YAKEEN BATCH

Ch- Kinematics

Lect-01

Today's Goal

Avg Speed and Avg Velocity

Instataneous Velocity and Acceleration

1. Average velocity= $\vec{V}_{avg} = \dfrac{\text{Total displacement}}{\text{Total time}}$

2. Average speed= $V_{avg} = \dfrac{\text{Total distance}}{\text{Total time}}$

Q1) A train travels from city A to City B with a Constant speed of 40 m/s and returns back to A with a constant speed of 60 m/s. Find its average speed and average velocity.

a) 50 m/s,0

b) 49 m/s,0

c) 48 m/s,0

d) 45 m/s,0

$A \longrightarrow B$

$A \longleftarrow B$

Total disp $= 0$

Total time $= t$

$$\text{Avg velocity} = \frac{0}{t} = 0$$

~~$$\text{Avg Speed} = \frac{40+60}{2} = 50$$~~

$$A \underset{V_2,\ x}{\overset{V_1,\ x}{\rightleftarrows}} B \qquad t_1 = \frac{x}{V_1} \qquad t_2 = \frac{x}{V_2}$$

$$\text{Avg speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{2x}{t_1 + t_2}$$

$$= \frac{2x}{\frac{x}{V_1} + \frac{x}{V_2}}$$

$$\text{Avg speed} = \frac{2\cancel{x}}{\cancel{x}\left(\frac{1}{V_1} + \frac{1}{V_2}\right)}$$

$$= \frac{2}{\frac{V_2 + V_1}{V_1 V_2}}$$

$$\text{Avg speed} = \frac{2V_1V_2}{V_1 + V_2}$$

$$= \frac{2 \times 40 \times 60}{40 + 60}$$

$$= 48 \text{ m/s}$$

Q2) A person travelling in a $\boxed{\text{straight line}}$ moves with a constant velocity v_1 for a certain distance 'x' and with a constant velocity 'v_2' for next equal distance. The average velocity v is given by the relation. [NEET 2019]

a) $v = \sqrt{v_1 v_2}$ ✗

b) $\dfrac{1}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$

c) $\dfrac{2}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$

d) $\dfrac{v}{2} = \dfrac{v_1 + v_2}{2}$

$t_1 = \dfrac{x}{V_1}$ $\xrightarrow[V_1]{x}$ $\xrightarrow[V_2]{x}$ $t_2 = \dfrac{x}{V_2}$

$$\text{Avg velocity} = \frac{\text{Total disp}}{\text{Total time}} = \frac{2x}{t_1 + t_2} = \frac{2x}{\frac{x}{V_1} + \frac{x}{V_2}}$$

$$V = \frac{2V_1V_2}{V_1 + V_2}$$

$$V = \frac{2V_1V_2}{V_1+V_2}$$

$$\frac{V_1+V_2}{V_1V_2} = \frac{2}{V}$$

$$\frac{1}{V_2} + \frac{1}{V_1} = \frac{2}{V}$$

Q3) A particle covers half of its total distance with speed v_1 and the rest of half distance with speed v_2. Its average speed during the complete journey is [CBSE AIPMT 2011]

a) $\frac{v_1+v_2}{2}$

b) $\frac{v_1 v_2}{v_1+v_2}$

c) $\frac{2v_1 v_2}{v_1+v_2}$

d) $\frac{v_1^2 v_2^2}{v_1^2+v_2^2}$

$$\xrightarrow[V_1]{x} \quad \xrightarrow[V_2]{x}$$

$$V_{avg} = \frac{2V_1V_2}{V_1+V_2}$$

① $A \underset{V_2\,,\,x}{\overset{V_1\,,\,x}{\rightleftarrows}} B$

② $\xrightarrow[x]{V_1} \; \xrightarrow[x]{V_2}$

$$V_{avg\ speed} = \frac{2V_1V_2}{V_1+V_2} = \frac{2V_1V_2}{V_1+V_2}$$

<u>Remember</u>

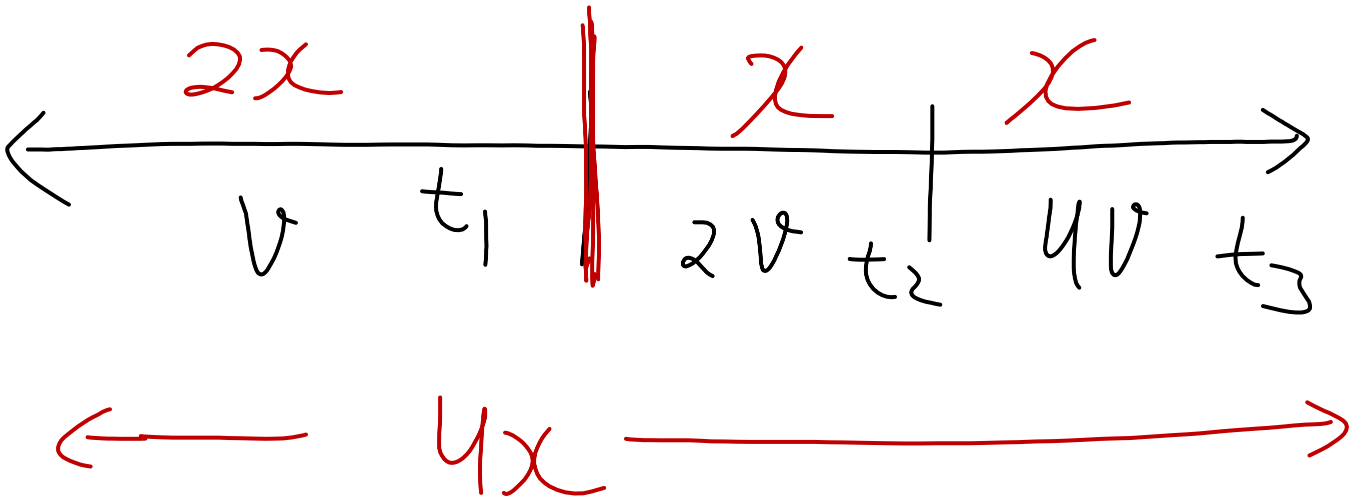
Q4) A particle covers half its journey with a constant speed of v, half the remaining part of journey with a constant speed of $2v$ and the rest of the journey with a constant speed of $4v$. Find its average speed during the entire journey.

a) $\dfrac{8}{11}v$

b) $\dfrac{11}{8}v$

c) $\dfrac{7}{13}v$

~~d)~~ $\dfrac{16}{11}v$



$$\text{Avg Speed} = \frac{4x}{t_1+t_2+t_3} = \frac{4x}{\frac{2x}{v}+\frac{x}{2v}+\frac{x}{4v}}$$

$$\text{Avg speed} = \frac{4x}{t_1 + t_2 + t_3}$$

$$= \frac{4x}{\frac{2x}{v} + \frac{x}{2v} + \frac{x}{4v}}$$

$$= \frac{4\cancel{x}}{\cancel{x}\left(\frac{2}{v} + \frac{1}{2v} + \frac{1}{4v}\right)}$$

$$\text{Avg speed} = \frac{4}{\frac{2\times 4 + 2 + 1}{4v}}$$

$$= \frac{4 \times 4v}{11}$$

$$= \frac{16}{11} v$$

Similar question $\longrightarrow$ 3–4 Q DPP

$\hookrightarrow$ One level up.

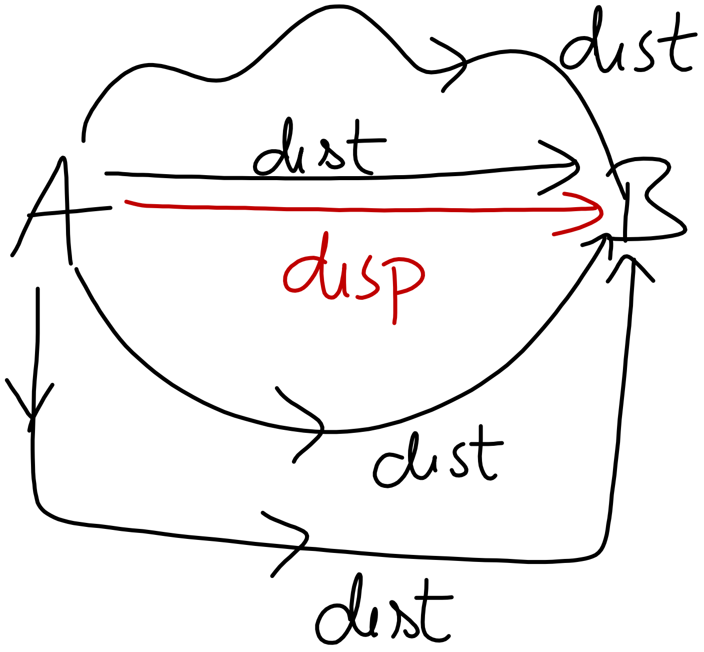
Q5) The numerical ratio of displacement to the distance covered is always

a) Less than one

b) Equal to one

c) Equal to or less than one

d) Equal to or greater than one



$$\text{disp} \leq \text{dist}$$

$$\frac{\text{disp}}{\text{dist}} \leq 1$$

Instantaneous velocity (V_{ins})

$\theta_2 > \theta_1 \Rightarrow V_Q > V_P$

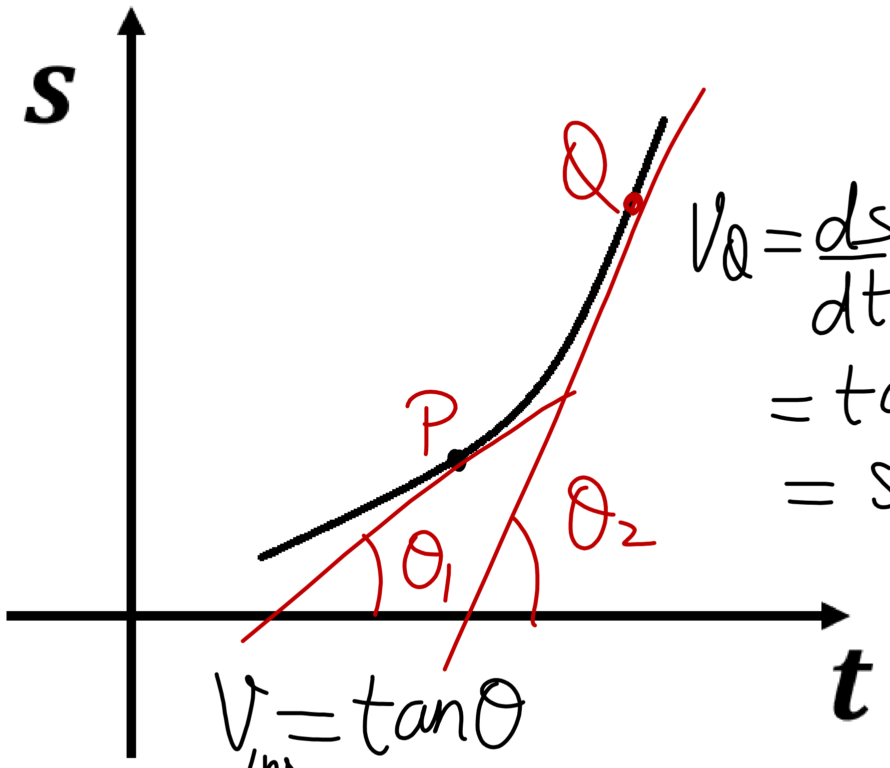
$V_P = \frac{ds}{dt} = \tan\theta_1$

$= \text{slope}$

$$V_{avg} = \frac{\text{total disp}}{\text{total time}}$$

$$V_{avg} = \frac{\Delta s}{\Delta t}$$

$$V_{ins} = \frac{ds}{dt}$$



$V_Q = \frac{ds}{dt}$

$= \tan\theta_2$

$= \text{slope}$

$V_{ins} = \tan\theta$

The displacement is given by X=2t^2+t+5 m. The velocity at t=2 s is :

$$X = 2t^2 + t + 5$$

$$v = \frac{dx}{dt} = 4t + 1 + 0$$

$$v = 4t + 1$$

$t = 2$

$$v = 8 + 1 = 9\, m/s$$

Q6) Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $X_P(t) = at + bt^2$ and $X_Q(t) = ft - t^2$. At what time do the cars have the same velocity? [NEET 2016]

a) $\dfrac{a-f}{1+b}$

b) $\dfrac{a+f}{2(b-1)}$

c) $\dfrac{a+f}{2(1+b)}$

d) $\dfrac{f-a}{2(1+b)}$

$\boxed{\text{CAR P}} \longrightarrow X_P = at + bt^2$

$\boxed{\text{CAR Q}} \longrightarrow X_Q = ft - t^2$

$V_P = V_Q$

$$v_P = v_Q$$

$$\frac{dx_P}{dt} = \frac{dx_Q}{dt}$$

$$a + 2bt = f - 2t$$

$$2bt + 2t = f - a$$

$$2t(b+1) = f - a$$

$$\boxed{t = \frac{f-a}{2(b+1)}}$$

Relevant
Sawall

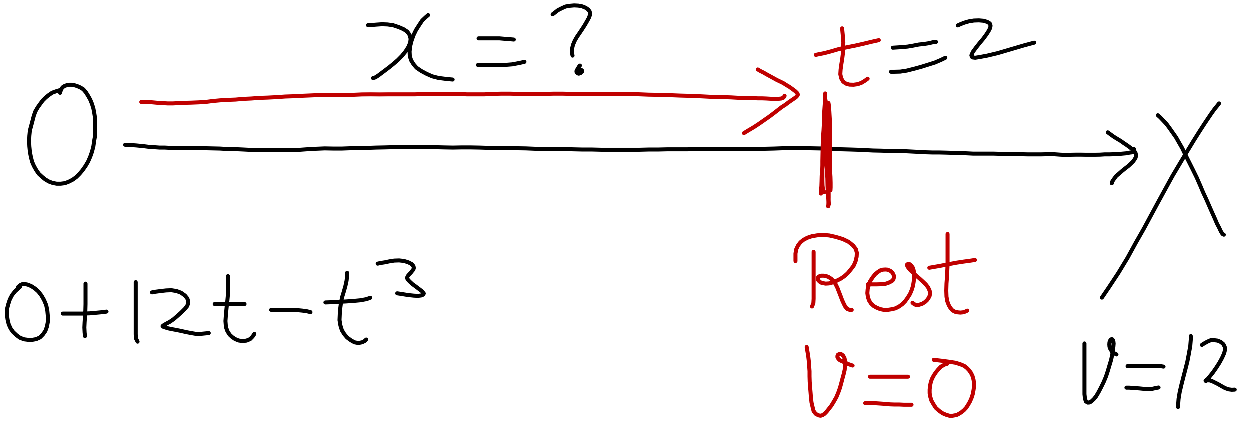
Q7) A particle moves along a straight line OX. At a time t (in second), the distance x (in meter) of the particle O is given by $x=40+12t-t^3$ How long would the particle travel before coming to rest? [CBSE AIPMT 2006]

a) 24 m

b) 40 m

c) 56 m

d) 16 m



$x = 40 + 12t - t^3$

$v = \frac{dx}{dt} = 0 + 12 - 3t^2$

$v = 12 - 3t^2$

$v = 12 - 3t^2$

$0 = 12 - 3t^2$

$\cancel{3}t^2 = \cancel{12}\,4$

$t = \pm 2$

Q8) The displacement-time graph of a moving particle is shown below. The instantaneous velocity of the particle is negative at the point

a) C

b) D

c) E

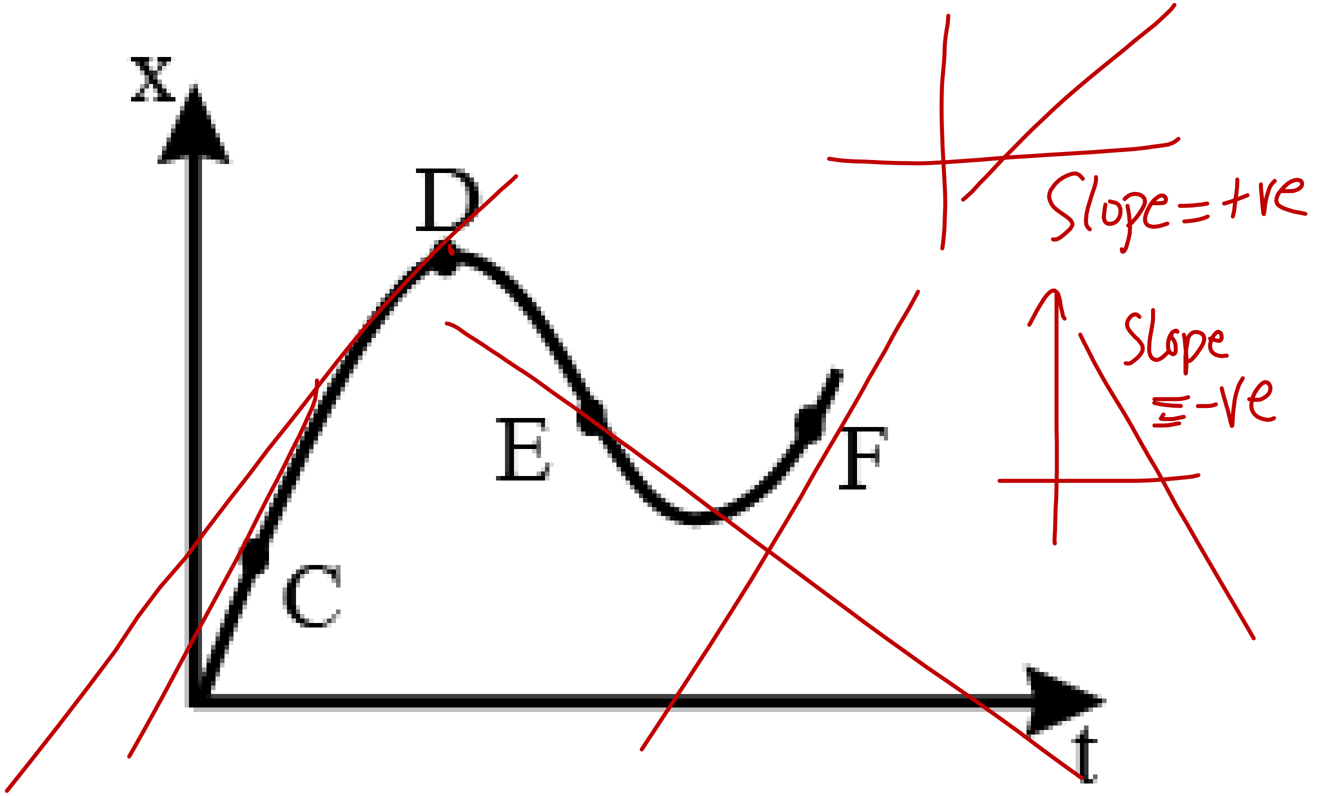
d) F

$V_{inst} =$ Slope $= \tan\theta$

S | A
T | C

$\theta > 90^\circ$

$\tan\theta = -ve$



ACCELERATION $\Rightarrow$ Rate of change of velocity

1. Average Acceleration

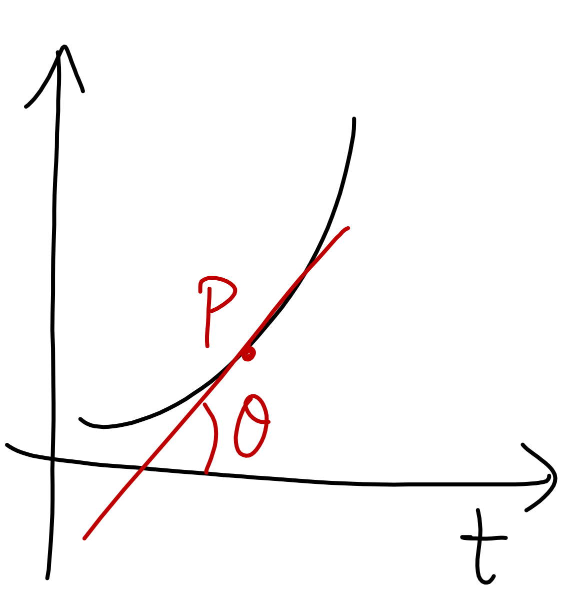
$$a_{avg} = \frac{\text{Change in velocity}}{\text{Change in time}} = \frac{V_2 - V_1}{t_2 - t_1}$$

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

2. Instantaneous Acceleration

$$\boxed{\vec{a}_{ins} = \frac{d\vec{v}}{dt}}$$

$$a_p = \tan\theta = \text{Slope} = \frac{dv}{dt}$$



3 . Retardation $=$ – acceleration

$\boxed{100\,\%\ \text{correct}}$

Q9) A particle is moving so that its displacement s is given as $s=t^3-6t^2+3t+4$ meter. Its velocity at the instant when its acceleration is zero will be-

a) **3 m/s**

b) **-12 m/s**

c) **42 m/s**

d) **-9 m/s**

$S = t^3 - 6t^2 + 3t + 4$

$V = ?$ when $a = 0$

$v = \frac{ds}{dt} = 3t^2 - 12t + 3$

$a = \frac{dv}{dt} = 6t - 12$

$a = 0$

$6t - 12 = 0$

$t = 2$

$V = ?$

$V_{t=2} = 3(2)^2 - 12(2) + 3$

$= -9$

Q10) The motion of a particle along a straight line is described by equation $x=8+12t-t^3$ where, x is in meter and t in sec. the retardation of the particle when its velocity becomes zero, is $= -a$ [CBSE AIPMT 2012]

a) $24\ ms^{-2}$

b) Zero

c) $6\ ms^{-2}$

d) $12\ ms^{-2}$

$$x = 8 + 12t - t^3$$

$a = ?$ when $v = 0$

$$v = \frac{dx}{dt} = 0 + 12 - 3t^2$$

$$a = \frac{dv}{dt} = 0 - 6t$$

retardation $= -a$

$= +12 m/s^2$

when $v = 0$

$12 - 3t^2 = 0$

$\cancel{3}t^2 = \cancel{12}\,4$

$t = \pm 2$

$a = -6(2) = -12$

$t = 2$

Q11) The position vector of a particle is given as $\vec{r} = (t^2 - 4t + 6)\hat{i} + (t^2)\hat{j}$. Find velocity vector, speed of particle & acceleration vector at t=3 sec.

→ Magnitude of Vel.

$$\vec{r} = (t^2 - 4t + 6)\hat{i} + (t^2)\hat{j}$$

$$x = t^2 - 4t + 6 \qquad y = t^2$$

$$V_x = \frac{dx}{dt} = 2t - 4 \qquad V_y = \frac{dy}{dt} = 2t$$

$$a_x = \frac{dV_x}{dt} = 2 \qquad a_y = \frac{dV_y}{dt} = 2$$

$$\vec{V} = V_x\hat{i} + V_y\hat{j}$$

$$\vec{V} = (2t-4)\hat{i} + 2t\hat{j}$$

at $t=3$

$$\vec{V} = 2\hat{i} + 6\hat{j}$$

$$\text{Speed} = |v|$$

$$= \sqrt{(2)^2 + (6)^2}$$

$$= \sqrt{40}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$\vec{a} = 2\hat{i} + 2\hat{j}$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt}$$

$$a_x = \frac{dv_x}{dt} \qquad a_y = \frac{dv_y}{dt}$$

Q12) The x and y co-ordinates of the particle at any time are $x=5t-2t^2$ and $y=10t$ respectively, where x and y are in meters and t in seconds. The acceleration of the particle at t=2 s is:

[NEET 2017]

a) 0

b) 5 m/s^2

c) -4 m/s^2 ✓

d) -8 m/s^2

$$x = 5t - 2t^2$$

$$v_x = \frac{dx}{dt} = 5 - 4t$$

$$a_x = \frac{dv_x}{dt} = 0 - 4$$

$$\vec{a} = -4\hat{i}$$

$$y = 10t$$

$$v_y = \frac{dy}{dt} = 10$$

$$a_y = \frac{dv_y}{dt} = 0$$

DPP one level up

$x =$ $y =$

$$a = a_x \hat{i} + a_y \hat{j}$$

$$|a| = \sqrt{a_x^2 + a_y^2}$$

Another expression for Instantaneous acceleration

imp

$$a = \frac{dv}{dt}$$ ✓

v 't' के Terms में हो

$a =$

$$a = \frac{dv}{dt} \times \frac{dx}{dx}$$

$$a = \left(\frac{dx}{dt}\right) \frac{dv}{dx}$$

$$\boxed{a = v\frac{dv}{dx}}$$

कब लगाना है

$v = x^2 + 1$

$v = 2x + 4$

v x के Terms में हो

$a =$

Q13) For motion of an object along the x-axis, the velocity v depends on the displacement x as $v=3x^2-2x$, then what is the acceleration at x=2 m

a) 48 ms^{-2}

b) 80 ms^{-2}

c) 18 ms^{-2}

d) 10 ms^{-2}

$v = 3x^2 - 2x$

$a = ?$

$x = 2$

$a = v\frac{dv}{dx} = (3x^2-2x)(6x-2)$

$a = (3x^2-2x)(6x-2)$

$x = 2$

$a = 8 \times 10 = 80$

Q14) A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x)=\beta x^{-2n}$, where β and n are constants and x is the position of the particle ~~as a function of x, is given by~~ $a = ?$ [2015]

a) $-2n\beta^2 x^{-2n-1}$ X

b) $-2n\beta^2 x^{-4n-1}$

c) $-2n\beta^2 x^{-2n+1}$

d) $-2n\beta^2 x^{-4n+1}$

$$V = \beta x^{-2n}$$

$$a = v\frac{dv}{dx} = \beta x^{-2n}\left(\beta\left(-2n\, x^{-2n-1}\right)\right)$$

$$a = -2n\beta^2 x^{-2n-2n-1} = -2n\beta^2 x^{-4n-1}$$

Reverse Process → Integration

अब तक

$S =$

$v = \frac{ds}{dt}$

$a = \frac{dv}{dt}$

अब आगे

$v = \frac{ds}{dt}$

$\int ds = \int v\,dt$

$\boxed{S = \int v\,dt}$

$a = \frac{dv}{dt}$

$\int dv = \int a\,dt$

$\boxed{v = \int a\,dt}$

Q15) If the velocity of a particle is $v = At + Bt^2$, where A and B are constants , then the distance travelled by it between 1s and 2s is [NEET 2016]

limits $1s \longrightarrow 2s$

a) $3A+7B$

b) $\frac{3}{2}A+\frac{7}{3}B$

c) $\frac{1}{2}A+\frac{1}{3}B$

d) $\frac{3}{2}A+4B$

$$V = At + Bt^2$$

$$S = ?$$

$$S = \int v\,dt = \int_1^2 (At + Bt^2)\,dt$$

$$V = \frac{ds}{dt}$$

$$\int ds = \int v\,dt$$

$$S = \int v\,dt$$

$$S = \int_1^2 (At + Bt^2)\,dt$$

$$= \left[\frac{At^2}{2} + \frac{Bt^3}{3}\right]_1^2$$

$$= \left[\frac{A(2)^2}{2} + \frac{B(2^3)}{3} - \left(\frac{A(1)}{2} + \frac{B(1)}{3}\right)\right]$$

$$S = 2A + \frac{8}{3}B$$

$$- \frac{A}{2} - \frac{B}{3}$$

$$= 2A - \frac{A}{2}$$

$$+ \frac{8}{3}B - \frac{B}{3}$$

$$= \frac{3}{2}A + \frac{7B}{3}$$

Q16) If velocity of particle is given as v=t+1, Find displacement of particle at the end of 4s, if initially particle is at origin.

$t = 0 \quad , \quad S = 0 \quad$ origin

$t = 0$

~~a) 12 m~~

b) 18 m

c) 24 m

d) 6 m

$V = t + 1$

$S \rightarrow$ at end of 4s.

limit not given $\rightarrow$ Indefinate Integration

$$S = \int v\,dt = \int (t+1)\,dt = \frac{t^2}{2} + t + C$$

$$S = \frac{t^2}{2} + t + C$$

$$\underline{\underline{C = ?}}$$

$t = 0, S = 0$

$$0 = \frac{0^2}{2} + 0 + C$$

$$C = 0$$

$$S = \frac{t^2}{2} + t$$

$t = 4$

$$S = \frac{4^2}{2} + 4$$

$$= \frac{16}{2} + 4$$

$$= 12m$$

Alternate

$$S = \int_0^4 v\,dt$$

$S \longrightarrow$ Velocity ko Integrate

$V \longrightarrow$ acceleration ko Integrate

$$V = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$

Q17) If acceleration of particle is given by $a=3t^2$, find velocity of particle at t=3s, if initially particle was at rest $\rightarrow t=0, v=0$

$t=0 \qquad v=0$

a) 9 m/s

b) 18 m/s

c) ~~27 m/s~~

d) 30 m/s

$\boxed{\text{Relevant}}$

$$a = 3t^2$$

$$v = \int a\,dt = \int 3t^2$$

$$= \frac{\cancel{3}t^3}{\cancel{3}} + C$$

$$v = t^3 + C$$

More accurate

$$a = \frac{dv}{dt}$$

$$\int dv = \int a\,dt$$

$$v = \int a\,dt$$

$v = t^3 + C$

$t = 0$
$v = 0$

$0 = 0 + C$

$C = 0$

$v = t^3$

$t = 3$

$v = 3^3$

$v = 27 m/s$

$v = \int_0^3 a\,dt$

Q18) A particle initially at rest moves along the x-axis. Its acceleration varies with time as a=4t. If it starts from the origin, the distance covered by it in 3 s is

a) 12 m

b) 18 m

c) 24 m

d) 36 m

$$a = 4t$$

$$v = \int a\,dt = \int 4t\,dt = \frac{4t^2}{2} + C = 2t^2 + C$$

Initially at rest

$t = 0 \qquad v = 0$

$$0 = 2(0)^2 + C$$

$$C = 0$$

$$\boxed{v = 2t^2}$$

$$V = 2t^2$$

$$S = \int v\,dt$$

$$= \int 2t^2\,dt$$

$$S = 2\frac{t^3}{3} + C_1$$

$$S = \frac{2t^3}{3} + C_1$$

$t = 0$ origin

$s = 0$

$$0 = \frac{2(0)^3}{3} + C_1$$

$$C_1 = 0$$

$$S = \frac{2t^3}{3}$$

$t = 3$

$$S = \frac{2(3)^3}{3}$$

$$= \frac{2 \times 27}{3}$$

$$= 18m$$

Q) $a = bt$ Particle starts from origin with initial velocity V_0. Find distance travelled in time t.

1995

a) $V_0 t + \frac{bt^2}{3}$

b) $V_0 t + \frac{bt^2}{2}$

c) $V_0 t + \frac{bt^3}{6}$

d) $V_0 t + \frac{bt^3}{3}$

$$V = \int a\,dt = \int bt\,dt = \frac{bt^2}{2} + C$$

initial velocity $= V_0$
$t = 0$

$$V_0 = \frac{b(0)}{2} + C$$

$$C = V_0$$

$$V = \frac{bt^2}{2} + V_0$$

$$V = \frac{bt^2}{2} + V_0$$

$$S = \int v\,dt$$

$$= \int \left(\frac{bt^2}{2} + V_0\right) dt$$

$$S = \frac{b}{2}\frac{t^3}{\times 3} + V_0 t + C_1$$

Starts from origin

$t = 0 \qquad S = 0$

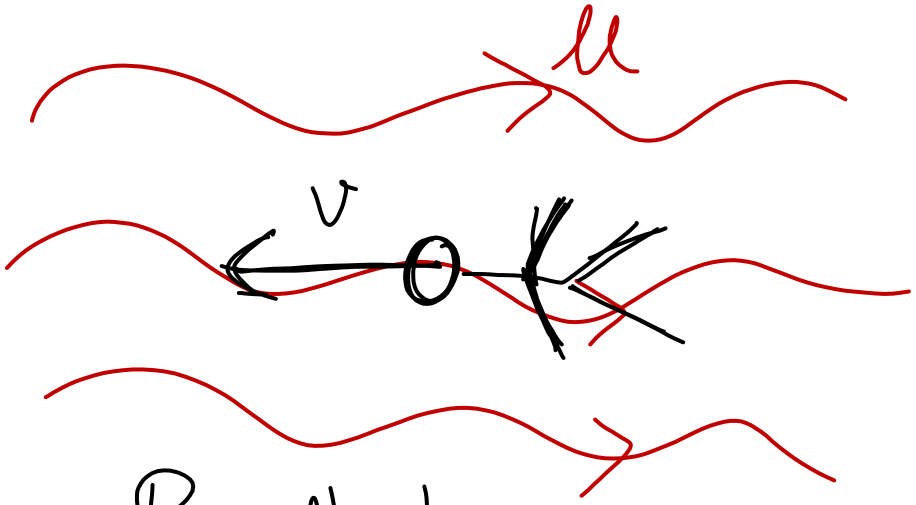
$$0 = (0) + (0) + C_1$$

$$C_1 = 0$$

$$S = \frac{bt^3}{6} + V_0 t$$

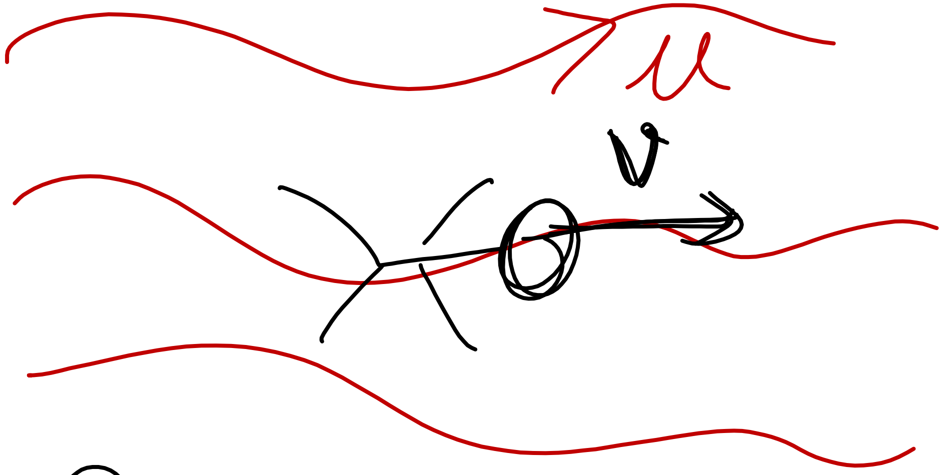
Upstream & Downstream

Upstream



Resultant speed $= v \sim u$

Downstream



Resultant speed $= v + u$

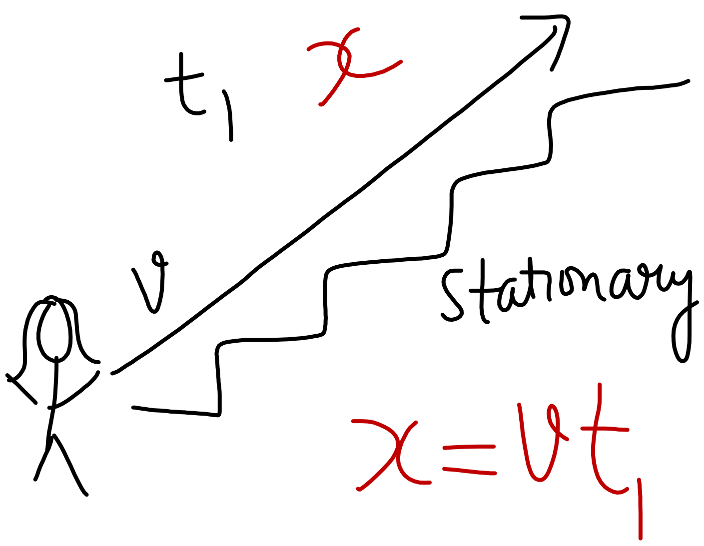
Q19) Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be: [NEET 2017]

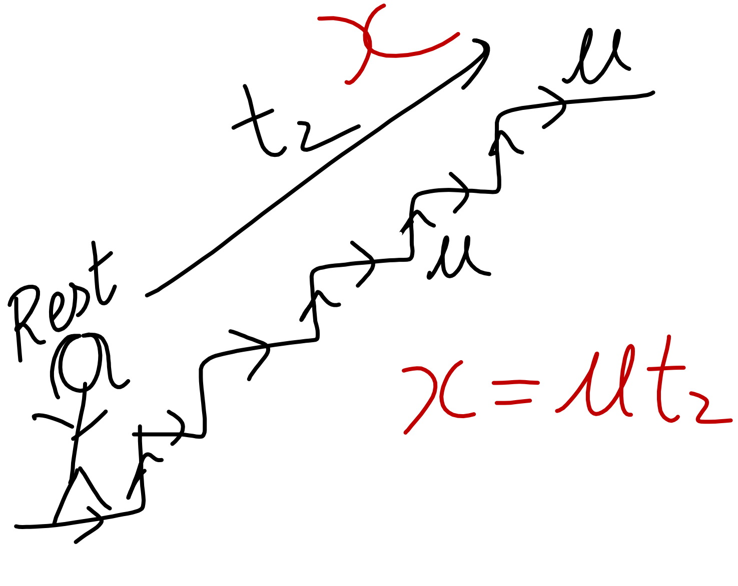
a) $\dfrac{t_1+t_2}{2}$

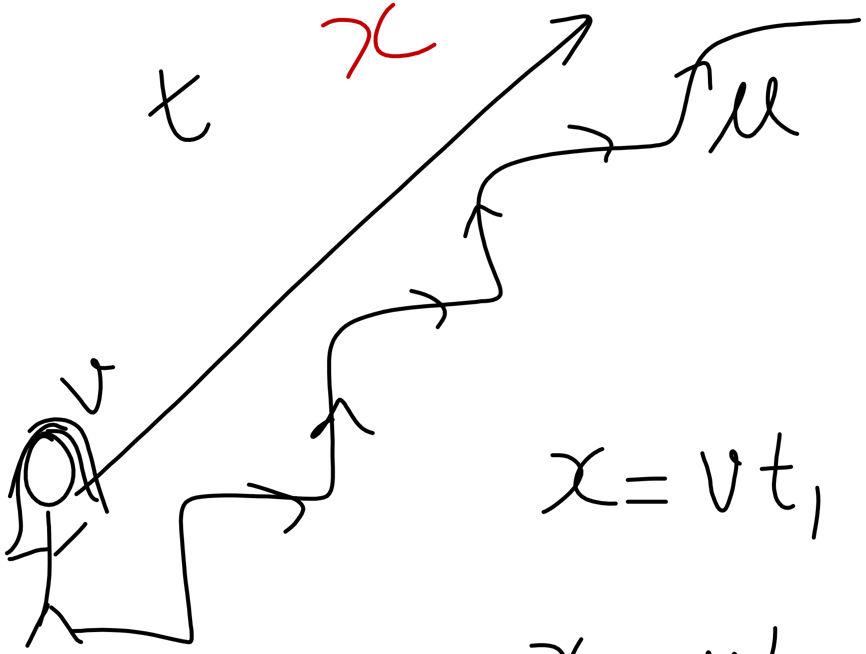
b) $\dfrac{t_1 t_2}{t_2-t_1}$

c) $\dfrac{t_1 t_2}{t_2+t_1}$

d) $t_1 - t_2$







$$x = vt_1$$

$$x = ut_2$$

Resultant speed $v+u$

$$x = (v+u)t$$

$$x = \left(\frac{x}{t_1} + \frac{x}{t_2}\right)t$$

$$1 = \left(\frac{1}{t_1} + \frac{1}{t_2}\right)t$$

$$1 = \left(\frac{t_1+t_2}{t_1 t_2}\right)t$$

$$t = \frac{t_1 t_2}{t_1+t_2}$$

Thank You

Download lecture notes of this lecture right after this session.