ARJUNA NEET BATCH

UNITS AND MEASUREMENTS

LECTURE - 04

TODAYS GOAL

- question on dimensional analysis
- Singnificant digit ✓
- Rounding off. ✓
- MOKSH

The frequency of vibrations f of a mass m suspended from a spring of spring constant K is given by a relation of type $f = cm^x K^y$, where c is a dimensionless constant. The values of x and y are:

(a) $x = \frac{1}{2}, y = \frac{1}{2}$

(b) $x = \frac{-1}{2}, y = \frac{-1}{2}$

(c) $x = \frac{1}{2}, y = \frac{-1}{2}$

(d) $x = \frac{-1}{2}, y = \frac{1}{2}$

$\frac{MR^2}{T^{-1}} = m^x (mT^{-2})^y$

spring const^n $= \frac{F}{y} = (mT^{-2})$

$f = c\, m^x K^y$

$T^{-1} = [M^1]^x (MT^{-2})^y$

$M^0 T^{-1} = M^{x+y} T^{-2y}$

$2y = 1$

$y = \frac{1}{2}$

$x + y = 0$

$x = -y = -\frac{1}{2}$

Dimensional const^n

Ex (G)

Dimensionless const^n

Ex - π

In equation $y = x^2 \cos^2 2\pi \beta\gamma/\alpha$, then units of x, α, β and m, s^{-1} and (ms^{-1}) respectively. The units of y and γ are

(a) m^2, ms^{-2} ✗

(b) m, ms^{-1} ✗ — are

(c) m^2, m^{-1} ✓

(d) m, ms^{-2} ✗

$$Y = x^2 \times \cos^2\left(\frac{2\pi\beta\gamma}{\alpha}\right)$$

$$Y = x^2$$

$$\boxed{Y = m^2}$$

$$\frac{\beta\gamma}{\alpha} = 1$$

$$\gamma = \frac{\alpha}{\beta} = \frac{s^{-1}}{m\,s^{-1}} = m^{-1}$$

If force F, area A and density D are taken as the fundamental units, the representation of Young's modulus 'Y' will be:

(a) $[F^{-1} A^{-1} D^{-1}]$ (b) $[F A^{-2} D^{2}]$

(c) $[F A^{-1} D]$ (d) $[F A^{-1} D^{0}]$

MR^x

$\gamma(M^2)$

$F(MLT^{-2})$

$A(L^2)$

$D(ML^{-3})$

$Y = \text{Pressure} = \frac{\text{force}}{\text{Area}}$

$Y(ML^{-1}T^{-2}) \propto F^x A^y D^z$

↳ Same as Pressure

$ML^{-1}T^{-2} = (MLT^{-2})^x (L^2)^y (ML^{-3})^z$

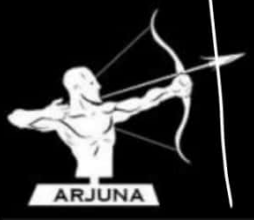
$ML^{-1}T^{-2} = M^{x+z} L^{x+2y-3z} T^{-2x}$

$-2x = -2$

$x = 1$

$1 = x + z$

$z = 0$



On the basis of dimension, decide which of the following relation for displacement of a particle is not correct [NCERT]

(a) $y = a\sin\left(\frac{2\pi t}{T}\right)$ → Correct

(b) $y = a\sin\left(\frac{vt}{\lambda}\right)$ ✓

(c) $y = \sqrt{2}a\sin\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$

(d) $y = \frac{x}{t}\sin(\lambda t)$

a = Position
λ = wavelength
T = time
v = velocity

Correct

$\rightarrow \theta = \frac{vt}{\lambda} = \frac{LT^{-1}T}{L}$

= no unit.

* Lecture → MRA Notes
* DPP
* PW-Mates
* NEET-Based Practice

Bio → NCERT

MCQ → MWT
↳ objective-1
objective-2
1-20 Question

ARJUNA

The speed of ligth C, gravitational constant G and plank constant h are taken as fundamental P.Q then the dimension of time in the new system of unit [JEE]

(a) $G^{1/2} h^{1/2} c^{-5/2}$ (b) $G^{-1/2} h^{1/2} c^{-1/2}$

(c) $G^{-1/2} h^{1/2} c^{-3/2}$ (d) $G^{-1/2} h^{1/2} c^{1/2}$

mR^x

$m^0 t = Gh$

समझो

$t \propto C^x G^y h^z$

$T^1 = (LT^{-1})^x (M^{-1}L^3T^{-2})^y (ML^2T^{-1})^z$

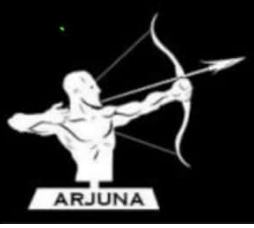
$M^0 L^0 T^1 = M^{z-y} L^{x+3y+2z} T^{-x-2y-z}$

$C = LT^{-1}$

$G = M^{-1}L^3T^{-2}$ *

$h = M^1L^2T^{-1}$ *

↳ find x,y,z.



Q. If surface Tension (S) Moment of Inertia (I) and Planks Constant (h) were to be taken as fundamental Unit the dimension of liner Momentum will be

(a) $S^{3/2} I^{1/2} h^0$

(b) $S^{1/2} I^{1/2} h^0$

(c) $S^{1/2} I^{3/2} h^{-1}$

(d) $S^{1/2} I^{1/2} h^{-1}$

MR*

$P = S^x I^y h^z$

↳ Eqn

$S(MT^{-2})$

$I(ML^2)$

$h(ML^2T^{-1})$

$\Rightarrow P = (mv) = (M)(L)^1 T^{-1}$ (JEE main)

MR* ↳ I should think about length.

Find dimension of $\frac{d^2y}{dx^2}$, where y = force and x=momentum.

$$\Rightarrow \frac{d^2y}{dx^2} = ?? \Rightarrow \frac{y}{x^2} = \frac{\text{force}}{(\text{momentum})^2} = \frac{MLT^{-2}}{(MLT^{-1})^2}$$

change in $x = x_f - x_i = 5m - 2m = 3m$

Momentum of object is given as $P = \alpha t\,[1 - \beta e^{\gamma x}]$ then find dimension of α, β and γ. Where t is times x is position.

$$P = \alpha t\,[1 - \beta e^{\gamma x}]$$

γx → Power dim^n

$\gamma x = M^0 L^0 T^0$

$\gamma = \frac{1}{x} = L^{-1}$

$P = \alpha t$

$[MLT^{-1}] = \alpha T$

$\boxed{\alpha = MLT^{-2}}$

$(1 - \beta \times 1)$

$[1 = \beta]$

$\beta = M^0 L^0 T^0$

Position of object Y is given Y = A sin(kx – wt) then. Find dimension of K; w and A.

$Y = A \sin(Kx - \omega t)$

$Y = A$

$A = L^1$

$\theta = Kx - \omega t$

$\theta = Kx = \omega t = M^0 L^0 T^0$

$Kx = 1 = \omega t$

$K = L^{-1}$ | $\omega = T^{-1}$

If $Y = A \sin\left[\frac{2\pi}{\lambda}(ct - x)\right]$ Find dimension of C.

$$Y = A \sin\left[\frac{2\pi}{\lambda}(ct - x)\right]$$

Sol^n

Why here it is Ers

$ct = x \leftarrow$ dimensions

$C = \frac{x}{T} = [LT^{-1}] =$ velocity

If unit of length is 4m, unit of Mass is 4 kg and unit of time is 2 sec then find value of 1J energy in this system of unit.

Sol^{n}

Ramlal's Unit

$l = 4m$

$m = 4kg$

$T = 2s$

Find value of 1 J energy =??

$nu = Cost^{n}$ → Apply for energy

$n_1u_1 = n_2u_2$

S.I — Ramlal

$1 \times 1kg\ m^2/s^2 = n_2\ \frac{4kg\ m^2}{s^2}$

$1kg\ m^2/s^2 = n_2\ \frac{4kg\ 16m^2}{4s^2}$

$n_2 = \frac{1}{16}$

If unit of length is 4 m, unit of mass is 4 kg and unit of time is 2 sec then, find unit of energy in this new system of unit.

Unit of energy in new system of unit ??

Ramlal's unit

$l = 4m$

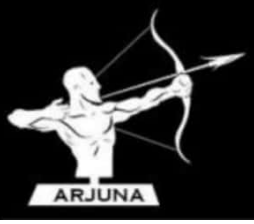
$m = 4kg$

$T = 2sec$

$$\text{Energy} = ML^2T^{-2} = \frac{ML^2}{T^2}$$

$$\text{Energy} = \frac{4kg \times (4m)^2}{(2s)^2} = 16J$$

$$\text{Energy} = \frac{4 \times 16}{4} \left(\frac{kg\,m^2}{s^2}\right)$$



If unit of length is xm. Then find value of $8m^3$ in this system of unit.

Volume

IIT

$m' =$ Length $= x\ m$

$nu =$ Costⁿ

Dim = ◯

$nu =$ (cost)

$[nu]_{SI\ unit} = [nu]_{Volume\ in\ new\ unit}$

or Volume

$8\ m^3 = n\ (m'^3)$

$8\ m^3 = n\ x^3\ m^3$

$n = \frac{8}{x^3}$ Ans

If unit of length and force becomes x_m and y_{newton} then unit of energy will be.

Unit of length = x_m

Unit of force = Y_{newton}

* On this question if I have to find value of 1 J in new system of unit then we have to use n1=(u1)'

$E = F^a L^b$

$E = (\text{Force}) \times \text{length}$

$E = (xy)\ \text{Joule}$

A2

Find value of universal gravitational constant G in C.G.S unit.

$F = G\frac{m_1 m_2}{r^2}$

$\frac{kg\,m}{s^2} = G\frac{kg^2}{m^2}$

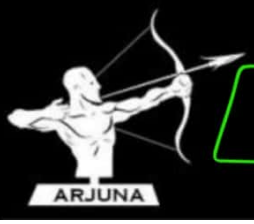
$n u = \text{Const}^n$ for 'G'

$[n_1 u_1]_{SI\ unit} = [n_2 u_2]_{C.G.S.}$

$6.67 \times 10^{-11}\ \frac{m^3}{s^2\,kg} = n_2\left[\frac{cm^3}{s^2\,gram}\right]$

$\frac{6.67 \times 10^{-11} \times 10^6\ cm^3}{s^2\ 10^3\ gm} = n_2\left[\frac{cm^3}{s^2\,gm}\right]$

$n_2 = 6.6 \times 10^{-8}$



In new system of unit mass becomes double. length becomes half and unit of time is made three times then what will be the unit of force.

??

$$\text{force} = \frac{ML}{T^2} = \frac{2ML}{2(3T)^2}$$

$$= \frac{1}{9} \text{ time}$$

unit of force becomes $\frac{1}{9}$ times *

Convert 54 km h^{-1} into ms^{-1}

(a) 5 ms^{-1}

(b) 10 ms^{-1}

(c) 15 ms^{-1}

(d) 20 ms^{-1}

$nu = \text{Cost}^n$

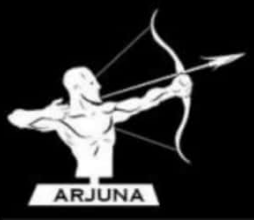
Check equation is dimensionally correct or not.

$K.E = \frac{1}{2}mv^2$ (a) $K.E = \frac{2}{5}mv^2$ (b) $K.E = mv^2 \sin\theta$ (c)

all are dimensionally correct :—

↳ but physically correct is only (a)

$K.E = \frac{1}{2}mv^2$



Check equation is physically correct or not.

$K.E = \frac{1}{2}mv$ (a)

$K.E = \frac{1}{2}m^2v$ (b)

Both are physically Incorrect

↳ Dimensionally both are Incorrect (Wrong)

$K.E = \frac{1}{2} mv^2$

→ Physically correct then dimensonaly must be corre

(D) → may or may not be physically correct

		Equation is physically correct	Equation is physically not correct
A	B		
Equation is Dimensionally not correct	Equation is Dimensionally correct	C	D

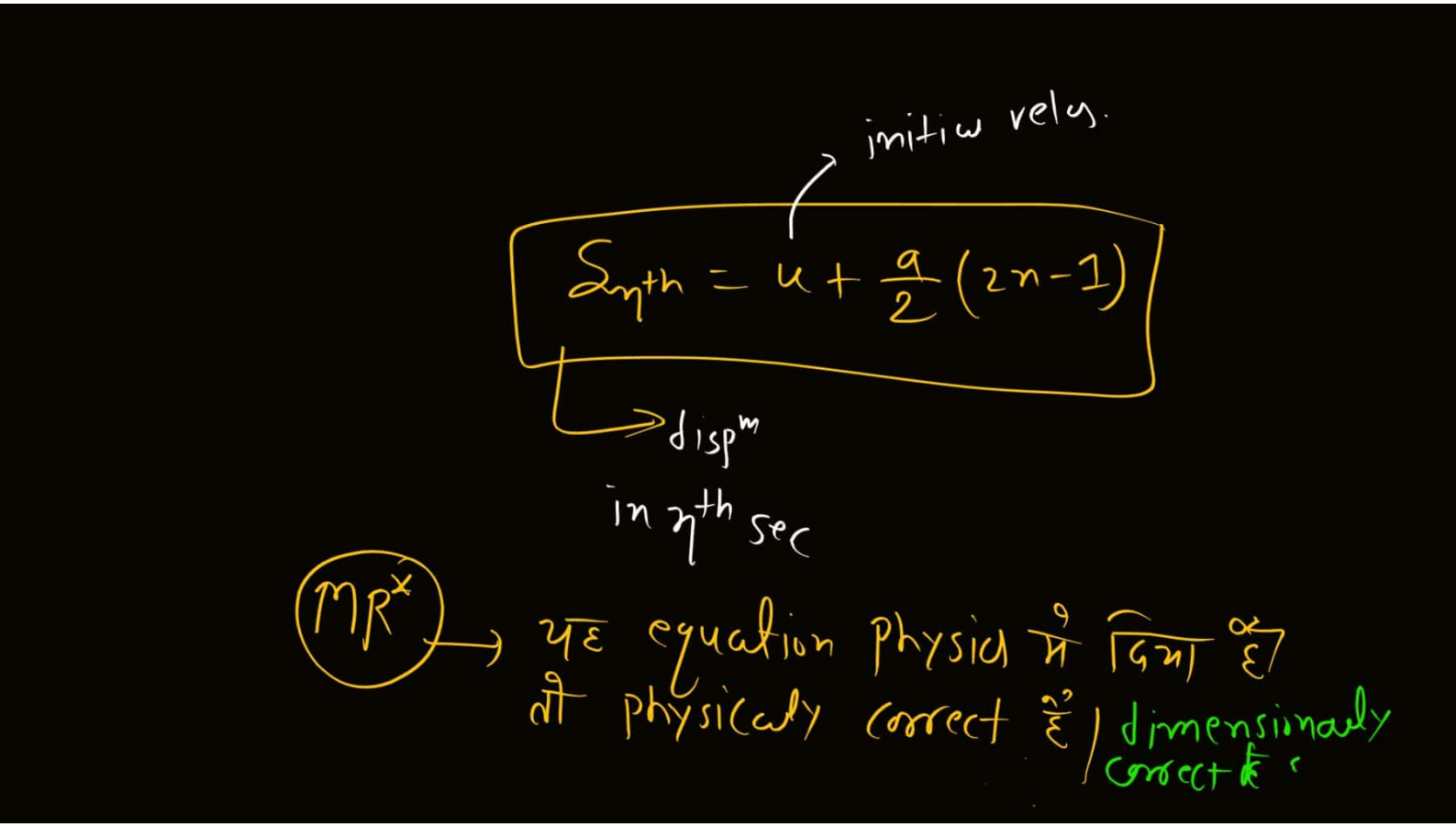
Physically

Dimn

Ans A → Physically must be not correct

B → Physical may or may not be correct. $K.E = \frac{3}{2}mv^2$ ✗

C → equation must be dimn correct.



Limitation of Dimensional Analysis

It Can't about physically correct

(1) It is not use to derive dimensionless proportional constant.

(2) It can not derive dimensionless function, like sinθ, cosθ, tanθ e^x etc.

(3) If physical quantity depends upon two P.Q. of same dimension

(4) It can not derive formula which have '+' and '–' term

Ex : $S = ut + \frac{1}{2}at^2$ $\quad S = ut = \frac{1}{2}at^2$

(5) We equate the power of M, L and T. So, it only work when quantity depends only on three physical quantity.

Ex : If force depends upon energy, velocity, time work.

$F = E^x V^y t^z W^p$

Which of the following equation can be derived dimensionally

(a) $s = vt - \frac{1}{2}at^2$

(b) $v^2 = u^2 - 2as$

(c) $h = \frac{\omega^2 r^2}{2g}$

(d) $v = \frac{d}{t}$

Which of the following equation can not be derived dimensionally

(a) $F = 6\pi r\eta$ ✗

(b) $\theta = \omega t$ ✓

(c) $\frac{d\theta}{dt} = \rho A v$

(d) $P = \rho g h$ ✓

Ans (a)

$T = 2\pi\sqrt{\frac{l}{g}}$ ✗

$T = \sqrt{\frac{l}{g}}$ ✓

Significant Figures

Significant Figures (Meaningful Digits

- All non-zero digits are significant

  Ex – 44 m ⇒ 2
- All zeros between non-zero digits are significant

  Ex – 405 m ⇒ 3
- All zeros on left side are non-significant.
- Exact number have infinite significant figure
- Power form are not considered as significant figure.
- using Dimension analysis we can't find equation is physically correct or not.