OSCILLATIONS (SHM)

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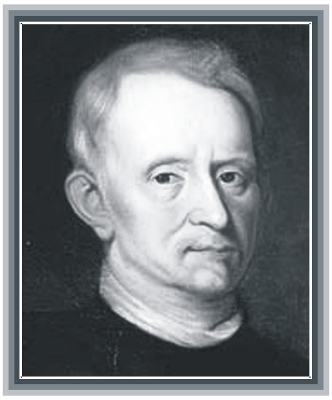
NEET SYLLABUS

Periodic motion-period, frequency, displacement as a function of time. Periodic functions. Simple harmonic motion(SHM) and its equation; phase; oscillations of a spring-restoring force and force constant; energy in SHM - Kinetic and potential energies; simple pendulum-derivation of expression for its time period; free, forced and damped oscillations (qualitative ideas only), resonance.

ROBERT HOOKE (1635 – 1703 A.D.)



Robert Hooke was born on July 18, 1635 in Freshwater, Isle of Wight. He was one of the most brilliant and versatile seventeenth century English scientists. He attended Oxford University but never graduated. Yet he was an extremely talented inventor, instrument-maker and building designer. He assisted Robert Boyle in the construction of Boylean air pump. In 1662, he was appointed as Curator of Experiments to the newly founded Royal Society. In 1665, he became Professor of Geometry in Gresham College where he carried out his astronomical observations. He built a Gregorian reflecting telescope; discovered the fifth star in the trapezium and an asterism in the constellation Orion; suggested that Jupiter rotates on its axis; plotted detailed sketches of Mars which were later used in the 19th century to determine the planet's rate of rotation; stated the inverse square law to describe planetary motion, which Newton modified later etc. He was elected Fellow of Royal Society and also served as the Society's Secretary from 1667 to 1682. In his series of observations presented in Micrographia, he suggested wave theory of light and first used the word 'cell' in a biological context as a result of his studies of cork. Robert Hooke is best known to physicists for his discovery of law of elasticity: Ut tensio, sic vis (This is a Latin expression and it means as the distortion, so the force). This law laid the basis for studies of stress and strain and for understanding the elastic materials.

HENDRIK ANTOON LORENTZ (1853 - 1928) DUTCH

Theoretical physicist, professor at Leiden. He Investigated the relationship between electricity, magnetism, and mechanics. In order to explain the observed effect of magnetic fields on emitters of light (Zeeman effect), he postulated the existence of electric charges. In the atom, for which he was awarded the Nobel Prize In 1902. He derived a set of transformation equations (known after him as Lorentz transformation equations) by some tangled mathematical arguments, but he was not aware that these equations hinge on a new concept of space and time.

OSCILLATIONS
(SHM, DAMPED AND FORCED OSCILLATIONS & RESONANCE)

1. PERIODIC MOTION AND ITS CHARACTERISTICS AND TYPES OF SHM

1.1 Periodic Motion

(i) Any motion which repeats itself after regular interval of time is called periodic motion or harmonic motion.

(ii) The constant interval of time after which the motion is repeated is called time period.

Examples : (i) Motion of planets around the sun.

(ii) Motion of the pendulum of wall clock.

1.2 Oscillatory Motion

(i) The motion of a body is said to be oscillatory or vibratory motion if it moves back and forth (to and fro) about a fixed point after certain interval of time.

(ii) The fixed point about which the body oscillates is called mean position or equilibrium position.

Examples : (i) Vibration of the wire of 'Sitar'.

(ii) Oscillation of the mass suspended from spring.

1.3 Harmonic Functions

The trigonometric function of constant amplitude and single frequency is define as harmonic function. (Among all the trigonometrical functions only "sin" and "cos" functions are taken as harmonic function in basic form")

$$\left.\begin{array}{l} y = A\sin\theta = A\sin\omega t \\ y = A\cos\theta = A\cos\omega t \end{array}\right\} \text{Harmonic function}$$

Note : The function will be non-harmonic if :

(i) It's amplitude is not constant.

(ii) It is basically formed by "tan", "cot", "sec", "cosec" functions.

1.4 Some basic terms

Mean Position

The point at which the restoring force on the particle is zero and potential energy is minimum, is known as its mean position.

Restoring Force

- The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force.
- This force is always directed towards the mean position.
- Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.
- It is given by $F = -kx$ and has dimension MLT^{-2}.

Amplitude

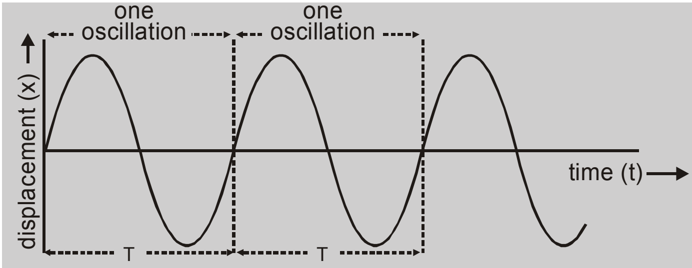
The maximum displacement of particle from mean position is define as amplitude.

Time period (T)

- The time after which the particle keeps on repeating its motion is known as time period.
- It is given by $T = \frac{2\pi}{\omega}$, $T = \frac{1}{n}$ where ω is angular frequency and n is frequency.

Oscillation or Vibration

When a particle goes on one side from mean position and returns back and then it goes to other side and again returns back to mean position, then this process is known as one oscillation.



Frequency (n or f)

(a) The number of oscillations per second is define as frequency.

(b) It is given by $n = \dfrac{1}{T}$, $n = \dfrac{\omega}{2\pi}$

(c) **SI UNIT :** Hertz (Hz)

1 Hertz = 1 cycle per second (cycle is a number not a dimensional quantity).

(d) **Dimension :** $[M^0 L^0 T^{-1}]$

Phase

(a) Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.

(b) $y = A\sin(\omega t + \phi)$

The quantity $(\omega t + \phi)$ represents the phase angle at that instant.

(c) The phase angle at time t = 0 is known as **initial phase or epoch.**

(d) The difference of total phase angles of two particles executing S.H.M. with respect to the mean position is known as phase difference.

(e) If the phase angles of two particles executing S.H.M. are $(\omega t + \phi_1)$ and $(\omega t + \phi_2)$ respectively, then the phase difference between two particles is given by

$$\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) \quad \text{or} \quad \Delta\phi = \phi_2 - \phi_1$$

(f) Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π, i.e., $\Delta\phi = 2N\pi \Rightarrow$ Same phase.

(g) Two vibrating particle are said to be in opposite phase if the phase difference between them is an odd multiple of π i.e., $\Delta\phi = (2N + 1)\pi \Rightarrow$ opposite phase.

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Angular frequency (ω)

(a) The rate of change of phase angle of a particle with respect to time is define as its angular frequency.

(b) **SI UNIT :** radian/second

Dimension : $[M^0 L^0 T^{-1}]$

Instantaneous displacement

(a) The displacement of the particle from mean position in a particular direction at any instant of time is known as instantaneous displacement.

(b) At time t the instantaneous displacement $x = A \sin(\omega t + \phi)$, where ϕ is initial phase and A is amplitude.

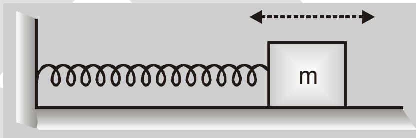
1.5 Simple harmonic motion (S.H.M.)

Simple harmonic motion is the simplest form of vibratory or oscillatory motion.

(i) S.H.M. are of two types

(a) Linear S.H.M.

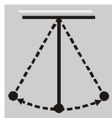
When a particle moves to and fro about a fixed point (called equilibrium position) along a straight line then its motion is called linear simple harmonic motion.



Example : Motion of a mass connected to spring.

(b) Angular S.H.M.

When a system oscillates angularly with respect to a fixed axis then its motion is called angular simple harmonic motion.



Example :- Motion of a bob of simple pendulum.

(ii) Necessary Condition to execute S.H.M.

(a) Motion of particle should be oscillatory.

(b) Total mechanical energy of particle should be conserved (Kinetic energy + Potential energy = constant)

(c) Extreme position should be well defined.

(d) In linear S.H.M.

The restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position

$\therefore \quad F \propto -y \quad$ or $\quad a \propto -y$

Negative sign shows that direction of force and acceleration is towards equilibrium position and y is displacement of particle from equilibrium position.

(e) In angular S.H.M.

The restoring torque (or angular acceleration) acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position

$\therefore \quad \tau \propto -\theta \quad$ or $\quad \alpha \propto -\theta$

(iii) Comparison between linear and angular S.H.M.

Linear S.H.M.	Angular S.H.M.
$F \propto -x$	$\tau \propto -\theta$
$F = -kx$	$\tau = -C\theta$
Where k is the restoring force constant	Where C is the restoring torque constant
$a = -\frac{k}{m}x$	$\alpha = -\frac{C}{I}\theta$
$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$	$\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$
It is known as differential equation of linear S.H.M.	It is known as differential equation of angular S.H.M.
$x = A \sin\omega t$	$\theta = \theta_0 \sin \omega t$
$a = -\omega^2 x$	$\alpha = -\omega^2\theta$
where ω is the angular frequency	
$\omega^2 = \frac{k}{m}$	$\omega^2 = \frac{C}{I}$
$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi n$	$\omega = \sqrt{\frac{C}{I}} = \frac{2\pi}{T} = 2\pi n$
where T is time period and n is frequency	
$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{I}{C}}$
$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$	$n = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$
This concept is valid for all types of linear S.H.M.	This concept is valid for all types of angular S.H.M.

GOLDEN KEY POINTS

- Oscillatory motion can be treated as simple harmonic motion only in the limit of small amplitude because in this limit the restoring force (or torque) becomes linear.
- Harmonic oscillations is that oscillations which can be expressed in terms of single harmonic function.

  (i.e. sine function or cosine function)
- The motion of the molecules of a solid, the vibration of the air columns and the vibration of string of musical instruments are either simple harmonic or superposition of simple harmonic motions.

Illustrations

Illustration 1

Which of the following functions represent SHM :–

(i) sin $2\omega t$ (ii) sin ωt + 2cos ωt (iii) sin ωt + cos $2\omega t$

Solution

(i) As $y = \sin 2\omega t \Rightarrow v = \frac{dy}{dt} = 2\,\omega\cos 2\omega t \Rightarrow$ Acceleration $= \frac{d^2y}{dt^2} = -4\omega^2\sin 2\omega t = -4\,\omega^2 y$

so $y = \sin 2\omega t$ represents S.H.M.

(ii) $y = \sin\omega t + 2\cos\omega t \Rightarrow v = \frac{dy}{dt} = \omega\cos\omega t - 2\omega\sin\omega t,$

Acceleration $= \frac{dv}{dt} = -\omega^2\sin\omega t - 2\omega^2\cos\omega t = -\omega^2(\sin\omega t + 2\cos\omega t) = -\omega^2 y$

$\therefore$ The given function represents SHM

(iii) $y = \sin\omega t + \cos 2\omega t \Rightarrow \frac{dy}{dt} = \omega\cos\omega t - 2\omega\sin 2\omega t,\ \frac{d^2y}{dt^2} = -\omega^2\sin\omega t - 4\omega^2\cos 2\omega t = -\omega^2(\sin\omega t + 4\cos 2\omega t)$

$\frac{d^2y}{dt^2} \not\propto (-y)$ (Oscillatory but S.H.M. not possible)

Illustration 2

If two S.H.M. are represented by equations $y_1 = 10\sin\left[3\pi t + \frac{\pi}{4}\right]$

and $y_2 = 5\left[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)\right]$ then find the ratio of their amplitudes and phase difference in between them.

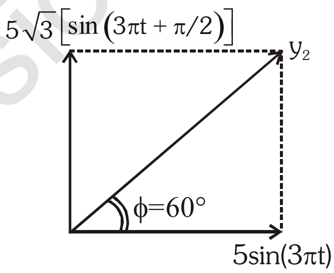
Solution

As $y_2 = 5\left[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)\right]$ ...(i)

So if $5 = A\cos\phi$ and $5\sqrt{3} = A\sin\phi$

Then $A = \sqrt{5^2 + (5\sqrt{3})^2} = 10$

and $\tan\phi = \frac{5\sqrt{3}}{5} = \sqrt{3}$ so $\phi = \frac{\pi}{3}$



$y_2 = 10\sin(3\pi t + \phi)$

the above equation (i) becomes

$y_2 = A\cos\phi\sin(3\pi t) + A\sin\phi\cos(3\pi t) \Rightarrow y_2 = A\sin(3\pi t + \phi)$

but $y_2 = 10\sin[3\pi t + (\frac{\pi}{3})]$

so, $\frac{A_1}{A_2} = \frac{10}{10} \Rightarrow A_1 : A_2 = 1 : 1,$

Phase difference $= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ rad.

BEGINNER'S BOX-1

1. Which of the following functions of time represent (a) SHM and (b) Periodic but not SHM (c) Non-periodic motion?

(i) $\sin \omega t + \cos \omega t$ (ii) $\sin \omega t + \cos 2\omega t + \sin 4\omega t$ (iii) $e^{-\omega t}$

(iv) $\log(\omega t)$ (v) $\sin \omega t - \cos \omega t$ (vi) $\sin^3 \omega t$

2. The equation of motion of a particle executing simple harmonic motion is $a+16\pi^2x = 0$. In this equation, a is the linear acceleration in m/s^2 of the particle at a displacement x in metre. Find the time period.

3. Displacement of a particle executing SHM is represented by $Y = 0.08\sin\left(3\pi t + \frac{\pi}{4}\right)$ metre. Then calculate:-

(a) Time period (b) Initial phase

(c) Displacement from mean position at $t = \frac{7}{36}$ sec.

2. SIMPLE HARMONIC MOTION (SHM) AND ITS EQUATION; VELOCITY, ACCELERATION AND PHASE

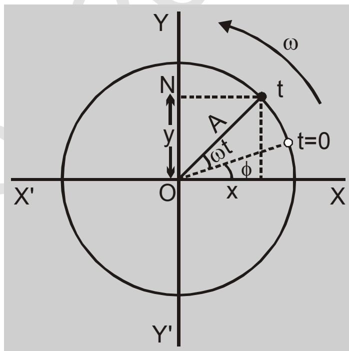
2.1 Geometrical meaning of S.H.M.

If a particle is moving with uniform speed along the circumference of a circle then the straight line motion of the foot of perpendicular drawn from the particle on the diameter of the circle is called S.H.M.

DESCRIPTION OF S.H.M. BASED ON CIRCULAR MOTION.

(a) Draw a circle, having radius equal to amplitude (A) of S.H.M.

(b) Suppose particle is moving with uniform speed with angular frequency ω along the circumference of the circle.

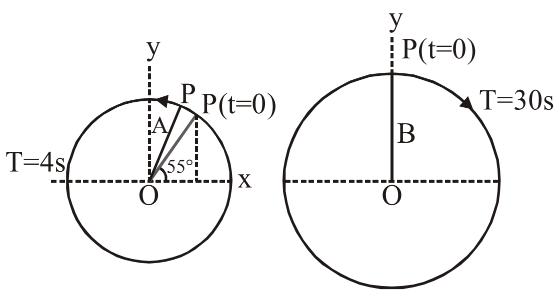


(c) Shadow (foot of the perpendicular from particle position) of particle performs S.H.M. on vertical and horizontal diameter of circle.

(d) Position of particle's shadow can be represented on diameter at t = 0 or any instant and position of particle performing circular motion can be determined by direction of velocity.

(e) By joining centre of circle to particle's position, angle θ is determined from horizontal or vertical diameter. After time t radius vector will turn ωt. so $\theta = \omega t$.

Ex. Depicts two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution are indicated on the figures. Obtain the simple harmonic motions of the x-projection of the radius vector of the rotating particle P in each case.



Sol. (a) At t = 0, OP makes an angle of 45° = π/4 rad with the (positive direction of) x-axis. After time t, it covers an $\frac{2\pi}{T}t$ in the anticlockwise sense, and makes an angle of $\frac{2\pi}{T}t+\frac{\pi}{4}$ with the x-axis.

The projection of OP on the x-axis at time t is given by,

$$x(t)=A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{4}\right)$$

For T = 4 s,

$$x(t)=A\cos\left(\frac{2\pi}{4}t+\frac{\pi}{4}\right)$$

which is a SHM of amplitude A, period 4 s, and an initial phase* = $\frac{\pi}{4}$.

(b) In this case at t = 0, OP makes an angle of 90° = $\frac{\pi}{2}$ with the x-axis. After a time t, it t covers an angle of $\frac{2}{T}t$ sense and makes an angle of $\frac{\pi}{2}-\frac{2}{T}t$ with the x-axis. The projection of OP on the x-axis at time t is given by

$$x(t)=B\cos\left(\frac{\pi}{2}-\frac{2}{T}t\right)=B\sin\left(\frac{2}{T}t\right)$$

For T = 30 s,

$$x(t)=B\sin\left(\frac{\pi}{15}t\right)$$

Writing this as $x(t)=B\sin\left(\frac{\pi}{15}-\frac{\pi}{2}\right)$ and comparing with Eq. (14.4). We find that this represents a SHM of amplitude B, period 30 s, and an initial phase of $\frac{\pi}{2}$.

2.2 Displacement, Velocity and Acceleration in S.H.M.

Displacement in S.H.M.

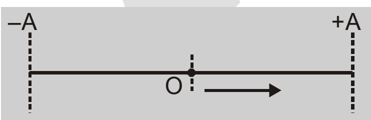
(i) The displacement of a particle executing linear S.H.M. at any instant is defined as the distance of the particle from the mean position at that instant.

(ii) It can be given by relation x = Asinωt or x = Acosωt.

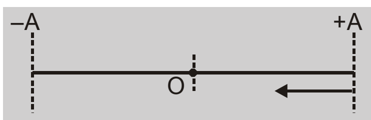
The first relation is valid when the time is measured from the mean position and the second relation is valid when the time is measured from the extreme position of the particle executing S.H.M. along a straight line path.

Ex. What will be the equation of displacement in the following different conditions ?

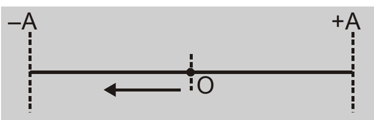
(i)



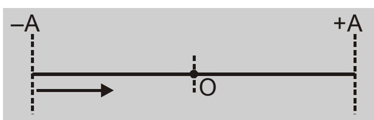
(ii)



(iii)



(iv)



Sol. (i) x = Asinωt

(ii) x = Asin(ωt + $\frac{\pi}{2}$) ⇒ x = Acosωt

(iii) x = Asin(ωt + π) ⇒ x = –Asinωt

(iv) x = Asin(ωt + $\frac{3\pi}{2}$) ⇒ x = –Acosωt

2.3 Velocity in S.H.M.

(i) It is define as the time rate of change of the displacement of the particle at a given instant.

(ii) Velocity in S.H.M. is given by $v = \frac{dx}{dt} = \frac{d}{dt}(A\sin\omega t)$ $\Rightarrow$ $v = A\omega\cos\omega t$

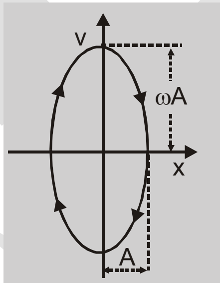
$$v = \pm A\omega\sqrt{1-\sin^2\omega t} \quad \Rightarrow \quad v = \pm A\omega\sqrt{1-\frac{x^2}{A^2}} \quad [\because\ x = A\sin\omega t]$$

$$v = \pm\omega\sqrt{(A^2-x^2)}$$

Squaring both the sides $v^2 = \omega^2(A^2 - x^2)$ $\Rightarrow$ $\frac{v^2}{\omega^2} = A^2 - x^2$

$$\frac{v^2}{\omega^2A^2} = 1-\frac{x^2}{A^2} \quad \Rightarrow \quad \frac{x^2}{A^2}+\frac{v^2}{A^2\omega^2} = 1$$

This is equation of ellipse. So curve between displacement and velocity of particle executing S.H.M. is ellipse.



(iii) The graph between velocity and displacement is shown in figure.

If particle oscillates with unit angular frequency (ω = 1) then curve between V and x will be circle.

Note: (i) The direction of velocity of a particle in S.H.M. is either towards or away from the mean position.

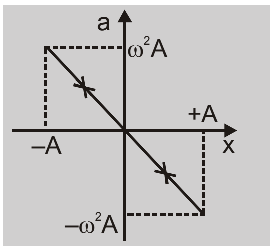
(ii) At mean position (x = 0), velocity is maximum (=$A\omega$) and at extreme position (x = $\pm$ A), the velocity of particle executing S.H.M. is zero.

2.4 Acceleration in S.H.M.

(i) It is define as the time rate of change of the velocity of the particle at given instant.

(ii) Acceleration in S.H.M. is given by $a = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\omega t)$

$a = -\omega^2 A\sin\omega t$ $\Rightarrow$ $a = -\omega^2 x$



(iii) The graph between acceleration and displacement is a straight line as shown in figure.

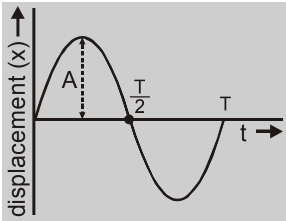
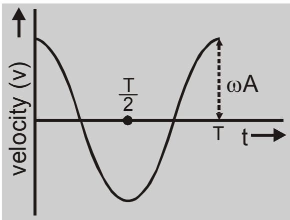
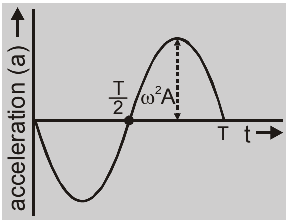
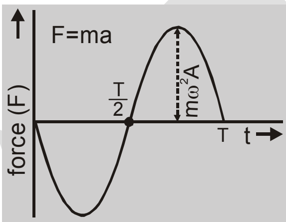
Note :

(i) The acceleration of a particle executing S.H.M. is always directed towards the mean position.

(ii) The acceleration of the particle executing S.H.M. is maximum at extreme position (= ω^2A) and minimum at mean position (= zero).

2.5 Graphical Representation

Graphical study of displacement, velocity, acceleration and force in S.H.M.

S. No.	Graph	In form of t	In form of x	Maximum value
1.	Displacement 	$x = A\sin\omega t$	$x = x$	$x = \pm A$
2.	Velocity 	$v = A\omega\cos\omega t$	$v = \pm\omega\sqrt{A^2 - x^2}$	$v = \pm\omega A$
3.	Acceleration 	$a = -\omega^2 A\sin\omega t$	$a = -\omega^2 x$	$a = \pm\omega^2 A$
4.	Force (F = ma) 	$F = -m\omega^2 A\sin\omega t$	$F = -m\omega^2 x$	$F = \pm m\omega^2 A$

GOLDEN KEY POINTS

- The direction of displacement is always away from the mean position whether the particle is moving from or coming towards the mean position.
- In linear S.H.M., the length of S.H.M. path = 2A
- In S.H.M., the total work done and displacement in one complete oscillation is zero but total travelled length is 4A.
- In S.H.M., the velocity and acceleration varies simple harmonically with the same frequency as displacement.
- Velocity is always ahead of displacement by phase angle $\frac{\pi}{2}$ radian
- Acceleration is ahead of displacement by phase angle π radian i.e., opposite to displacement.
- Acceleration leads the velocity by phase angle $\frac{\pi}{2}$ radian.

Illustrations

Illustration 3

An object performs S.H.M. of amplitude 5 cm and time period 4 s. If timing is started when the object is at the centre of the oscillation i.e., x = 0 then calculate –

(i) Frequency of oscillation

(ii) The displacement at 0.5 sec.

(iii) The maximum acceleration of the object.

(iv) The velocity at a displacement of 3 cm.

Solution

(i) Frequency $f = \frac{1}{T} = \frac{1}{4} = 0.25$ Hz

(ii) The displacement equation of object x = Asinωt

so at t = 0.5 s $\quad x = 5\sin(2\pi \times 0.25 \times 0.5) = 5\sin\frac{\pi}{4} = \frac{5}{\sqrt{2}}$ cm

(iii) Maximum acceleration $a_{max} = \omega^2 A = (0.5\,\pi)^2 \times 5 = 12.3$ cm/s^2

(iv) Velocity at x = 3 cm is $v = \pm\omega\sqrt{A^2 - x^2} = \pm 0.5\pi\sqrt{5^2 - 3^2} = \pm 6.28$ cm/s

Illustration 4

A particle executes S.H.M. from extreme position and covers a distance equal to half of its amplitude in 1 s. Determine the time period of motion.

Solution

For particle starting S.H.M. from extreme position

$y = A\cos\omega t \quad \Rightarrow \quad \frac{A}{2} = A\cos(\omega \times 1)$

$\Rightarrow \quad \cos\omega = \cos\frac{\pi}{3} \quad \Rightarrow \quad \omega = \frac{\pi}{3} \quad \Rightarrow \quad T = \frac{2\pi}{\omega} = \frac{2\pi \times 3}{\pi} = 6\,s$

Illustration 5

Amplitude of a harmonic oscillator is A, when velocity of particle is half of maximum velocity, then determine position of particle.

Solution

$v = \omega\sqrt{A^2 - x^2} \quad$ but $\quad v = \frac{v_{max}}{2} = \frac{A\omega}{2}$

$\frac{A\omega}{2} = \omega\sqrt{A^2 - x^2} \quad \Rightarrow \quad A^2 = 4[A^2 - x^2]$

$\Rightarrow \quad x^2 = \frac{4A^2 - A^2}{4}$

$\Rightarrow \quad x = \pm\frac{\sqrt{3}A}{2}$

Illustration 6

The velocity of a particle in S.H.M. at position x_1 and x_2 are v_1 and v_2 respectively. Determine value of time period and amplitude.

Solution

$v = \omega\sqrt{A^2 - x^2} \quad \Rightarrow \quad v^2 = \omega^2 (A^2 - x^2)$

At position x_1 velocity $v_1^2 = \omega^2 (A^2 - x_1^2)$ ... (i)

At position x_2 velocity $v_2^2 = \omega^2 (A^2 - x_2^2)$ ... (ii)

Subtracting (ii) from (i) $v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2) \Rightarrow \omega = \sqrt{\dfrac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$

Time period $T = \dfrac{2\pi}{\omega} \Rightarrow T = 2\pi\sqrt{\dfrac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

Dividing (i) by (ii) $\dfrac{v_1^2}{v_2^2} = \dfrac{A^2 - x_1^2}{A^2 - x_2^2} \Rightarrow v_1^2A^2 - v_1^2x_2^2 = v_2^2A^2 - v_2^2x_1^2$

So $A^2(v_1^2 - v_2^2) = v_1^2x_2^2 - v_2^2x_1^2 \Rightarrow A = \sqrt{\dfrac{v_1^2x_2^2 - v_2^2x_1^2}{v_1^2 - v_2^2}}$

Illustration 7

A particle executing S.H.M. having amplitude 0.01 m and frequency 60 Hz. Determine maximum acceleration of particle.

Solution

Maximum acceleration $a_{max.} = \omega^2A = 4\pi^2n^2A$

$= 4\pi^2(60)^2 \times (0.01)$

$= 144\ \pi^2\ m/s^2$

Illustration 8

A particle performing SHM is found at its equilibrium position at t = 1 sec and it is found to have a speed of 0.25 m/s at t = 2 sec. If the period of oscillation is 8 sec. Calculate the amplitude of oscillations.

Solution

$x = A\sin(\omega t + \phi)$

at t = 1 sec. particle at mean position

$0 = A\sin\left(\dfrac{2\pi}{8} \times 1 + \phi\right) \Rightarrow \boxed{\phi = -\dfrac{\pi}{4}}$

at t = 2 sec. velocity of particle is 0.25 m/s

$0.25 = A\omega\cos\left(\dfrac{\pi}{4} \times 2 - \dfrac{\pi}{4}\right)$

$0.25 = \dfrac{A\omega}{\sqrt{2}} \Rightarrow \boxed{A = \dfrac{\sqrt{2}}{\pi}}$

BEGINNER'S BOX-2

1. A particle executing simple harmonic motion completes 1200 oscillations per minute and passes through the mean position with a velocity of $3.14ms^{-1}$. Determine the maximum displacement of the particle from its mean position. Also obtain the displacement equation of the particle if its displacement be zero at the instant t = 0.

2. A particle oscillates along the x–axis according to equation $x = 0.05 \sin\left(5t - \frac{\pi}{6}\right)$ where x is in metre and t is in second. Find its velocity at t = 0 second.

3. A particle is executing SHM given by $x = A \sin(\pi t + \phi)$. The initial displacement of particle is 1 cm and its initial velocity is π cm/sec. Find the amplitude of motion and initial phase of the particle.

4. A body executing S.H.M. has its velocity 10 cm/sec and 7 cm/sec when its displacement from the mean positon are 3 cm and 4 cm respectively. Calculate the length of the path.

5. A particle is executing S.H.M. of time period 4s. What is the time taken by it to move from the

(a) Mean position to half of the amplitude.

(b) Extreme position to half of the amplitude.

6. A particle undergoes simple harmonic motion having time period T. Find the time taken to complete $\frac{3}{8}$ oscillation.

7. The displacement of a particle executing simple harmonic motion is given by $y = 10\sin(6t + \frac{\pi}{3})$. Here y is in metre and t is in second. Find initial displacement & velocity of the particle.

8. For a particle executing S.H.M. In which part of a complete oscillation

(a) Acceleration supports the velocity. (b) Acceleration opposes the velocity.

9. A particle is in linear simple harmonic motion between two points A and B, 10 cm apart. Take the direction from A to B as the positive direction and given the signs of velocity, acceleration and force on the particle when it is

(a) at the end A.
(b) at the end B.
(c) at the mid-point of AB going towards A.
(d) at 2 cm away from B going towards A.
(e) at 3 cm away from A going towards B, and
(f) at 4 cm away from A going towards A.

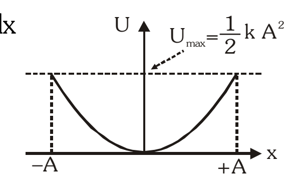
10. The piston in the cylinder head of a locomotive has a stroke (twice of the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min., what is its maximum speed ?

3. ENERGY IN SHM – POTENTIAL & KINETIC ENERGIES

3.1 Potential Energy (U or P.E.)

(i) In terms of displacement

The potential energy is related to force by the relation $F = -\frac{dU}{dx} \Rightarrow \int dU = -\int F dx$

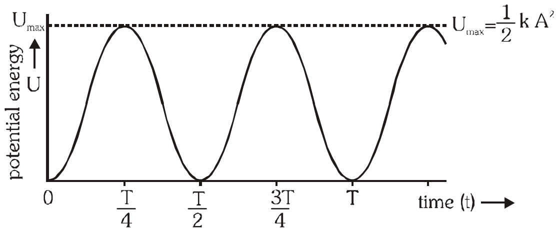


For S.H.M. $F = -kx$ so $\int dU = -\int(-kx)dx = \int kx\,dx \Rightarrow U = \frac{1}{2}kx^2 + C$

At $x = 0$, $U = U_0 \Rightarrow C = U_0$ So $U = \frac{1}{2}kx^2 + U_0$

Where the potenital energy at equilibrium position = U_0 when $U_0 = 0$ then $U = \frac{1}{2}kx^2$

(ii) In terms of time



Since $x = A\sin(\omega t + \phi), U = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$

If initial phase (ϕ) is zero then $U = \frac{1}{2}kA^2\sin^2\omega t = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

Note :

(i) In S.H.M. the potential energy is a parabolic function of displacement, the potential energy is minimum at the mean position (x = 0) and maximum at extreme position ($x = \pm A$)

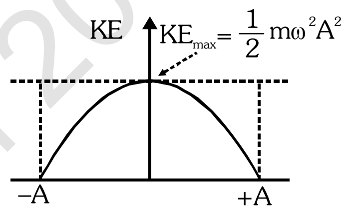
(ii) The potential energy is the periodic function of time.

It is minimum at $t = 0, \frac{T}{2}, T, \frac{3T}{2}...$ and maximum at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}...$

3.2 Kinetic Energy (K)

(i) In terms of displacement

If mass of the particle executing S.H.M. is m and Its velocity is v then kinetic energy at any instant.



$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$$

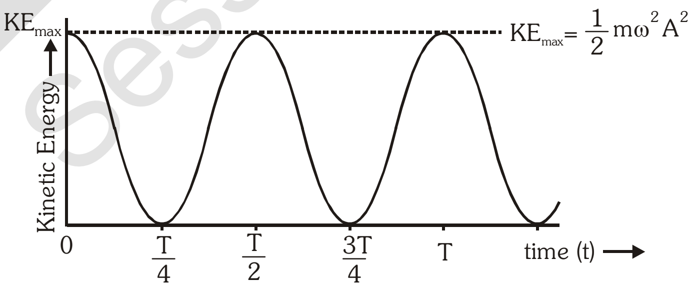
(ii) In terms of time

$\because v = A\omega\cos(\omega t + \phi)$

$\therefore K = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$

If initial phase ϕ is zero

$K = \frac{1}{2}m\omega^2A^2\cos^2\omega t$



Note :

(i) In S.H.M. the kinetic energy is a inverted parabolic function of displacement. The kinetic energy is maximum ($\frac{1}{2}kA^2$) at mean position (x = 0) and minimum (zero) at extreme position ($x = \pm A$)

(ii) The kinetic energy is the periodic function of time. It is maximum at $t = 0, \frac{T}{2}, T, \frac{3T}{2}$ ....and minimum at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}...$

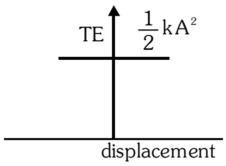
3.3 Total energy (E)

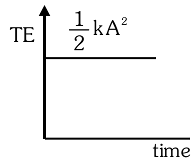
Total energy in S.H.M. is given by ; E = potential energy + kinetic energy = U + K

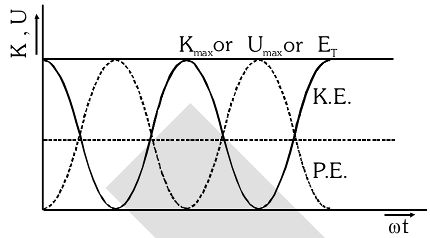
(i) w.r.t. position $E = \frac{1}{2}kx^2 + \frac{1}{2}k(A^2 - x^2) \quad \Rightarrow \quad E = \frac{1}{2}kA^2 = \text{constant}$

(ii) w.r.t. time

$$E = \frac{1}{2}m\omega^2A^2 \sin^2\omega t + \frac{1}{2}m\omega^2A^2\cos^2\omega t = \frac{1}{2}m\omega^2A^2(\sin^2\omega t + \cos^2\omega t) = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2 = \text{constant}$$







Note :

(i) Total energy of a particle in S.H.M. is same at all instant and at all displacement.

(ii) Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.

3.4 Average energy in S.H.M.

(i) The time average of P.E. and K.E. over one cycle is

(a) $\langle KE \rangle_t = \left\langle \frac{1}{2}m\omega^2A^2\cos^2\omega t \right\rangle = \frac{1}{2}m\omega^2A^2\langle \cos^2\omega t \rangle = \frac{1}{2}m\omega^2A^2\left(\frac{1}{2}\right) = \frac{1}{4}m\omega^2A^2 = \frac{1}{4}kA^2$

(b) $\langle PE \rangle_t = \left\langle \frac{1}{2}m\omega^2A^2\sin^2\omega t \right\rangle = \frac{1}{2}m\omega^2A^2\langle \sin^2\omega t \rangle = \frac{1}{2}m\omega^2A^2\left(\frac{1}{2}\right) = \frac{1}{4}m\omega^2A^2 = \frac{1}{4}kA^2 + U_0$

(c) $\langle TE \rangle_t = \left\langle \frac{1}{2}m\omega^2A^2 + U_0 \right\rangle = \frac{1}{2}m\omega^2A^2 + U_0 = \frac{1}{2}kA^2 + U_0$

GOLDEN KEY POINTS

- The frequency of oscillation of potential energy and kinetic energy is twice as that of displacement or velocity or acceleration of a particle executing S.H.M.
- Frequency of total energy is zero because it remains constant.

Illustrations

Illustration 9

In case of simple harmonic motion –

(a) What fraction of total energy is kinetic and what fraction is potential when displacement is one half of the amplitude.

(b) At what displacement the kinetic and potential energies are equal.

Solution

In S.H.M.

K. E. $= \frac{1}{2}k(A^2 - x^2)$ P. E. $= \frac{1}{2}kx^2$ T.E. $= \frac{1}{2}kA^2$

(a) $f_{K.E.} = \frac{K.E.}{T.E.} = \frac{A^2 - x^2}{A^2}$ $f_{P.E.} = \frac{P.E.}{T.E} = \frac{x^2}{A^2}$

at $x = \frac{A}{2}$ $f_{K.E.} = \frac{A^2 - A^2/4}{A^2} = \frac{3}{4}$ and $f_{P.E.} = \frac{A^2/4}{A^2} = \frac{1}{4}$

(b) K. E. = P. E. $\Rightarrow \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2 \quad \Rightarrow \quad 2x^2 = A^2 \quad \Rightarrow \quad x = \pm\frac{A}{\sqrt{2}}$

Illustration 10

A particle starts oscillating simple harmonically from its equilibrium position with time period T. Determine ratio of K.E. and P.E. of the particle at time t = $\frac{T}{12}$.

Solution

at t = $\frac{T}{12}$ $\qquad x = A\sin\frac{2\pi}{T}\times\frac{T}{12} = A\sin\frac{\pi}{6} = \frac{A}{2}$

so $\quad$ K.E. = $\frac{1}{2}k(A^2 - x^2) = \frac{3}{4}\times\frac{1}{2}kA^2$ $\quad$ and $\quad$ P.E. = $\frac{1}{2}kx^2 = \frac{1}{4}\times\frac{1}{2}kA^2$

$\therefore \quad \frac{K.E.}{P.E.} = \frac{3}{1}$

Illustration 11

The potential energy of a particle executing S.H.M. is 2.5 J, when its displacement is half of the amplitude, then determine total energy of particle.

Solution

P.E. = $\frac{1}{2}kx^2 \Rightarrow \quad \frac{1}{2}k\frac{A^2}{4} = 2.5 \quad \Rightarrow \quad$ Total energy = $\frac{1}{2}kA^2 = 2.5 \times 4 = 10$ J

Illustration 12

A harmonic oscillator of force consant 4×10^6 Nm^{-1} and amplitude 0.01 m has total energy 240 J. What is maximum kinetic energy and minimum potential energy ?

Solution

k = 4×10^6 N/m, a = 0.01 m, T.E. = 240 J, $\qquad$ As $\omega^2 = \frac{k}{m}$

Maximum kinetic energy = $\frac{1}{2}m\omega^2a^2 = \frac{1}{2}ka^2 = \frac{1}{2}\times4\times10^6\times(0.01)^2 = 200$ J

Minimum potential energy = Total energy – Maximum kinetic energy = 40 J

Illustration 13

The potential energy of a particle oscillating on x-axis is U = 20 + $(x - 2)^2$. Here U is in joules and x in meters. Total mechanical energy of the particle is 36 J.

(a) State whether the motion of the particle is simple harmonic or not.

(b) Find the mean position.

(c) Find the maximum kinetic energy of the particle.

Solution

(a) $\quad F = -\frac{dU}{dx} = -2(x - 2)$ $\qquad$ By assuming x – 2 = X, we have F = –2X

Since, $F \propto -X$ $\qquad$ The motion of the particle is simple harmonic

(b) The mean position of the particle is X = 0 $\Rightarrow$ x – 2 = 0, which gives x = 2m

(c) Maximum kinetic energy of the particle is, $K_{max} = E - U_{min} = 36 - 20 = 16$ J

Note : U_{min} is 20 J at mean position or at x = 2m.

BEGINNER'S BOX-3

1. A point particle of mass 0.1 Kg is executing SHM with amplitude of 0.1m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} Joule. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.
2. In case of SHM what fraction of total energy is kinetic and what fracton is potential, when displacement is one fourth of amplitude.
3. A particle execute S.H.M. with frequency f. Find frequency with which its kinetic energy oscillates?
4. A particle of mass 10g is placed in potential field given by V = $(50x^2 + 100)$ erg/g. What will be frequency of oscillation of particle ?

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4. OSCILLATIONS OF A SPRING BLOCK SYSTEM

4.1 Spring Block System

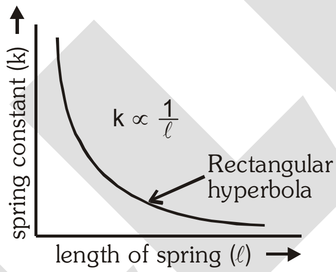
(i) When spring is given small displacement by stretching or compressing it, then restoring elastic force is developed in it because it obeys Hook's law.

$F \propto -x \qquad \Rightarrow F = -kx \qquad$ Here k is spring constant

(ii) Spring is assumed massless, so restoring elastic force in spring is assumed same everywhere.

(iii) Spring constant (k) depends on length (ℓ), radius and material of wire used in spring.

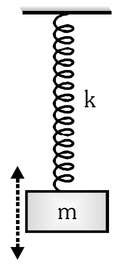
For spring, $k\ell$ = constant



4.2 Spring Pendulum

(i) When a small mass is suspended from a mass-less spring then this arrangement is known as spring pendulum. For small linear displacement the motion of spring pendulum is simple harmonic.

(ii) For a spring pendulum



$$F = -kx \Rightarrow m\frac{d^2x}{dt^2} = -kx \quad [\because F = ma = m\frac{d^2x}{dt^2}]$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

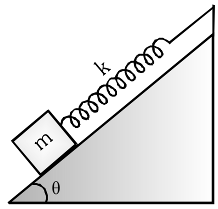
$$\because \frac{d^2x}{dt^2} = -\omega^2 x \Rightarrow \omega^2 = \frac{k}{m}$$

This is standard equation of linear S.H.M.

Time period $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$, Frequency $n = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

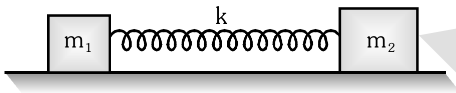
(iii) Time period of a spring pendulum is independent of acceleration due to gravity. This is why a clock based on oscillation of spring pendulum will keep proper time everywhere on a hill or moon or in a satellite or different places of earth.

(iv) If a spring pendulum oscillates in a vertical plane is made to oscillate on a horizontal surface or on an inclined plane then time period will remain unchanged.



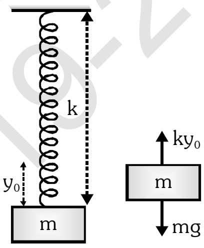
(v) By increasing the mass, time period of spring pendulum increases $(T \propto \sqrt{m})$, but by increasing the force constant of spring (k), Its time period decreases $\left[T \propto \frac{1}{\sqrt{k}}\right]$ whereas frequency increases $(n \propto \sqrt{k})$

(vi) If two masses m_1 and m_2 are connected by a spring and made to oscillate then time period $T = 2\pi\sqrt{\frac{\mu}{k}}$



Here, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass

(vii) If the stretch in a vertically loaded spring is y_0 then for equilibrium of mass m.

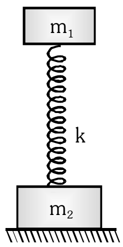


$ky_0 = mg$ i.e., $\frac{m}{k} = \frac{y_0}{g}$

So, time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$

But remember time period of spring pendulum is independent of acceleration due to gravity.

(viii) If two particles are attached with spring in which only one is oscillating then the

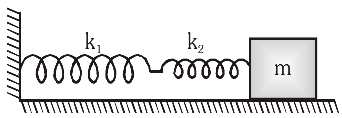


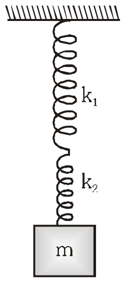
Time period $= 2\pi\sqrt{\frac{\text{mass of oscillating particle}}{\text{force constant}}} = 2\pi\sqrt{\frac{m_1}{k}}$

4.3 Various Spring Arrangements

• **Series combination of springs**

In series combination same restoring force exerts in all springs but extension will be different.





Total displacement $x = x_1 + x_2$

Force acting on both springs $F = -k_1x_1 = -k_2x_2$

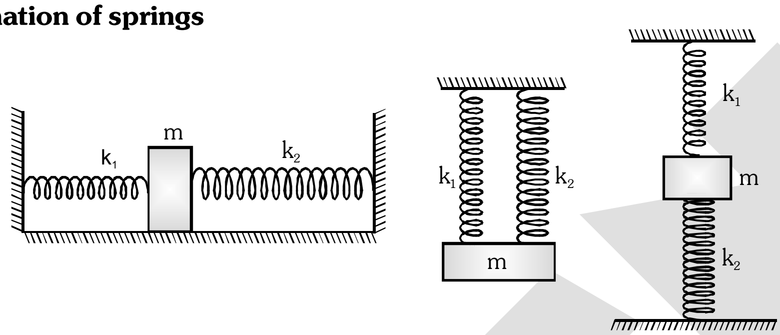
$\because\ x_1 = -\dfrac{F}{k_1}$ and $x_2 = -\dfrac{F}{k_2}$ $\quad\therefore\quad x = -\left[\dfrac{F}{k_1}+\dfrac{F}{k_2}\right]$ ....(i)

If equivalent force constant is k_s then $F = -k_sx$

so by equation (i) $\quad -\dfrac{F}{k_s} = -\dfrac{F}{k_1}-\dfrac{F}{k_2} \Rightarrow \dfrac{1}{k_s} = \dfrac{1}{k_1}+\dfrac{1}{k_2} \Rightarrow\ k_s = \dfrac{k_1k_2}{k_1+k_2}$

Time period $T = 2\pi\sqrt{\dfrac{m}{k_s}}$ $\qquad$ Frequency $n = \dfrac{1}{2\pi}\sqrt{\dfrac{k_s}{m}}$, Angular frequency $\omega = \sqrt{\dfrac{k_s}{m}}$

- **Parallel Combination of springs**



In parallel combination displacement on each spring is same but restoring force is different.

Force acting on the system $F = F_1 + F_2 \Rightarrow\ F = -k_1x - k_2x$ ...(i)

If equivalent force constant is k_P then, $F = -k_Px$, so by equation (i) $-k_Px = -k_1x - k_2x \Rightarrow\ k_P = k_1 + k_2$

Time period $\quad T = 2\pi\sqrt{\dfrac{m}{k_P}} = 2\pi\sqrt{\dfrac{m}{k_1+k_2}}$; Frequency $n = \dfrac{1}{2\pi}\sqrt{\dfrac{k_P}{m}}$; Angular frequency $\omega = \sqrt{\dfrac{k_1+k_2}{m}}$

GOLDEN KEY POINTS

- If the length of the spring is made n times then effective force constant becomes $\dfrac{1}{n}$ times and the time period becomes $\sqrt{n}$ times.
- If a spring of spring constant k is divided into n equal parts, the spring constant of each part becomes nk and time period becomes $\dfrac{1}{\sqrt{n}}$ times.
- In case of a loaded spring the time period comes out to be the same in both horizontal and vertical arrangement of spring system.
- The force constant k of a stiffer spring is higher than that of a soft spring. So the time period of a stiffer spring is less than that of a soft spring.

Illustrations

Illustration 14

A body of mass m attached to a spring which is oscillating with time period 4 seconds. If the mass of the body is increased by 4 kg, its timer period increases by 2 sec. Determine value of initial mass m.

Solution

In I*st case* : $T = 2\pi\sqrt{\dfrac{m}{k}} \Rightarrow\ 4 = 2\pi\sqrt{\dfrac{m}{k}}$ ...(i) $\qquad$ and in II*nd case:* $6 = 2\pi\sqrt{\dfrac{m+4}{k}}$ ...(ii)

Divide (i) by (ii) $\dfrac{4}{6} = \sqrt{\dfrac{m}{m+4}} \Rightarrow \dfrac{16}{36} = \dfrac{m}{m+4} \Rightarrow\ m = 3.2$ kg

Illustration 15

One body is suspended from a spring of length ℓ, spring constant k and has time period T. Now if spring is divided in two equal parts which are joined in parallel and the same body is suspended from this arrangement then determine new time period.

Solution

Spring constant in parallel combination k' = 2k + 2k = 4k

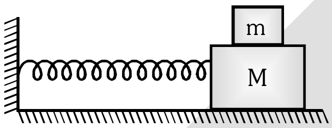
$$\therefore T' = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{4k}} = 2\pi\sqrt{\frac{m}{k}} \times \frac{1}{\sqrt{4}} = \frac{T}{\sqrt{4}} = \frac{T}{2}$$

Illustration 16

A block of mass m is on a horizontal slab of mass M which is moving horizontally and executing S.H.M. The coefficient of static friction between block and slab is μ. If block is not separated from slab then determine angular frequency of oscillation.

Solution

If block is not separated from slab then restoring force due to S.H.M. should be less than frictional force between slab and block.



$$F_{\text{restoring}} \le F_{\text{friction}} \Rightarrow m\, a_{max.} \le \mu mg \Rightarrow a_{max.} \le \mu g \Rightarrow \omega^2 A \le \mu g \Rightarrow \omega \le \sqrt{\frac{\mu g}{A}}$$

Illustration 17

A block of mass m is attached from a spring of spring constant k and dropped from its natural length. Find the amplitude of S.H.M.

Solution

Let amplitude of S.H.M. be A then by work energy theorem W = ΔKE

$$mgx_0 - \frac{1}{2}kx_0^2 = 0 \Rightarrow x_0 = \frac{2mg}{k}$$

So amplitude $A = \frac{mg}{k}$

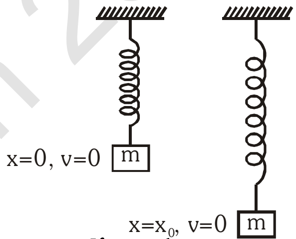


Illustration 18

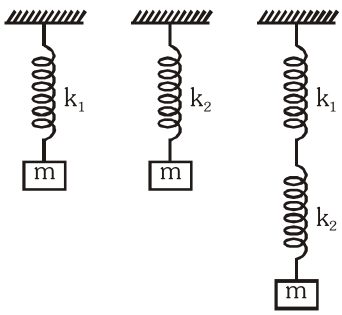
Periodic time of oscillation T_1 is obtained when a mass is suspended from a spring. If another spring is used with same mass then periodic time of oscillation is T_2. Now if this mass is suspended from series combination of above springs then calculate the time period.

Solution

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}} \Rightarrow T_1^2 = 4\pi^2\frac{m}{k_1} \Rightarrow k_1 = \frac{4\pi^2 m}{T_1^2} \text{ and } T_2 = 2\pi\sqrt{\frac{m}{k_2}} \Rightarrow T_2^2 = 4\pi^2\frac{m}{k_2} \Rightarrow k_2 = \frac{4\pi^2 m}{T_2^2}$$

$$K_{eq.} = \frac{4\pi^2 m}{T_{eq.}^2}$$

In series combination –



$$\frac{1}{K_{eq.}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{T_{eq.}^2}{4\pi^2 m} = \frac{T_1^2}{4\pi^2 m} + \frac{T_2^2}{4\pi^2 m}$$

$$T_{eq.} = \sqrt{T_1^2 + T_2^2}$$

Illustration 19

Infinite springs with force constants k, 2k, 4k, 8k, ... respectively are connected in series. Calculate the effective force constant of the spring.

Solution

$$\frac{1}{k_{eff}}=\frac{1}{k}+\frac{1}{2k}+\frac{1}{4k}+\frac{1}{8k}+\ldots\ldots\ldots\ldots\infty \text{ (For infinite G.P. } S_\infty=\frac{a}{1-r} \text{ where a = First term, r = common ratio)}$$

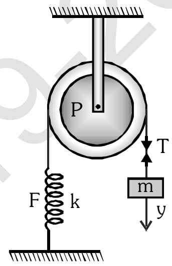
$$\frac{1}{k_{eff}}=\frac{1}{k}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\ldots\right]=\frac{1}{k}\left[\frac{1}{1-\frac{1}{2}}\right]=\frac{2}{k} \text{ so } k_{eff}=k/2$$

Illustration 20

Figure shows a system consisting of a massless pulley, a spring of force constant k = 4000 N/m and a block of mass m = 1 kg. If the block is slightly displaced vertically down from its equilibrium position and released find the frequency of its vertical oscillation in given cases.

Solution

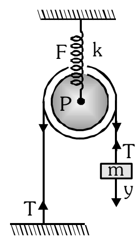
Case (A) :



As the pully is fixed and string is inextensible, if mass m is displaced by y the spring will stretch by y, and as there is no mass between string and spring (as pully is massless) F = T = ky i.e., restoring force is linear and so motion of mass m will be linear simple harmonic with frequency

$$n_A=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{4000}{1}}\approx 10 \text{ Hz}$$

Case (B) :

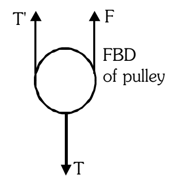


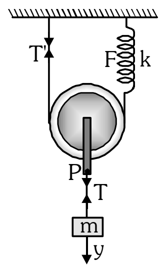
The pulley is movable and string inextensible, so if mass m moves down a distance y, the pulley will move down by (y/2). So the force in the spring F = k(y/2). Now as pully is massless F = 2T, ⇒ T = F/2 = (k/4)y. So the restoring force

on the mass m $T=\frac{1}{4}ky=k'y \Rightarrow k'=\frac{1}{4}k$

So $n_B=\frac{1}{2\pi}\sqrt{\frac{k'}{m}}=\frac{1}{2\pi}\sqrt{\frac{k}{4\pi}}=\frac{n_A}{2}=5 \text{ Hz}$

Case (C) :





In this situation if the mass m moves by y the pully will also move by y and so the spring will stretch by 2y (as string is inextensible) and so T' = F = 2ky. Now as pulley is massless so T = F + T'= 4ky, i.e., the restoring force on the mass m

$$T=4ky=k'y \Rightarrow k'=4k$$

so $n_C=\frac{1}{2\pi}\sqrt{\frac{k'}{m}}=\frac{1}{2\pi}\sqrt{\frac{4k}{m}}=2n_A=20\text{Hz}$

Illustration 21

Frequency of oscillation of a body is 6 Hz when force F_1 is applied and 8 Hz when F_2 is applied. If both forces F_1 & F_2 are applied together then find out the frequency of oscillation. **[AIPMT 2004]**

Solution

According to question $F_1 = -K_1 x$ & $F_2 = -K_2 x$

so $n_1 = \frac{1}{2\pi}\sqrt{\frac{K_1}{m}} = 6\text{Hz}$; $n_2 = \frac{1}{2\pi}\sqrt{\frac{K_2}{m}} = 8$ Hz

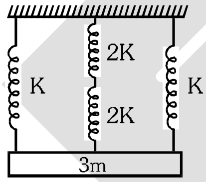
Now $F = F_1 + F_2 = -(K_1 + K_2)x$ Therefore $n = \frac{1}{2\pi}\sqrt{\frac{K_1 + K_2}{m}}$

$$\Rightarrow \quad n = \frac{1}{2\pi}\sqrt{\frac{4\pi^2 n_1^2 m + 4\pi^2 n_2^2 m}{m}} = \sqrt{n_1^2 + n_2^2} = \sqrt{8^2 + 6^2} = 10 \text{ Hz}$$

BEGINNER'S BOX-4

1. A man with a wrist watch on his hand falls from the top of a tower. Does the watch give correct time during the free fall ?

2. Calculate the time period of following system.



3. A body of mass 'm' is suspended from a vertical spring of length 'L'. It execute S.H.M. with a time period T. Find the time period if

(a) Length of spring is made half.

(b) Mass of body is made half.

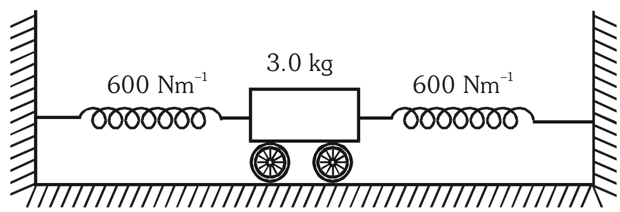
4. When a mass of 1 kg is suspended from a vertical spring, its length increases by 0.98 m. If this mass is pulled downwards and then released, what will be periodic time of vibration of spring ? ($g=9.8$ m/s^2).

5. A block of mass 'm' moving with velocity (v) collides perfectly inelastically with another identical block attached to spring of force constant K. What will be the amplitude of resulting SHM ?

6. A spring having a spring constant 1200 Nm^{-1} is mounted on a horizontal table. A mass of 3kg is attached to the free end of the spring. The mass is then pulled to a distance of 2.0 cm and released. Determine Maximum acceleration of mass.

7. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

8. A trolley of mass 3.0 kg, as shown in Fig., is connected to two springs, each of spring has spring constant 600 Nm^{-1}. If the trolley is displaced from its equilibrium position by 5.0 cm and released, what is (a) the period of ensuing oscillations, and (b) the maximum speed of the trolley ? How much energy is dissipated as heat by the time the trolley comes to rest due to damping forces ?

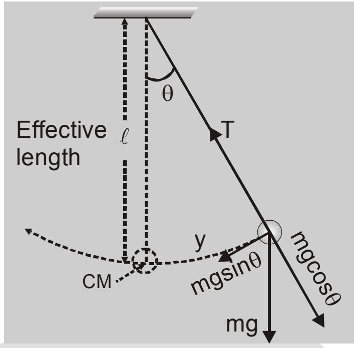


5. SIMPLE PENDULUM

If a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum

Expression for time period

Method-I (Force Method)



For small angular displacement, $\sin\theta \approx \theta$, so that

$F = -mg\sin\theta$

$\quad = -mg\,\theta$

$\quad = -\left(\dfrac{mg}{\ell}\right)y$

$\quad = -ky \quad \Rightarrow \quad K = \dfrac{mg}{\ell}$

(because $y = \ell\theta$), Thus, the time period of the simple pendulum is

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \Rightarrow \quad \text{or } T = 2\pi\sqrt{\frac{\ell}{g}}$$

Method-II (Torque Method)

Bob of pendulum moves along the arc of circle in vertical plane. Here motion involved is angular and oscillatory where restoring torque is provided by gravitational force.

$\tau = -(mg)(\ell\sin\theta)$ (Negative sign shows opposite direction of τ and angular displacement)

$\tau = -mg\ell\theta$ (If angular displacement is small, then $\sin\theta \simeq \theta$)

$I\alpha = -mg\ell\theta$

$\alpha = -\dfrac{mg\ell}{I}\theta$ (where I = moment of inertia of bob about point of suspension so $I = m\ell^2$)

Now $\alpha = -\dfrac{mg\ell}{m\ell^2}\theta \Rightarrow \alpha = -\dfrac{g}{\ell}\theta \Rightarrow \dfrac{d^2\theta}{dt^2} + \dfrac{g}{\ell}\theta = 0$

It is differential equation of angular SHM of simple pendulum.

Comparing with standard differential equation of angular SHM $\left(\dfrac{d^2\theta}{dt^2} + \omega^2\theta = 0\right)$:-

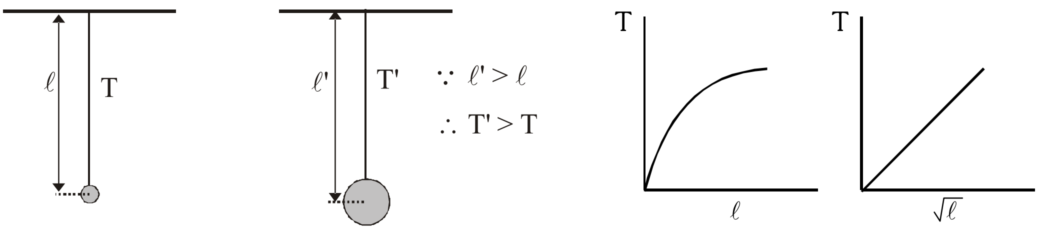
Therefore $\quad T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{\ell}{g}}$

Note : Simple pendulum is the example of SHM but only when its angular displacement is very small.

Important points :

1. The time period of simple pendulum is independent from mass of the bob but it depends on size of bob (position of centre of mass). So in simple pendulum when a solid iron bob is replaced by light aluminium bob of same radius then time period remains unchanged.

2. Time period of simple pendulum is directly proportional to square root of length.



When a person sitting on an oscillating swing comes in standing position then centre of mass raises upwards and length decreases, so time period decreases and frequency increases means swing oscillates faster.

3. When a hollow spherical bob of simple pendulum is completely filled with water and a small hole is made in bottom of it, then as water drainout, at first its time period increases, after that it decrease and when sphere becomes empty then finally it becomes as before (T).

4. If simple pendulum is shifted to poles, equator or hilly areas, then its time period may be different $\left(T \propto \frac{1}{\sqrt{g}}\right)$

5. If a clock based on oscillation of simple pendulum is shifted from earth to moon then it becomes slow because its time period increases and becomes $\sqrt{6}$ times compare to earth. $\frac{g_M}{g_E} = \frac{1}{6} \Rightarrow T_M = \sqrt{6}T_E$

6. **Periodic time of simple pendulum in reference system**

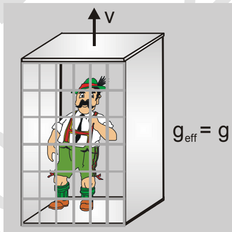
$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$

where, $g_{eff.}$ = effective gravity acceleration in reference system

or total downward acceleration.

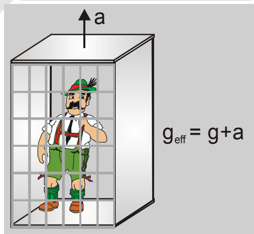
(a) If reference system is lift

(i) If velocity of lift v = constant



acceleration a = 0 and $g_{eff.} = g$ $\therefore$ $T = 2\pi\sqrt{\frac{\ell}{g}}$

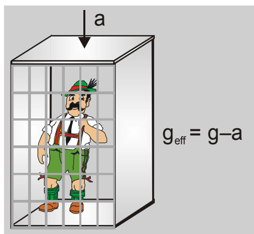
(ii) If lift is moving upwards with acceleration a



$g_{eff.} = g + a$

$T = 2\pi\sqrt{\frac{\ell}{g+a}}$ $\Rightarrow$ T decreases

(iii) If lift is moving downwards with acceleration a



$g_{eff.} = g - a$

$\therefore$ $T = 2\pi\sqrt{\frac{\ell}{g-a}}$ $\Rightarrow$ T increases

(iv) If lift falls downwards freely

$g_{eff.} = g - g = 0$ $\Rightarrow$ $T = \infty$ simple pendulum will not oscillate

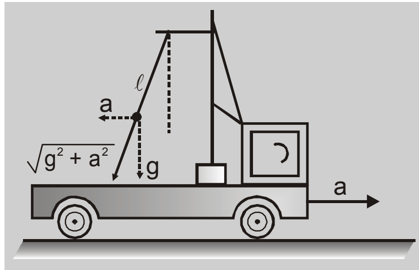
If simple pendulum is shifted to the centre of earth, freely falling lift, in artificial satellite then it will not oscillate and its time period is infinite ($\because g_{eff.} = 0$).

(b) A simple pendulum is mounted on a moving truck

(i) If truck is moving with constant velocity, no pseudo force acts on the pendulum and time period remains same $T = 2\pi\sqrt{\frac{\ell}{g}}$

(ii) If truck accelerates forward with acceleration f then a pseudo force acts in opposite direction.

So effective acceleration, $g_{eff.} = \sqrt{g^2 + a^2}$ and $T' = 2\pi\sqrt{\dfrac{\ell}{g_{eff.}}}$

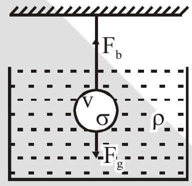


Time period $T' = 2\pi\sqrt{\dfrac{\ell}{\sqrt{g^2 + a^2}}}$ $\Rightarrow T'$ decreases

7. If a simple pendulum of density σ is made to oscillate in a liquid of density ρ then its time period will increase as compare to that of air and is given by

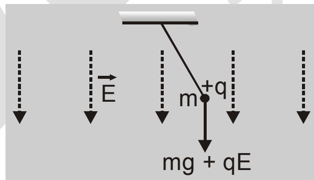
$F_{net} = F_g - F_b$

$$\frac{mg_{net}}{m} = \frac{mg}{m} - \frac{V\rho g}{m}, \qquad g_{net} = g - \frac{V\rho g}{V\sigma} = g\left(1 - \frac{\rho}{\sigma}\right)$$



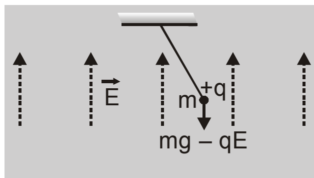
$$T = 2\pi\sqrt{\frac{\ell}{\left[1 - \frac{\rho}{\sigma}\right]g}}$$

8.(a) If the bob of simple pendulum has positive charge q and pendulum is placed in uniform electric field which is in downward direction then time period decreases



$$T = 2\pi\sqrt{\frac{\ell}{g + \frac{qE}{m}}}$$

(b) If the bob of simple pendulum has positive charge q and is made is oscillate in uniform electric field acting in upward direction then time period increases



$$T = 2\pi\sqrt{\frac{\ell}{g - \frac{qE}{m}}}$$

9. $T = 2\pi\sqrt{\dfrac{\ell}{g}}$ is valid when length of simple pendulum (ℓ) is negligible as compare to radius of earth (ℓ << R) but if ℓ is comparable to radius of earth

then time period $T = 2\pi\sqrt{\dfrac{R}{\left[1 + \frac{R}{\ell}\right]g}} = 2\pi\sqrt{\dfrac{\ell}{g\left[1 + \frac{\ell}{R}\right]}} = 2\pi\sqrt{\dfrac{1}{\left[\frac{1}{\ell} + \frac{1}{R}\right]g}}$

The time period of oscillation of simple pendulum of infinite length

$T = 2\pi\sqrt{\dfrac{R}{g}} \simeq 84.6$ minute $\approx 1\dfrac{1}{2}$ hour It is maximum time period.

10. Second's pendulum

If the time period of a simple pendulum is 2 second then it is called second's pendulum. Second's pendulum take one second to go from one extreme position to other extreme position.

For second's pendulum, time period T = 2 = $2\pi\sqrt{\frac{\ell}{g}}$

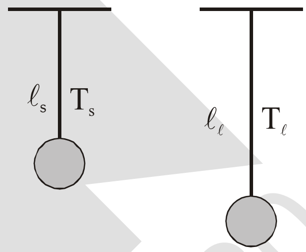
At the surface of earth $g = 9.8\ m/s^2 \approx \pi^2 m/s^2$,

So length of second pendulum at the surface of earth $\ell \approx 1$ metre

11. When a long and short pendulum start oscillation simultaneously then both will be in same phase in minimum time when short pendulum complete one more oscillation compare to long pendulum.

After starting, if long pendulum completes N oscillation to come in same phase in minimum time then short will complete (N+1) oscillation.

$$t = NT_\ell = (N+1)T_S$$



$$N\left(2\pi\sqrt{\frac{\ell_\ell}{g}}\right) = (N+1)\left(2\pi\sqrt{\frac{\ell_s}{g}}\right)$$

$$\boxed{N\sqrt{\ell_\ell} = (N+1)\sqrt{\ell_s}}$$

GOLDEN KEY POINTS

- Simple pendulum performs angular S.H.M. but due to small angular displacement, it is considered as linear S.H.M.
- If time period of clock based on simple pendulum increases then clock will be slow and if time period decreases then clock will be fast.
- If $\Delta\ell$ is change in length and Δg is the change in acceleration then for small variation (up to 5%) change in time period (ΔT) will be

$$\frac{\Delta T}{T} \times 100 = \left[\frac{1}{2}\frac{\Delta\ell}{\ell} - \frac{1}{2}\frac{\Delta g}{g}\right] \times 100$$

- Due to change in shape of earth (not spherical but elliptical) gravitational acceleration is different at different places. So time period of simple pendulum varies with variation of g.
- The time period of simple pendulum is independent of mass of bob.

Illustrations

Illustration 22

A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest, its time period is T. With what acceleration should lift be accelerated upwards in order to reduce its time period to $\frac{T}{2}$?

Solution

In stationary lift $T = 2\pi\sqrt{\frac{\ell}{g}}$ ...(i)

In accelerated lift $\frac{T}{2} = T' = 2\pi\sqrt{\frac{\ell}{g+a}}$ ...(ii)

Divide (i) by (ii) $2 = \sqrt{\frac{g+a}{g}}$ or $g + a = 4g$ or $a = 3g$

Illustration 23

The length of a second's pendulum at the surface of earth is 1m. Determine the length of second's pendulum at the surface of moon.

Solution

For second's pendulum at the surface of earth $2 = 2\pi\sqrt{\frac{\ell_e}{g_e}}$ ...(i)

For second's pendulum at the surface of moon $2 = 2\pi\sqrt{\frac{\ell_m}{g_m}}$ ...(ii)

From (i) and (ii) $\frac{\ell_e}{g_e} = \frac{\ell_m}{g_m} \Rightarrow \ell_m = \left[\frac{g_m}{g_e}\right]\ell_e$

$\Rightarrow \ell_m = \frac{\ell_e}{6}$ $\left[\because \quad g_m = \frac{g_e}{6}\right]$

Illustration 24

If length of a simple pendulum is increased by 4%. Then determine percentage change in time period.

Solution

Percentage change in time period $\frac{\Delta T}{T}\times 100\% = \frac{1}{2}\frac{\Delta\ell}{\ell}\times 100$ $[\because \quad \Delta g = 0]$

According to question $\frac{\Delta\ell}{\ell}\times 100 = 4\%$

$\therefore \quad \frac{\Delta T}{T}\times 100\% = \frac{1}{2}\times 4\% = 2\%$

Illustration 25

A bob of simple pendulum is suspended by a metalic wire. If α is the coefficient of linear expansion and $d\theta$ is the change in temperature then prove that percentage change in time period is $50\alpha d\theta$.

Solution

With change in temperature $d\theta$, the effective length of wire becomes $\ell' = \ell(1 + \alpha\, d\theta)$

$T' = 2\pi\sqrt{\frac{\ell'}{g}}$ and $T = 2\pi\sqrt{\frac{\ell}{g}}$

Hence $\frac{T'}{T} = \sqrt{\frac{\ell'}{\ell}} = (1 + \alpha\, d\theta)^{1/2} = 1 + \frac{1}{2}\alpha d\theta$

$\therefore$ Percentage increase in time period $= \left[\frac{T'-T}{T}\right]\times 100 = \left[\frac{T'}{T}-1\right]\times 100$

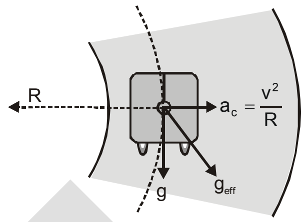
$= \left[1+\frac{\alpha d\theta}{2}-1\right]\times 100 = 50\,\alpha\, d\theta$

Illustration 26

A simple pendulum of length L and mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes oscillation in a radial direction about its equilibrium position, then calculate its time period .

Solution

Centripetal acceleration $a_c = \frac{v^2}{R}$ & Acceleration due to gravity = g



So $g_{eff} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2} \Rightarrow$ Time period $T = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$

BEGINNER'S BOX-5

1. The angle made by the string of a simple pendulum with the vertical depends upon time as $\theta = \frac{\pi}{90}\sin \pi t$. Find the length of the pendulum if $g = \pi^2\ ms^{-1}$

2. The bob of a simple pendulum is a hollow sphere filled with water. Explain how will the period of oscillation change if the water begins to drain out of the hollow sphere through a small hole in the bottom ?

3. A girl is swinging on the swing in the sitting position. What shall be effect on the frequency of oscillation if she stands up ?

4. Why a pendulum clock does not work during free fall or in an artificial satellite ?

5. Find the time period and frequency of a simple pendulum of 1·000 m at a location where g = 9·800 m/s^2.

6. A simple pendulum has time period T_1 when it is on earth's surface and T_2 when it is taken at height 2R above the earth surface. Calculate $\frac{T_2}{T_1}$. (R is the radius of earth).

7. What is the length of a simple pendulum whose time period of oscillation for small amplitudes equals 2·0 seconds

8. What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity ?

6. DIFFERENT TYPES OF OSCILLATIONS

Different types of oscillations

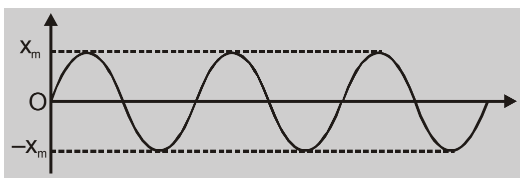
Free, Damped, Forced oscillations and Resonance

(a) Free oscillation

(i) The oscillations of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations.

(ii) The amplitude, frequency and energy of oscillations remain constant.

(iii) The oscillator which keeps on oscillating with constant amplitude for infinite time is known as free oscillator.



(b) Damped oscillations :

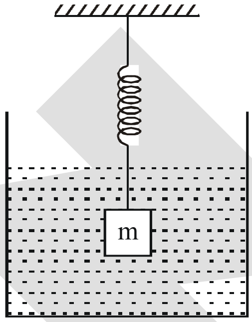
(i) The oscillations in which the amplitude decreases gradually with the passage of time are called damped oscillations.

(ii) In many real systems, non-conservative forces such as friction retard the motion. Consequently the mechanical energy of the system diminishes in time, and the motion is said to be damped. The lost mechanical energy is transformed into internal energy of the object & the retarding medium.

(iii) The retarting force can be expressed as $\vec{F} = -b\vec{v}$ (where b is a constant called the damping coefficient) and restoring force on the system is –kx, we can write Newton's second law as

$$\sum f_x = -kx - bv = ma$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$



This is the differential equation of damped oscillation, solution of this equation is given by

$$x = Ae^{(-b/2m)t}\cos(\omega' t + \phi)$$

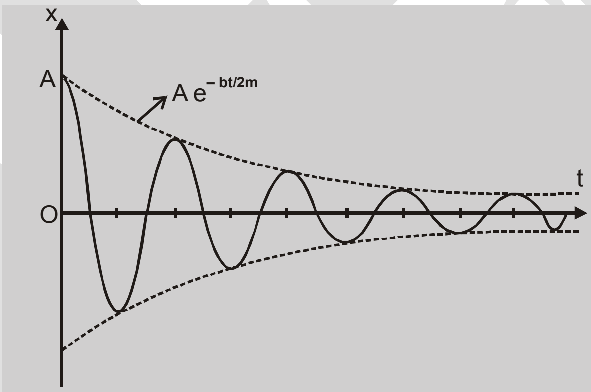
where angular frequency of oscillation is

$$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega^2 - \left(\frac{b}{2m}\right)^2}$$

where $\omega = \sqrt{\frac{k}{m}}$ represents the angular frequency in the absence of retarting force (the undamped oscillator) & is called natural frequency.

(c) Effect of damped oscillation :

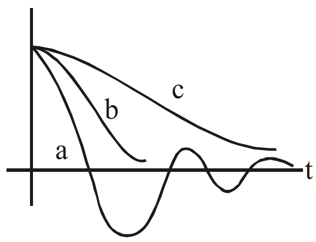
(i) When the retarding force is small, the oscillatory character of the motion is preserved but amplitude decreases in time, and it decays exponentially with time



(a) When the magnitude of retarding force is small such that b/2m < ω, the system is said to be underdamped

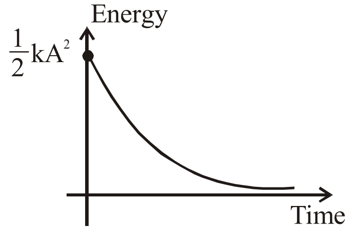
(b) When b reaches a critical value b_c such that $\frac{b_c}{2m} = \omega$, then system doesnot oscillate & is said to be critically damped.

(c) When retarding force is large as compared to restoring force, i.e., $\frac{b}{2m} > \omega$ then system is overdamped.



a = underdamped oscillator
b = critically damped oscillator
c = overdamped oscillator

(ii) Mechanical energy of undamped oscillator is $\frac{1}{2}kA^2$. For a damped oscillator amplitude is not constant but depend on time, so total energy is



$$E(t) = \frac{1}{2}k(Ae^{-bt/2m})^2 = \frac{1}{2}kA^2e^{-bt/m}$$

(d) Forced oscillations :

(i) All free oscillations eventually die out because of ever present damping force, However, an external agency can maintain these oscillations. These are called forced or driven oscillations.

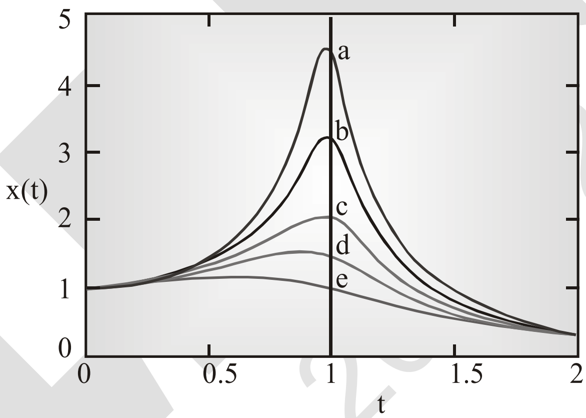
(ii) Under forced periodic oscillation, system does not oscillate with its natural frequency (ω) but with driven frequency (ω_d).

(iii) Suppose an external force F(t) of amplitude F_0 that varies periodically with time is applied to a damped oscillator. Such a force is $F(t) = F_0 \cos \omega_d t$

The equation of particle under combined force is

$$ma(t) = -kx - bv + F_0 \cos \omega_d t$$

$$\frac{md^2x}{dt^2} + \frac{bdx}{dt} + kx = F_0 \cos \omega_d t$$



After solving

$$x = A' \cos(\omega_d t + \phi)$$

where

$$A' = \frac{F_0}{m\sqrt{(\omega_d^2 - \omega^2)^2 + \left(\frac{b\omega_d}{m}\right)^2}}$$

(ω is natural frequency, $\omega = \sqrt{\frac{k}{m}}$)

(iv) The amplitude of driven oscillator decreases due to damping forces but on account of the energy gained from external source (driver) it remains constant.

(v) The amplitude of forced vibration is determined by the difference between the frequency of applied force & the natural frequency. If difference between frequency is small then amplitude will be large.

Resonance :

(i) For small damping, the amplitude is large when the frequency of the driving force is near the natural frequency of oscillation ($\omega_d \approx \omega$). The increase in amplitude near the natural frequency is called resonance, & the natural frequency ω is also called the resonance frequency of the system.

(ii) In the state of resonance, there occurs maximum transfer of energy from the driver to the driven.

GOLDEN KEY POINTS

- When a tunning fork is struck against a rubber pad, the prongs begin to execute free vibration.
- When the stem of vibrating tunning fork is pressed against the top of tabla, then the tabla will suffer forced vibration.
- Soldiers are asked to break step while crossing a bridge, If the soldiers march in step, there is a possibility that the frequency of the foot steps may match the natural frequency of the bridge. Due to resonance, the bridge may start oscillating violently, thereby damaging itself.
- The oscillations of simple pendulum in air are damped oscillations.

Illustrations

Illustration 27

The amplitude of a damped oscillator becomes half in one minute. The amplitude after 3 minutes will be $\frac{1}{x}$ times of the original. Determine the value of x.

Solution

Amplitude of damped oscillation is $A = A_0 e^{-\gamma t}$ [from $x = x_m e^{-\gamma t}$]

at t = 1 min $A = \frac{A_0}{2}$ so $\frac{A_0}{2} = A_0 e^{-\gamma}$ or $e^{\gamma} = 2$

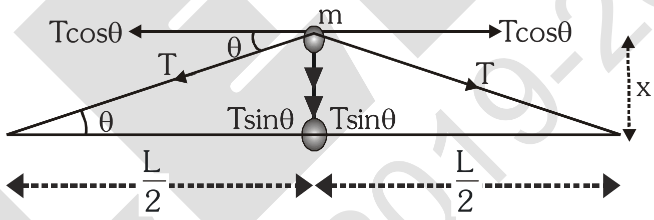
After 3 minutes $A = \frac{A_0}{x}$ so $\frac{A_0}{x} = A_0 e^{-\gamma \times 3}$ or $x = e^{3\gamma} = (e^{\gamma})^3 = 2^3 = 8$

Illustration 28

A ball of mass m kept at the centre of a string of length L is pulled from center in perpendicular direction and released. Prove that motion of ball is simple harmonic and determine time period of oscillation

Solution

Restoring force $F = -2T\sin\theta$



When θ is small $\sin\theta \approx \tan\theta \approx \theta = \frac{x}{L/2}$

$$m\frac{d^2x}{dt^2} = -2T\sin\theta = -2T\theta = -2T\frac{x}{L/2}$$

$$\frac{d^2x}{dt^2} = -\frac{4T}{mL}x \Rightarrow \frac{d^2x}{dt^2} \propto -x$$

So motion is simple harmonic $\omega = \frac{2\pi}{T} = \sqrt{\frac{4T}{mL}} \Rightarrow T = 2\pi\sqrt{\frac{mL}{4T}}$

Illustration 29

In damped oscillations, the amplitude after 50 oscillations is $0.8\ a_0$, where a_0 is the initial amplitude, then determine amplitude after 150 oscillations **[AIPMT 2008]**

Solution

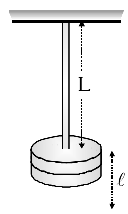
The amplitude, a, at time t is given by $a = a_0 \exp(-\alpha t)$

$a_{50} = a_0 \exp(-\alpha \times 50T) = 0.80\ a_0$ where T is the period of oscillation

$a_{150} = a_0 \exp(-\alpha \times 150T) = a_0 (0.8)^3 = 0.512\ a_0$

7. EXAMPLES OF SIMPLE HARMONIC MOTION (These are for students practice)

- If a mass m is suspended from a wire of length L, cross section A and young's modulus Y and is pulled along the length of the wire then restoring force will be developed by the elasticity of the wire.



$$Y = \frac{\text{stress}}{\text{strain}} \Rightarrow \frac{F/A}{\ell/L} \Rightarrow = \frac{FL}{\ell A} \Rightarrow F = -\frac{YA}{L}\ell$$

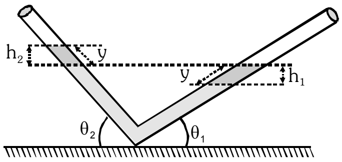
Restoring force is linear so motion is linear simple harmonic with force constant

$$k = \frac{YA}{L} \quad \text{i.e.,} \quad n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

- **Motion of a liquid in a V-shape tube when it is slightly depressed and released**

Here cross-section of the tube is uniform and the liquid is incompressible and non viscous. Initially the level of liquid in the two limbs will be at the same height. If the liquid is pressed by y in one limb, it will rise by y along the length of the tube in the other limb so the restoring force will developed by hydrostatic pressure difference, i.e.,

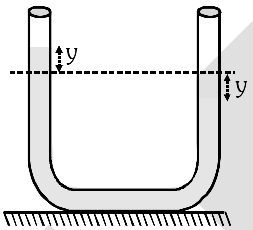
$$F = -\Delta P \times A = -(h_1 + h_2)gdA \Rightarrow \quad F = -Agd(\sin\theta_1 + \sin\theta_2)y$$



As the restoring force is linear, motion will be linear simple harmonic.

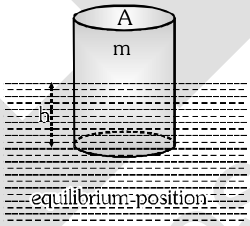
Force constant (k) = Agd ($\sin\theta_1 + \sin\theta_2$) So $T = 2\pi\sqrt{\dfrac{m}{Adg(\sin\theta_1 + \sin\theta_2)}}$

Note : If the tube is a U-tube and liquid is filled to a height h



$\theta_1 = \theta_2 = 90°$ and $m = hAd \times 2$ So time period $T = 2\pi\sqrt{\dfrac{h}{g}}$

- **When a partially submerged floating body is slightly pressed and released :**

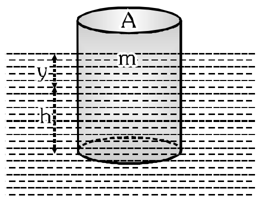


If a body of mass m and cross section A is floating in a liquid of density σ with height h inside the liquid then

$$mg = \text{Thrust} = Ah\sigma g, \quad \text{i.e., } m = Ah\sigma \quad \ldots(i)$$

Now from this equilibrium position if it is pressed by y, restoring force will developed due to extra thrust i.e.

$$F = -A\sigma g y$$



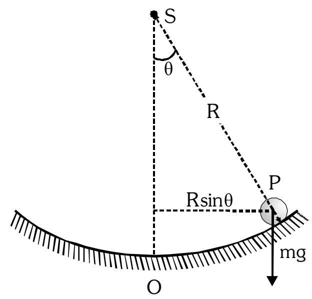
As restoring force is linear, motion will be linear simple harmonic with force constant $k = A\sigma g$,

$$\text{So } T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{A\sigma g}}$$

From this expression it is clear that if density of liquid decreases, time period will increase and vice-versa.

And also as from eq^n. (i) $m = Ah\sigma$, $T = 2\pi\sqrt{\dfrac{h}{g}}$ where h is the height of the body inside the liquid.

- **Motion of a ball in a bowl**



If a small steel ball of mass m is placed at a small distance from O inside a smooth concave surface of radius R and released, it will oscillate about O. The restoring torque here will be due to the force of gravity mg on the ball i.e., $\tau = -mg\,(R\sin\theta) = -mgR\theta$ [As θ is small]

Now as restoring torque is angular so motion will be angular simple harmonic. And as by definition.

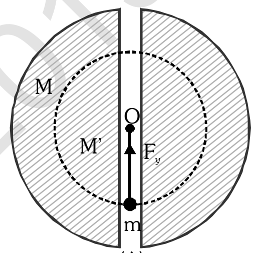
$$\tau = I\alpha = mR^2\left[\frac{d^2\theta}{dt^2}\right] \quad [\text{as } I = mR^2 \text{ and } \alpha = \frac{d^2\theta}{dt^2}]$$

so, $$mR^2\frac{d^2\theta}{dt^2} = -mgR\theta \quad \text{i.e., } \frac{d^2\theta}{dt^2} = -\omega^2\theta \Rightarrow \omega^2 = \frac{g}{R} \text{ so } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

- **Motion of a ball in a tunnel through the earth**

Case I :

If the tunnel is along a diameter and a ball is released from the surface. If the ball at any time is at a distance y from the centre of earth, then the restoring force will act on the ball due to gravitation between ball and earth. But from theory of gravitation we know that force that acts on a particle inside the earth at a distance y from its centre is only due to mass M' of the earth that lies within sphere of radius y. (the portion of the earth that lies out side this sphere does not exert any net force on the particle) so $F = \frac{-GmM'}{y^2}$



(A)

But as $M = \frac{4}{3}\pi R^3\rho$ and $M' = \frac{4}{3}\pi y^3\rho$, i.e., $M' = M\left[\frac{y}{R}\right]^3$

$$F = \frac{-Gm}{y^2} \times M\left[\frac{y^3}{R^3}\right] = -\frac{GMm}{R^3}y$$

Restoring force is linear so the motion is linear SHM with force constant .

$$k = \frac{GMm}{R^3} \text{ so } T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{R^3}{GM}}$$

Further more as $g = \frac{GM}{R^2} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$

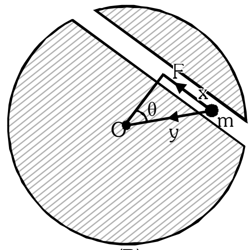
Which is same as that of a simple pendulum of infinite length and is equal to 84.6 minutes.

***Case II** :*

If the tunnel is along a chord and ball is released from the surface and if the ball at any time is at a distance x from the centre of the tunnel. The restoring force will be :

$$F' = F\sin\theta = \left[-\frac{GMm}{R^3}y\right]\left[\frac{x}{y}\right] = -\frac{GMm}{R^3}x$$

Which is again linear with same force constant $k = \dfrac{GMm}{R^3}$



(B)

So that motion is linear simple harmonic with same time period -

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ minutes}$$

Note : In SHM $v_{max} = \omega A$

(i) In I case and IInd case time period will be same but v_{max} will be different.

(ii) If ball is dropped from height h it will perform oscillatory motion not SHM $[F \propto \frac{1}{r^2}$ and not $F \propto (-r)]$

(iii) If a ball is released in a tunnel along the chord from one side then it will take 42.3 minute to reach the other side of chord.

Illustrations

Illustration 30

A liquid of mass m is set into oscillations in a U-tube of cross section A. Its time period recorded is T, where $T = 2\pi\sqrt{\frac{\ell}{2g}}$, here ℓ is the length of liquid column. If the liquid of same mass is set into oscillations in U-tube of cross section A/16 then determine time period of oscillation.

Solution

mass is constant volume × density = constant $\Rightarrow V_1 d = V_2 d$

$$(A\ell)d = \left[\frac{A}{16}\ell'\right]d \Rightarrow \ell' = 16\,\ell$$

$$\because T = 2\pi\sqrt{\frac{\ell}{2g}} \qquad \therefore \frac{T'}{T} = \sqrt{\frac{\ell'}{\ell}} = \sqrt{\frac{16\ell}{\ell}} = 4 \Rightarrow T' = 4T$$

Illustration 31

Which agency provides the restoring force in the following case ?

(a) A spring compressed and then free to vibrate.

(b) Water pressed in U tube and then left free to vibrate.

(c) Ball released in a diametric tunnel of earth.

(d) Pendulum pulled from its mean position and released.

Solution

(a) Elasticity of spring

(b) Hydrostatic pressure of water

(c) Gravitational force of earth.

(d) Weight of bob of pendulum

ANSWERS

BEGINNER'S BOX-1

1. (i) – (a), (ii) – (b), (iii) – (c), (iv) – (c), (v) – (a), (vi) – (b)

2. 0.5 s

3. (a) 0.67 s, (b) $\frac{\pi}{4}$rad , (c) 0.04 metre

BEGINNER'S BOX-2

1. 0.025m, y = 0.025 sin (40πt) m

2. $\frac{\sqrt{3}}{8}$ ms^{-1} **3.** $\sqrt{2}$ cm and $\frac{\pi}{4}$ rad

4. 9.52 cm **5.** (a) $\frac{1}{3}$ s, (b) $\frac{2}{3}$s

6. $\frac{5T}{12}$ **7.** $5\sqrt{3}$ m,30 m/s

8. (a) When particle moves from extreme to mean position

(b) When particle moves from mean to extreme position

9. (a) 0, +, +; (b) 0, –, –; (c) –, 0, 0;

(d) –, –, – ; (e) +, +, +; (f) –, +, +

10. 100 m/min

BEGINNER'S BOX-3

1. $Y = 0.1\sin\left[4t + \frac{\pi}{4}\right]$

2. $\frac{15}{16}$ and $\frac{1}{16}$ **3.** 2f **4.** $\frac{5}{\pi}$ s^{-1}

BEGINNER'S BOX-4

1. watch will give correct time because it depends on spring action and does not depend on gravity.

2. $2\pi\sqrt{\frac{m}{K}}$ **3.** (a) $T' = \frac{T}{\sqrt{2}}$, (b) $T' = \frac{T}{\sqrt{2}}$

4. 2s **5.** $A = v\sqrt{\frac{m}{2K}}$ **6.** 8 m/s

7. 218.7 N **8.** (a) 0.314 s, (b) 1 m/s, 1.5 J

BEGINNER'S BOX-5

1. 1 m

2. The time period will increase at first, then decrease until the sphere is empty acquire its initial value.

3. increase

4. In both cases effective acceleration due to gravity becomes zero. In absence of 'g_{eff}' there is no restoring force and pendulum does not oscillate.

5. 2 s, 0.5 Hz **6.** 3

7. 0·99 m **8.** zero

EXERCISE-I (Conceptual Questions)

Build Up Your Understanding

PERIODIC MOTION AND ITS CHARACTERISTICS

1. A particle of mass m is executing S.H.M. If amplitude is a and frequency n, the value of its force constant will be :

(1) mn^2 (2) $4mn^2a^2$
(3) ma^2 (4) $4\pi^2mn^2$

2. The equation of motion of a particle executing S.H.M. where letters have usual meaning is :

(1) $\frac{d^2x}{dt^2} = -\frac{k}{m}x$ (2) $\frac{d^2x}{dt^2} = +\omega^2x$

(3) $\frac{d^2x}{dt^2} = -\omega^2x^2$ (4) $\frac{d^2x}{dt^2} = -kmx$

3. The equation of motion of a particle executing SHM is $\left(\frac{d^2x}{dt^2}\right) + kx = 0$. The time period of the particle will be :

(1) $2\pi/\sqrt{k}$ (2) $2\pi/k$

(3) $2\pi k$ (4) $2\pi\sqrt{k}$

4. Which of the following equation does not represent a simple harmonic motion :

(1) $y = a\sin\omega t$
(2) $y = b\cos\omega t$
(3) $y = a\sin\omega t + b\cos\omega t$
(4) $y = a\tan\omega t$

SIMPLE HARMONIC MOTION (SHM) AND ITS EQUATION

5. The displacement of a particle in S.H.M. is indicated by equation $y = 10\sin(20t + \pi/3)$ where y is in metres. The value of time period of vibration will be (in seconds) :

(1) $10/\pi$ (2) $\pi/10$
(3) $2\pi/10$ (4) $10/2\pi$

6. The value of phase at maximum displacement from the mean position of a particle in S.H.M. is :

(1) $\pi/2$ (2) π
(3) Zero (4) 2π

7. The equation of a simple harmonic motion is $x = 0.34\cos(3000t + 0.74)$. Where x and t are in mm and sec. respectively. The frequency of the motion is :

(1) 3000 (2) $3000/2\pi$
(3) $0.74/2\pi$ (4) $3000/\pi$

8. The acceleration of a particle executing S.H.M. is
(1) Always directed towards the equillibrium position
(2) Always towards the one end
(3) Continuously changing in direction
(4) Maximum at the mean position

9. The distance covered by a particle executing SHM, in one time period is equal to :
(1) Four times the amplitude
(2) Two times the amplitude
(3) One times the amplitude
(4) Eight times the amplitude

10. The phase of a particle in S.H.M. is $\pi/2$, then :
(1) Its velocity will be maximum.
(2) Its acceleration will be minimum.
(3) Restoring force on it will be minimum.
(4) Its displacement will be maximum.

11. The displacement of a particle in S.H.M. is indicated by equation $y = 10\sin(20t + \pi/3)$ where y is in metres. The value of maximum velocity of the particle will be :

(1) 100 m/sec. (2) 150 m/sec.
(3) 200 m/sec. (4) 400 m/sec.

12. In the above question, the value of phase constant will be :

(1) Zero (2) 45° (3) 60° (4) 30°

13. The phase of a particle in SHM at time t is $\pi/6$. The following inference is drawn from this:
(1) The particle is at $x = a/2$ and moving in + X-direction
(2) The particle is at $x = a/2$ and moving in – X-direction
(3) The particle is at $x = -a/2$ and moving in + X-direction
(4) The particle is at $x = -a/2$ and moving in – X-direction

14. Two particles execute S.H.M. along the same line at the same frequency. They move in opposite direction at the mean position. The phase difference will be :

(1) 2π (2) $2\pi/3$ (3) π (4) $\pi/2$

15. The displacement from mean position of a particle in SHM at 3 seconds is $\sqrt{3}/2$ of the amplitude. Its time period will be :

(1) 18 sec. (2) $6\sqrt{3}$ sec.

(3) 9 sec. (4) $3\sqrt{3}$ sec.

16. A particle executes SHM of type $x = a\sin\omega t$. It takes time t_1 from $x = 0$ to $x = \frac{a}{2}$ and t_2 from $x = \frac{a}{2}$ to $x = a$. The ratio of $t_1 : t_2$ will be :

(1) 1 : 1 (2) 1 : 2 (3) 1 : 3 (4) 2 : 1

17. The time taken by a particle in SHM for maximum displacement is :

(1) T/8 (2) T/6 (3) T/2 (4) T/4

18. A particle executes SHM with periodic time of 6 seconds. The time taken for traversing a distance of half the amplitude from mean position is :

(1) 3 sec. (2) 2 sec. (3) 1 sec. (4) 1/2 sec.

19. The phase difference between the displacement and acceleration of particle executing S.H.M. in radian is :

(1) $\pi/4$ (2) $\pi/2$ (3) π (4) 2π

20. The phase difference in radians between displacement and velocity in S.H.M. is :

(1) $\pi/4$ (2) $\pi/2$ (3) π (4) 2π

21. If the maximum velocity of a particle in SHM is v_0. then its velocity at half the amplitude from position of rest will be :

(1) $v_0/2$ (2) v_0

(3) $v_0\sqrt{3/2}$ (4) $v_0\sqrt{3}/2$

22. At a particular position the velocity of a particle in SHM with amplitude a is $\sqrt{3}/2$ that at its mean position. In this position, its displacement is :

(1) a/2 (2) $\sqrt{3}\,a/2$ (3) $a\sqrt{2}$ (4) $\sqrt{2a}$

23. The acceleration of a particle in SHM at 5 cm from its mean position is 20 cm/sec^2. The value of angular frequency in radians/sec will be :

(1) 2 (2) 4 (3) 10 (4) 14

24. The amplitude of a particle in SHM is 5 cms and its time period is π. At a displacement of 3 cms from its mean position the velocity in cms/sec will be :

(1) 8 (2) 12 (3) 2 (4) 16

25. The maximum velocity and acceleration of a particle in S.H.M. are 100 cms/sec and 157 cm/sec^2 respectively. The time period in seconds will be :

(1) 4 (2) 1.57 (3) 0.25 (4) 1

26. If the displacement, velocity and acceleration of a particle in SHM are 1 cm, 1cm/sec, 1cm/sec^2 respectively its time period will be (in seconds) :

(1) π (2) 0.5π (3) 2π (4) 1.5π

27. The particle is executing S.H.M. on a line 4 cms long. If its velocity at mean position is 12 cm/sec, its frequency in Hertz will be :

(1) $\frac{2\pi}{3}$ (2) $\frac{3}{2\pi}$ (3) $\frac{\pi}{3}$ (4) $\frac{3}{\pi}$

28. Which of the following statement is incorrect for an object executing S.H.M. :

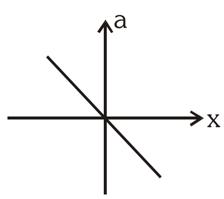
(1) The value of acceleration is maximum at the extreme points

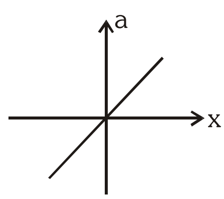
(2) The total work done for completing one oscillation is zero.

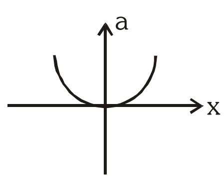
(3) The energy changes from one form to another

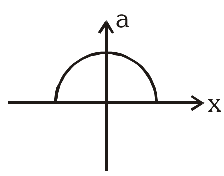
(4) The velocity at the mean position is zero

29. The variation of acceleration (a) and displacement (x) of the particle executing SHM is indicated by the following curve :

(1) 

(2) 

(3) 

(4) 

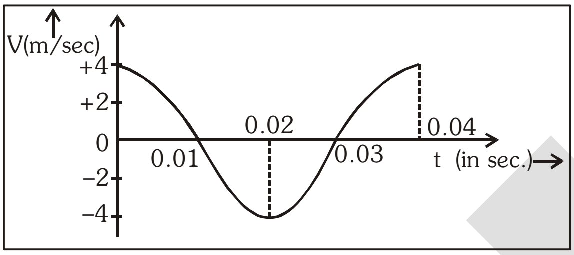
30. The time period of an oscillating body executing SHM is 0.05 sec and its amplitude is 40 cm. The maximum velocity of particle is :

(1) $16\pi\ ms^{-1}$ (2) $2\pi\ ms^{-1}$
(3) $3.1\ ms^{-1}$ (4) 4π m/s

31. A body of mass 5 gm is executing S.H.M. about a point with amplitude 10 cm. Its maximum velocity is 100 cm/sec. Its velocity will be 50 cm/sec at a distance from mean position :

(1) 5 cm (2) $5\sqrt{2}$ cm
(3) $5\sqrt{3}$ cm (4) $10\sqrt{2}$ cm

32. The velocity-time diagram of a harmonic oscillator is shown in the adjoining figure. The frequency of oscillation is :



(1) 25 Hz (2) 50 Hz
(3) 12.25 Hz (4) 33.3 Hz

33. A particle is executing S.H.M. of frequency 300 Hz and with amplitude 0.1 cm. Its maximum velocity will be :

(1) 60π cm/s (2) 0.6π cm/s
(3) 0.50π cm/s (4) 0.05π cm/s

34. If amplitude of the particle which is executing S.H.M., is doubled, then which quantity will become double ?

(1) Frequency (2) Time period
(3) Energy (4) Maximum velocity

35. Which one of the following statements is true for the speed 'v' and the acceleration 'a' of a particle executing simple harmonic motion

(1) Value of 'a' is zero, whatever may be the value of 'v'
(2) When 'v' is zero, 'a' is zero
(3) When 'v' is maximum, 'a' is zero
(4) When 'v' is maximum, 'a' is maximum

36. For a particle executing simple harmonic motion which of the following statement is not correct:

(1) The total energy of particle always remains the same
(2) The restoring force is always directed towards a fix point.
(3) The restoring force is maximum at the extreme positions.
(4) The acceleration of particle is maximum at the equilibrium positions.

37. In SHM velocity is maximum:

(1) At extreme position
(2) When displacement is half of amplitude
(3) At the central position
(4) When Displacement is $\frac{1}{\sqrt{2}}$ of amplitude

38. A body oscillates with SHM according to the equation $x = 5.0 \cos(2\pi t + \pi)$. At time t = 1.5 s, its displacement, speed and acceleration respectively is :

(1) $0, -10\pi, +20\pi^2$ (2) $5, 0, -20\pi^2$
(3) $2.5, +20\pi, 0$ (4) $-5.0, +5\pi, -10\pi^2$

39. A particle executing simple harmonic motionof amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of oscillation is :

(1) 1 Hz (2) 3 Hz (3) 2 Hz (4) 4 Hz

40. The maximum velocity of simple harmonic motion represented by $y = 3\sin\left(100t + \frac{\pi}{6}\right)$ is given by

(1) 300 (2) $\frac{3\pi}{6}$ (3) 100 (4) $\frac{\pi}{6}$

41. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm is 4.4 m/s. The period of oscillation is :

(1) 100 s (2) 0.01 s
(3) 10 s (4) 0.1 s

42. Average velocity of a particle performing SHM in one time period is :-

(1) Zero (2) $\frac{A\omega}{2}$
(3) $\frac{A\omega}{2\pi}$ (4) $\frac{2A\omega}{\pi}$

43. A particle is executing S.H.M. with amplitude A and Time period T. Time taken by the particle to reach from extreme position to $\frac{A}{2}$

(1) $\frac{T}{6}$ (2) $\frac{T}{12}$ (3) $\frac{T}{3}$ (4) $\frac{T}{4}$

44. Total work done on a simple pendulum in one complete oscillation will be :-

(1) $\frac{1}{2}kx^2$ (2) $\frac{1}{2}kA^2$

(3) kA^2 (4) Zero

45. In S.H.M. Which one of the following quantities has constant ratio with acceleration :-

(1) Time (2) Displacement

(3) Velocity (4) Mass

46. The displacement y of a particle varies with time t, in seconds, as

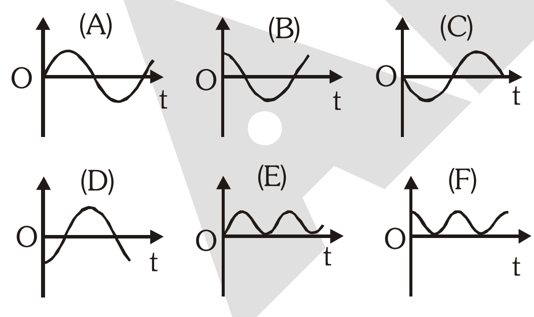
$y = 2\cos(\pi t + \pi/6)$.

The time period of the oscillations is

(1) 2 s (3) 4 s (3) 1 s (4) 0.5 s

ENERGY IN SHM-KINETIC AND POTENTIAL ENERGIES

47. The displacement of a particle in S.H.M. is $x = a\sin\omega t$. Which of the following graph between displacement and time is correct :



(1) A (2) B (3) C (4) D

48. In Question 47 which of the graph between velocity and time is correct ?

(1) A (2) B (3) C (4) D

49. In Question 47 which of the graph between kinetic energy and time is correct ?

(1) A (2) B (3) E (4) F

50. In Question 47 which of the graph between potential energy and time is correct ?

(1) A (2) B (3) E (4) F

51. In Question 47 which of the graph between acceleration and time is correct ?

(1) A (2) B (3) C (4) D

52. In Question 47 if the displacement of a particle executing SHM is $x = a\cos\omega t$, which of the graph between displacement and time is correct ?

(1) A (2) B (3) C (4) D

53. In question 52 which of the graph between velocity and time is correct ?

(1) A (2) B (3) C (4) D

54. In question 52 which of the graph between acceleration and time is correct ?

(1) A (2) B (3) C (4) D

55. In question 52 which of the graph between K.E. and time is correct ?

(1) A (2) B (3) E (4) F

56. In question 52 which of the graph between P.E. and time is correct ?

(1) A (2) B (3) E (4) F

57. The energy of SHM at the mean position of a pendulum will be :

(1) Zero

(2) Partial P.E. and partial K.E.

(3) Totally K.E.

(4) Totaly P.E.

58. The total energy of a particle executing SHM is directly proportional to the square of the following quantity :

(1) Acceleration (2) Amplitude

(3) Time period (4) Mass

59. The total energy of a vibrating particle in SHM is E. If its amplitude and time period are doubled, its total energy will be :

(1) 16E (2) 8E (3) 4E (4) E

60. The total vibrational energy of a particle in S.H.M. is E. Its kinetic energy at half the amplitude from mean position will be :

(1) E/2 (2) E/3 (3) E/4 (4) 3E/4

61. If total energy of a particle in SHM is E, then the potential energy of the particle at half the amplitude will be :
(1) E/2 (2) E/4 (3) 3E/4 (4) E/8

62. A particle executes SHM on a line 8 cm long. Its K.E. and P.E. will be equal when its distance from the mean position is :
(1) 4 cm (2) 2 cm
(3) $2\sqrt{2}$ cm (4) $\sqrt{2}$ cm

63. The average P.E. of a body executing S.H.M. is :
(1) $\frac{1}{2}ka^2$ (2) $\frac{1}{4}ka^2$ (3) ka^2 (4) Zero

64. The value of total mechanical energy of a particle in S.H.M. is :
(1) Always constant (2) Depend on time
(3) $\frac{1}{2}kA^2\cos^2(\omega t+\phi)$ (4) $\frac{1}{2}mA^2\cos^2(\omega t+\phi)$

65. The maximum K.E. of a oscillating spring is 5 joules and its amplitude 10 cm. The force constant of the spring is :
(1) 100 Newton/m.
(2) 1000 Newton–m
(3) 1000 Newton/m.
(4) 1000 watts.

66. The force acting on a 4gm mass in the energy region $U = 8x^2$ at $x = -2$cm is :
(1) 8 dyne (2) 4 dyne
(3) 16 dyne (4) 32 dyne

67. Displacement between max. P.E. position and max. K.E. position for a particle executing simple harmonic motion is :
(1) $\pm\frac{a}{2}$ (2) $+ a$ (3) $\pm a$ (4) -1

68. A particle is describing SHM with amplitude 'a'. When the potential energy of particle is one fourth of the maximum energy during oscillation, then its displacement from mean position will be:
(1) $\frac{a}{4}$ (2) $\frac{a}{3}$ (3) $\frac{a}{2}$ (4) $\frac{2a}{3}$

69. The ratio of K.E. of the particle at mean position to the point when distance is half of amplitude will be:
(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) $\frac{3}{2}$

70. A particle is executing S.H.M., If its P.E. & K.E. is equal then the ratio of displacement & amplitude will be :
(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$ (3) $\frac{1}{2}$ (4) $\frac{3}{2}$

71. Which of the following is constant during SHM:
(1) Velocity (2) Acceleration
(3) Total energy (4) Phase

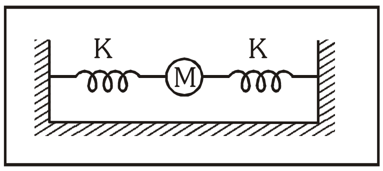
72. If <E> and <V> denotes the average kinetic and average potential energies respectively of mass describing a simple harmonic motion over one period then the correct relation is:
(1) <E> = <V> (2) <E> = 2<V>
(3) <E> = −2<V> (4) <E> = − <V>

73. The elongation of spring is 1 cm and its potential energy is U. If the spring is elongated by 3cm then potential energy will be :-
(1) 3U (2) $\frac{U}{3}$ (3) 9U (4) $\frac{U}{9}$

74. The potential energy of a spring when stretched by a distance x is E. The energy of the spring when stretched by x/2 is
(1) E (2) E/2 (3) E/4 (4) E/6

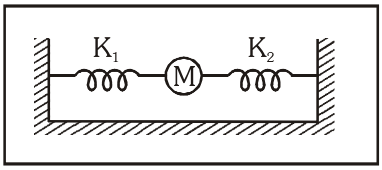
OSCILLATIONS OF A SPRING

75. On suspending a mass M from a spring of force constant K, frequency of vibration f is obtained. If a second spring as shown in the figure, is arranged then the frequency will be :



(1) $f\sqrt{2}$ (2) $f/\sqrt{2}$ (3) 2f (4) f

76. In the adjoining figure the frequency of oscillation for a mass M will be proportional to :



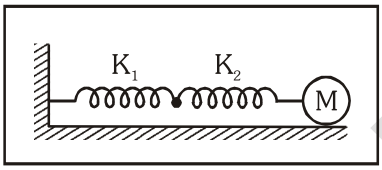
(1) K_1K_2 (2) $K_1 + K_2$

(3) $\sqrt{K_1 + K_2}$ (4) $\sqrt{1/K_1 + K_2}$

77. An object of mass m is suspended from a spring and it executes S.H.M. with frequency υ. If the mass is increased 4 times, the new frequency will be :

(1) 2υ (2) $\upsilon/2$ (3) υ (4) $\upsilon/4$

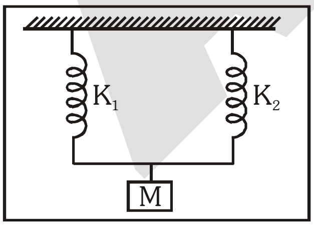
78. As shown in the figure, two light springs of force constant K_1 and K_2 oscillate a block of mass M. Its effective force constant will be :



(1) K_1K_2 (2) $K_1 + K_2$

(3) $\frac{1}{K_1} + \frac{1}{K_2}$ (4) $\frac{K_1K_2}{K_1 + K_2}$

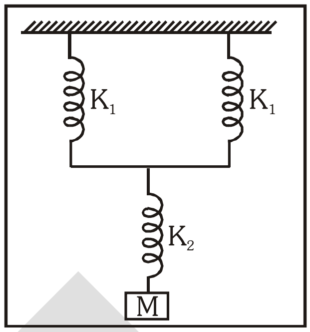
79. The spring constants of two springs of same length are K_1 and K_2 as shown in figure. If an object of mass M is suspended and set vibration, the time period will be :



(1) $2\pi\sqrt{\frac{MK_1}{K_2}}$ (2) $2\pi\sqrt{\frac{M}{K_1K_2}}$

(3) $2\pi\sqrt{\frac{M}{K_1 - K_2}}$ (4) $2\pi\sqrt{M/(K_1 + K_2)}$

80. The total spring constant of the system as shown in the figure will be :



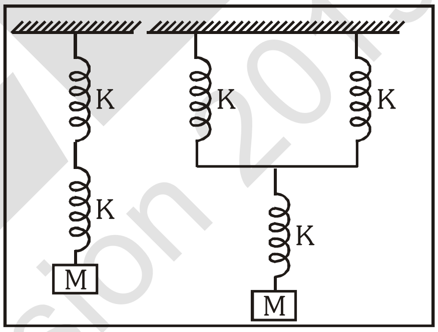
(1) $\frac{K_1}{2} + K_2$

(2) $\left[\frac{1}{2K_1} + \frac{1}{K_2}\right]^{-1}$

(3) $\frac{1}{2K_1} + \frac{1}{K_2}$

(4) $\left[\frac{2}{K_1} + \frac{1}{K_2}\right]^{-1}$

81. Some springs are combined in series and parallel arrangement as shown in the figure and a mass M is suspended from them. The ratio of their frequencies will be :



(1) 1 : 1 (2) 2 : 1

(3) $\sqrt{3}$: 2 (4) 4 : 1

82. A spring is made to oscillate after suspending a mass m from one of its ends. The time period obtained is 2 seconds. On increasing the mass by 2 kg, the period of oscillation is increased by 1 second. The initial mass m will be :

(1) 2 kg (2) 1 kg

(3) 0.5 kg (4) 1.6 kg

83. The time period of a spring pendulum on earth is T. If it is taken on the moon, and made to oscillate, the period of vibration will be :

(1) Less than T (2) Equal to T

(3) More than T (4) None of these

84. On loading a spring with bob, its period of oscillation in a vertical plane is T. If this spring pendulum is tied with one end to the a friction less table and made to oscillate in a horizontal plane, its period of oscillation will be :

(1) T

(2) 2T

(3) T/2

(4) will not execute S.H.M.

85. In a winding (spring) watch, the energy is stored in the form of :

(1) Kinetic energy (2) Potential energy

(3) Electrical energy (4) None of these

86. In an artificial satellite, the object used is :

(1) Spring watch

(2) Pendulum watch

(3) Watches of both spring and pendulum

(4) None of these

87. Mass 'm' is suspended from a spring of force constant K. Spring is cut into two equal parts and same mass is suspended from it, then new frequency will be:

(1) 2ν (2) $\sqrt{2}\,\nu$ (3) ν (4) $\frac{\nu}{2}$

88. The spring constant of two springs are K_1 and K_2 respectively springs are stretch up to that limit when potential energy of both becomes equal. The ratio of applied force (F_1 and F_2) on them will be :

(1) $K_1 : K_2$ (2) $K_2 : K_1$

(3) $\sqrt{K_1} : \sqrt{K_2}$ (4) $\sqrt{K_2} : \sqrt{K_1}$

89. Force constant of a spring is K. If one fourth part is detach then force constant of remaining spring will be :

(1) $\frac{3}{4}K$ (2) $\frac{4}{3}K$ (3) K (4) 4K

90. The spring constant of a spring is K. When it is divided into n equal parts, then what is the spring constant of one part :

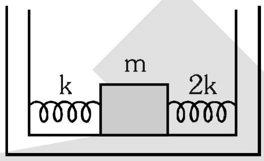
(1) nK (2) K/n

(3) $\frac{nK}{(n+1)}$ (4) $\frac{(n+1)K}{n}$

91. The time period of a mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be

(1) $\frac{T}{4}$ (2) T (3) $\frac{T}{2}$ (4) 2T

92. Two springs of force constant k and 2k are connected to a mass as shown below. The frequency of oscillation of the mass is :



(1) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$

93. Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by :

(1) $\frac{(k_1 + k_2)}{2}$ (2) $k_1 + k_2$

(3) $\frac{k_1 k_2}{(k_1 + k_2)}$ (4) $\sqrt{k_1 k_2}$

94. A mass of 10g is connected to a massless spring then time period of small oscillation is 10 second. If 10 g mass is replaced by 40 g mass in same spring, then its time period will be :-

(1) 5s (2) 10s (3) 20s (4) 40s

SIMPLE PENDULUM

95. The mass of a bob, suspended in a simple pendulum, is halved from the initial mass, its time period will :

(1) Be less (2) Be more

(3) Remain unchanged (4) None of these

96. If the amplitude of a simple pendulum is doubled, how many times will the value of its maximum velocity be that of the maximum velocity in initial case :

(1) $\frac{1}{2}$ (2) 2 (3) 4 (4) $\frac{1}{4}$

97. The length of a simple pendulum is $39.2/\pi^2$ m. If $g = 9.8$ m/sec^2, the value of time period is:
(1) 4 s (2) 8 s
(3) 2 s (4) 3 s

98. The length of a simple pendulum is increased four times of its initial value, its time period with respect to its previous value will :
(1) Become twice
(2) Not be different
(3) Be halved
(4) Be $\sqrt{2}$ times

99. The time taken for a second pendulum from one extreme point to another is :
(1) 1 s (2) 2 s
(3) 1/2 s (4) 4 s

100. The length of a seconds pendulum is (approximately) :
(1) 1 m (2) 1 cm
(3) 2 m (4) 2 cm

101. The acceleration due to gravity at height R above the surface of the earth is g/4. The periodic time of a simple pendulum in an artificial satellite at this height will be :
(1) $T = 2\pi\sqrt{2l/g}$
(2) $T = 2\pi\sqrt{l/2g}$
(3) Zero
(4) Infinity

102. In an artificial satellite, the use of a pendulum watch is discarded, because :
(1) The satellite is in a constant state of motion
(2) The effective value of g becomes zero in the artificial satellite
(3) The periodic time of the pendulum watch is reduced
(4) None of these

103. An oscillating pendulum stops, because its energy
(1) Changes into kinetic energy
(2) Change into potential energy
(3) Change into heat energy
(4) Is destroyed

104. Simple pendulum of large length is made equal to the radius of the earth. Its period of oscillation will be :
(1) 84.6 min. (2) 59.8 min.
(3) 42.3 min. (4) 21.15 min.

105. The maximum time period of oscillation of a simple pendulum of large length is:
(1) Infinity (2) 24 hours
(3) 12 hours (4) 1½ hours

106. In a simple oscillating pendulum, the work done by the string in one oscillation will be:
(1) Equal to the total energy of the pendulum
(2) Equal to the K.E. of the pendulum
(3) Equal to the P.E. of the pendulum
(4) Zero

107. A lift is ascending with acceleration g/3. What will be the time period of a simple pendulum suspended from its ceiling if its time period in stationary lift is T ?
(1) $\frac{T}{2}$ (2) $\frac{\sqrt{3}T}{2}$
(3) $\frac{\sqrt{3}T}{4}$ (4) $\frac{T}{4}$

108. A child swinging on a swing in sitting position, stands up, then the period of the swing will be:
(1) Increase
(2) Decrease
(3) Remain same
(4) Increase if child is long and decrease if child is short

109. A simple pendulum is suspended from the ceiling of a vehicle, its time period is T. Vehicle is moving with constant velocity, then time period of simple pendulum will be :
(1) Less than T
(2) Equal to T
(3) More than T
(4) Cannot predict

SUPERPOSITION OF SHMs, FREE, FORCED AND DAMPED OSCILATIONS, RESONANCE

110. The vibrations taking place in the diaphragm of a microphone will be :-
(1) free vibrations
(2) damped vibrations
(3) forced vibrations
(4) electrically maintained vibrations

111. In the case of sustained forced oscillations the amplitude of oscillations :-
(1) decreases linearly
(2) decreases sinusoidally
(3) decreases exponentially
(4) always remains constant

112. Two sources of sound are in resonance when :-
(1) they look alike
(2) they are situated at a particular distance from each other
(3) they produce the sound of same intensity
(4) they are excited by the same exciting device

113. When a tuning fork is vibrated, another in the neighbourhood begins to vibrate. This is due to the phenomenon of :-
(1) gravitation
(2) Newton's III law
(3) Resonance
(4) None

114. The amplitude of a SHM reduces to 1/3 in first 20 second then in first 40 second its amplitude becomes:

(1) $\frac{1}{3}$ (2) $\frac{1}{9}$ (3) $\frac{1}{27}$ (4) $\frac{1}{\sqrt{3}}$

115. Amplitude of vibrations remains constant in case of
(i) free vibrations
(ii) damped vibrations
(iii) maintained vibrations
(iv) forced vibrations
(1) i, iii, iv (2) ii, iii
(3) i, ii, iii (4) ii, iv

EXERCISE-I (Conceptual Questions) **ANSWER KEY**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	**4**	**1**	**1**	**4**	**2**	**1**	**2**	**1**	**1**	**4**	**3**	**3**	**1**	**3**	**1**
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	**2**	**4**	**4**	**3**	**2**	**4**	**1**	**1**	**1**	**1**	**3**	**4**	**4**	**1**	**1**
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	**3**	**1**	**1**	**4**	**3**	**4**	**3**	**2**	**1**	**1**	**2**	**1**	**1**	**4**	**2**
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	**1**	**1**	**2**	**4**	**3**	**3**	**2**	**3**	**4**	**3**	**4**	**3**	**2**	**4**	**4**
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	**2**	**3**	**2**	**1**	**3**	**4**	**3**	**3**	**3**	**1**	**3**	**1**	**3**	**3**	**1**
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans.	**3**	**2**	**4**	**4**	**2**	**3**	**4**	**2**	**1**	**2**	**1**	**2**	**3**	**2**	**1**
Que.	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
Ans.	**3**	**3**	**3**	**3**	**3**	**2**	**1**	**1**	**1**	**1**	**4**	**2**	**3**	**2**	**4**
Que.	106	107	108	109	110	111	112	113	114	115					
Ans.	**4**	**2**	**2**	**2**	**3**	**4**	**4**	**3**	**2**	**1**					

EXERCISE-II (Previous Year Questions) AIPMT/NEET & AIIMS (2006-2018)

AIPMT 2006

1. The potential energy of a long spring when stretched by 2cm is U. If the spring is stretched by 8 cm the potential energy stored in it is :-

(1) 4 U (2) 8 U (3) 16 U (4) $\frac{U}{4}$

AIPMT 2007

2. The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is :-

(1) Zero (2) 0.5 π (3) π (4) 0.707π

3. The particle executing simple harmonic motion has a kinetic energy $K_o \cos^2 \omega t$. The maximum values of the potential energy and the total energy are respectively :-

(1) K_o and K_o (2) 0 and $2K_o$

(3) $\frac{K_o}{2}$ and K_o (4) K_o and $2K_o$

4. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is :-

(1) T/2 (2) T/4 (3) T/8 (4) T/12

AIPMT 2008

5. Two simple Harmonic Motions of angular frequency 100 and 1000 rad s^{-1} have the same displacement amplitude. The ratio of their maximum accelerations is:-

(1) 1 : 10^3 (2) 1 : 10^4 (3) 1 : 10 (4) 1: 10^2

6. A point performs simple harmonic oscillation of period T and the equation of motion is given by $x = a \sin(\omega t + \pi/6)$. After the elapse of what fraction of the time period the velocity of the point will be equal to half of its maximum velocity ?

(1) T/3 (2) T/12 (3) T/8 (4) T/6

AIPMT 2009

7. A simple pendulum performs simple harmonic motion about x = 0 with an amplitude a and time period T. The speed of the pendulum at x = a/2 will be :-

(1) $\frac{\pi a\sqrt{3}}{T}$ (2) $\frac{\pi a\sqrt{3}}{2T}$ (3) $\frac{\pi a}{T}$ (4) $\frac{3\pi^2 a}{T}$

8. Which one of the following equations of motion represents simple harmonic motion :-

(1) Acceleration = kx

(2) Acceleration = $-k_0x+k_1x^2$

(3) Acceleration = $-k(x + a)$

(4) Acceleration = $k(x + a)$

Where k, k_0, k_1 and a are all positive

AIPMT (Pre) 2010

9. The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be :-

(1) $\sqrt{2}T$ (2) T (3) $\frac{T}{\sqrt{2}}$ (4) 2T

AIPMT (Pre) 2011

10. Out of the following functions representing motion of a particle which represents SHM :

(A) $y = \sin\omega t - \cos \omega t$

(B) $y = \sin^3\omega t$

(C) $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$

(D) $y = 1 + \omega t + \omega^2 t^2$

(1) Only (A)

(2) Only (D) does not represent SHM

(3) Only (A) and (C)

(4) Only (A) and (B)

AIPMT (Mains) 2011

11. Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite directions when their displacement is half of the amplitude. The phase difference is:-

(1) $\frac{\pi}{6}$ (2) 0 (3) $\frac{2\pi}{3}$ (4) π

AIPMT 2014

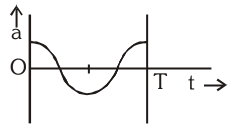
12. The oscillation of a body on a smooth horizontal surface is represented by the equation,

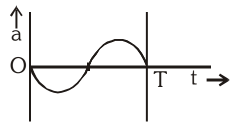
$X = A \cos(\omega t)$

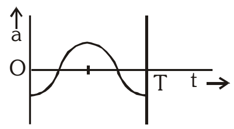
where X = displacement at time t

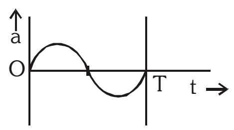
ω = frequency of oscillation

Which one of the following graphs shows correctly the variation 'a' with 't' ?

(1) 

(2) 

(3) 

(4) 

AIPMT 2015

13. When two dispalcements represented by $y_1 = a\sin(\omega t)$ and $y_2 = b\cos(\omega t)$ are superimposed the motion is :

(1) simple harmonic with amplitude $\frac{a}{b}$

(2) simple harmonic with amplitude $\sqrt{a^2 + b^2}$

(3) simple harmonic with amplitude $\frac{(a+b)}{2}$

(4) not a simple harmonic

14. A particle is executing SHM along a straight line. Its velocities at distances x_1 and x_2 from the mean position are V_1 and V_2, respectively. Its time period is :-

(1) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$ (2) $2\pi\sqrt{\frac{V_1^2 + V_2^2}{x_1^2 + x_2^2}}$

(3) $2\pi\sqrt{\frac{V_1^2 - V_2^2}{x_1^2 - x_2^2}}$ (4) $2\pi\sqrt{\frac{x_1^2 + x_2^2}{V_1^2 + V_2^2}}$

15. Two similar springs P and Q have spring constants K_P and K_Q, such that $K_P > K_Q$. The are stretched, first by the same amount (case a,) then by the same force (case b). The work done by the springs W_P and W_Q are related as, in case (a) and case (b), respectively :

(1) $W_P = W_Q$; $W_P = W_Q$

(2) $W_P > W_Q$; $W_Q > W_P$

(3) $W_P < W_Q$; $W_Q < W_P$

(4) $W_P = W_Q$; $W_P > W_Q$

RE-AIPMT 2015

16. A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β. Then its time period of vibration will be :-

(1) $\frac{2\pi\beta}{\alpha}$ (2) $\frac{\beta^2}{\alpha^2}$ (3) $\frac{\alpha}{\beta}$ (4) $\frac{\beta^2}{\alpha}$

AIIMS 2015

17. In damped oscillation mass is 1 kg and spring constant = 100 N/m, damping coefficient = 0.5 kg s^{-1}. If the mass is displaced by 10 cm from its mean position than what will be the value of its mechanical energy after 4 seconds ?

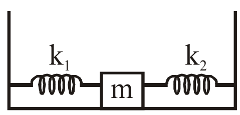
(1) 0.67 J (2) 0.067 J (3) 6.7 J (4) 0.5 J

NEET-II 2016

18. A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5 s. The value of m in kg is :-

(1) $\frac{16}{9}$ (2) $\frac{9}{16}$ (3) $\frac{3}{4}$ (4) $\frac{4}{3}$

AIIMS 2016

19. 

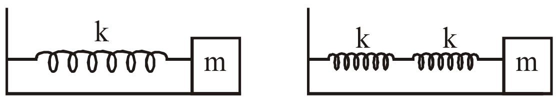
When mass m oscillates with spring of spring constant k_1 its frequency is 3Hz and with spring of spring constant k_2 its frequency is 6Hz. Then frequency of oscillations when mass is connected with springs as shown in figure is :-

(1) 6.7 (2) 4.5 (3) 9 (4) 3

20. In an angular SHM angular amplitude of oscillation is π rad and time period is 0.4 sec then calculate its angular velocity at angular displacement $\pi/2$ rad.

(1) 34.3 rad/sec (2) 42.7 rad/sec
(3) 22.3 rad/sec (4) 50.3 rad/sec

21. Spring is oscillating with frequency 4Hz having spring constant k. An identical spring is connected in series in new system as shown in figure. Find new frequency.



(1) 2Hz (2) $2\sqrt{2}$Hz
(3) 4Hz (4) 8Hz

NEET(UG) 2017

22. A spring of force constant k is cut into lengths of ratio 1 : 2 : 3. They are connected in series and the new force constant is k'. Then they are connected in parallel and force constant is k''. Then k' : k'' is:-

(1) 1 : 9 (2) 1 : 11
(3) 1 : 14 (4) 1 : 16

23. A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is :-

(1) $\frac{\sqrt{5}}{2\pi}$ (2) $\frac{4\pi}{\sqrt{5}}$ (3) $\frac{2\pi}{\sqrt{3}}$ (4) $\frac{\sqrt{5}}{\pi}$

AIIMS 2017

24. A body of mass 600 gm is attached to a spring of spring constant k = 100 N/m and it is performing damped oscillations. If damping constant is 0.2 and driving force is $F = F_0 \cos(\omega t)$, where $F_0 = 20$ N. Find the amplitude of oscillation at resonance.

(1) 4.1 m (2) 0.57 m
(3) 7.7 m (4) 0.98 m

NEET(UG) 2018

25. A pendulum is hung from the roof of a sufficiently high building and is moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is 20 m/s^2 at a distance of 5 m from the mean position. The time period of oscillation is :-

(1) 2π s (2) π s (3) 2 s (4) 1 s

AIIMS 2018

26. An oscillator of mass 10 gram is oscillating with natural frequency of 100 Hz. Under slight damped conditions, a periodic force F = 100 cos20πt is applied on it. Amplitude of oscillation is approximately :-

(1) 0.025 cm (2) 2.5 cm
(3) 0.25 cm (4) 25 cm

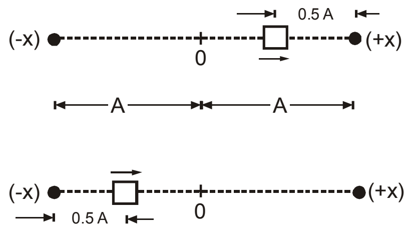
EXERCISE-II (Previous Years Questions) ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	**3**	**2**	**1**	**4**	**4**	**2**	**1**	**3**	**1**	**3**	**3**	**3**	**2**	**1**	**2**
Que.	16	17	18	19	20	21	22	23	24	25	26				
Ans.	**1**	**2**	**2**	**1**	**2**	**2**	**2**	**2**	**3**	**2**	**2**				

EXERCISE-III (Analytical Questions)

Check Your Understanding

1. Two bodies performing S.H.M. have same amplitude and frequency. Their phases at a certain instant are as shown in the figure. The phase difference between them is



(1) $\frac{11}{6}\pi$ (2) π (3) $\frac{\pi}{3}$ (4) $\frac{3}{5}\pi$

2. A particle performing S.H.M. is found at its equilibrium at t = 1 s and it is found to have a speed of 0.25 m/s at t = 2 s. If the period of oscillation is 6s. Calculate amplitude of oscillation

(1) $\frac{3}{2\pi}$ m (2) $\frac{3}{4\pi}$ m (3) $\frac{6}{\pi}$ m (4) $\frac{3}{8\pi}$ m

3. The period of a particle is 8s. At t = 0 it is at the mean position. The ratio of distance covered by the particle in first second and second will be-

(1) $\frac{\sqrt{2}-1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}-1}$ (4) $\sqrt{2}-1$

4. A man of mass 60 kg standing on a plateform executing S.H.M. in the vertical plane. The displacement from the mean position varies as $y = 0.5 \sin(2\pi ft)$. The minimum value of f, for which the man will feel weightlessness at the highest point is : (y is in metres)

(1) $\frac{g}{4\pi}$ (2) $4\pi g$

(3) $\frac{\sqrt{2g}}{2\pi}$ (4) $2\pi\sqrt{2g}$

5. Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos 100\pi t$. The phase difference of the velocity of particle 1, with respect to the velocity of particle 2 is-

(1) $\frac{-\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{-\pi}{3}$ (4) $\frac{\pi}{6}$

6. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as- $a = A \cos(\omega t + \delta)$, then-

(1) $A = x_0$, $\delta = -\pi/4$ (2) $A = x_0\omega^2$, $\delta = \pi/4$

(3) $A = x_0\omega^2$, $\delta = -\pi/4$ (4) $A = x_0\omega^2$, $\delta = 3\pi/4$

7. The potential energy of a simple harmonic oscillator at mean position is 3 joules. If its mean K.E. is 4 joules, its total energy will be :

(1) 7J (2) 8J (3) 10J (4) 11J

8. The total energy of a harmonic oscillator of mass 2kg is 9 joules. If its potential energy at mean position is 5 joules, its K.E. at the mean position will be :

(1) 9J (2) 14J (3) 4J (4) 11J

9. A horizontal spring is connected to a mass M. It executes simple harmonic motion. When the mass M passes through its mean position, an object of mass m is put on it and the two move together. The ratio of frequencies before and after will be-

(1) $\left(1+\frac{m}{M}\right)^{1/2}$ (2) $\left(1+\frac{m}{M}\right)$

(3) $\left(\frac{M}{M+m}\right)^{1/2}$ (4) $\left(\frac{M}{M+m}\right)$

10. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities during oscillations are equal, the ratio of amplitudes of A and B is-

(1) $\sqrt{k_1/k_2}$ (2) k_1/k_2

(3) $\sqrt{k_2/k_1}$ (4) k_2/k_1

11. The period of oscillation of simple pendulum of length L suspended from the roof of the vehicle which moves without friction, down on an inclined plane of inclination α, is given by :-

(1) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (2) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$

(3) $2\pi\sqrt{\frac{L}{g}}$ (4) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

12. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$, $(K = 1\ m/s^2)$ where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is : $(g = 10\ m/s^2)$

(1) $\frac{6}{5}$ (2) $\frac{5}{6}$ (3) 1 (4) $\frac{4}{5}$

13. A particle describes SHM in a straight line about O.

If the time period of the motion is T then its kinetic energy at P be half of its peak value at O, if the time taken by the particle to travel from O to P is

(1) $\frac{1}{2}T$ (2) $\frac{1}{4}T$ (3) $\frac{1}{2\sqrt{2}}T$ (4) $\frac{1}{8}T$

14. An oscillator consists of a block attached to spring (k = 400 N/m), At some time t, the position (measured from the system's equilibrium location), velocity and acceleration of the block are x = 0.100m, v = – 15.0 m/s, and a = – 90 m/s^2. The amplitude of the motion and the mass of the block are :-

(1) 0.2 m, 0.84 kg (2) 0.3 m, 0.76 kg
(3) 0.4 m, 0.54 kg (4) 0.5 m, 0.44 kg

EXERCISE-III (Analytical Questions) **ANSWER KEY**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	3	1	3	3	1	4	4	3	1	3	1	1	4	4

EXERCISE-IV (Assertion & Reason) Target AIIMS

Directions for Assertion & Reason questions

These questions consist of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses.

(A) If both Assertion & Reason are True & the Reason is a correct explanation of the Assertion.

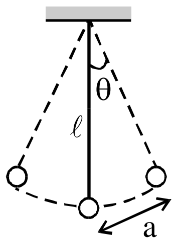
(B) If both Assertion & Reason are True but Reason is not a correct explanation of the Assertion.

(C) If Assertion is True but the Reason is False.

(D) If both Assertion & Reason are false.

1. ***Assertion :*** The motion of a simple pendulum is simple harmonic only for a << ℓ.

Reason : Motion of a simple pendulum is SHM for small angular displacement.



(1) A
(2) B
(3) C
(4) D

2. ***Assertion :*** Pendulum clocks go slow in summer and fast in winter.

Reason : The length of the pendulum used in clock increases in summer.

(1) A (2) B (3) C (4) D

3. ***Assertion*** : A simple pendulum is mounted on a truck which move with constant velocity. The time period of pendulum will increase.

Reason : The effective length of pendulum will decrease.

(1) A (2) B (3) C (4) D

4. ***Assertion :*** SHM is not a periodic motion.

Reason : Periodic motion does not repeat its position after certain interval of time.

(1) A (2) B (3) C (4) D

5. ***Assertion :*** When a particle is at extreme position performing SHM, its momentum is equal to zero.

Reason : At extreme position the velocity of particle performing SHM is equal to zero.

(1) A (2) B (3) C (4) D

6. ***Assertion :*** Acceleration is proportional to the displacement; this condition is not sufficient for motion in simple harmonic.

Reason : In simple harmonic motion, direction of displacement is also considered.

(1) A (2) B (3) C (4) D

7. ***Assertion :*** When a girl sitting on a swing stand up, the periodic time of the swing will increase.

Reason : In standing position of a girl, the effective length of the swing will decrease.

(1) A (2) B (3) C (4) D

8. ***Assertion :*** A heavy metal sphere is suspended by a spring. When pulled down a little and released, it oscillates up and down with a frequency f. If it is taken to the moon where the acceleration due to gravity is g/6, the frequency of oscillation is $\sqrt{6}f$.

Reason : The frequency of oscillation of the spring depends only on the mass of the sphere and spring constant.

(1) A (2) B (3) C (4) D

9. ***Assertion :-*** A undamped spring-mass system is simplest free vibration system. **[AIIMS 2013]**

Reason :- It has three degrees of freedom.

(1) A (2) B (3) C (4) D

10. ***Assertion :-*** In SHM acceleration is always directed towards the mean position.

Reason :- The body in SHM stops momentary at the extreme positions and then moves back to mean position.

(1) A (2) B (3) C (4) D

11. ***Assertion :-*** The graph between velocity and displacement for a harmonic oscillation is a straight line.

Reason :- Velocity changes uniformly with displacement in simple harmonic motion.

(1) A (2) B (3) C (4) D

12. ***Assertion :-*** In SHM, kinetic energy is zero when potential energy is maximum.

Reason :- In SHM, the kinetic and potential energies become equal when the displacement is $\frac{1}{\sqrt{2}}$ times the amplitude.

(1) A (2) B (3) C (4) D

13. ***Assertion :-*** The oscillating particle and the driving force have different phase.

Reason :- It depends on the initial condition of the driven.

(1) A (2) B (3) C (4) D

14. ***Assertion :-*** Time average KE and time average PE are not exactly equal in simple pendulum.

Reason :- Friction is not negligible in simple pendulum. **[AIIMS 2018]**

(1) A (2) B (3) C (4) D

15. ***Assertion :-*** A harmonic oscillator has two degrees of freedom. **[AIIMS 2018]**

Reason :- It has both kinetic energy and potential energy.

(1) A (2) B (3) C (4) D

EXERCISE-IV (Assertion & Reason) **ANSWER KEY**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	1	4	4	1	1	4	4	3	2	4	2	3	1	1