DEFINITION OF CIRCULAR MOTION

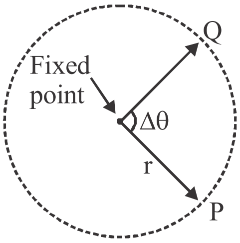
- When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point.
  That fixed point is called centre and the distance is called radius of circular path.

- The vector joining the centre of the circle and the center of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction.

KINEMATICS OF CIRCULAR MOTION

Angular Displacement

- Angle traced by position vector of a particle moving w.r.t. some fixed point is called angular displacement.



$\Delta\theta$ = angular displacement

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}} \quad \text{or} \quad \Delta\theta = \frac{\text{Arc PQ}}{r}$$

- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \quad \text{But } \vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$$

- It is dimensionless and has S.I. unit "Radian" while other units are degree or revolution.

$$2\pi \text{ radian} = 360° = 1 \text{ revolution}$$

Frequency (n) : Number of revolution describes by particle per second is its frequency. Its unit is revolution per second (r.p.s.) or revolution per minute (r.p.m.)

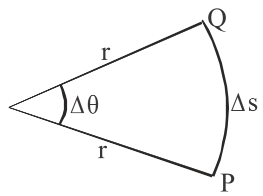
Time Period (T) : It is time taken by particle to complete one revolution. $T = \frac{1}{n}$

Angular Velocity (ω) : It is defined as the rate of change of angular displacement of moving particle

$$\omega = \frac{\text{Angle traced}}{\text{Time taken}} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Relation between linear and Angular velocity

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}} \text{ or } \frac{\Delta s}{r} \qquad \Delta\theta = \frac{\Delta s}{r} \text{ or } \Delta s = r\Delta\theta$$



$$\therefore \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} \text{ if } \Delta t \to 0 \text{ then } \frac{ds}{dt} = r\frac{d\theta}{dt} \qquad \boxed{v = \omega r}$$

Average Angular Velocity (ω_{av})

$$\omega_{av} = \frac{\text{total angle of rotation}}{\text{total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_2} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi n$$

where θ_1 and θ_2 are angular position of the particle at instant t_1 and t_2.

Instantaneous Angular Velocity

The angular velocity at some particular instant $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ or $\vec{\omega} = \frac{d\vec{\theta}}{dt}$

Ex. The angular velocity of a particle is given by $\omega = 1.5\ t - 3t^2 + 2$, Find the time when its angular acceleration becomes zero.

Sol. $\alpha = \frac{d\omega}{dt} = 1.5 - 6\ t = 0 \Rightarrow t = 0.25$ s.

Ex. A disc starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$. Calculate the angular velocity after 2 s.

Sol. $\frac{d\omega}{dt} = 3t - t^2 \Rightarrow \int_0^{\omega} d\omega = \int_0^{t} (3t - t^2)\,dt \Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} \Rightarrow$ at $t = 2$ s, $\quad \omega = \frac{10}{3}$ rad/s

Angular Acceleration (α)

- Rate of change of angular velocity is called angular acceleration. $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$ or $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$
- Unit $\to$ rad/s^2

Tangential Acceleration

Magnitude of tangential acceleration in case of circular motion.

$$a_T = \alpha r$$

As $\vec{a}_T$ is along the direction of motion (in the direction of $\vec{v}$) so that $\vec{a}_T$ is responsible for change in speed of the particle. Its magnitude is rate of change of speed of the particle. If particle is moving on a circular path with constant speed then tangential acceleration is zero.

Centripetal acceleration (pls copy circular motion pdf page 6)

Magnitude of centripetal acceleration, $a_C = \omega v = \frac{v^2}{r} = \omega^2 r$.

- Centripetal acceleration is always perpendicular to the velocity or displacement at each point.

Net linear Acceleration

$\vec{a} = \vec{a}_T + \vec{a}_C$ and $\vec{a}_T \perp \vec{a}_C$ so that $\boxed{|\vec{a}| = \sqrt{a_T^2 + a_C^2} \Rightarrow |\vec{a}| = \sqrt{(R\,\alpha)^2 + (R\,\omega^2)^2}}$

ABOUT UNIFORM CIRCULAR MOTION (THEORY COPY FROM NLM)

- Position vector $(\vec{r})$ is always perpendicular to the velocity vector $(\vec{v})$ i.e. $\vec{r}.\vec{v}=0$
- velocity vector is always perpendicular to the acceleration. $\vec{v}.\vec{a}=0$
- $\because |\vec{v}|=\text{constant} \Rightarrow a_t=0$ (tangential acceleration) $\Rightarrow f_t=0$ (tangential force)
- Important difference between the projectile motion and uniform circular motion :
  In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes.

DYNAMICS OF CIRCULAR MOTION

If a particle moves with constant speed in a circle, motion is called uniform circular. In uniform circular motion a resultant non-zero force acts on the particle. The acceleration is due to the change in direction of the velocity vector. In uniform circular motion tangential acceleration (a_t) is zero. The acceleration of the particle is towards the centre and its magnitude is $\frac{v^2}{r}$. Here, v is the speed of the particle and r the radius of the circle. The direction of the resultant force F is therefore towards centre and its magnitude

is $F=\frac{mv^2}{r}=mr\omega^2$ (as $v=r\omega$)

Here, ω is the angular speed of the particle. This force F is called the centripetal force. Thus, a centripetal force of magnitude $\frac{mv^2}{r}$ is needed to keep the particle moving in a circle with constant speed. This force is provided by some external source such as friction, magnetic force, coulomb force, gravitational, tension, etc.

CIRCULAR TURNING OF ROADS

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways:

• By friction only. • By banking of roads only. • By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both.

- **BY FRICTION ONLY**

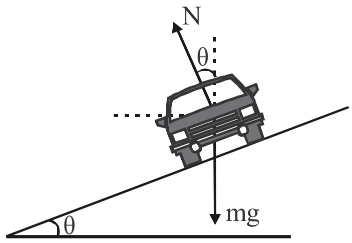
Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards centre.

Thus, $\because f_{max}=\mu N=\mu mg$

Therefore, for a safe turn without sliding $\frac{mv^2}{r}\le f_{max} \Rightarrow \frac{mv^2}{r}\le \mu mg \Rightarrow v\le\sqrt{\mu rg}$

- **By Banking of Roads only**

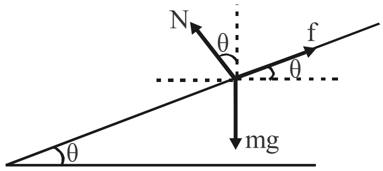
Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.



$$N \sin\theta = \frac{mv^2}{r} \text{ and } N\cos\theta = mg \Rightarrow \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$$

- **FRICTION AND BANKING OF ROAD BOTH**

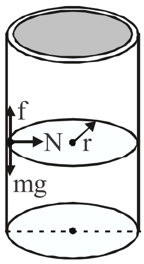
If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed in magnitude and direction. The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force, i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_{max} = \mu N$). So, the magnitude of normal reaction N and direction plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the centre.



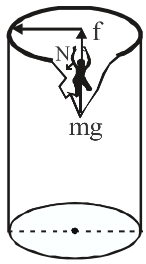
'DEATH WELL' OR ROTOR

In case of 'death well' a person drives a motorcycle on a vertical surface of a large wooden well while in case of a rotor at a certain angular speed of rotor a person hangs resting against the wall without any support from the bottom. In death well walls are at rest and person revolves while in case of rotor person is at rest and the walls rotates.

In both cases friction balances the weight of person while reaction provides the centripetal force for circular motion,



Death well



Rotor

$$\text{i.e., } f = mg \text{ and } N = \frac{mv^2}{2} = mr\omega^2$$

Ex. Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4 [acceleration due to gravity = 10 m/s^2]

Sol. Here centripetal force is provided by friction so

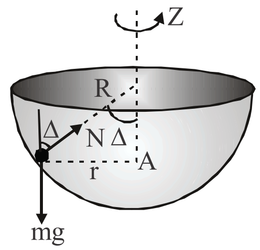
$$\frac{mv^2}{2} \le \mu mg \Rightarrow v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 \text{ ms}^{-1}$$

Ex. For traffic moving at 60 km/hr, if the radius of the curve is 0.1 km, what is the correct angle of banking of the road ? (g = 10 m/s2)

Sol. In case of banking $\tan\theta = \frac{v^2}{rg}$ Here $v = 60$ km/hr $= 60 \times \frac{5}{18}\,\text{ms}^{-1} = \frac{50}{3}\,\text{ms}^{-1}$ $r = 0.1$ km $= 100$ m

So $\tan\theta = \frac{50/3 \times 503}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{18}\right)$

Ex. A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.

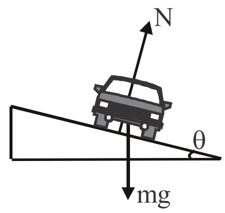


Sol. $N\cos\alpha = mg$ and $N\sin\alpha = mr\omega^2$ but $r = R\sin\alpha$

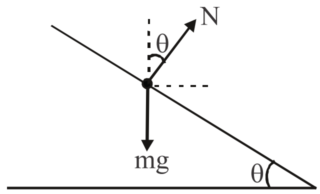
$\Rightarrow N\sin\alpha = mR\sin\alpha\omega^2 \Rightarrow N = mR\omega^2$

$\Rightarrow (mR\omega^2)\cos\alpha = mg \qquad \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$

Ex. A car is moving along a banked road laid out as a circle of radius r. (a) What should be the banking angle θ so that the car travelling at speed v needs no frictional force from the tyres to negotiate the turn? (b) The coefficients of friction between tyres and road are $\mu_k = 0.9$ and $\mu_k = 0.8$. At what maximum speed can a car enter the curve without sliding towards the top edge of the banked turn?



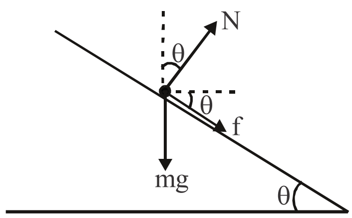
Sol. (a) $N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg \Rightarrow \tan\theta = \frac{v^2}{rg}$



Note : In above case frction does not play any role in negotiating the turn.

(b) If the driver moves faster than the speed mentioned above, a friction force must act parallel to the road, inward towards centre of the turn.

$\Rightarrow F\cos\theta + N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg + f\sin\theta$



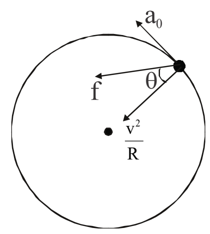
For maximum speed of $f = \mu N$

$\Rightarrow N(\mu\cos\theta + \sin\theta) = \frac{mv^2}{r}$ and $N(\cos\theta - \mu\sin\theta) = mg$

$\Rightarrow \frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} \Rightarrow v = \sqrt{\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) rg}$

Ex. A car starts from rest with a constant tangential acceleration a_0 in a circular path of radius r. At time t, the car skids, find the value of coefficient of friction.

Sol. The tangential and centripetal acceleration is provided only by the frictional force.



$$\text{Thus, } f\sin\theta = ma_0,\ f\cos\theta = \frac{mv^2}{r} = \frac{m(a_0t)^2}{r}$$

$$\Rightarrow f = m\sqrt{a_0^2 + \frac{(a_0t)^4}{r^2}} = ma_0\sqrt{1+\frac{a_0^2t^4}{r^2}} = f_{max}$$

$$\mu mg = ma_0\sqrt{1+\frac{a_0^2t^4}{r^2}} \Rightarrow \mu = \frac{a_0}{g}\sqrt{1+\frac{a_0^2t^4}{r^2}}$$

CENTRIFUGAL FORCE

Centrifugal force is a pseudo force which an observer needs to consider while making observations in a rotating frame. This force is non physical and arises from kinematics and not due to physical interactions. Centrifugal force is directed away from axis of rotation of rotating frame and its value is $m\omega^2r$, where ω is angular speed of rotating frame where observer has kept himself fixed and r is distance of object of mass m from axis of rotation.

EXERCISE

EXERCISE (S-1)

1. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

2. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

3. A particle moves in a circle of radius 1.0 cm at a speed given by v = 2.0 t where v is in cm/s and t in seconds.
(a) Find the radial acceleration of the particle at t = 1s.
(b) Find the tangential acceleration at t = 1s.
(c) Find the magnitude of the acceleration at t = 1 s.

4. A particle is travelling in a circular path of radius 4m. At a certain instant the particle is moving at 20m/s and its acceleration is at an angle of 37° from the direction to the centre of the circle as seen from the particle
(i) At what rate is the speed of the particle increasing?
(ii) What is the magnitude of the acceleration?

5. In a park there are three concentric circular running tracks. Radius of 2nd track is double of first and of 3rd track is triple of first. Three runners are running on these tracks with constant speed. When the runner in the first track completes one round, the runner in 2nd has completed half round and the runner in third track has completed quarter round. If the accelerations of the runners are in ratio $\alpha : \beta : \gamma$, where α, β & γ are least integers, then find the value of $\frac{\alpha+\beta+\gamma}{3}$.

6. A stone is thrown horizontally with the velocity 15m/s. Determine the tangential and normal accelerations of the stone in 1 second after it begins to move.

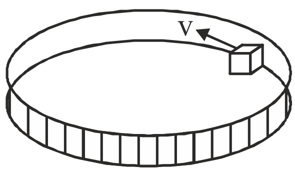
7. A particle moves in the x-y plane with the velocity $\vec{v} = a\hat{i} + bt\hat{j}$. At the instant $t = a\sqrt{3}/b$ the magnitude of tangential, normal and total acceleration are _____, _______, & _________.

8. A body is projected with a velocity 10 ms^{-1} at an inclination 45° to the horizontal. Minimum radius of curvature of the trajectory described by the particle is ____________.

HCV Exercises (Chapter No. 7 - 1,2,3)

HCV Worked out Examples (Chapter No. 7 - 2,3,4,6,8,9,10,11,12,13)

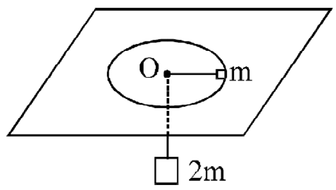
9. A block of mass m moves with speed v against a smooth, fixed vertical circular groove of radius r kept on smooth horizontal surface.
Find :
(i) normal reaction of the floor on the block.
(ii) normal reaction of the vertical wall on the block.



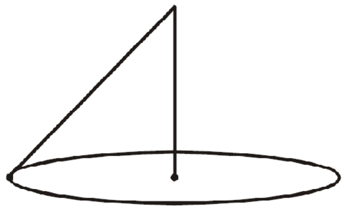
10. A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The coefficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn ?

11. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev./min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?

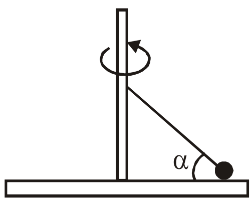
12. A mass m rotating freely in a horizontal circle of radius 1 m on a frictionless smooth table supports a stationary mass 2m, attached to the other end of the string passing through smooth hole O in table, hanging vertically. Find the angular velocity of rotation.



13. Consider a conical pendulum having bob of mass m is suspended from a ceiling through a string of length L. The bob moves in a horizontal circle of radius r. Find (a) the angular speed of the bob and (b) the tension in the string.



14. A circular platform rotates around a vertical axis with angular velocity $\omega = 10$ rad/s. On the platform is a ball of mass 1 kg, attached to the long axis of the platform by a thin rod of length 10 cm ($\alpha = 30°$). Find normal force exerted by the ball on the platform (in newton). Friction is absent.



15. A mosquito is sitting on an L.P. record of a gramophone disc rotating on a turn table at $33\frac{1}{3}$ revolution per minute. The distance of the mosquito from the centre of the disc is 10 cm. Show that the friction coefficient between the record and the mosquito is greater than $\pi^2/81$. Take $g = 10$ m/s^2.

16. A scooter weighing 150 kg together with its rider moving at 36 km/hr is to take a turn of radius 30 m. What horizontal force on the scooter is needed to make the turn possible?

17. If the horizontal force needed for the turn in the previous problem is to be supplied by the normal force by the road, what should be the proper angle of banking ?

18. A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed ?

19. An aircraft executes a horizontal loop at a speed of 720 km/h with its wings banked at 15°. What is the radius of the loop ?

20. A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the

(a) optimum speed of the race- car to avoid wear and tear on its tyres, and

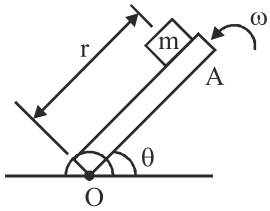
(b) maximum permissible speed to avoid slipping ?

21. A block of mass m = 20 kg is kept at a distance R = 1m from central axis of rotation of a round turn table (A table whose surface can rotate about central axis). Table starts from rest and rotates with constant angular acceleration, $\alpha = 3$ rad/sec^2. The friction coefficient between block and table is $\mu = 0.5$. At time $t = \frac{x}{3}$ sec from starting of motion (i.e. t = 0 sec) the block is just about to slip. Find the value of x.

HCV Exercises (Chapter No. 7 -8,11,13,16,17,18,19,20,21,22,23,24,26,27,28,29,30)

EXERCISE (S-2)

1. The member OA rotates in vertical plane about a horizontal axis through O with a constant counter clockwise velocity $\omega = 3$ rad/sec . As it passes the position $\theta = 0$, a small mass m is placed upon it at a radial distance $r = 0.5$ m . If the mass is observed to slip at $\theta = 37^0$, the co–efficient of friction between the mass & the member is ______



2. A stone is launched upward at 45° with speed v_0. A bee follows the trajectory of the stone at a constant speed equal to the initial speed of the stone.

(i) Find the radius of curvature at the top point of the trajectory.

(ii) What is the acceleration of the bee at the top point of the trajectory? For the stone, neglect the air resistance.

3. Two blocks of mass m_1=10kg and m_2=5kg connected to each other by a massless inextensible string of length 0.3m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rad/sec about a vertical axis passing through its centre. The masses are placed along the diameter of the table on either side of the centre O such that m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table.

(i) Calculate the frictional force on m1

(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position.

(iii) How should the masses be placed with the string remaining taut, so that there is no frictional force required.

4. A particle is moving along a circular path of radius R in such a way that at any instant magnitude of radial acceleration & tangential acceleration are equal. If at t = 0 velocity of particle is V_0. Find the velocity as a function of time.

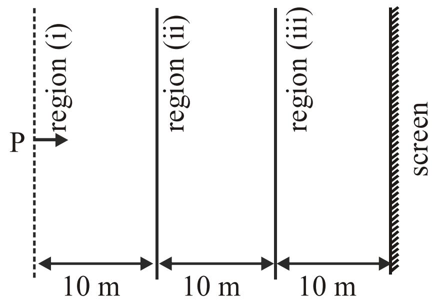
5. A small particle initially at point p starts moving from rest. The whole space where particle will move is divided into three regions as shown in figure. In region

(i) particle accelerates through ($5\ m/s^2$) where direction of acceleration is along the normal of the screen while in region

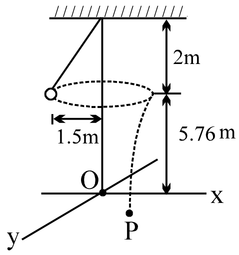
(ii) the acceleration acts in such a way that it is always perpendicular to the direction of motion resulting the particle to move on a circular track having radius $\frac{20}{\sqrt{3}}$ m. There is uniform acceleration in region

(iii) in such a manner that velocity of particle become thrice (without change in direction) when it just reach the screen.

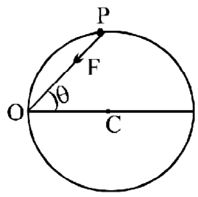
Find the average speed of a particle (in m/s).



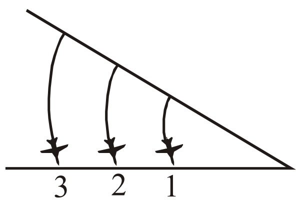
6. A particle suspended from the ceiling by inextensible light string is moving along a horizontal circle of radius 1.5 m as shown. The string traces a cone of height 2 m. The string breaks and the particle finally hits the floor (which is xy plane 5.76 m below the circle) at point P. Find the distance OP in cm.



7. A particle P is moving on a circle under the action of only one force acting always towards fixed point O on the circumference. Find ratio of $\frac{d^2\theta}{dt^2}$ & $\left(\frac{d\theta}{dt}\right)^2$.



8. Three aircraft make a turn in the horizontal plane at uniform speed, moving along concentric circular trajectories that are shown in figure. The aircraft move such that they are at constant distance of 600 m from each other at any time. The aircraft closest to the center moves in a circle of radius R = 600 m. The aircraft 2 is moving at a speed of v_2 = 720 km/h. Find the acceleration of third aircraft (in m/s^2)



9. The speed of an object undergoing uniform circular motion is 4 m/s. The magnitude of the change in the velocity during 0.5 sec is also 4 m/s. Find the minimum possible centripetal acceleration (in m/s^2) of the object. [Take π= 25/8]

10. A block of mass m moves on a horizontal circle agains the wall of a cylindrical room of radius R. The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is μ. The block is given an initial speed υ_0. As a function of the speed υ write (a) the normal force by the wall on the block, (b) the frictional force by the wall and (c) the tangential acceleration of the block. (d) Integerate the tangential acceleration $\left(\frac{d\upsilon}{dt} = \upsilon\frac{d\upsilon}{ds}\right)$ to obtain the speed of the block after one revolution.

11. A thin circular loop of radius R rotates about its vertical diameter with an angular frequency a). Show that a small bead on the wire loop remains at its lowermost point for $\omega \le \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{2g/R}$? Neglect friction.

EXERCISE (O-1)

1. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be

(A) 2π & 0 mm/s (B) $22\sqrt{\pi}$ & 4.44 mm/s

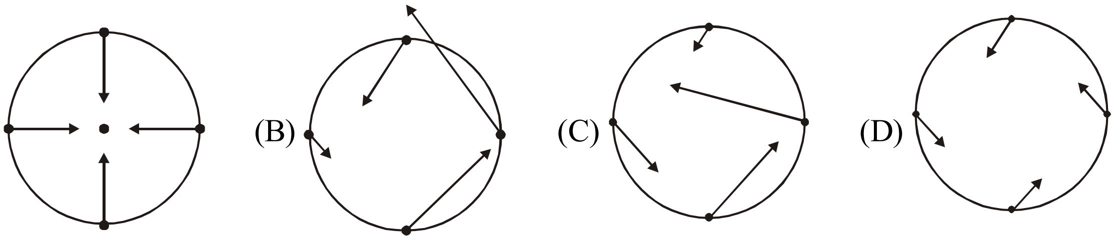
(C) 2 & $2\sqrt{\pi}$ mm/s (D) 2π & $2\sqrt{2}\pi$ mm/s

2. A car is moving with speed 27 km/h. The driver applies brakes as he approaches a circular turn on the road of radius 80 m and his speed reduces at the constant rate of 0.50 m/s every second. The magnitude of net acceleration is :

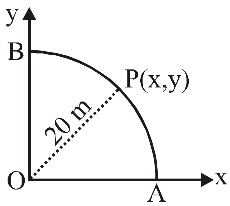
(A) 20 ms^{-2} (B) 0.86 ms^{-2} (C) 100 ms^{-2} (D) None of these

3. A car speeds up with constant magnitude of tangential acceleration in circular path moving in anticlockwise direction. Which of the following figure represents acceleration of the car?

(A) (B) (C) (D)



4. A point P moves in counter clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^2 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 5\sqrt{\frac{3}{10}}$ seconds is nearly :

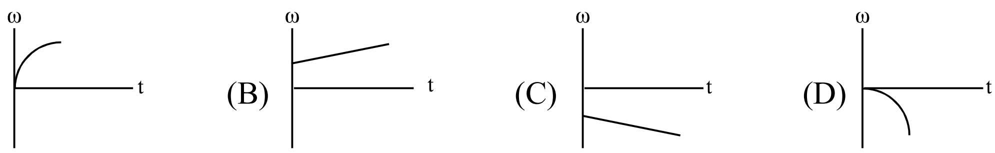


(A) 2 m/s^2 (B) 1.5 m/s^2 (C) 2.5 m/s^2 (D) 3 m/s^2

5. A particle is moving in a circle :
(A) the resultant force on the particle must be towards the centre
(B) the cross product of the tangential acceleration and the angular velocity will be zero
(C) the direction of the angular acceleration and the angular velocity must be the same
(D) the resultant force may be towards the centre

6. The graphs below show angular velocity as a function of time. In which one is the magnitude of the angular acceleration constantly decreasing?

(A) (B) (C) (D)



7. One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v the net force on the particle (directed towards the centre) is :

(A) T (B) $T - \frac{mv^2}{\ell}$ (C) $T + \frac{mv^2}{\ell}$ (D) 0

8. A particle of mass m is tied to a light string and rotated with a speed v along a circular path of radius r. If T = tension in the string and mg = gravitational force on the particle then the actual forces acting on the particle are :

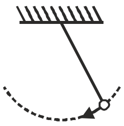
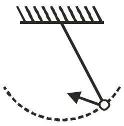
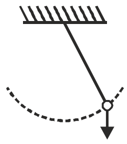
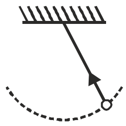
(A) mg and T only

(B) mg, T and an additional force of $\frac{mv^2}{r}$ directed inwards.

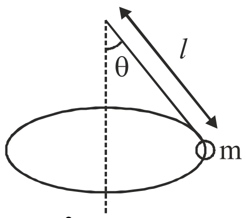
(C) mg, T and an additional force of $\frac{mv^2}{r}$ directed outwards.

(D) only a force $\frac{mv^2}{r}$ directed outwards.

9. Which vector in the figures best represents the acceleration of a pendulum mass at the intermediate point in its swing?

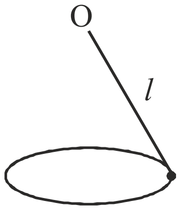
(A)  (B)  (C)  (D) 

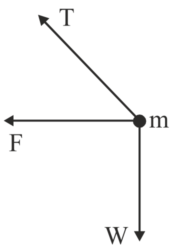
10. A conical pendulum is moving in a circle with angular velocity ω as shown. If tension in the string is T, which of following equations are correct ?

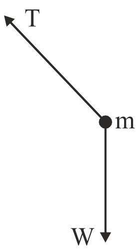


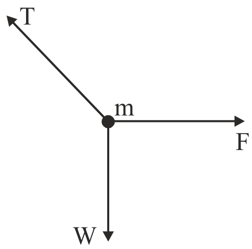
(A) $T = m\omega^2 l$ (B) $T \sin\theta = m\omega^2 l$ (C) $T = mg\cos\theta$ (D) $T = m\omega^2 l \sin\theta$

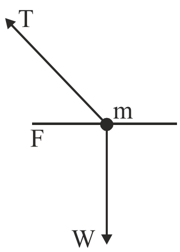
11. A point mass m is suspended from a light thread of length ℓ, fixed at O, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are :



(A) 

(B) 

(C) 

(D) 

MULTIPLE CORRECT TYPE QUESTIONS

12. A car runs around a curve of radius 10 m at a constant speed of 10 ms^{-1}. Consider the time interval for which car covers a curve of 120° arc :

(A) Resultant change in velocity of car is $10\sqrt{3}$ ms^{-1}

(B) Instantaneous acceleration of car is 10 ms^{-2}

(C) Average acceleration of car is $\frac{5}{24}$ ms^{-2}

(D) Instantaneous and average acceleration are same for the given period of motion.

13. A particle of mass 1 kg slides in a horizontal circle of radius 20 cm with a constant speed of 1 m/s. The only forces in the vertical direction acting on the particle are its weight and the normal reaction, however no information is available about the forces in the horizontal plane. The coefficient of friction is $\mu = 0.5$. Then

(A) the magnitude of frictional force due to ground acting on particle must be 5 N.
(B) the frictional force due to ground must be in tangential direction.
(C) the frictional force due to ground must be towards the centre.
(D) no comment can be made about the direction or magnitude of friction force due to ground based on the given data.

14. A car is moving with constant speed on a rough banked road.

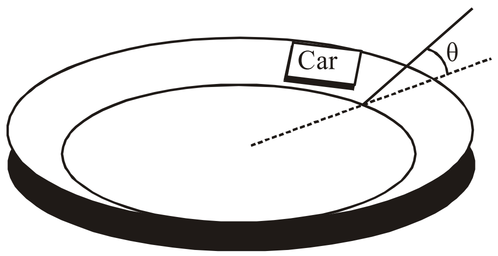
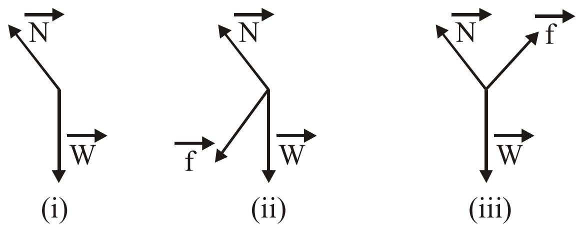


Figure (i), (ii) and (iii) show the free body diagram of car A, B & C respectively:-



(A) Car A has more speed than car C
(B) Car A has less speed than car B
(C) FBD for car A is not possible
(D) If $\mu > \tan\theta$ the FBD for car C is not possible

15. A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60° with the vertical. Then

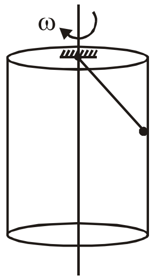
(A) its period of revolution is $\frac{4\pi}{7}$ sec.

(B) the tension in the string is double the weight of the particle

(C) the velocity of the particle = $2.8\sqrt{3}$ m/s

(D) the centripetal acceleration of the particle is $9.8\sqrt{3}$ m/s^2.

16. In the shown figure inside a fixed hollow cylinder with vertical axis a pendulum is moving conically with its axis same as that of the cylinder with uniform angular velocity. Radius of cylinder is 30 cm, length of string is 50 cm and mass of bob is 400 gm. The bob makes contact with the inner frictionless wall of the cylinder while moving :



(A) The minimum value of angular velocity of the bob so that it does not leave contact is 5 rad/s
(B) Tension in the string is 5N for all values of angular velocity
(C) For angular velocity of 10 rad/s the bob pushes the cylinder with a force of 9N
(D) For angular velocity of 10 rad/s, tension in the string is 20N

Paragraph for question nos. 17 to 19

Ram has just learnt driving and he is driving on a wet straight road ($\mu_S = 0.1$, $\mu_k = 0.05$) with a speed of 108 km/hr. He sees his friend Shyam ahead travelling at a constant speed of 36km/hr in the same direction. His horn fails, he is at a distance of 102 m from Shyam. He applies brakes just hard enough to prevent slipping, yet providing for maximum deceleration.

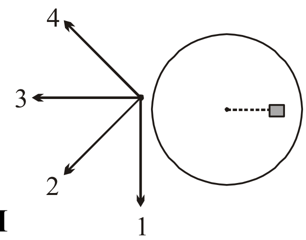
17. What is his speed at the time of hitting Shyam.
(A) 24 m/s (B) 21 m/s (C) 18 m/s (D) 26 m/s

18. If horn had worked and Shyam started accelerating
(A) Accident could have been avoided only if Shyam accelerated at maximum possible rate too.
(B) Accident could not have been avoided.
(C) Accident could have been avoided even if Shyam did not accelerate at maximum possible rate.
(D) Accident could have been avoided even if Shyam did not accelerate at all, but moved at same speed as before.

19. If instead of braking Ram decides to take a turn, what is the minium possible radius of the turn ?
(A) 400 m (B) 225 m (C) 900 m (D) 625 m

MATRIX MATCH TYPE QUESTION

20. A block is placed on a horizontal table which can rotate about its axis.The block is placed at a certain distance from centre as shown in figure. Table rotates such that particle does not slide. Select possible direction of net acceleration of block at the instant shown in figure.



	Column-I	**Column-II**	
(A)	When rotation is clockwise with constant ω	(P)	1
(B)	When rotation is clock wise with decreasing ω	(Q)	2
(C)	When rotation is clockwise with increasing ω	(R)	3
(D)	Just after clockwise rotation begins from rest	(S)	4

EXERCISE (O-2)

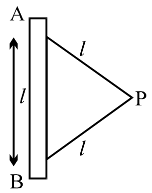
1. A ring of radius r and mass per unit length m rotates with an angular velocity ω in free space. The tension in the ring is :

(A) zero (B) $\frac{1}{2}$ $m\omega^2r^2$ (C) $m\omega^2r^2$ (D) $mr\omega^2$

2. The magnitude of displacement of a particle moving in a circle of radius a with constant angular speed ω varies with time t as

(A) 2a sin ωt (B) 2a sin$\frac{\omega t}{2}$ (C) 2a cos ωt (D) 2a cos$\frac{\omega t}{2}$

3. A particle P of mass m is attached to a vertical axis by two strings AP and BP of length l each. The separation AB=l. P rotates around the axis with an angular velocity ω. The tensions in the two strings are T_1 and T_2



(A) T_1=T_2 (B) T_1+T_2=$m\omega^2l$

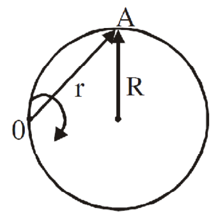
(C) T_1-T_2=2mg (D) BP will remain taut only if $\omega \geq \sqrt{\frac{2g}{\ell}}$

4. A particle is moving in a circular path. The acceleration and momentum of the particle at a certain moment are $\vec{a} = \left(4\hat{i} + 3\hat{j}\right)$ m/s^2 and $\vec{p} = \left(8\hat{i} - 6\hat{j}\right)$kg–m/s. The motion of the particle is

(A) uniform circular motion (B) accelerated circular motion

(C) de–accelerated circular motion (D) we can not say anything with $\vec{a}$ and $\vec{p}$ only

5. A particle A moves along a circle of radius R=50 cm so that its radius vector r relative to the point O (figure) rotates with the constant angular velocity ω=0.40 rad/s. Then modulus of the velocity of the particle, and the modulus of its total acceleration will be



(A) v= 0.4 m/s, a = 0.4 m/s^2

(B) v = 0.32 m/s, a = 0.32 m/s^2

(C) v = 0.32 m/s, a = 0.4 m/s^2

(D) v = 0.4 m/s, a = 0.32 m/s^2

6. A rod of length L is pivoted at one end and is rotated with a uniform angular velocity in a horizontal plane. Let T_1 and T_2 be the tensions at the points L/4 and 3L/4 away from the pivoted ends.

(A) $T_1 > T_2$

(B) $T_2 > T_1$

(C) $T_1 = T_2$

(D) The relation between T_1 & T_2 depends on whether the rod rotates clockwise or anticlockwise

MULTIPLE CORRECT TYPE QUESTIONS

7. For a curved track of radius R, banked at angle θ (Take $v_0 = \sqrt{Rg\tan\theta}$)

(A) a vehicle moving with a speed v_0 is able to negotiate the curve without calling friction into play at all

(B) a vehicle moving with any speed $V > V_0$ is always able to negotiate the curve, with friction called into play

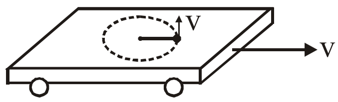
(C) a vehicle moving with any speed $V < V_0$ must have the force of friction into play

(D) the minimum value of the angle of banking for a vehicle parked on the banked road can stay there without slipping, is given by $\theta = \tan^{-1}\mu_0$ (μ_0 = coefficient of static friction)

8. On a train moving along east with a constant speed v, a boy revolves a bob with string of length ℓ on smooth surface of a train, with equal constant speed v relative to train. Mark the correct option(s).

(A) Maximum speed of bob is 2 v in ground frame.

(B) Tension in string connecting bob is $\dfrac{4mv^2}{\ell}$ at an instant.



(C) Tension in string is $\dfrac{mv^2}{\ell}$ at all the moments.

(D) Minimum speed of bob is zero in ground frame.

9. Let $\vec{v}(t)$ be the velocity of a particle at time t. Then :

(A) $|d\vec{v}(t)/dt|$ and $d|\vec{v}(t)|/dt$ are always equal

(B) $|d\vec{v}(t)/dt|$ and $d|\vec{v}(t)|/dt$ may be equal

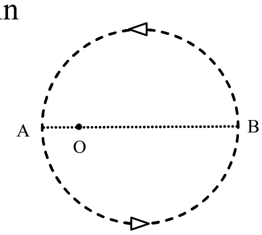
(C) $d|\vec{v}(t)|/dt$ can be zero while $|d\vec{v}(t)/dt|$ is not zero

(D) $d|\vec{v}(t)|/dt \neq 0$ implies $|d\vec{v}(t)/dt| \neq 0$

10. Which of the following statements is /are true for a particle moving in a circle with a constant angular speed?

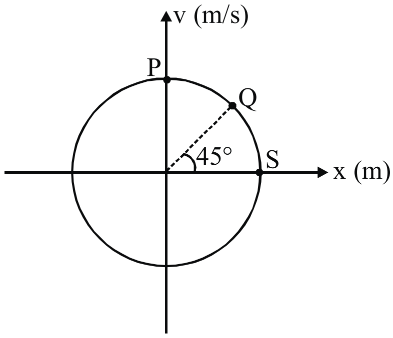
(A) The velocity vector is tangent to the circle.
(B) The acceleration vector is tangent to the circle.
(C) The velocity and acceleration vectors are perpendicular to each other.
(D) The acceleration vector points to the centre of circle.

11. An object moves counter–clockwise along the circular path shown. As it moves along the path, its acceleration vector continuously points towards point O. In the figure, line AB is a diameter.



(A) The object speeds up from A to B and slows down from B to A.
(B) The object slows down from A to B and speeds up from B to A.
(C) The object has maximum speed at A and minimum speed at B.
(D) The object has minimum speed at A and maximum speed at B.

12. A particle is in motion on the x-axis. The variation of its velocity with position is as shown. The graph is circle and its equation is $x^2 + v^2 = 1$, where x is in m and v in m/s. The **CORRECT** statement(s) is/are



(A) When x is positive, acceleration is negative.

(B) When x is negative, acceleration is positive.

(C) At Q, acceleration has magnitude $\frac{1}{\sqrt{2}}$ m/s^2

(D) At S, acceleration is infinite.

13. A particle is attached to an end of a rigid rod. The other end of the rod is hinged and the rod rotates always remaining horizontal. It's angular speed is increasing at constant rate. The mass of the particle is m. The force exerted by the rod on the particle is $\vec{F}$, then

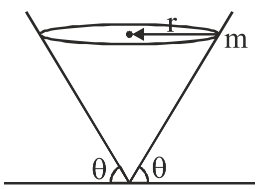
(A) $F \geq mg$

(B) F is constant

(C) The angle between $\vec{F}$ and horizontal plane decreases.

(D) The angle between $\vec{F}$ and the rod decreases.

14. A ball of mass 'm' is rotating in a circle of radius 'r' with speed v inside a smooth cone as shown in figure. Let N be the normal reaction on the ball by the cone, then choose the correct option.



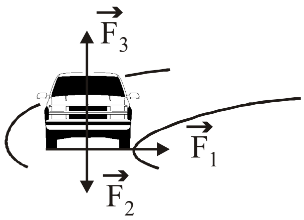
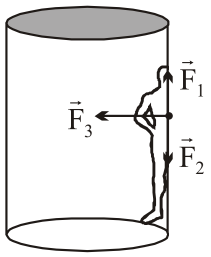
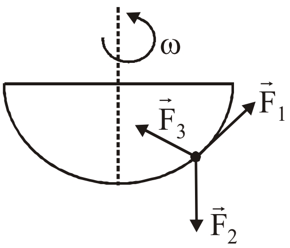
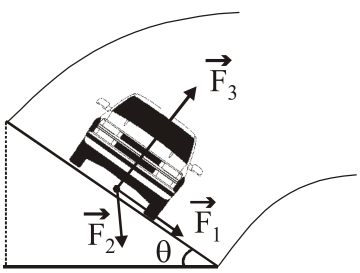
(A) $N = mg\cos\theta$

(B) $g\sin\theta = \frac{v^2}{r}\cos\theta$

(C) $N\sin\theta - \frac{mv^2}{r} = 0$

(D) None of these

15. Column-I shows certain situations and column-2 shows information about forces.

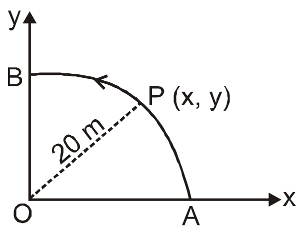
Column - I		**Column - II**
	Situation	
(A)	 Front view of a car rounding a curve with constant speed.	(P) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ is centripetal force.
(B)	 Passengers in a rotor not sliding relative to rotor wall cylindrical rotor is rotating with constant angular velocity about its symmetry axis.	(Q) $\vec{F}_1$ is static friction.
(C)	 Particle kept on rough surface of a bowl, no relative motion of particle in bowl, bowl has constant angular velocity	(R) $\vec{F}_1$ can be in direction opposite to that shown in figure.
(D)	 Car moving on a banked road with constant speed, no sideways skidding	(S) $\vec{F}_1 + \vec{F}_2 = \vec{0}$
		(T) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$

EXERCISE (JM)

1. Which of the following statements is false for a particle moving in a circle with a constant angular speed ?
(1) The velocity vector is tangent to the circle **[AIEEE - 2004]**
(2) The acceleration vector is tangent to the circle
(3) The acceleration vector point to the center of the circle
(4) The velocity and acceleration vectors are perpendicular to each other

2. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that **[AIEEE - 2004]**
(1) its velocity is constant (2) its acceleration is constant
(3) its kinetic energy is constant (4) it moves in a straight line

3. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2 s is nearly. **[AIEEE - 2010]**



(1) 13 m/s^2 (2) 12 m/s^2 (3) 7.2 m/s^2 (4) 14 m/s^2

4. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (Here θ is measured from the x-axis) **[AIEEE - 2010]**

(1) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$ (2) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

(3) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$ (4) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

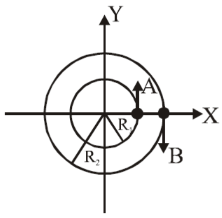
5. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :
[AIEEE - 2012]
(1) $m_1 r_1 : m_2 r_2$ (2) $m_1 : m_2$ (3) $r_1 : r_2$ (4) 1 : 1

6. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is : **[JEE Main-2019]**
(1) 10.3 m (2) 2.5 m (3) 2.8 m (4) 5.1 m

7. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change is velocity of the particle, when it moves through an angle of 60° round the centre of the circle? **[JEE Main-2019]**

(1) $10\sqrt{2}$m/s (2) 10 m/s (3) $10\sqrt{3}$m / s (4) zero

8. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure :



The relative velocity $\vec{\upsilon}_A - \vec{\upsilon}_B$ at $t = \frac{\pi}{2\omega}$ is given by : **[JEE Main-2019]**

(1) $-\omega(R_1 + R_2)\hat{i}$ (2) $\omega(R_1 + R_2)\hat{i}$ (3) $\omega(R_2 - R_1)\hat{i}$ (4) $\omega(R_1 - R_2)\hat{i}$

9. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length *l*. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in spring is: **[JEE Main-2020]**

(1) $\frac{ml\omega^2}{k - m\omega^2}$ (2) $\frac{ml\omega^2}{k + m\omega}$ (3) $\frac{ml\omega^2}{k + m\omega^2}$ (4) $\frac{ml\omega^2}{k - \omega m}$

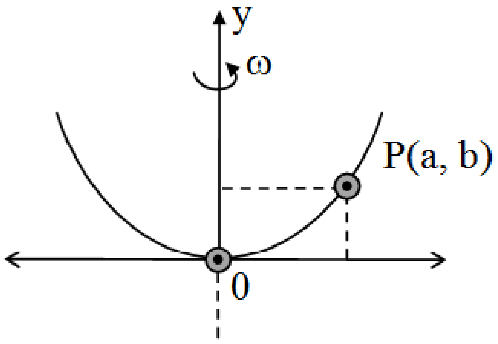
10. A particle moves such that its position vector $\vec{r}(t) = \cos\omega t\hat{i} + \sin\omega t\hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle: **[JEE Main-2020]**

(1) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin
(2) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$
(3) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$
(4) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

11. A spring mass system (mass m, spring constant k and natural length *l*) rests in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system rotates about it's axis with an angular velocity ω, (k >>mω^2) the relative change in the length of the spring is best given by the option: **[JEE Main-2020]**

(1) $\frac{2m\omega^2}{k}$ (2) $\frac{m\omega^2}{3k}$ (3) $\frac{m\omega^2}{k}$ (4) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$

12. A bead of mass m stays at point P(a, b) on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of is (neglect friction) : **[JEE Main-2020]**

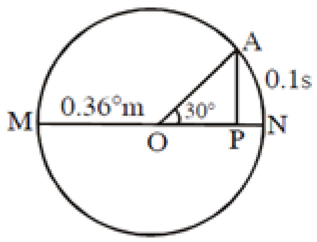


(1) $\sqrt{\frac{2gC}{ab}}$ (2) $2\sqrt{gC}$ (3) $2\sqrt{2gC}$ (4) $\sqrt{\frac{2g}{C}}$

13. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of: **[JEE Main-2020]**

(1) 10^{-3} (2) 10^{-2} (3) 10^{-4} (4) 10^{-1}

14. The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1 s. The perpendicular projection 'P' from 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration force per unit mass when P touches M will be : **[JEE Main-2021]**



(1) 100 N (2) 0.49 N (3) 50 N (4) 9.87 N

15. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R3. Its time period of revolution will be given by : **[JEE Main-2021]**

(1) $T \propto R^2$ (2) $T \propto R^{\frac{3}{2}}$ (3) $T \propto R^{\frac{5}{2}}$ (4) $T \propto R^{\frac{4}{3}}$

EXERCISE (JA)

1. A long horizontal rod has a bead which can slide along its length and is initially placed at a distanceL from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration, α. If the coefficient of friction between the rod and bead is μ, and gravity is neglected,then the time after which the bead starts slipping is **[IIT-JEE 2000]**

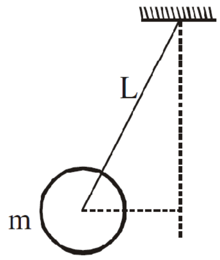
(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

2. **Statement-1 :** For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary. **[IIT-JEE 2008]**
and

Statement-2 : If the observer and the object are moving at velocities $\vec{V}_1$ and $\vec{V}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{V}_2 - \vec{V}_1$.

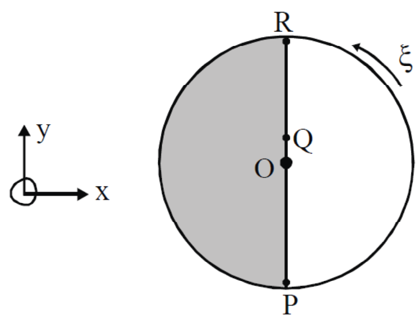
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

3. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is **[IIT-JEE-2011]**



(A) 9 (B) 18 (C) 27 (D) 36

4. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then **IIT-JEE-2012]**



(A) P lands in the shaded region and Q in the unshaded region.
(B) P lands in the unshaded region and Q in the shaded region.
(C) Both P and Q land in the unshaded region.
(D) Both P and Q land in the shaded region.

ANSWER KEY

EXERCISE - (S-1)

1.	$9.89, -\hat{r}$	**2.**	$a_C = 62.5$ m/sec^2	**3.**	4 cm/s^2, 2cm/s^2, $\sqrt{20}\,\text{cm/s}^2$
4.	(i) 75 m/s^2, (ii) 125 m/s^2	**5.**	9	**6.**	$a_t = \frac{2g}{\sqrt{13}}, a_n = \frac{3g}{\sqrt{13}}$
7.	$\frac{\sqrt{3}b}{2}, \frac{b}{2}, b$	**8.**	5m	**9.**	(i) mg (ii) $\frac{mv^2}{r}$
10.	yes	**11.**	T = 6.66, V = 34.6 m/s	**12.**	$\sqrt{2g}$ rad / s
13.	(a) $w = \sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$, (b) $T = \frac{mgL}{\sqrt{L^2 - r^2}}$			**14.**	5
15.	$\mu \ge \frac{\pi^2}{81}$	**16.**	500N	**17.**	$\theta = \tan^{-1}\left(\frac{1}{3}\right)$
18.	w = 4.72 rad/s	**19.**	r = 14.98 km	**20.**	28.3 m/s, 38.5 m/s
21.	2				

EXERCISE - (S-2)

1.	0.1875	**2.**	(i) $r = \frac{V^2}{2g}$ (ii) 2g		
3.	(i) 36N, (ii) 11.78 rad/s (iii) $r_1 = 0.1$m, $r_2 = 0.2$m			**4.**	$\frac{1}{V} = \frac{1}{V_0} - \frac{t}{R}$
5.	10	**6.**	3.9	**7.**	$2\tan\theta$
8.	50 m/s^2	**9.**	8.33		
10.	(a) $\frac{mv^2}{R}$ (b) $\mu\frac{mv^2}{r}$ (c) $-\frac{\mu v^2}{r}$ (d) $V = V_0 e^{-\frac{\mu S}{R}}$			**11.**	$\theta = 60°$

EXERCISE - (O-1)

1.	C	**2.**	B	**3.**	C	**4.**	C	**5.**	D
6.	A	**7.**	A	**8.**	A	**9.**	B	**10.**	A
11.	C	**12.**	A,B	**13.**	A,B	**14.**	A,B	**15.**	A,B,C,D
16.	A,B,C	**17.**	A	**18.**	C	**19.**	C		

20. $A \to R, B \to S, C \to Q, D \to P$

EXERCISE (O-2)

1.	D	**2.**	B	**3.**	B,C,D	**4.**	B	**5.**	D
6.	A	**7.**	A,C	**8.**	A,C,D	**9.**	B,C,D	**10.**	A,C,D
11.	B,C	**12.**	A,B,C	**13.**	A,C,D	**14.**	B,C		

15. $A \to PQ, B \to PQS, C \to PQR, D \to PQR$

EXERCISE (JM)

1.	2	**2.**	3	**3.**	4	**4.**	3	**5.**	3	**6.**	3	**7.**	2
8.	3	**9.**	1	**10.**	4	**11.**	3	**12.**	3	**13.**	1		

EXERCISE (JA)

1.	B	**2.**	D	**3.**	C or D	**4.**	C