$$

Newton's
2nd Law

$$\vec{P} = m\vec{v}$$

$$\vec{F} = \frac{d(m\vec{v})}{dt}$$

$$\boxed{\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}}$$

Case- 1

If mass = Constⁿ

$$\left(\frac{dm}{dt}\right) = 0$$

$$\boxed{\vec{F} = m\vec{a}}$$ → this is not a newton's 2nd law

Case -2

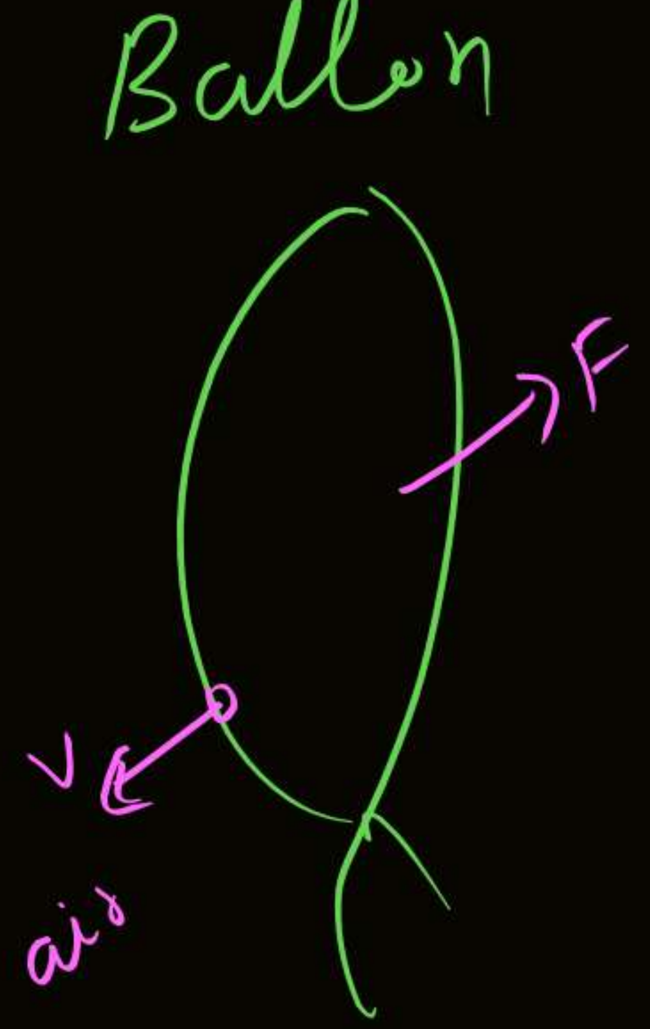
$\vec{V}$ = Constⁿ; m = variable, then

Force ≠ 0

$$\vec{F}_{ma} = \vec{v}\frac{dm}{dt}$$

$$\boxed{\vec{F}_{Avg} = V\frac{\Delta m}{\Delta t}}$$ ✓

$\boxed{\text{If } \vec{V} = \text{Const}^n\text{; F must be zero}}$ ✗ True



Rocket Prop^{n}

Example of variable mass

System

m = variable, $\vec{v}$ = const

$$\vec{F} = \vec{v}\frac{dm}{dt}$$

A cricketer catches a ball of mass 150 g in 0.1 s moving with speed 20 m/s, then the experiences force of

(a) 300 N

(b) 30 N

(c) 3 N

(d) 0.3 N

$$\left(\vec{Force}\right)_{\text{ग़स}} = \frac{dp}{dt}$$

$$\vec{F}_{Avg} = \frac{\Delta P}{\Delta t} = \frac{0 - \frac{150}{1000} \times 20}{0.1} = \frac{15 \times 2 \times \cancel{10}}{\cancel{10}}$$

ARJUNA

A force of 6 N acts on a body at rest and of mass 1 kg. During this time, the body attains a velocity of 30 m/s. The time for which the force acts on the body is

(a) 7 second
(b) 5 second
(c) 10 second
(d) 8 second

$F = 6\,N$ $\quad \vec{P_i} = 0$ $\quad m = 1\,kg$

$V_F = 30\,m/s$

$\vec{F}_{Avg} = \dfrac{\vec{P_F} - \vec{P_i}}{\Delta t} = \dfrac{30 - 0}{\Delta t}$ $\quad \vec{P_F} = mV_F = 30\,kg\,m/s$

$6 = \dfrac{\cancel{30}\,5}{\Delta t}$

$\boxed{\Delta t = 5\,sec}$

A body of mass 3 kg is acted on by a force which varies as shown in the graph below. The momentum acquired is given by :

(a) Zero (b) 5 N-s

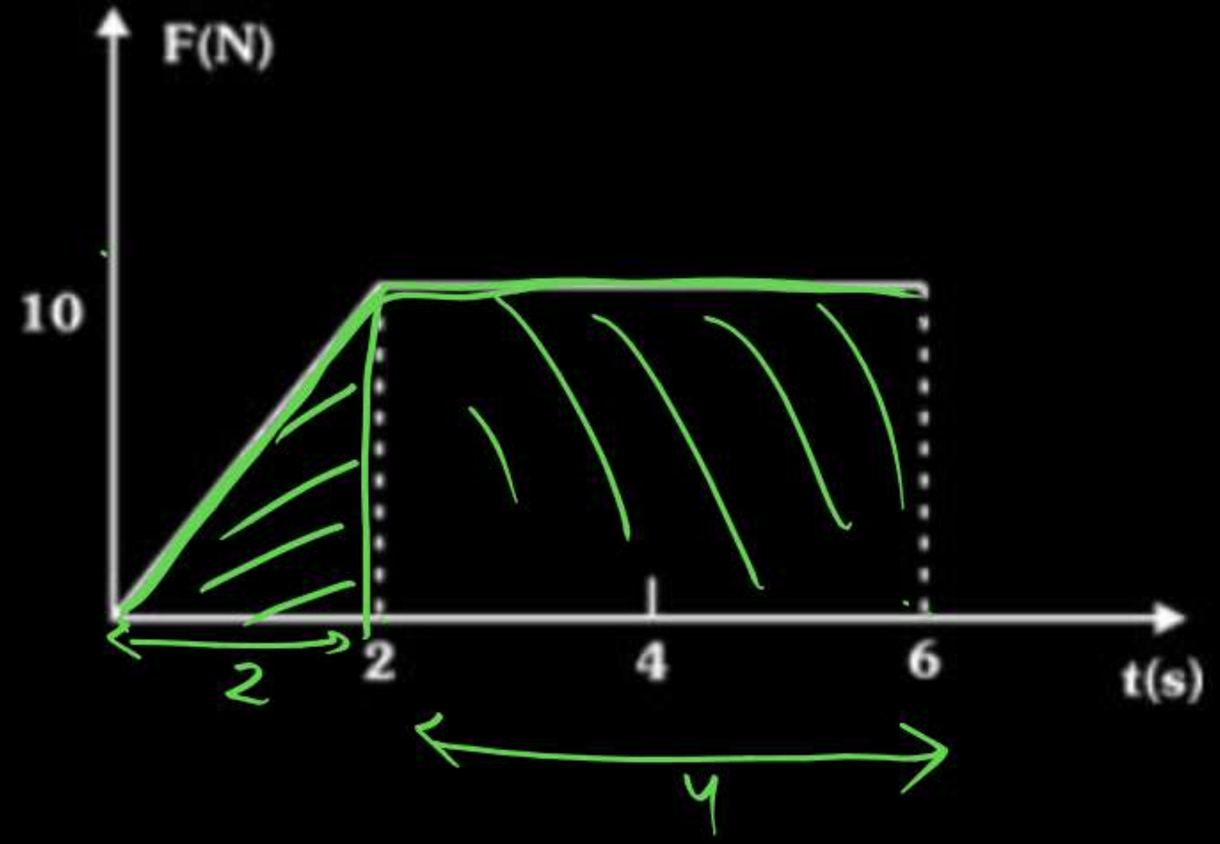
(c) 30 N-s (d) 50 N-s

$\Delta P =$ Area of F-t graph

$= \frac{1}{\cancel{2}} \times 10 \times \cancel{2} + 4 \times 10$

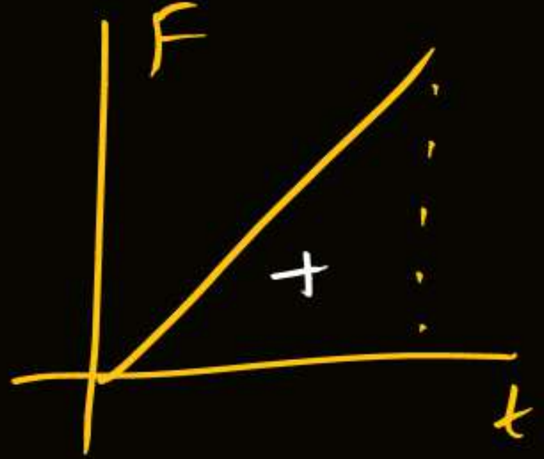
$= 10 + 40$

$= 50$ N-s

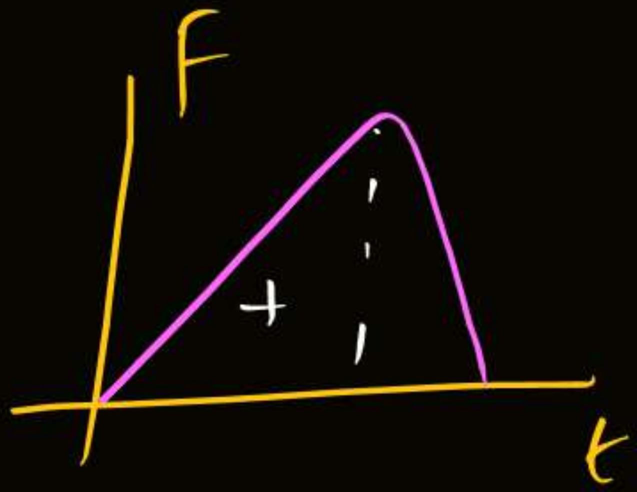


(Q) In which Graph Change in Momentum is zero

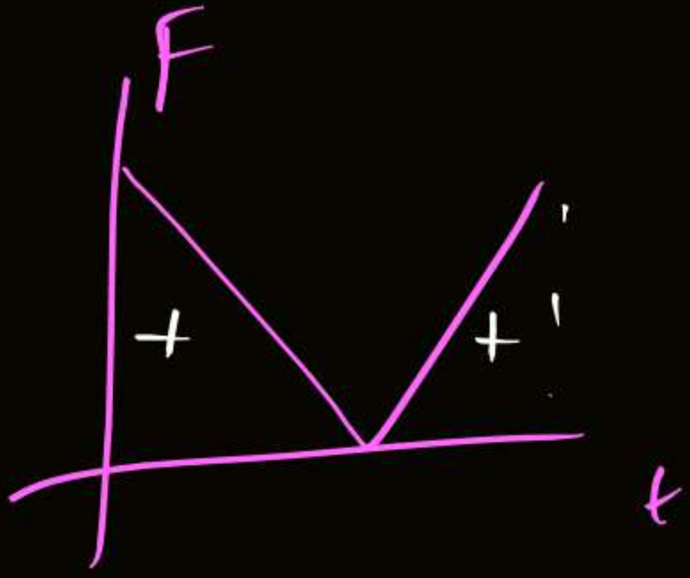
(a)



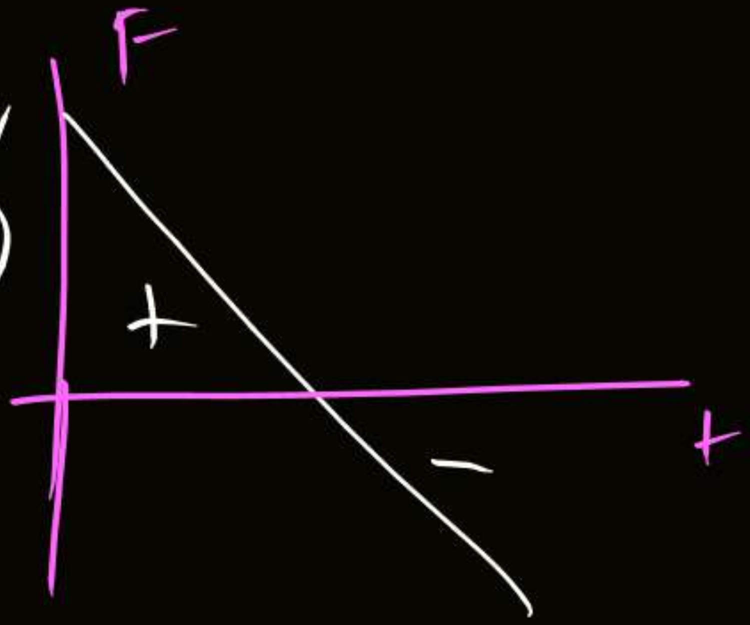
(b)



(c)



✓(d)



Sol^n

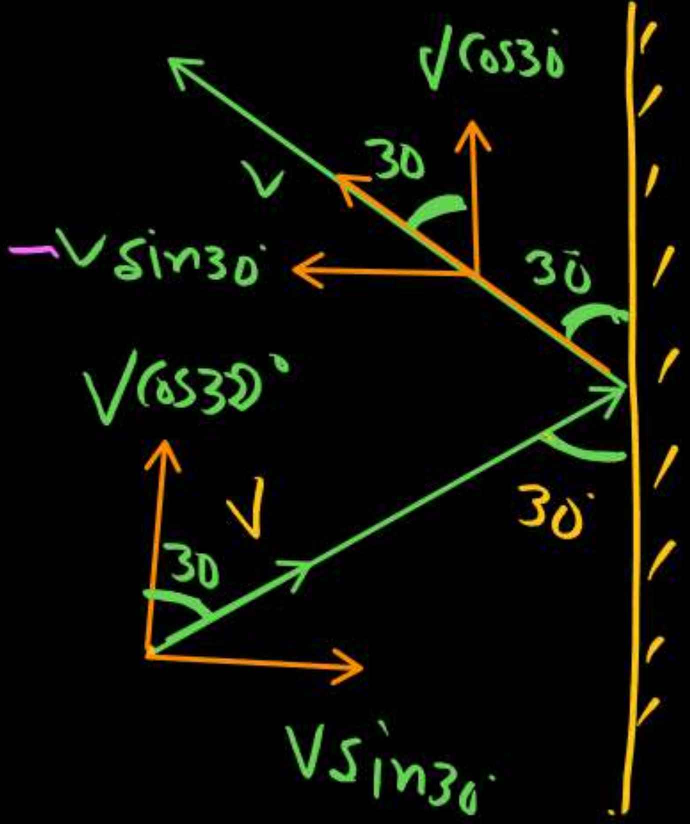
Area of force-time graph is change in momentum

$$\boxed{\Delta p \propto \text{Area}}$$

A 0.5 kg ball moving with a speed of 12 m/s strikes a hard wall at an angle of 30° with the wall. It is reflected with the same speed and at the same angle. If the ball is in contact with the wall for 0.25 s, the average force acting on the wall is: [AIPMT (Prelims)-2006]

(a) 48 N
(b) 24 N
(c) 12 N
(d) 96 N

Sol^n

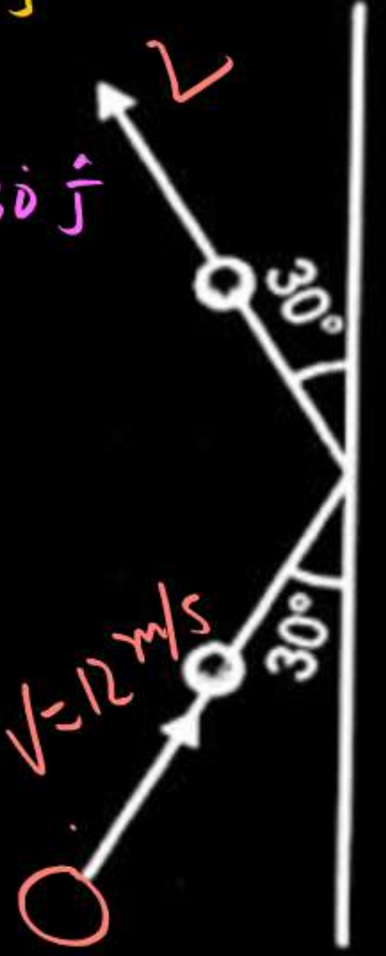


$\vec{P_i} = mv\sin 30\,\hat{i} + mv\cos 30\,\hat{j}$

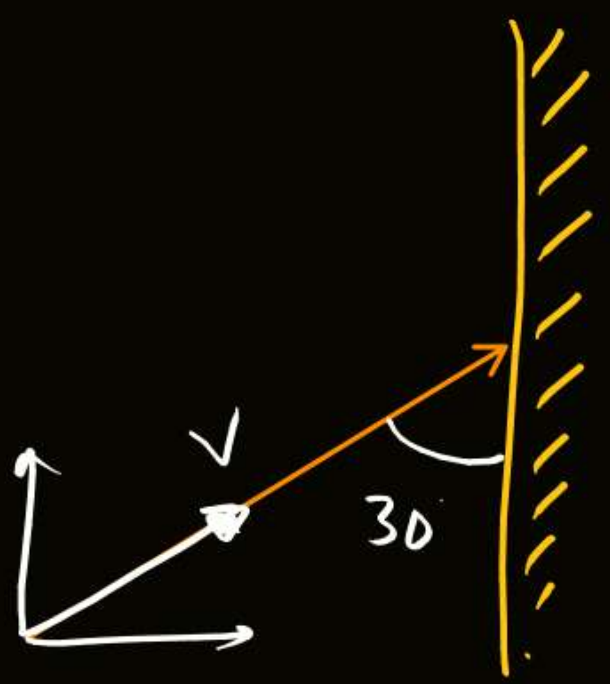
$\vec{P_f} = -mv\sin 30\,\hat{i} + mv\cos 30\,\hat{j}$

$\vec{\Delta P} = \vec{P_f} - \vec{P_i}$

$\vec{\Delta P} = -mv\sin 30\,i + \cancel{mv\cos 30}\,\hat{j}$
$\quad - mv\sin 30\,i - \cancel{mv\cos 30}$



MR^{x}

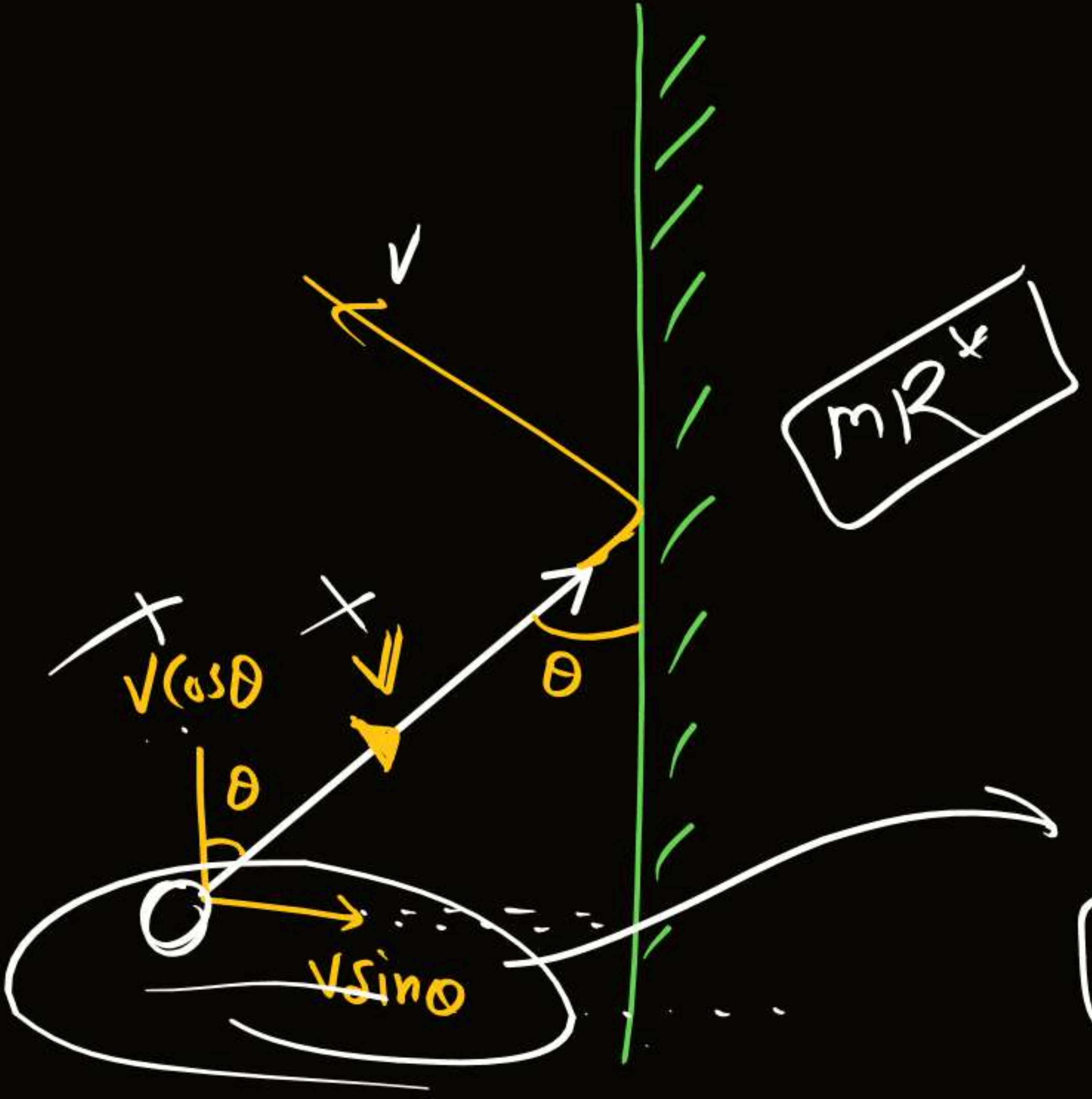


$$\vec{\Delta p} = -2mv\sin 30^\circ \, \hat{i}$$

$$|\vec{\Delta p}| = 2mv\sin 30^\circ$$

$$\vec{F}_{Avg} = \frac{\Delta p}{\Delta t} = \frac{2mv\sin 30^\circ}{\Delta t}$$

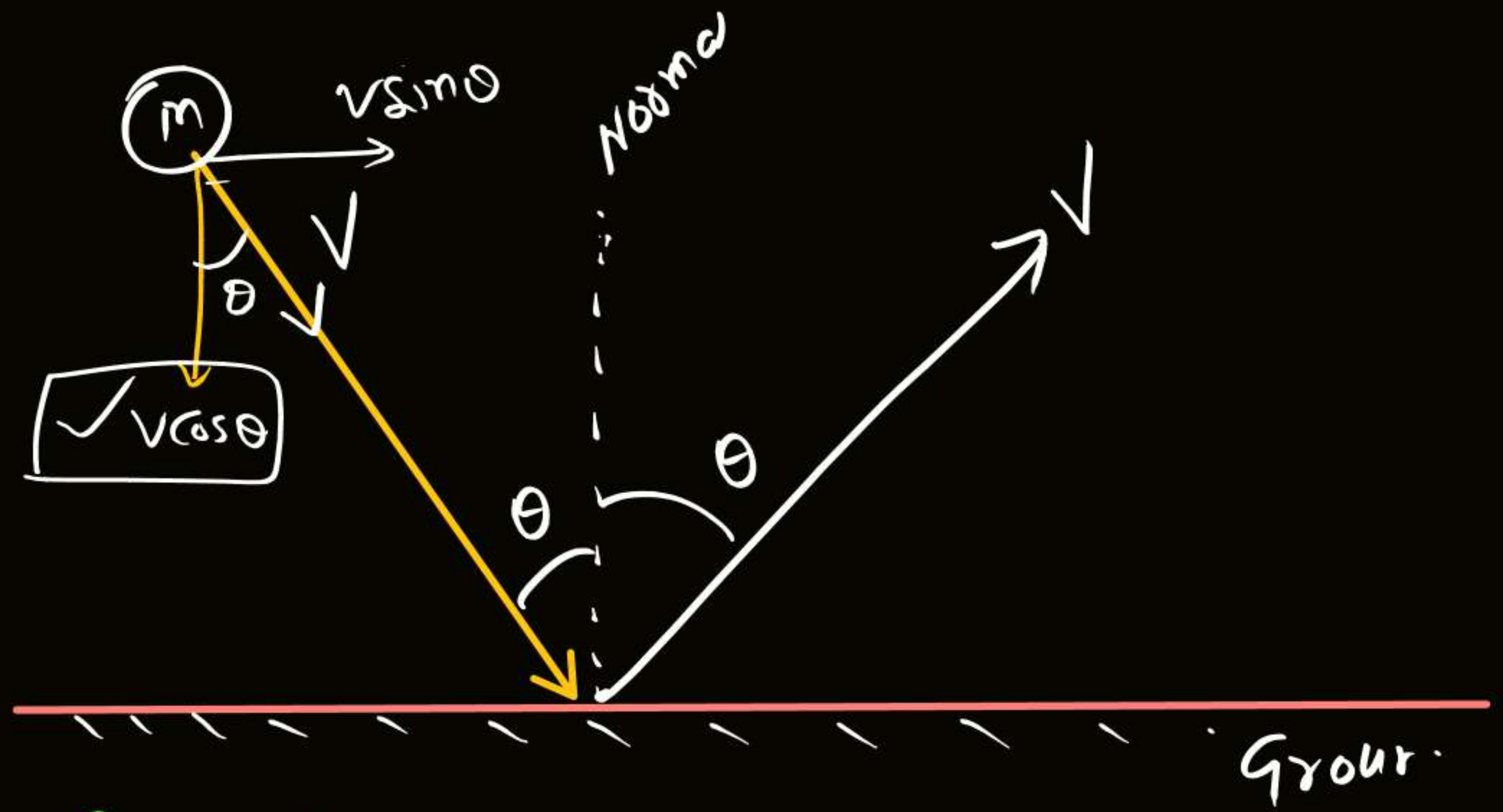
$$= \frac{2\times\cancel{\frac{1}{2}}\times 12\times\cancel{\frac{1}{2}}}{\cancel{\frac{1}{4}}} = 24\,N$$



$\Delta v = 2V\sin\theta$

$\boxed{\Delta P = 2mv\sin\theta}$

$F = \dfrac{2mv\sin\theta}{\Delta t}$ Ans



(Q) for given figure find change in momentum ??

$$\Delta P = 2mv\cos\theta$$

Δt = time of contact

$$\boxed{F = \frac{\Delta P}{\Delta t}}$$

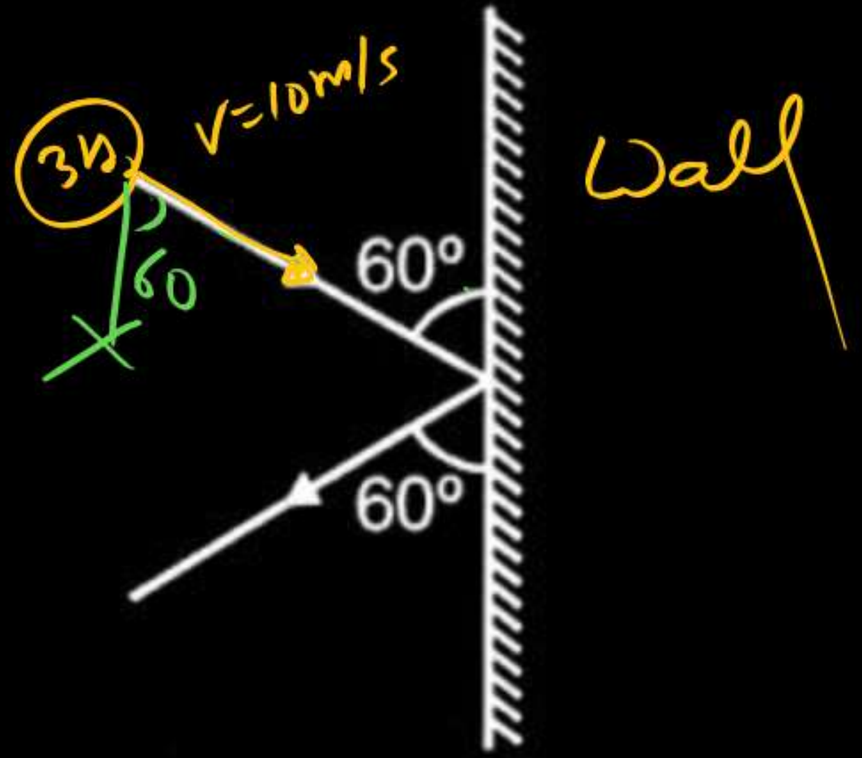
A body of mass 3 kg moving with velocity 10 m/s hits a wall at an angle of 60° and returns at the same angle. The impact time was 0.2 s. Calculate the force exerted on the wall.

(a) $150\sqrt{3}$ N
(b) $50\sqrt{3}$ N
(c) 100 N
(d) 75 N

$$\vec{F} = \frac{\Delta p}{\Delta t} = \frac{2mv\sin 60}{\Delta t}$$

$$= \frac{\cancel{2}\times 3\times 10\times \sqrt{3}}{\cancel{2}\times \frac{\cancel{2}}{\cancel{10}\,5}}$$

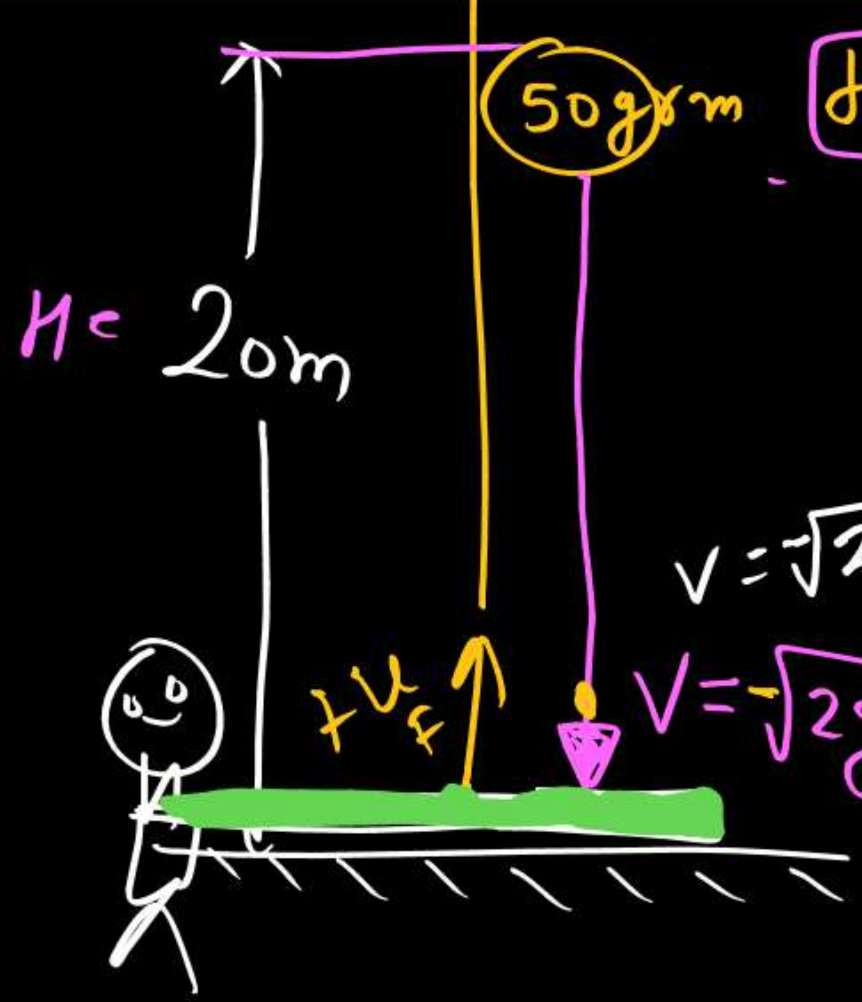
$$= 150\sqrt{3}$$



A ball of mass 50 g is dropped from a height of 20 m. A boy on the ground hits the ball vertically upwards with a bat with an average force of 200 N, so that it attains a vertical height of 45 m. The time for which the ball remains in contact with the bat is [Take $g = 10\ \text{m/s}^2$]

(a) $1/20^{\text{th}}$ of a second
(b) $1/40^{\text{th}}$ of a second
(c) $1/80^{\text{th}}$ of a second
(d) $1/120^{\text{th}}$ of a second

$H = 45m$



$\boxed{\text{drop } (v=0)}$

$v = -\sqrt{2\times10\times20} = -20$

$V = -\sqrt{2gH}$ (Just before Collision)

$\vec{P_i} = -mV$
$= -m(20)$ ✓

$\vec{V_F}$ (Just after collision) = ??

$H = \frac{v^2}{2g}$ $\quad V_F = \sqrt{2gH}$
$= \sqrt{2\times10\times45}$

$\vec{V_F} = +30$ m/s

$m\vec{v_F} = m30$

$$F = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

$$\Delta t = \frac{30m - (-20m)}{F} = \frac{\boxed{\frac{5\cancel{0}}{1000}} \times 5\cancel{0}}{\times 2\cancel{0}\cancel{0}}$$

$$\boxed{\Delta t = \left(\frac{1}{80}\right) sec}$$

$\boxed{50\,gm = 50\left(\frac{1}{1000}\right)kg}$

$$\boxed{S_{1s} : S_{2s} : S_{3s} = 5 : 20 : 45}$$

20m/s

30m/s

A stone of mass 1 kg is thrown with a velocity of 20 m/s across the frozen surface of a lake and it comes to rest after travelling a distance of 50 m. What is the magnitude of the force opposing the motion of the stone?

50m

$v_f = 0$

$u_i = 20\,m/s$

$$v_f^2 - v_i^2 = 2as$$

$$0 - (20)^2 = 2a \times 50$$

$$\vec{a} = \frac{-4\cancel{00}}{1\cancel{00}} = -4\,m/s^2$$

$$\vec{F} = ma$$

$$= 1(-4)$$

$$\vec{F} = -4\ \text{Newt}$$

$$|\vec{F}| = 4\ \text{Newton}$$

ARJUNA

A disc of mass 1.0 kg kept floating horizontally in air by firing bullets of mass 0.05 kg each vertically at it, at the rate of 10 per second. If the bullets rebound with the same speed, the speed with which these are fired will be-

(a) 0.098 m/s
(b) 0.98 m/s
(c) 9.8 m/s
(d) 98.0 m/s

$\downarrow \vec{F}_{on\ bullet} = \vec{F}_{on\ Disc} \uparrow$

F.B.D OF Disc

rest

$\uparrow F = \frac{2nmv}{t} = Mg$

$\downarrow Mg$

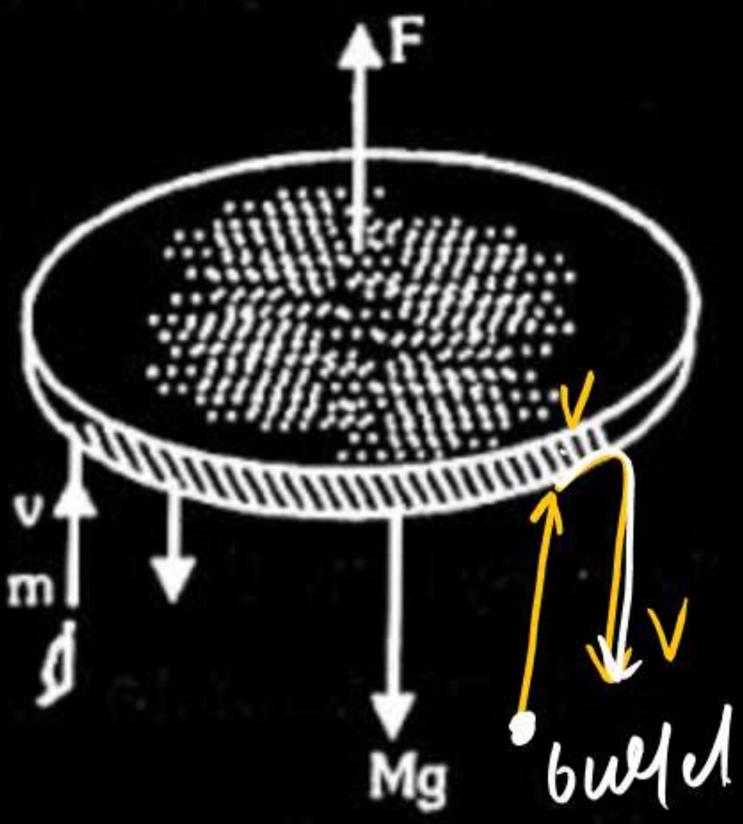
$\frac{2\times 10\times 5\, v}{100\times 1} = 1\times 10$

$\boxed{V = 10\ m/s}$

$(\vec{\Delta P})_{1\,bullet} = m\Delta V = 2mV$

$(\vec{\Delta P})_{n-bullet} = 2nmV$

$\vec{F}_{bullet} = \frac{\Delta P}{\Delta t} = \frac{2nmV}{t}$



A force $\vec{F} = (2t\hat{i} + 3t^2\hat{j})N$ acts on an object moving in xy plane. Find magnitude of change in momentum of the object in time interval $t = 0$ to $t = 2s$

$$\vec{F} = \underbrace{2t}\,\hat{i} + \underbrace{3t^2}\,\hat{j}$$

$$\frac{dp}{dt} = 2t\,\hat{i} + 3t^2\,\hat{j}$$

$$\int dp = \int_0^2 2t\,dt\,\hat{i} + \int_0^2 3t^2\,dt\,\hat{j}$$

$$\Delta p = (t^2)\,\hat{i} + (t^3)\,\hat{j} = \boxed{4\hat{i} + 8\hat{j}} = \sqrt{16+64} = \sqrt{80} = 9\ \text{kg m/s}$$

IMPULSE

- **Impulse** is defined as the change in momentum. It is measured as the product of the average force and time for which the force acts. It is a vector quantity directed along the direction of force.

$$\vec{I}_{Avg} = \Delta \vec{P}$$

$$\vec{I}_{int} = d\vec{P}$$

$$\Delta \vec{P} = \vec{P}_F - \vec{P}_i$$

$$= m\vec{v}_F - m\vec{v}_i$$

$$\vec{I} = m(\vec{v}_F - \vec{v}_i)$$

fugri

$$\vec{F} = \frac{dP}{dt}$$

$$\int dP = \int \vec{F} \cdot dt$$

$$\boxed{I = \int \vec{F} \cdot dt} = \text{Impule}$$

ARJUNA

The momentum p (in kg m/s) of a particle is varying with tie t (in s) as $p = 2 + 3t^2$. The force acting on the particle at $t = 3$ s will be

(a) 18 N (b) 54 N

(c) 9 N (d) 15 N

$$\vec{P} = 2 + 3t^2$$

$$\vec{F} = \frac{dp}{dt} = \frac{d(2+3t^2)}{dt} = 0 + 3(2t)$$

$$\vec{F} = 6t$$

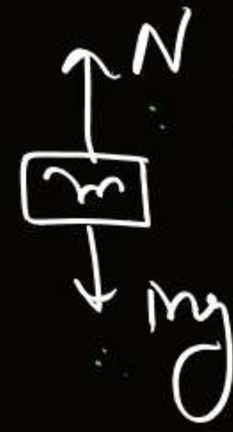
$$|\vec{F}|_{t=3} = 6 \times 3 = \underline{\underline{18\,N}}$$

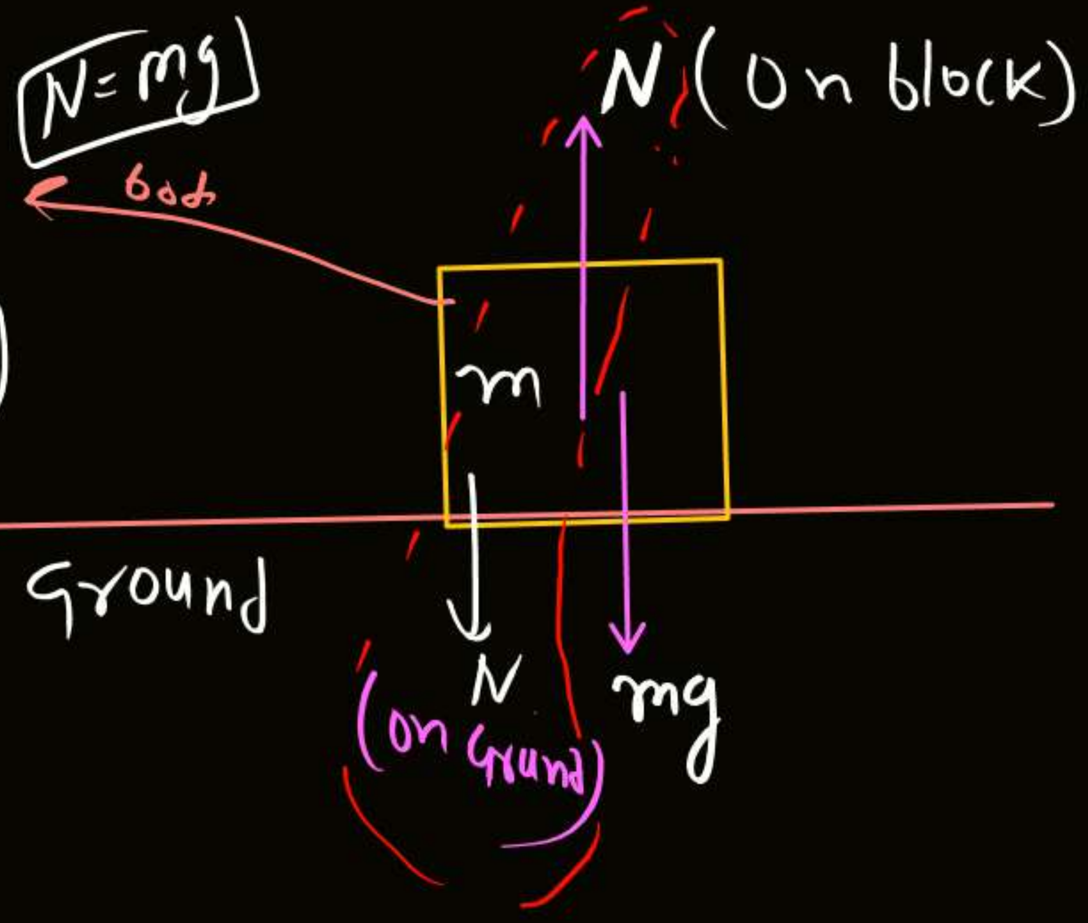
Newtons 3rd Law

↳ every Action have equal and opposite reaction. Action and reaction must be of same magnitud, opposite direction, on diffrent body at same time

Q) Normal reaction (N) and gravitational force (mg) forms action-reaction Pair??

Ans → No





Action - Reaction Pair → have same magnitude, opposite diretion at same time on different body. → $\boxed{\text{of a same nature}}$

MRT

SIGNIFICANCE OF NEWTON'S LAWS

- The first law talks about the natural state of motion of a body, i.e., motion along a straight line with constant speed ✓
- The second law says that if a body is not following it natural state of motion, then there has to be a net unbalanced external force acting on the body.
- The third law talks about the nature of the force, i.e., force exist in pairs

- Can we say that first law can be derived from second law? No, three laws are independent.
- Can we say that action occurs before the reaction? No, both occur at the same time.
- Can we say that action and reaction act on the same body? No, they always act on different bodies.

ARJUNA

If impulse I varies with time t as $I(\text{kg ms}^{-1}) = 20t^2 - 40t$. The change in momentum is minimum at

(a) $t = 2s$

(b) $t = 1s$

(c) $t = 1/2s$

(d) $t = 3/2\ s$

H.W

Conservation of Momentum {Introduction}

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

$$\boxed{\text{If } F_{ext} = 0}$$

$$\frac{d\vec{P}}{dt} = 0$$

$$\boxed{\vec{P}_{body/system} = \text{Const}^n}$$

$$\boxed{\vec{P}_i = \vec{P}_f}$$

$$\left.\begin{aligned} &\vec{P}_{initial} = 0 \\ &\vec{P}_f = \vec{P}_P + \vec{P}_R = 0 \end{aligned}\right\} \rightarrow \vec{F}_{ext} = 0$$

$$\vec{P}_R = -\vec{P}_{Photon}$$

Ramlal

$$\vec{F} = \frac{d\vec{P}}{dt}$$

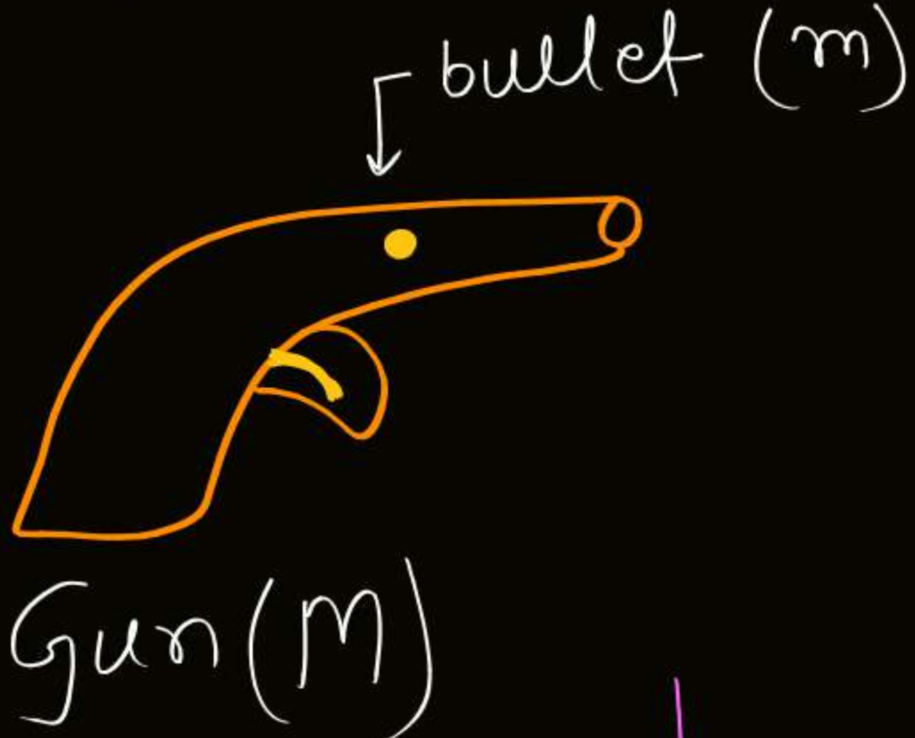
If $\vec{F}_{ext} = 0$ then

$$\frac{dP}{dt} = 0$$

$$\vec{P} = \text{Cost}^n$$

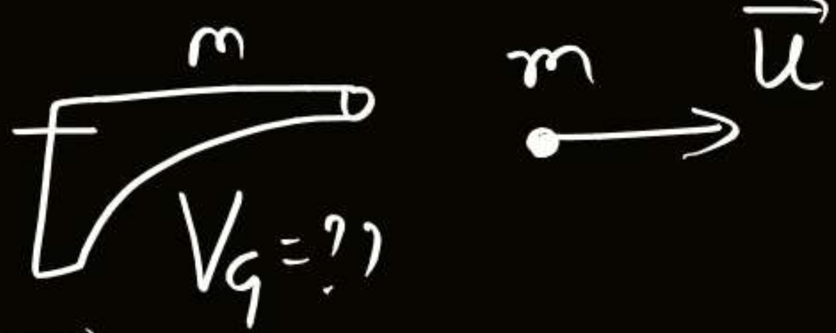
$$\vec{P_i} = \vec{P_F}$$ (Law of Conservation of Momentum)

Gun bullet Problem [find recoil velocity of Gun]



Initial

$\vec{P}_i = 0 + 0 = 0$



$\boxed{F_{ext} = 0}$

$\vec{P}_F = \vec{P}_B + \vec{P}_G$

Law of Conservation of Momentum

$\vec{P}_i = \vec{P}_f$

$0 = \vec{P}_B + \vec{P}_G$

$\vec{P}_G = -\vec{P}_B$

$M\vec{V}_G = -m\vec{u}_B$

V

$$\boxed{\vec{V_g} = \frac{m u_B}{M}}$$

→ recoil velocity of gun

#

rest

$\vec{P_i} = 0$

$$M\vec{u_g} = -m\vec{u_g}$$

A bullet of mass 50 g is fired from a gun with initial velocity of 35 m/s. If mass of the gun is 4 kg, then calculate the recoil velocity of the gun.

$$\vec{v}_g = \frac{m v_g}{M} = \frac{50 \times 35}{1000 \times 4}$$

A bullet of mass 40 g is fired from a gun of mass 10 kg. If velocity of bullet is 400 m/s, then the recoil velocity of the gun will be:

(a) 1.6 m/s in the direction of bullet

(b) 1.6 m/s opposite to the direction of bullet

(c) 1.8 m/s in the direction of bullet

(d) 1.8 m/s opposite to the direction of bullet

$\boxed{\vec{V}_g = \frac{m u_g}{M}}$

ARJUNA

A machine gun fires a bullet of mass 65 g with a velocity of 1300 m/s. The man holding it can exert a maximum force of 169 N on the gun. The number of bullets he can fire per second will be:

(a) 1
(b) 2
(c) 3
(d) 4

F_{man} (अधिकतम) $= 169\,N$

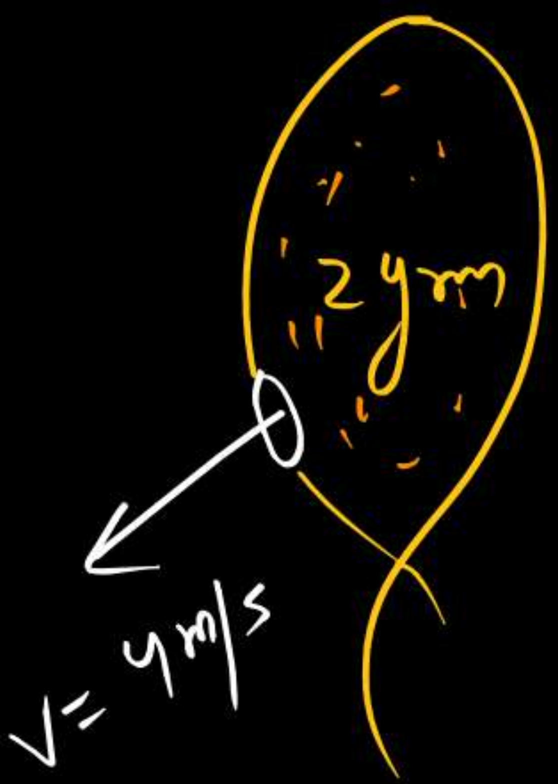
$$\vec{F}_{on\ Gun} = -\vec{F}_{Bullet}$$

$$\vec{F}_{\substack{on\ Gun\\ By\ man}} = \vec{F}_{on\ Gun} = \left(\frac{dP}{dt}\right)_{n\text{-}Bul} = \left(\frac{mu-0}{t}\right)n = \frac{n\,m_B u_B}{t}$$

$$169 = \frac{n\,65 \times 1300}{1000 \times t}$$

Q) A balloon has $2g$ of air. A small hole is pierced into it. The air comes out with a velocity of 4 m/s. If the balloon shrinks completely in 2.5 s. The average force acting on the balloon is:

(a) 0.008 N
(b) 0.0032 N
(c) 8 N
(d) 3.2 N



$$\vec{F}_{Avg} = v\frac{\Delta m}{\Delta t} = \left(4 \times \frac{2}{1000 \times \frac{5}{2}}\right)$$

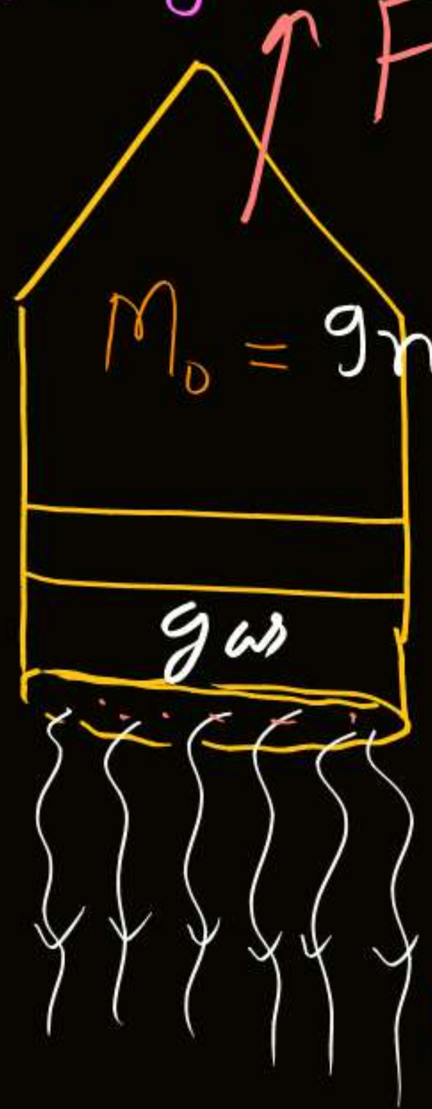
$$\vec{F}_{\text{force on air}} = \vec{F}_{\text{Force on bal}}$$

Rocket Propulsion

"$\vec{F}$ (thrust force on rocket) = force on air due to Rocket."

$\left(\frac{dm}{dt}\right)$ = Rate at which fule is burning $= \alpha$ kg/sec

F (up-thrust)



M_0 = initial mass of Rocket

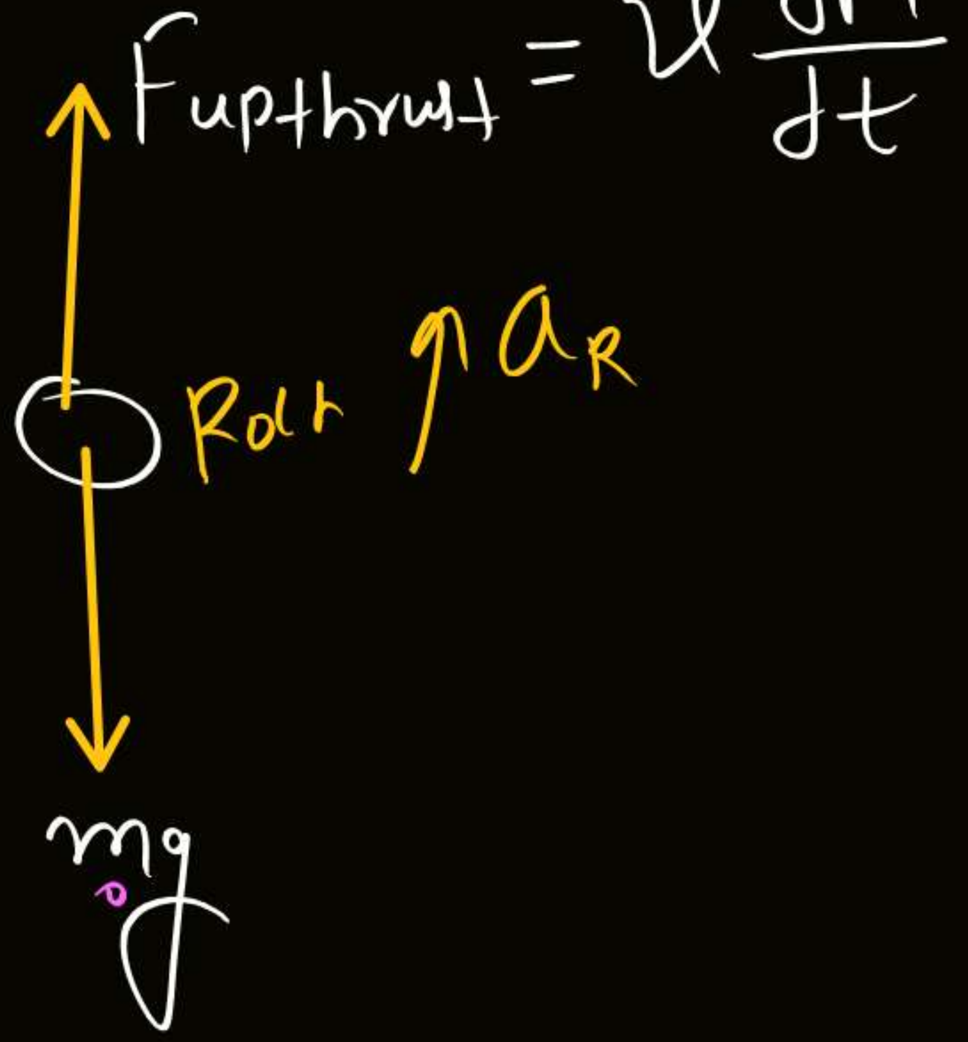
u_{gas} = Velocity of air by whic it is eJecting from Rocket.

u_{gas} = Coll

$$F_{thru} = \left\{ u \frac{dm}{dt} \right\}$$

Varible mass System

F.B.D of Rocket $(t=0)$



$$m\vec{a} = \vec{F}_{net}\ (t=0)$$

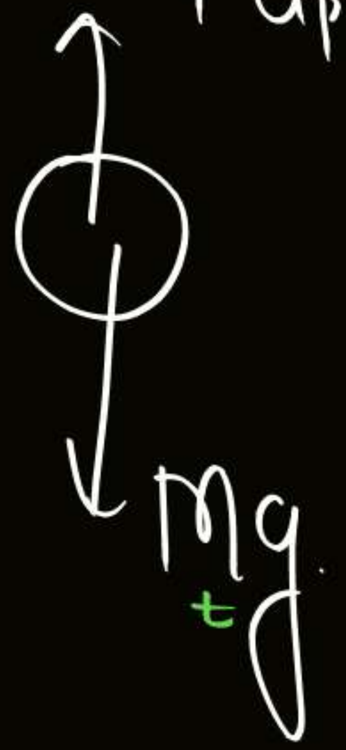
$$m_0\vec{a} = u\frac{dm}{dt} - m_0 g$$

$$\boxed{\vec{a}_{t=0} = \frac{v\frac{dm}{dt}}{m_0} - g}$$

after time 't'

Mass at time t

$$\boxed{m_t = m_0 - \frac{dm}{dt}t}$$



$$F_{upthrust} = u\frac{dm}{dt} = \text{Const}^n$$

$$\vec{F}_{net} = u\frac{dm}{dt} - m_t g$$

$$m_t a_t = u\frac{dm}{dt} - m_t g$$

$$\boxed{\vec{a}_t = \frac{u\frac{dm}{dt}}{m_t} - g}$$

$$\# \quad \boxed{\vec{a}_t = \frac{u\frac{dm}{dt}}{m_0 - \left(\frac{dm}{dt}\right)t} - g}$$

MR*

Rocke

$$\vec{F} = \left(u_r \frac{dm}{dt}\right)$$

at time t=0

$$\boxed{a_{t=0} = \frac{\left(u \frac{dm}{dt}\right)}{m_0} - g}$$

$$\boxed{\vec{a}_t = \frac{\left(u \frac{dm}{dt}\right)}{m_0 - \frac{dm}{dt}t} - g}$$

A cracker rocket is ejecting gases at a rate of 0.05 kg/s with a velocity 400 m/s. The accelerating force on the rocket is:

(a) 20 dyns

(b) 20 N

(c) 200 N

(d) Zero

H.W

A rocket of mass 5700 kg ejected mass at a constant rate of 15 kg/s with constant speed of 12 km/s. The acceleration of the rocket 1 minute after the blast is ($g = 10\ \mathrm{m/s^2}$)

(a) $34.9\ \mathrm{m/s^2}$

(b) $27.5\ \mathrm{m/s^2}$

(c) $3.50\ \mathrm{m/s^2}$

(d) $13.5\ \mathrm{m/s^2}$

H.W

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If the force on a rocket, that releases the exhaust gases with a velocity of 300 m/s is 210 N, then the rate of combustion of the fuel is:

(a) 0.07 kg/s
(b) 1.4 kg/s
(c) 0.7 kg/s
(d) 10.7 kg/s

H.W

A 800 kg rocket is fired from earth so that exhaust speed is 1200 m/s. Then calculate mass of fuel burning per second, to provide initial thurst to overcome its weight. ($g = 10\ m/s^2$)

H.W

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THANK YOU ☺

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