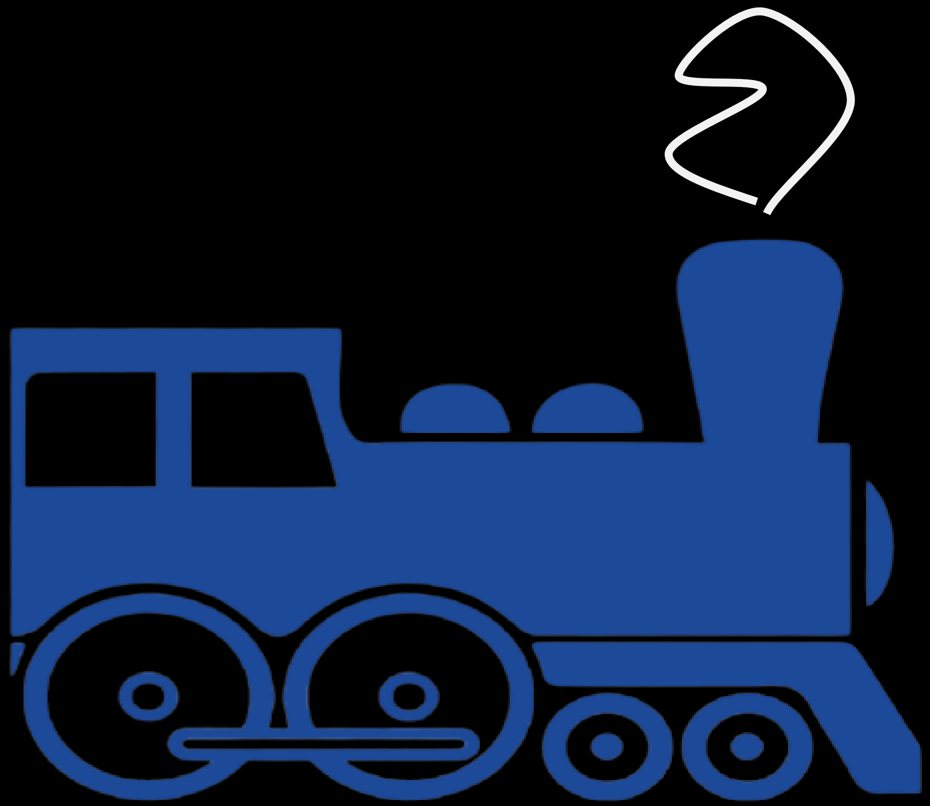
BACKLOG EXPRESS



By: Praveen Vijay Sir

LECTURE GOAL

PHYSICAL QUANTITY

UNITS

DIMENSIONS AND DIMENSIONAL ANALYSIS

CLASS 11-ARJUNA NEET

PHYSICAL QUANTITY

→ Those qty which can be measured Exa → Time, Length etc, mass

Base or Fundamental Quantity

Derived Quantity

Supplementary Quantity

TYPES OF PHYSICAL QUANTITY

numerical value

5 Kg

5

kg

10 m

$\boxed{Q = nu}$

$\boxed{n_1 u_1 = n_2 u_2}$

5kg

1 Year = 12 Months

(1) Base Qty ⇒ which are defined

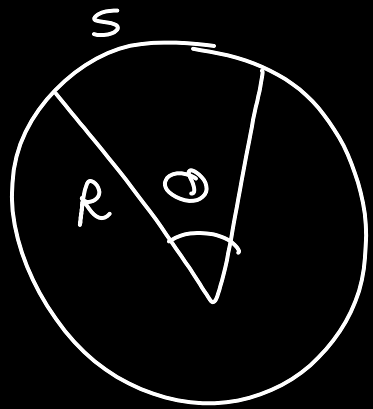
Qty	
Mass	Kg
Length	m
Time	sec
Temp	K
Amt of substance	mole
Luminous intensity	Cd
Current	Amp (A)

② **Derived qty** ⇒ qty which are derived from fundamental qty.

Exa → $\dfrac{\text{Length}}{\text{time}}$ $\dfrac{m}{sec}$

③ **Supp qty** ⇒ qty which are neither fundamental nor derived

Exa :→ ① **Angle** ⇒ $\theta = \dfrac{\text{Arc}}{\text{Radius}}$



Unit → Radian

② **Solid Angle** ⇒ $\omega = \dfrac{A}{R^2}$

Unit → St. Radian

UNITS

- Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen and internationally accepted reference standard ,called as Units.

$$175m = 175 \times 1m$$

$$5kg = 5 \times 1kg$$

PROPERTIES OF UNITS

1. Accepted to all
2. invariable
3. Accessible

TYPES OF UNITS

- FUNDAMENTAL
- DERIVED
- SUPPLEMENTARY

SYSTEM OF UNITS

1. S.I
2. M.K.S → Meter, Kg, sec
3. C G S → Centimeter, Grame, sec.
4. F.P.S → Foot, Pound, Sec

DIMENSIONS

When a physical qty is expressed in terms of fundamental qty, then the Power of exponent appears in that expression, is known as Dimensions.

Acc. $\rightarrow$ m/sec^2 $\quad \frac{L}{T^2} = LT^{-2}$

Force $\rightarrow$ kg $\frac{m}{sec^2}$ $\quad [MLT^{-2}]$

QUANTITIES	DIMENSIONAL EQN.
Distance , displacement , length ,depth/ thickness , wavelength	$[M^0L^1T^0]$
Mass , inertia , inertial mass , gravitational mass	$[M^1L^0T^0]$
Speed , velocity , velocity of sound, velocity of light	$[M^1L^0T^{-1}]$
Acceleration(a) , acceleration due to gravity(g)	$[M^1L^0T^{-2}]$
angular velocity , velocity gradient, decay constant (λ) , linear frequency, activeness	$[M^0L^0T^{-1}]$
Wavenumber , propagation constant(K), Rydberg constant	$[M^0L^{-1}T^0]$
Gravitational constant	$[M^{-1}L^3T^{-2}]$
Force, weight, tension, centripetal force	$[M^1L^1T^{-2}]$
Work, energy, torque, moment of couple, heat	$[M^1L^2T^{-2}]$
Linear momentum, impulse	$[M^1L^1T^{-1}]$
Surface tension	$[M^1L^0T^{-2}]$
Pressure, coefficient of elasticity, young modulus, bulk modulus, stress	$[M^1L^{-1}T^{-2}]$
Planck constant, angular momentum	$[M^1L^2T^{-1}]$
Viscosity coefficient	$[M^1L^{-1}T^{-1}]$

Dimensions in electricity-:

QUANTITIES	DIMENSIONAL EQN.
Charge	$[T^1 A^1]$
Current	$[A^1]$
Potential gradient , electric field , intensity of electric field	$[M^1 L^1 T^{-3} A^{-1}]$
Electrical capacitance	$[M^{-1} L^{-2} T^4 A^2]$
Potential , potential difference , potential energy , electromotive force	$[M^1 L^2 T^{-3} A^{-1}]$
Electric permittivity of free space	$[M^{-1} L^{-3} T^4 A^2]$
Resistance , reactance , impedance	$[M^1 L^2 T^{-3} A^{-2}]$
Electrical conductance , admittance , susptance	$[M^{-1} L^{-2} T^3 A^2]$
Electric flux	$[M^1 L^3 T^{-3} A^{-1}]$
Specific resistance	$[M^1 L^3 T^{-3} A^{-2}]$

Dimensions of magnetic quantity-:

QUANTITIES	DIMENSIONAL EQN.
Magnetic field , magnetic induction	$[M^1L^0T^{-2}A^{-1}]$
Permeability of magnet(μ)	$[M^1L^1T^{-2}A^{-2}]$
Momentum of magnet(M) , bohr magneton (μ_s)	$[M^0L^2T^0A^1]$
Self inductance , mutual inducatance	$[M^1L^2T^{-2}A^{-2}]$

Dimensionless quantities:

Quantities	Quantities
Efficiency(η)	Coefficient of amplification factor
Form factor	Power coefficient
Relative dielectric permittivity	Refractive index
Poison ratio	Mec. Coefficient of heat (J)
Angular displacement	Q-factor
Strain	Angle /Solid angle

QUANTITIES	DIMENSIONAL EQN.
Temperature	$[M^0L^0T^0\theta^0]$
Latent heat	$[M^0L^2T^{-2}\theta^0]$
Specific heat	$[M^0L^2T^{-2}\theta^{-1}]$
Coefficient of thermal expansion	$[M^0L^0T^0\theta^{-1}]$
Coefficient of thermal conductivity	$[M^1L^1T^{-3}\theta^{-1}]$
Mechanical equivalent (j)	$[M^0L^0T^0]$
Stephan constant	$[M^1L^0T^{-3}\theta^{-4}]$
Wein's constant	$[M^0L^1T^0\theta^1]$
Boltzmann constant , gas constant , solar constant , intensity of radiation	$[M^1L^2T^{-2}\theta^{-1}]$
Energy flux , pointing vector	$[M^1L^{-1}T^{-2}]$

DIMENSIONAL ANALYSIS

→ checking the dimensions of a physical qty or group of physical qty.

APPLICATION OF DIMENSIONAL ANALYSIS

1. To find the dimensions of an unknown physical quantity.

FIND THE DIMENSIONS OF THE FOLLOWING QUESTIONS

1. **Charge(Q)** $\Rightarrow$ $I = \frac{Q}{t} \Rightarrow Q = It \Rightarrow [Q] = [A^1 T^1]$

2. **Potential (V)** $\rightarrow$ $V = \frac{PE}{q} = \frac{N \times m}{q} = \frac{(\text{Mass} \times \text{Acc.})\ \text{Length}}{[A^1 T^1]}$

$$= \frac{[M^1 L^1 T^{-2}][L^1]}{[A^1 T^1]} = [M^1 L^2 T^{-3} A^{-1}]$$

3. **Resistance(R)** $\Rightarrow$ $V = IR$

$$R = \frac{V}{I} \Rightarrow [R] = \frac{[M^1 L^2 T^{-3} A^{-1}]}{[A^1]} = [M^1 L^2 T^{-3} A^{-2}]$$

2 FOR CONVERSION OF UNITS

$$n_1 U_1 = n_2 U_2$$

$$n_1 \left[M_1^x L_1^y T_1^z \right] = n_2 \left[M_2^x L_2^y T_2^z \right]$$

Q. CONVERT NEWTON INTO DYNE ?

SI CGS

801

$$1N = 1\,kg\,\frac{m}{sec^2}$$

$$= \frac{1\,(10^3 gm)\,(10^2 cm)}{(sec^2)}$$

$$= 10^5 \left(gm\text{-}\frac{cm}{sec^2}\right)$$

$$\boxed{1N = 10^5\ Dyne}$$

SI CGS

Q. CONVERT 1 JOULE TO ERGS ?

Sol^n

$$1\,J = 1\,(N \times m)$$

$$= 1\left(kg\text{-}\frac{m}{sec^2}\right) m$$

$$= 1\,kg\text{-}\frac{m^2}{sec^2}$$

$$= 1\,(10^3 gm)\,\frac{(10^2 cm)^2}{sec^2}$$

$$= 10^3 \times 10^4 \times \frac{gm\text{-}cm^2}{sec^2} = 10^7\,\frac{gm\text{-}cm^2}{sec^2}$$

$$\boxed{1\,J = 10^7\,ergs}$$

The value of g is 9.8 m/ sec^2 in M.K.S. system. Find its value in F.P.S. system.

{1 ft = 0.3048 m}

$$g = 9.8 \frac{m}{sec^2}$$

$$= \frac{9.8 \left(\frac{1}{0.3048} ft\right)}{sec^2}$$

$$= \frac{9.8}{0.3048} \frac{ft}{sec^2}$$

$$= 32 \frac{ft}{sec^2}$$

ft

$1 ft = 0.3048 m$

$1 m = \frac{1}{0.3048} \cancel{m} ft$

Q. In a system, fundamental units for mass, length & time is 1 ton, 1 km & 1 min respectively. Calculate the value of G, (Gravitational Constant) in this system.

Sol^n

$$G = 6.67 \times 10^{-11} \frac{N\text{-}m^2}{kg^2}$$

$$= 6.67 \times 10^{-11} \frac{\cancel{kg}\text{-}m}{sec^2} \times \frac{m^2}{kg^{\cancel{2}}}$$

$$= 6.67 \times 10^{-11} \frac{m^3}{kg\text{-}sec^2}$$

$$= 6.67 \times 10^{-11} \times \frac{(y \times 1\,km)^3}{(x \times 1\,ton)(z \times 1\,min)^2}$$

$$= 6.67 \times 10^{-11} \times \frac{y^3}{x \times z^2} \frac{km^3}{ton\text{-}(min)^2}$$

$1\,kg = x \times 1\,ton$

$1\,\cancel{kg} = x \times 10^3\,\cancel{kg} \Rightarrow x = 10^{-3}$

$1\,m = y \times 1\,km$

$1\,\cancel{m} = y \times 10^3\,\cancel{m} \Rightarrow y = 10^{-3}$

$1\,sec = z \times 1\,min$

$1\,\cancel{sec} = z \times 60\,\cancel{sec}$

$z = 1/60$

$$G = 6.67 \times 10^{-11} \times \frac{10^{-9}}{10^{-3} \times \left(\frac{1}{60}\right)^2}$$

$$G = 6.67 \times 3600 \times 10^{-17}$$

$$= 36 \times 6.67 \times 10^{-15} \frac{km^3}{ton\text{-}min^2}$$

A calorie is a unit of heat (energy in transit) and it equals about $4.2\ J$ where $1J = 1\ kg\ m^2 s^{-2}$. Suppose we employ a system of units in which the unit of mass equals $\alpha\ kg$, the unit of length equals $\beta\ m$, the unit of time is $\gamma\ s$. Show that a calorie has a magnitude of $4.2\ \alpha^{-1}\beta^{-2}\gamma^{2}$ in terms of the new units.

$$1\ Cal = 4.2\ J$$

$$= 4.2\ \frac{kg \cdot m^2}{sec^2}$$

$$= 4.2\ \frac{(1\ kg)(1\ m)^2}{(1\ sec)^2}$$

$$= 4.2\ \frac{\left(\frac{\alpha}{\alpha} \times 1\ kg\right)\left(\frac{\beta}{\beta} \times 1\ m\right)^2}{\left(\frac{\gamma}{\gamma}\ sec\right)^2}$$

$$= 4.2\ \frac{\left(\frac{\alpha\ kg}{\alpha}\right)\left(\frac{\beta m}{\beta}\right)^2}{\left(\frac{\gamma\ sec}{\gamma}\right)^2}$$

$$= 4.2\ \alpha^{-1}\beta^{-2}\gamma^{2}\left(\frac{(\alpha\ kg)(\beta m)^2}{(\gamma\ sec)^2}\right)$$

Checking the dimensional consistency of equations

PRINCIPLE OF HOMOGENEITY

Principle Of Homogeneity

The principle of homogeneity states that the dimensions of each the term of a dimensional equation on both sides are the same. Using this principle this given equation will have the same dimensions on both sides.

$$\underline{5m} + \underline{2m} = 7\underline{m}$$

$$5m + 2kg = \times$$

Power given to a particle is

$$P = F \cdot A + \frac{B}{t + C}$$

If F is force and t is time then which quantities are represented by A,B and C

$t + c$

$[C] = [T^1]$

time

$[P] = [F \ A]$

$\frac{\text{work}}{\text{time}} = \text{force}(A)$

$\frac{\cancel{\text{Force}}(\text{disp})}{\text{time}} = \cancel{\text{force}}(A)$

Velocity or speed

$A = \frac{\text{DISP}}{\text{time}} = L^1 T^{-1}$

$P = \frac{B}{t + C} = \frac{B}{\text{समय}}$

$\frac{\text{work}}{\cancel{\text{समय}}} = \frac{B}{\cancel{\text{समय}}}$

$B = \text{work}$

$[B] = [M^1 L^2 T^{-2}]$

NOTE-:

All trigonometry functions $\{\sin\theta, \cos\theta\}$,
logarithmic functions $\{\log_e x, \log_{10} x^2\}$ and
exponential functions $\{e^x, 2^x\}$ etc. are dimensionless and x is also dimensionless.

If equation $y = A\sin(BX) + D\cos(ET)$ is correct, then calculate dimensions of A,B,D and E where X and Y are distance and T is time

Use expansion of $sinx$ to explain this

$$Y = A\sin(BX) + D\cos(ET)$$

$[B][X] = M^0L^0T^0$

$[B][L^1] = M^0L^0T^0$

$[B] = \frac{M^0L^0T^0}{L^1}$

$= L^{-1}$

$[E][T] = M^0L^0T^0$

$[E] = [T^{-1}]$

$[Y] = [A] + [D]$

$[A] = [D] = [L^1]$

Deducing Relation among the Physical Quantities

The method of dimensions can sometimes be used to deduce relation among the physical quantities. For this we should know the dependence of the physical quantity on other quantities (up to three physical quantities or linearly independent variables) and consider it as a product type of the dependence.

Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.

Sol^n

$T \propto l^x m^y g^z$

$T = K l^x m^y g^z$

K → Dimensionless Const

$[T] = [l^x m^y g^z]$

$[M^0 L^0 T^1] = [L^x M^y [LT^{-2}]^z]$

$[M^0 L^0 T^1] = [L^x M^y L^z T^{-2z}]$

$M^0 L^0 T^1 = M^y L^{x+z} T^{-2z}$

Comp ⇒ $y = 0$

$x + z = 0 \Rightarrow x = -z = \frac{1}{2}$

$-2z = 1 \Rightarrow z = -\frac{1}{2}$

$T = K l^{1/2} m^0 g^{-1/2}$

$\boxed{T = K\sqrt{\frac{l}{g}}}$

If velocity (V), force (F) and time (T) are chosen as fundamental quantities. Express (i) Mass (ii) Energy
In terms of V , F and T

H. ω

Sol^n

$$Mass \propto V^x F^y T^z$$

$$[M^1 L^0 T^0] = [L^1 T^{-1}]^x [M^1 L^1 T^{-2}]^y [T^1]^z$$

$$[M^1 L^0 T^0] = [M^y L^{x+y} T^{-x-2y+z}]$$

$$y = 1$$

$$x + y = 0 \Rightarrow x = -y = -1$$

$$0 = -x - 2y + z$$

$$= -(-1) - 2(1) + z \Rightarrow z = 1$$

$$m = K V^{-1} F^1 T^1$$

$$m = K \frac{FT}{V}$$

Limitations of dimensional analysis

$[L^1 T^{-1}]$

1. If the dimensions are given then the physical quantity may not be unique.
2. We can't find proportionality We can't find proportionality constant by dimensional analysis.
3. We can derive those formulas which are in other than the product form also which contains exponential log functions etc. $V = u \pm at$
4. If a physical quantity depends on three other physical quantity out of which two have same dimensions. Then we cannot derive those formulae's.
5. The formula for a physical quantity depending on more than three other physical quantities cannot be derived it can be checked only

THANK YOU

PADHO AUR KHUB AAGE BADHO