ARJUNA NEET BATCH

MOTION IN A PLANE

= 2-D motion

LECTURE - 01

NEET

Todays goal

- Basic concept Of Motion in a plan
- Projectile motion

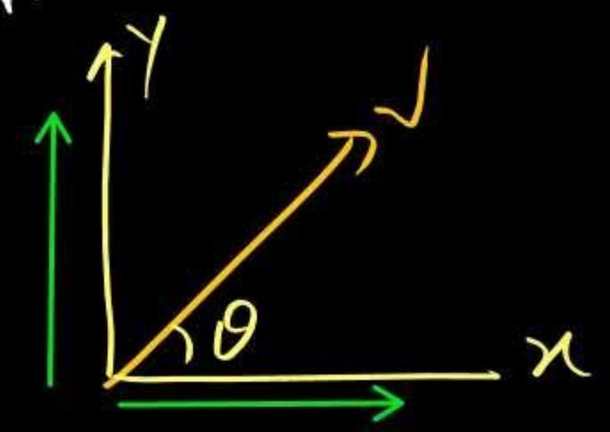
2-D MOTION

effect

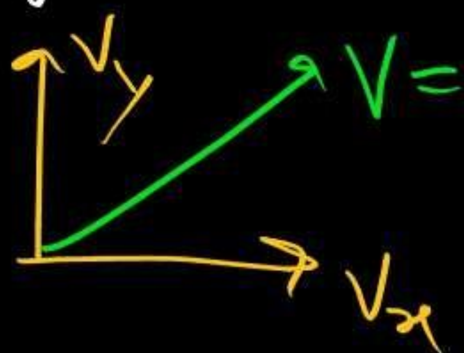
VECTOR

- **Component of vector** → net vector को Break करना
- **Magnitude of vector** → effect (component को Add कर के main vector निकाला)

$V_y = v\sin\theta$



$V_x = V\cos\theta$



$V = \sqrt{V_x^2 + V_y^2}$

चमक रहा है।

$$\text{2-D Motion} = \underbrace{[\text{1-D}]_{x\text{-axis}}}_{\#} + \underbrace{[\text{1-D}]_{y\text{-axis}}}_{\#}$$

ARJUNA

2-D motion

1. Break in 1-D + 1-D using concept of component
2. Solve $[1\text{-}D]_x$ axis then $[1\text{-}D]_y$ axis Seperatly.
3. then add using magnitude of vector

POSITION VECTOR IN PLANE

$$\boxed{\vec{r} = x\,\hat{i} + y\,\hat{j}}$$

$\Downarrow$

diff^n w.r.t time

$$\frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}\right)\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\boxed{\vec{V} = V_x\,\hat{i} + V_y\,\hat{j}}$$

$\Downarrow$

Magnitude of velocity (speed) $= \sqrt{V_x^2 + V_y^2}$

$\rightarrow$ diff^n w.r.t time

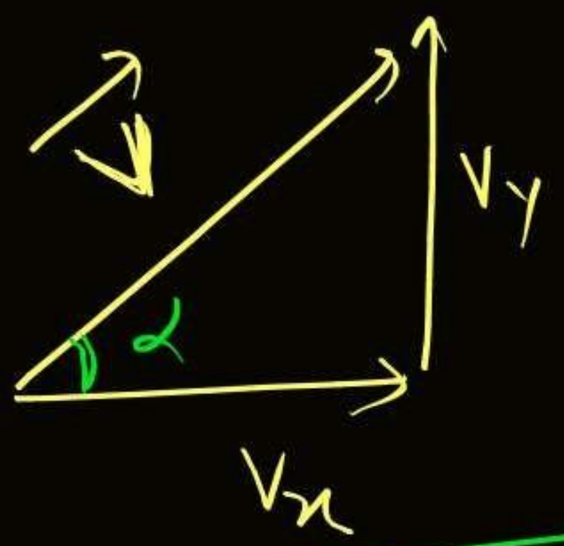
$$\frac{d\vec{V}}{dt} = \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j}$$

$$\vec{a} = a_x\,\hat{i} + a_y\,\hat{j}$$

$\hookrightarrow$ magnitude of acc^n

$$a = \sqrt{a_x^2 + a_y^2}$$

$$\vec{V} = \vec{V_x}\,\hat{i} + \vec{V_y}\,\hat{j}$$



$\boxed{\text{direction of motion}}$

→ direction of inst. velocity

→ Angle b/w x-axis $\vec{v}$

$$\boxed{\tan\alpha = \frac{V_y}{V_x}}$$

$\boxed{\cos\alpha = \frac{V_x}{V}}$

$\cos\beta = \frac{V_y}{V}$

$\cos\gamma = \frac{V_z}{V}$

EQUATION OF MOTION IN PLANE

PLANE $\rightarrow$ 2-D

$\boxed{1^{st} \text{ equation of motion}}$

$$\Longrightarrow \vec{V}_x = \vec{u}_x + \vec{a}_x t$$

$$\Rightarrow \vec{V}_y = \vec{u}_y + \vec{a}_y t$$

$$\vec{V} = \vec{V}_x \hat{i} + \vec{V}_y \hat{j}$$

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow V_x^2 - u_x^2 = 2 a_x x$$

$$\Rightarrow V_y^2 - u_y^2 = 2 a_y y$$

ARJUNA

If initial velocity of object $\vec{u} = 3\,\hat{i} + 4\,\hat{j}$ and after some time its velocity $v = 4\,\hat{i} + 3\,\hat{j}$ then find

(i) magnitude of change in velocity

(ii) change in magnitude of velocity.

change in velocity $= \vec{v_f} - \vec{u_i} = (4\hat{i} + 3\hat{j}) - (3\hat{i} + 4\hat{j})$

$= \hat{i} - \hat{j}$

$|\Delta\vec{v}| = \sqrt{2}$

(ii) magnitude of veloci

$|u_i| = 5$

$|v_f| = 5$

$\boxed{\Delta = 0}$

Position of object $\vec{r} = 3t^2\,\hat{i} + (6t - t^2)\,\hat{j}$, then find velocity and acceleration at $t = 1$ sec.

AIPMT-2014

$$\vec{r} = 3t^2\,\hat{i} + (6t - t^2)\,\hat{j}$$

diffⁿ

$$\vec{V} = 3[2t]\hat{i} + (6 - 2t)\hat{j}$$

$$\vec{V}_{t=1} = 6\hat{i} + 4\hat{j}$$

$$|\vec{V}|_{t=1} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52}$$

$$\vec{V} = 6t\,\hat{i} + (6-2t)\hat{j}$$

$\downarrow$ diffⁿ

$$\frac{d\vec{V}}{dt} = \boxed{\vec{a} = 6\hat{i} - 2\hat{j}}$$

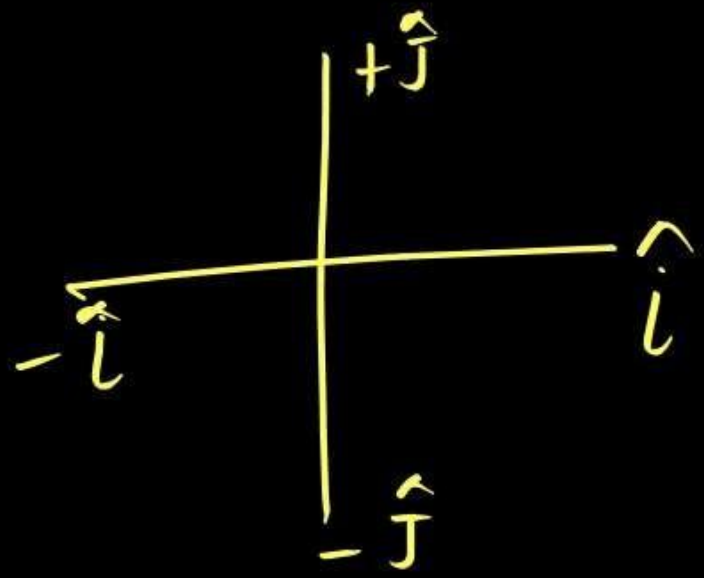
$\Downarrow$

Constⁿ

Object is moving in west with 5 m/s after 2 sec its velocity is 5 m/s in north then find acceleration.

Sol^n

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

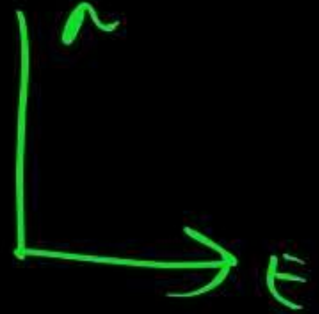


$$= \frac{5\hat{j} - (-5\hat{i})}{2}$$

$$= \frac{5}{2}(\hat{j} + \hat{i})$$

$$|\vec{a}| = \frac{5}{\sqrt{2}}$$

north east



Initial velocity of object is 5 m/s in east and acceleration of object is 2.5 m/s^2 north then find speed of object at $t = 2$ sec.

east

$u = 5\hat{i}$

$a = 0 \rightarrow v = \text{Cos.}$

$v_f = 5\hat{i}$

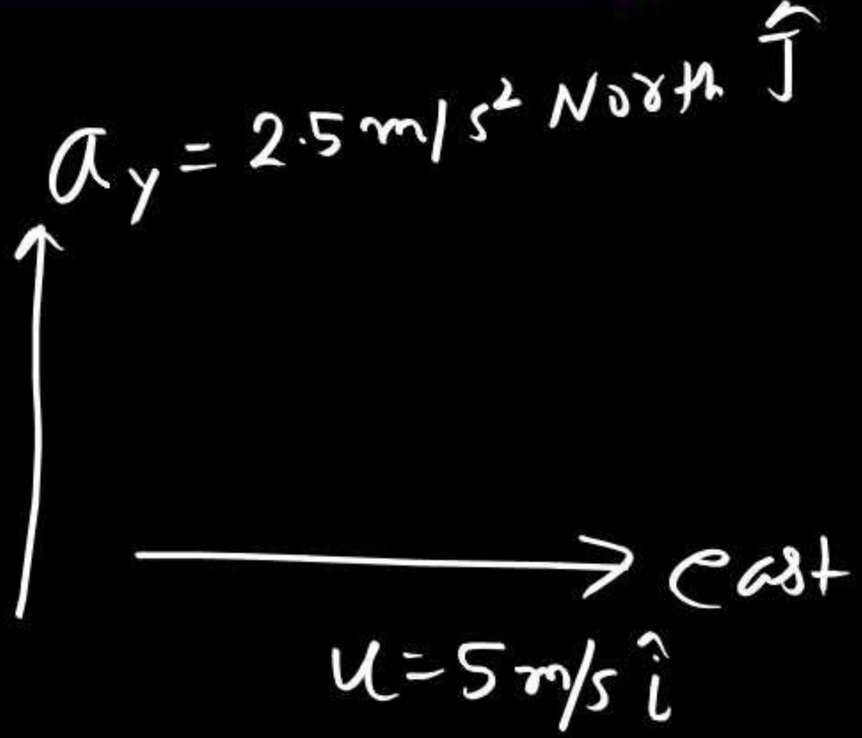
North

$u_y = 0$

$a = \frac{5}{2}\hat{j}$

$v_y = 0 + \frac{5}{\cancel{2}} \times \cancel{2}$

$v_y = 5\hat{j}$



$\vec{V} = 5\hat{i} + 5\hat{j}$

$V = 10\,m/s$ ✗

$|\vec{V}| = 5\sqrt{2}\,m/s$

Acceleration of object $a = 2\,\hat{\imath} + 3\,t^2\,\hat{\jmath}$, then find velocity at $t = 1$ sec it initial velocity of object is zero.

diffⁿ वाली

$$\vec{a} = 2\hat{i} + 3t^2\hat{j}$$

$$\int_0^v dv = \int_{t=0}^{1s} 2\,dt\,\hat{i} + \int_{t=0}^{1s} 3t^2\,dt\,\hat{j}$$

$$V = (2t)_0^1\,\hat{i} + \cancel{3}\left(\frac{t^3}{\cancel{3}}\right)_0^1\hat{j}$$

$$\boxed{V = 2\hat{i} + 1\hat{j}}$$

$$|\vec{v}| = \sqrt{(2)^2 + (1)^2} = \sqrt{5}\ \text{m/s}$$

object is moving such that initial velocity $\boxed{\vec{u} = (3\hat{i} + 4\hat{j})}$ and $\boxed{\vec{a} = (0.4\hat{i} + 0.3\hat{j})}$ then find speed of object at $t = 10$ sec.

NEET-17/18

$v = u + at$ *

$(v_f)_x = 3 + 0.4 \times 10$

$= 7\hat{i}$

$(v_f)_y = 4 + 0.3 \times 10$

$= (4 + 3) = 7\hat{j}$

$\vec{v} = 7\hat{i} + 7\hat{j}$

$|\vec{v}| = 7\sqrt{2}$

Q) object is moving in a plane such that $\boxed{x = 4t}$ and $\boxed{y = 2t^2 - 6t}$ then find acceleration of object [NEET]

Soln

$v_y = (4t - 6)\hat{j}$

$v_x = 4\hat{i}$

$\Downarrow$

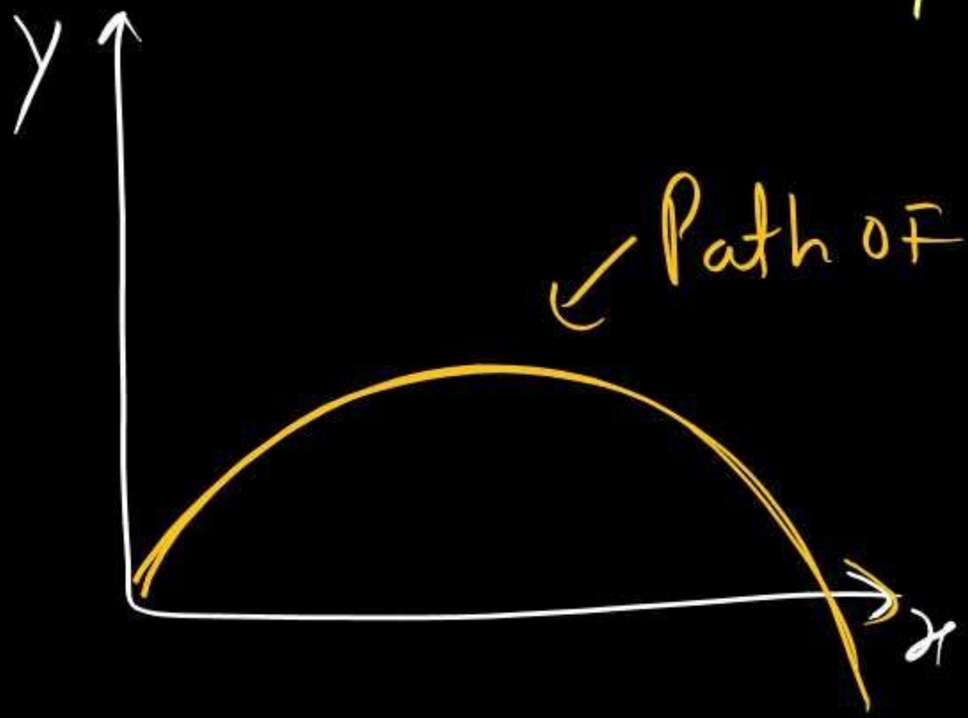
$\boxed{a_x = 0}$

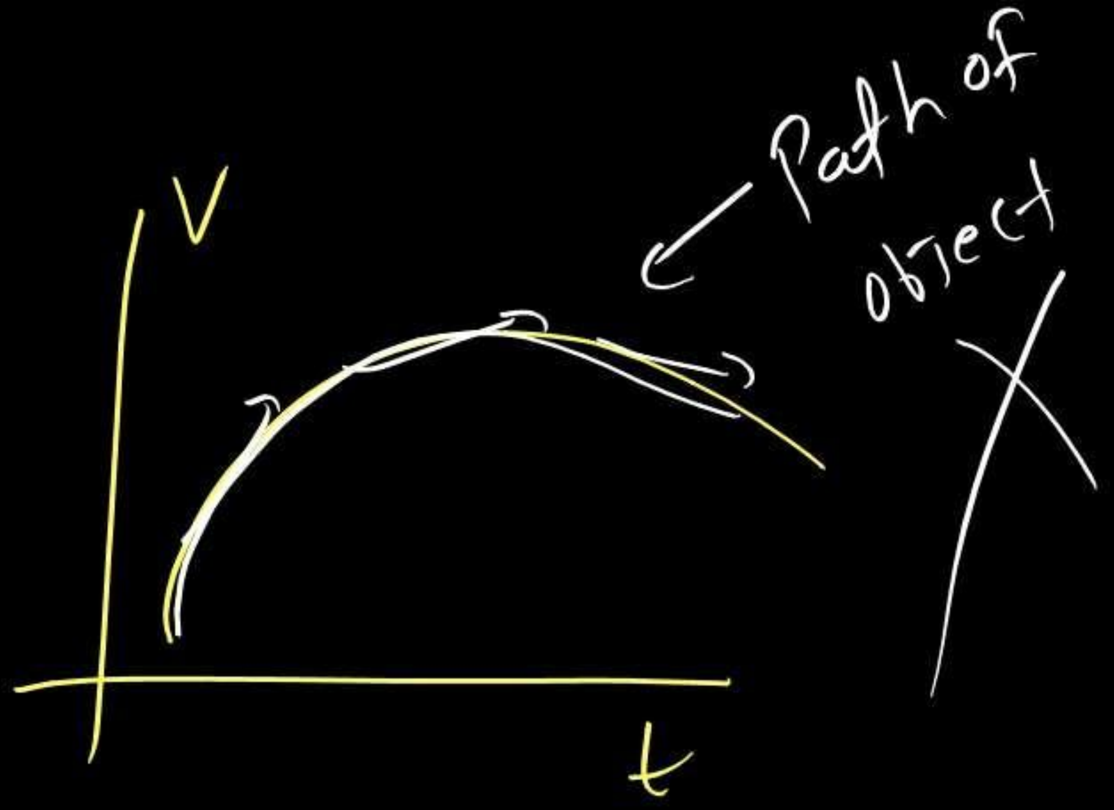
$a_y = [4(1) - 0]\hat{j} = 4\hat{j}$

$\vec{a} = 4\hat{j}$

EQUATION OF TRAJECTORY

- Path followed by object
- relation between 'x' and 'y' position.





Position of object at time 't' $\vec{r} = 2\,t\,\hat{i} + 4\,t^2\,\hat{j}$, then find equation of trajectory.

or Path ??

$x = 2t$ $\qquad$ $y = 4t^2$

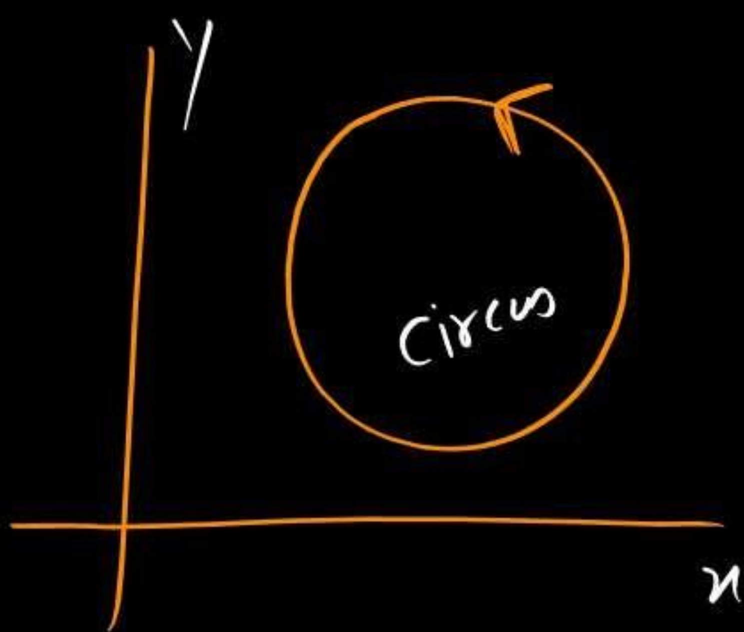
$t = \frac{x}{2}$

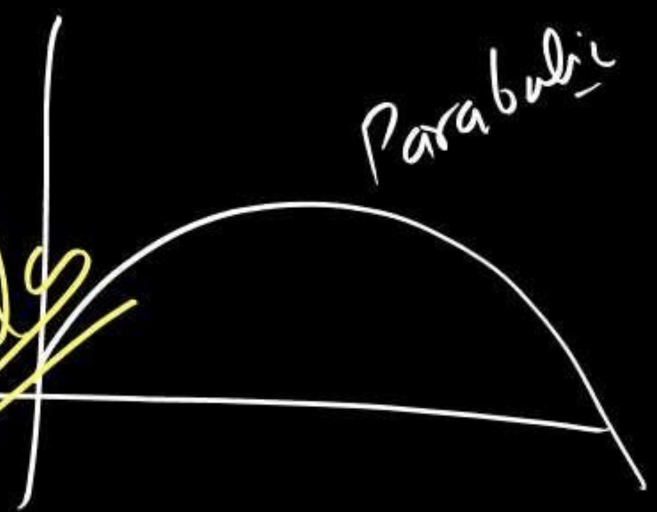
$y = 4\left(\frac{x}{2}\right)^2$

$y = \cancel{4}\,\frac{x^2}{\cancel{4}}$

$\boxed{y = x^2}$

Parabola





If position of object $\vec{r} = 3\sin(\omega t)\,\hat{i} + 3\cos(\omega t)\,\hat{j}$, then object is moving on.??

NEET

$$\vec{r} = \underbrace{3\sin(\omega t)}_{x}\,\hat{i} + \underbrace{3\cos(\omega t)}_{y}\,\hat{j}$$

$x = 3\sin(\omega t)$ —(i)

$y = 3\cos(\omega t)$ —(ii)

$(i)^2 + (ii)^2$

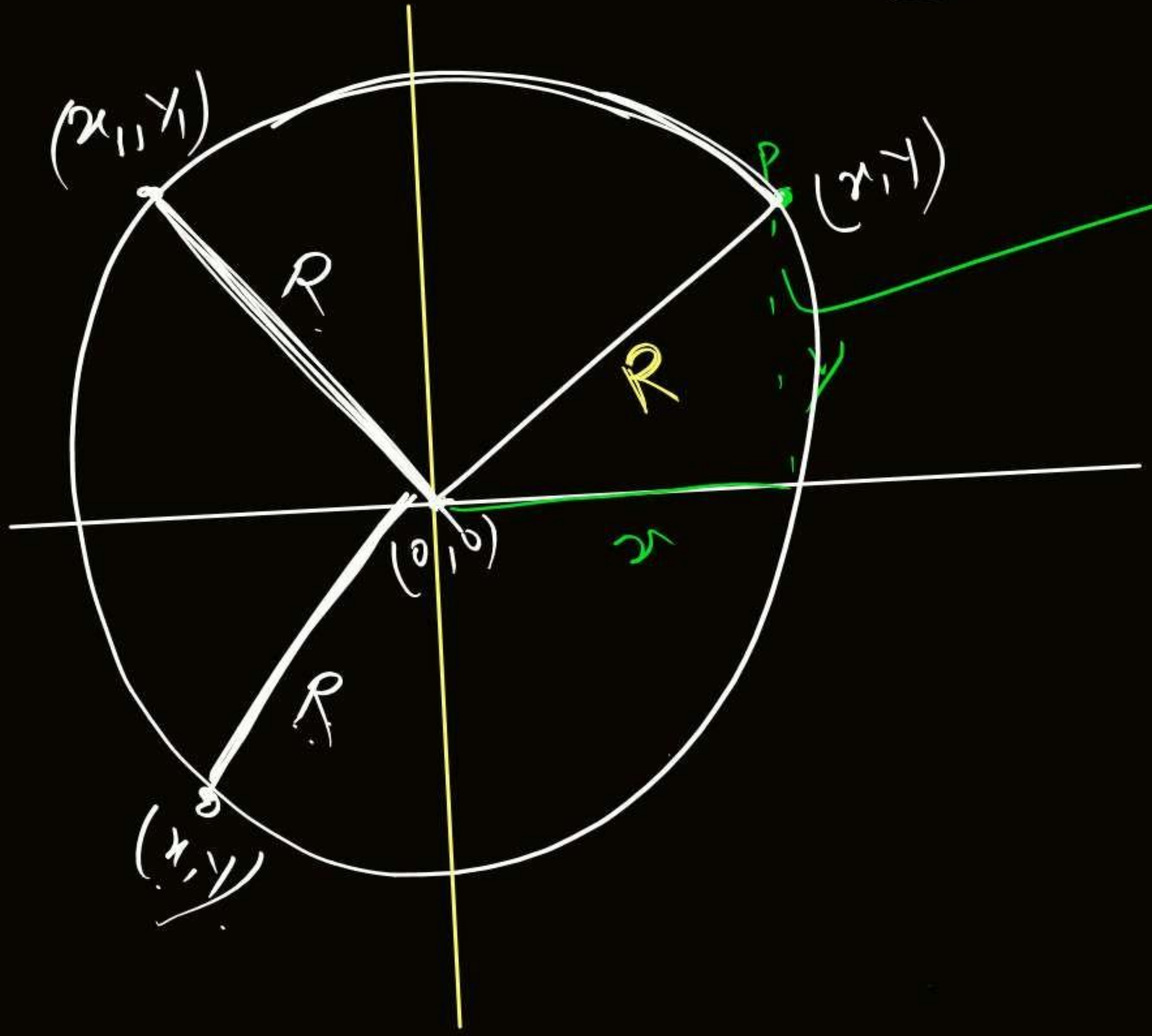
$$x^2 + y^2 = (3)^2\left[\sin^2(\omega t) + \cos^2(\omega t)\right]$$

$\sin^2(\omega t) + \cos^2(\omega t) = 1$

$$\boxed{x^2 + y^2 = (3)^2}$$

eqn of circle

$\boxed{x^2+y^2=R^2} \rightarrow$ eq^n of circle.



$$\text{dist}^n = \sqrt{(x-0)^2+(y-0)^2}$$

$$\text{dist}^n = \sqrt{x^2+y^2}$$

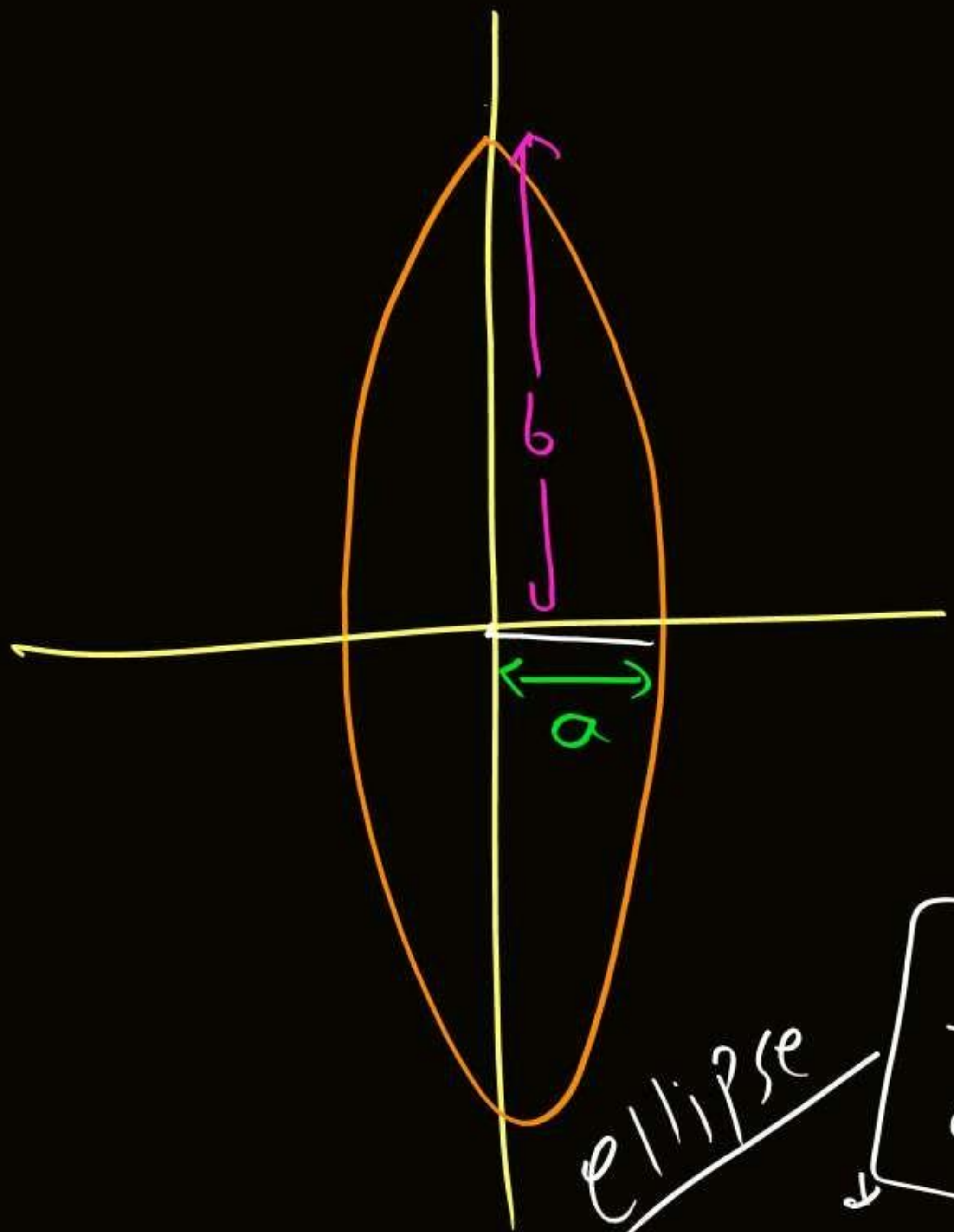
$$R = \sqrt{x^2+y^2}$$

$$\boxed{R^2 = x^2+y^2} \checkmark$$

$\rightarrow$ Ratta.

eq^n of circle

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1} \longrightarrow \text{eqn of ellipse}$$



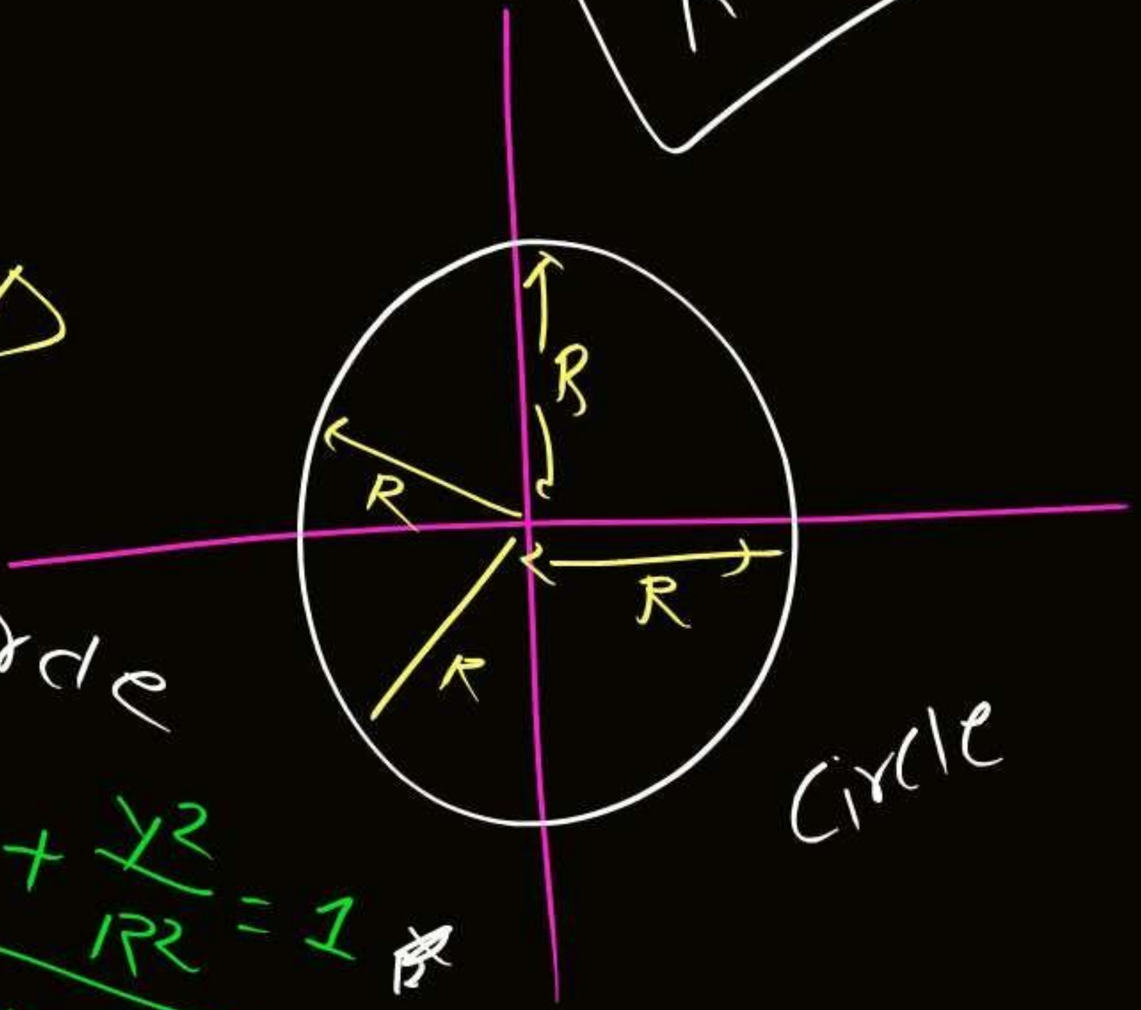
$$\boxed{\frac{x^2}{R^2}+\frac{y^2}{R^2}=1}$$

If $a=b=R$

then ellipse $\Longrightarrow$ circle

ellipse $\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1} \Longrightarrow \frac{x^2}{R^2}+\frac{y^2}{R^2}=1$

$$\#\ \boxed{x^2+y^2=R^2}\ \#$$



circle

If position of object $\vec{r} = 4\sin(\omega t)\,\hat{i} + 3\cos(\omega t)\,\hat{j}$.

$x = 4\sin(\omega t)$ — (i)

$y = 3\cos(\omega t)$

$\frac{x}{4} = \sin(\omega t)$ — (i)

$\frac{y}{3} = \cos(\omega t)$ — (ii)

$(i)^2 + (ii)^2$

$$\frac{x^2}{(4)^2} + \frac{y^2}{(3)^2} = \sin^2(\omega t) + \cos^2(\omega t)$$

$$\boxed{\frac{x^2}{(4)^2} + \frac{y^2}{(3)^2} = 1}$$

Ans Ellipse

If position of object $\vec{r} = \underline{4 \sin(\omega t)}\ \hat{i} + 3 \sin(\omega t)\ \hat{j}$.

$$\frac{x = 4\cancel{\sin}(\omega t) \quad -(i)}{y = 3\cancel{\sin}(\omega t) \quad -(ii)}$$

$$\frac{x}{y} = \frac{4}{3}$$

$$\boxed{y = \frac{3}{4}x}$$

→ straight line

Physics??

Velocity of object $\vec{V} = 2\,\hat{\imath} + x\,\hat{\jmath}$ then equation of trajectory.

AIEEE-2010

$$\vec{V} = \underbrace{2\hat{i}}_{} + \underbrace{x\hat{j}}_{}$$

$V_x = 2$

$$\boxed{\frac{dx}{dt} = 2} \quad \text{— (I)}$$

$$\int dx = \int 2\,dt$$

$$\boxed{x = 2t} \quad \text{— (I)}$$

$V_y = x$

$$\boxed{\frac{dy}{dt} = x} \quad \text{— (II)}$$

गजब

$$dy = x\,dt$$

$$\int dy = \int 2t\,dt$$

$$y = \cancel{2}\frac{t^2}{\cancel{2}} = t^2$$

$$y = t^2$$

using eqn (I)

$x = 2t$

$$t = \frac{x}{2}$$

$$\boxed{y = \frac{x^2}{4}} \rightarrow \text{Parabola}$$

ARJUNA

$$\frac{dx}{dt} = 2 \quad \text{(i)} \qquad \frac{dy}{dt} = x \quad \text{(ii)}$$

$$\frac{\frac{dy}{\cancel{dt}}}{\frac{dx}{\cancel{dt}}} = \frac{x}{2}$$

समझा ??

$$\int dy = \int \frac{x}{2}\, dx$$

$$\boxed{y = \frac{x^2}{4}}$$

THANK YOU ☺

ARJUNA