ARJUNA NEET BATCH

MOTION IN A PLANE

LECTURE - 03

Today Goal

PROJECTILE MOTION GROUND TO GROUND

QUESTION ON PROJECTILE MOTION

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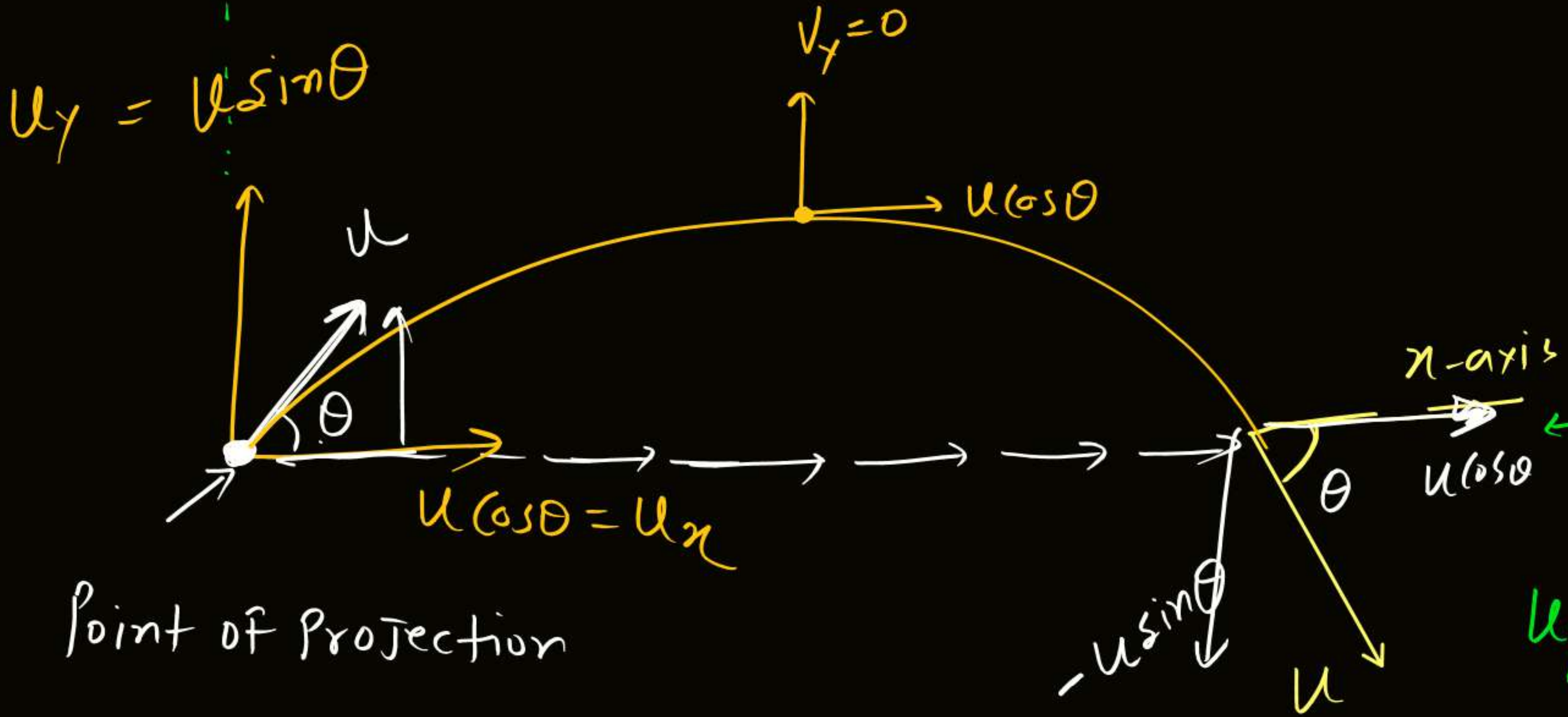
$\boxed{\text{air friction} = 0}$



angle of Projection $= \frac{u_y}{u_x}$

$$\boxed{\tan\theta = \frac{u_y}{u_x}}$$

x-axis

$u_x = u\cos\theta$

$a_x = 0$

$V_x = u\cos\theta\,\hat{i}$

$x = (u\cos\theta)\,t$

y-axis

$u_y = u\sin\theta\,\hat{j}$ $\quad a_x = -g\,\hat{j}$

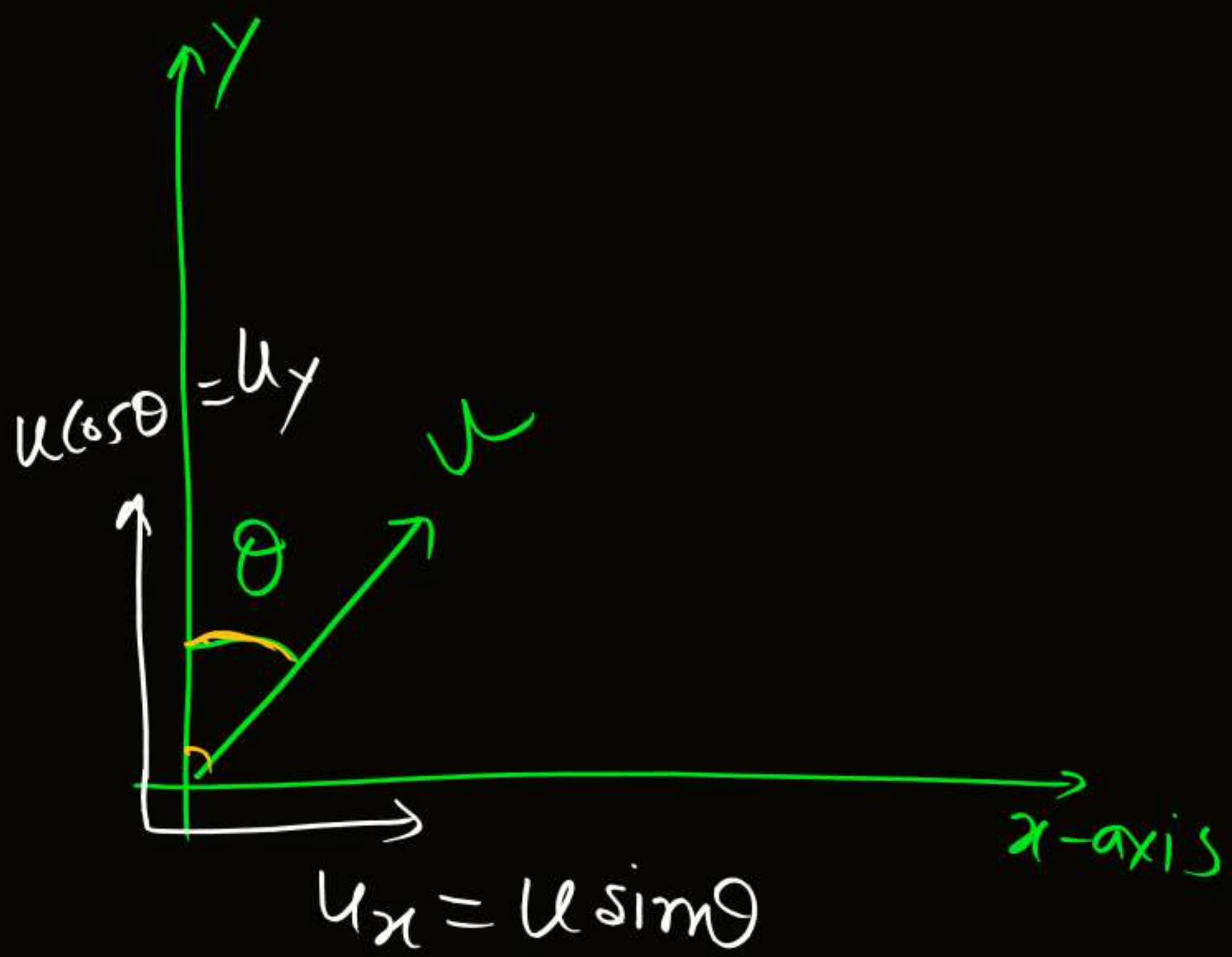
$V_t = (u\sin\theta - gt)\,\hat{j}$

min^m speed is at = max^m height

Time of flight $= \frac{2u_y}{g} = \boxed{\frac{2u\sin\theta}{g}}$

Max^m Height $= \frac{u_y^2}{2g} = \boxed{\frac{(u\sin\theta)^2}{2g}}$

Range $= u_x T_f = \frac{2u_x u_y}{g} = \boxed{\frac{u^2 \sin 2\theta}{g}}$



$$T_f = \frac{2u_y}{g}^{*} = \frac{2u\cos\theta}{g}$$

$$H_{max} = \frac{u_y^2}{2g}^{*} = \frac{u^2\cos^2\theta}{2g}$$

$$R = \underline{u_x} T_f^{*} = \boxed{\frac{u^2\sin 2\theta}{g}}$$

$$\boxed{R = \frac{2u_x u_y}{g}}$$

Ball is Projected with velocity $\vec{u} = \alpha\hat{i} + \beta\hat{j}$ then find H, T, R

IIT-2014

Sol^n

$$T_f = \frac{2u_y}{g} = \frac{2\beta}{g} \qquad R = \frac{2u_x u_y}{g} = \frac{2\alpha\beta}{g}$$

$$H = \frac{u_y^2}{2g} = \frac{\beta^2}{2g}$$

Q. Position of Projected object at any time t is $\vec{r} = \underbrace{3t}\,\hat{i} + \underbrace{(4t - 5t^2)}\,\hat{j}$ then find

(i) Angle of Projection (ii) T_f (iii) H_{max} (iv) Range

Sol^n

$x = 3t$

$\frac{dx}{dt} = u_x = 3$

$\boxed{u_x = 3}$

$y = 4t - 5t^2$

$\frac{dy}{dt} = 4 - 10t$

$u_y = 4 - 10t$

$(u_y)_{t=0} = 4$

$\tan\theta = \frac{u_y}{u_x} = \frac{4}{3} = 53^\circ$

$T_f = \frac{2u_y}{g} = \frac{2 \times 4}{g} = 0.8$ sec

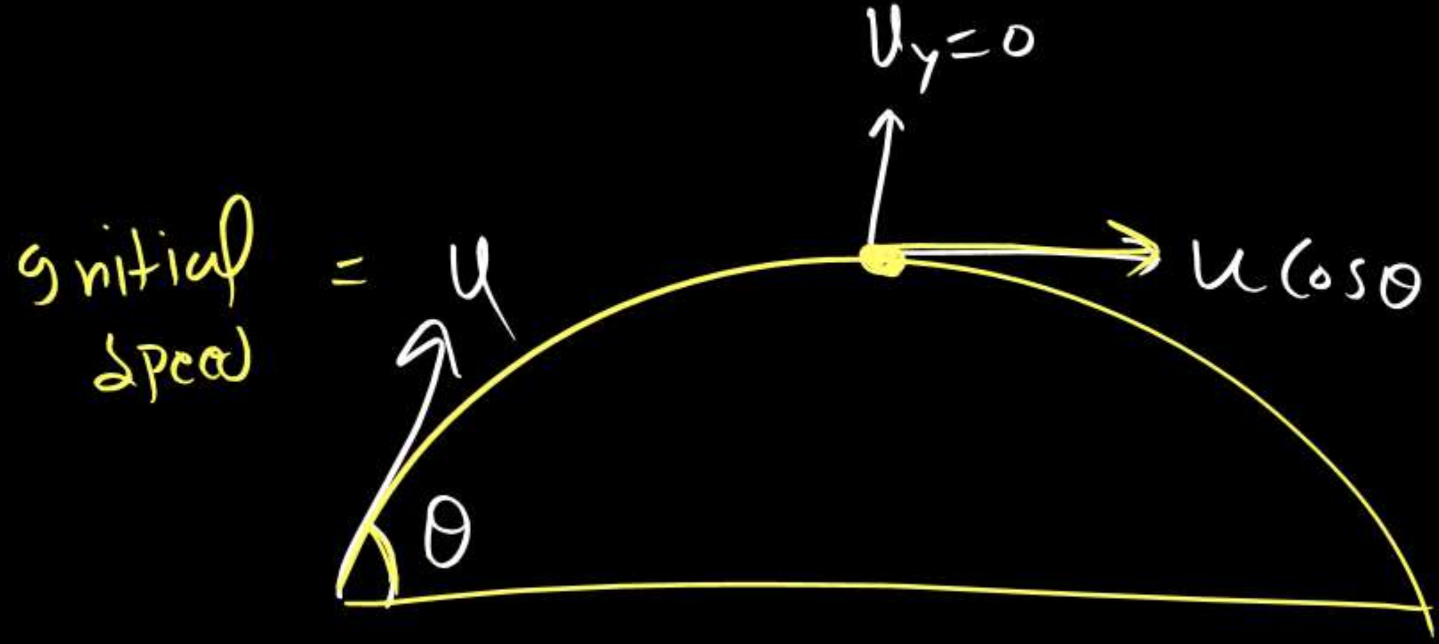
$H_m = \frac{(4)^2}{2g} = \frac{16}{20} = 0.8$ m

$R = \frac{2 \times 4 \times 3}{10} = 2.4$ m

The speed of a projectile at its maximum height is half of its initial speed. The angle of projection is:- [AIPMT 2010]

(a) 15° (b) 30°

(c) 45° (d) 60°



$\cancel{u}\cos\theta = \frac{\cancel{u}}{2}$

$\cos\theta = \frac{1}{2}$

$\boxed{\theta = 60^\circ}$

Two bodies are thrown up at angles of 45° and 60° respectively, with the horizontal. If both bodies attain same vertical height, then the ratio of velocities with which these are thrown is :

(A) $\sqrt{\frac{2}{3}}$

(B) $\frac{2}{\sqrt{3}}$

(C) $\sqrt{\frac{3}{2}}$

(D) $\frac{\sqrt{3}}{2}$

$$\Rightarrow \left(H_{max}\right)_1 = \left(H_{max}\right)_2$$

$$\frac{u_1^2 \sin^2 45^\circ}{\cancel{2g}} = \frac{u_2^2 \sin^2 60^\circ}{\cancel{2g}}$$

$$\left(\frac{u_1}{u_2}\right)^2 = \left(\frac{\sin 60}{\sin 45^\circ}\right)^2 = \frac{\sqrt{3}}{\sqrt{2}\,\cancel{2}} \times \frac{1}{\cancel{\sqrt{2}}}$$

$$\frac{u_1}{u_2} = \sqrt{\frac{3}{2}}$$

The range of a projectile when fired at 75° with the horizontal is 0.5 km. what will be its range when fired at 45° with same speed :-

(a) 0.5 km
(b) 1.0 km
(c) 1.5 km
(d) 2.0 km

$$\frac{R_1 = \frac{\cancel{u^2} \sin(2\times 75^\circ)}{2\cancel{g}}}{R_2 = \frac{\cancel{u^2} \sin(2\times 45^\circ)}{2\cancel{g}}} = \frac{\sin(150^\circ)}{\sin 90^\circ} = \frac{1}{2}$$

$$\boxed{R_2 = 2R_1}$$

The speed of the maximum height of a projectile is $\sqrt{3}/2$ times of its initial speed 'u' of projection. Its range on the horizontal plane :-

(a) $\dfrac{\sqrt{3}u^2}{2g}$

(b) $\dfrac{u^2}{2g}$

(c) $\dfrac{3u^2}{2g}$

(d) $\dfrac{3u^2}{g}$

AIPMT

$$\boxed{R = \frac{u^2 \sin(2\theta)}{g} = \frac{2u_x u_y}{g} = u_x T_f}$$

Sol^n

$$\cancel{u}\cos\theta = \frac{\sqrt{3}}{2}\cancel{u}$$

$$\boxed{\theta = 30^\circ}$$

$$R = \frac{u^2}{g}\sin 60^\circ = \frac{\sqrt{3}u^2}{2g}$$

RELATION BETWEEN HORIZONTAL RANGE AND MAXIMUM HEIGHT

$$R = \frac{2u_x u_y}{\cancel{g}}$$

$$H = \frac{u_y^2}{2\cancel{g}}$$

$$\frac{R}{H} = \frac{2u_x \times 2}{u_y}$$

$$\frac{R}{H} = \frac{4u_x}{u_y} = \frac{4\cancel{u}\cos\theta}{\cancel{u}\sin\theta}$$

$$\frac{R}{H} = \frac{4}{\tan\theta}$$

$$* \boxed{H = \frac{R\tan\theta}{4}}$$

Ratta

The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is : [AIPMT Pre. 2012]

(a) $\theta = \tan^{-1}(2)$ (b) $\theta = 45°$

(c) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (d) $\theta = \tan^{-1}(4)$

$H = \frac{R\tan\theta}{4}$

$\tan\theta = 4$

$\boxed{\theta = \tan^{-1}(4)}$

$H = R$

$\frac{u^2 \sin^2\theta}{2g} = \frac{u^2\, 2\sin\theta\cos\theta}{g}$

$\boxed{\tan\theta = 4}$

The horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Its angle of projectile will be:

(a) 45° (b) 60°

(c) 90° (d) 30°

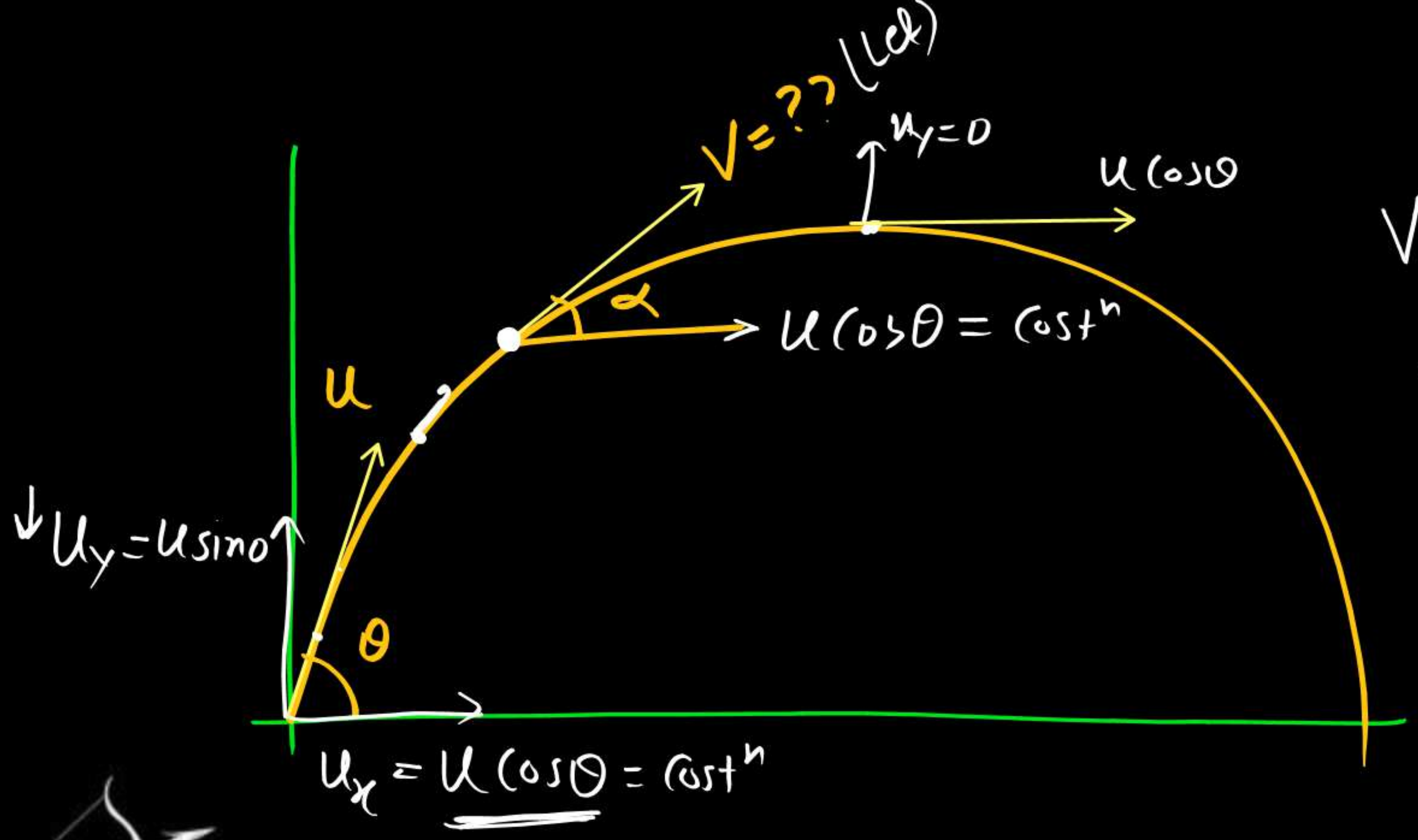
$$R = 4\sqrt{3}H \rightarrow x$$

$$H = \frac{R\tan\theta}{4}$$

$$\cancel{H} = \frac{\cancel{4}\sqrt{3}\cancel{H}\tan\theta}{\cancel{4}}$$

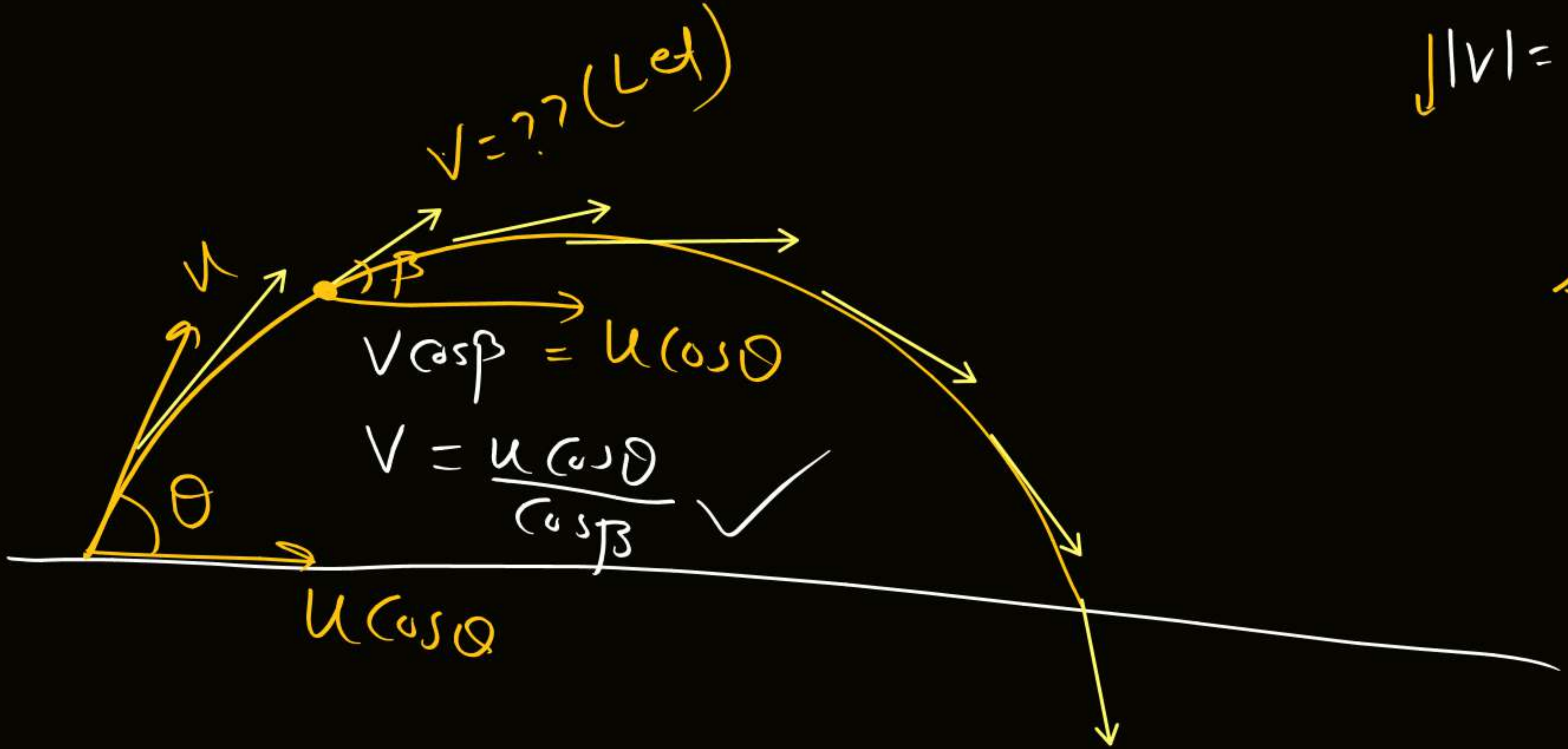
$$\tan\theta = \frac{1}{\sqrt{3}}$$

Speed of object when object is moving at an angle α from horizontal direction. if it was projected with speed u at an angle θ



$$V\cos\alpha = u\cos\theta$$

$$V = \frac{u\cos\theta}{\cos\alpha} = u\cos\theta.\sec\alpha$$



$$V\cos\beta = u\cos\theta$$

$$V = \frac{u\cos\theta}{\cos\beta}$$ ✓

$$\vec{V} = \underset{u\cos\theta}{v_x}\,\hat{i} + v_y\,\hat{j}$$

$$|V| = \sqrt{v_x^2 + v_y^2}$$

CONDITION OF MAXIMUM HORIZONTAL RNAGE

$$R_{max} = \frac{u^2 |\sin 2\theta|_{max}}{g}$$

$u = \text{const}^n$
$g = \text{const}^n$

$$|\sin 2\theta|_{max} = 1$$

$2\theta = 90^\circ$

$$\boxed{\theta = 45^\circ}$$

R will be max^m when $\frac{dR}{d\theta} = 0$

$$\boxed{(R_{max})_{\text{at } \theta = 45^\circ} = \frac{u^2}{g}}$$

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$$R = \frac{u^2 \sin(2\theta)}{g}$$

If $u = const^n$ (fixed) same

$$R_{max} = \frac{u^2}{2g} \text{ at } \boxed{\theta = 45^\circ}$$

A missile is fired for maximum range with an initial velocity of 20 m/s. If $g = 10\ m/s^2$, the range of the missile is :- [AIPMT 2011]

(a) 40 m (b) 50 m

(c) 60 m (d) 20 m

$\theta = 45^\circ$

$$R = \frac{u^2 \sin(2\theta)}{g} = \frac{(20)^2}{10} = \frac{40\cancel{0}}{1\cancel{0}} = 40\ m$$

If R is the maximum horizontal range of a particle, then the greatest height attained by it is :

(A) R (B) $2R$

(C) $R/2$ (D) $R/4$

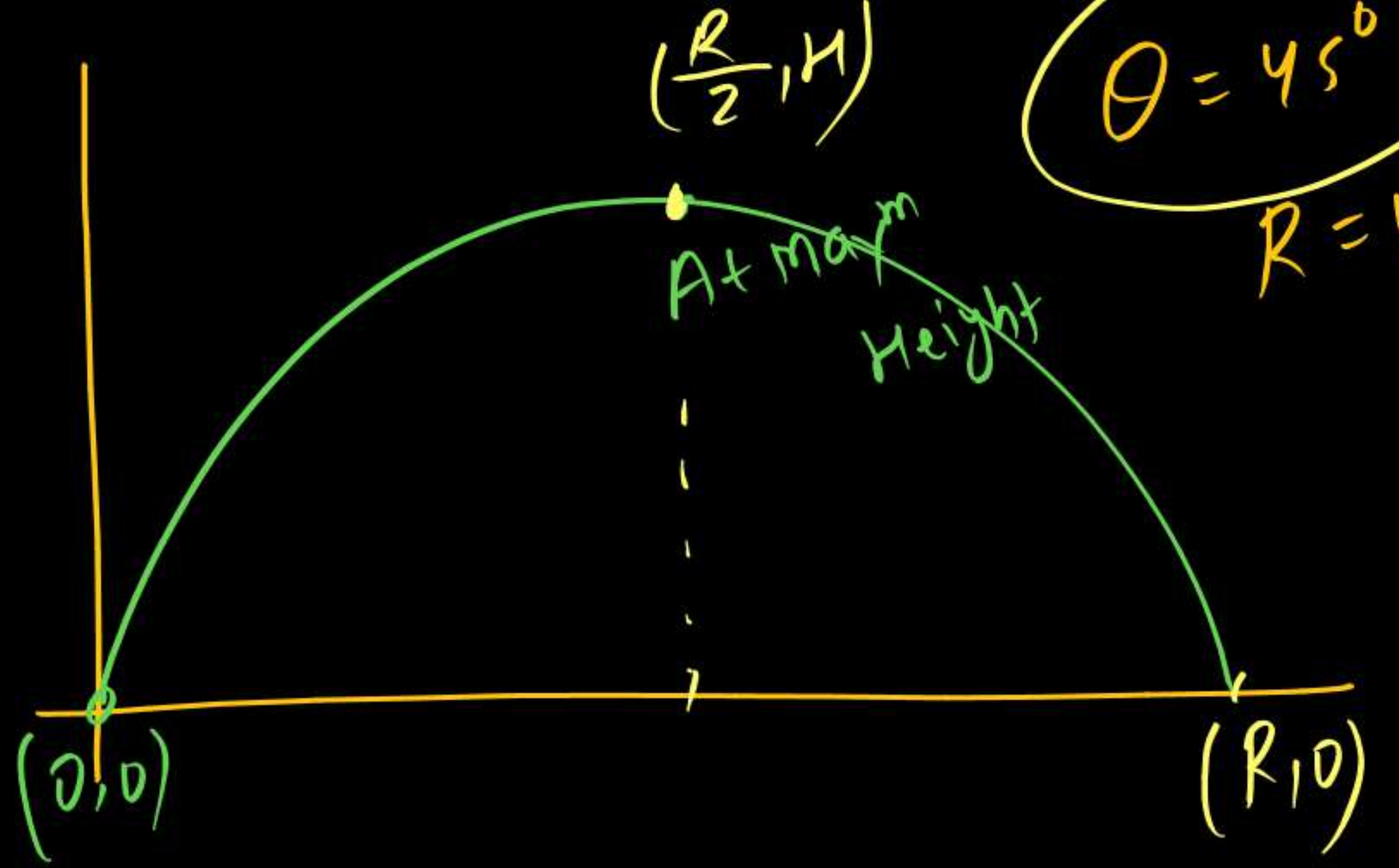
$\underline{\text{imp}^{*}}$ $\boxed{\theta = 45^\circ}$

$$H = \frac{R\tan\theta}{4} = \frac{R}{4}$$

A projectile is thrown into space so as to have the maximum possible horizontal range equal to 400m. Taking the point of projection as the origin, the coordinates of the point where the velocity of the projectile is minimum are :-

(a) (400, 100)   (b) (200, 100)

(c) (400, 200)   (d) (200, 200)



$\theta = 45^\circ$ ✓

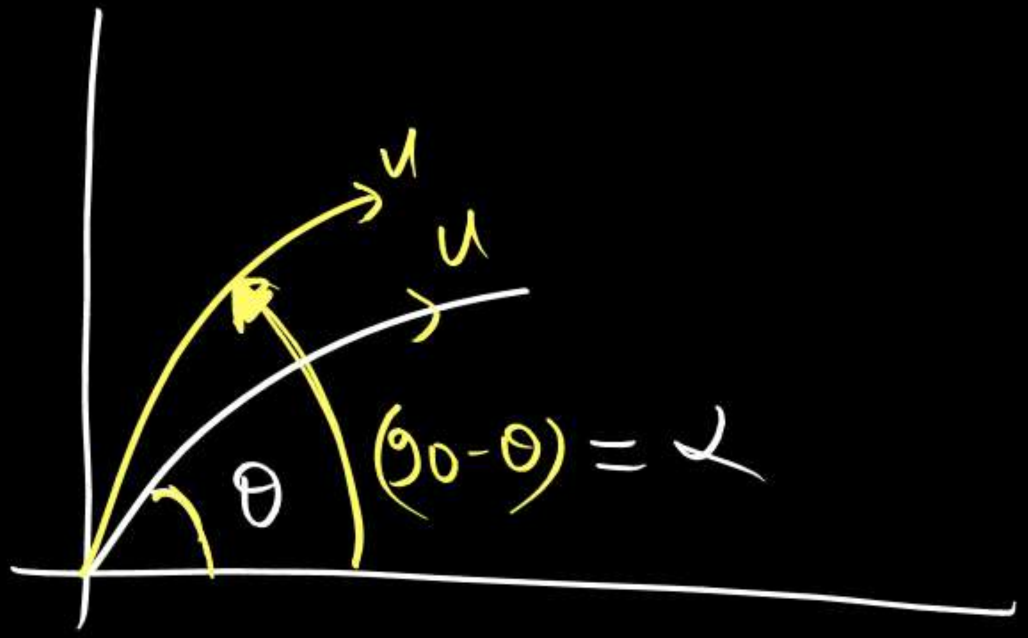
$R = 400\,m$

$\Rightarrow \left(\frac{R}{2}, H\right) = \left(200, \frac{R}{4}\right)$

$= (200, 100)$

Ball is projected with same speed at two different angle θ and (90 – θ) then prove that range will be same on these two different angle.

$(\theta < 90)$



$$R_1 = \frac{u^2 \sin(2\theta)}{g} \quad -(1)$$

$$R_2 = \frac{u^2 \sin 2(90-\theta)}{g} = \frac{u^2 \sin(180-2\theta)}{g}$$

$$R_2 = \frac{u^2 \sin(2\theta)}{g}$$

$\sin(180-\alpha)$

$= \sin\alpha$

$\boxed{R_1 = R_2}$ Hence Prove

$R_1 = R_2$

$\alpha = \theta$	$\beta = 90 - \theta$
30°	60
20°	70
10°	80°
75°	15°

??

Q) Ball is Projected with spe... 41° & 49° then find Relation of Range at these two angle

Solⁿ

$R_{41^\circ} > R_{49^\circ}$ ✗

$R_{41^\circ} < R_{49^\circ}$ ✗

$R_{41^\circ} = R_{49^\circ}$ ✓

Two projectiles of same mass and with same velocity are thrown at an angle 60° & 30° with the horizontal, then which quantity will remain same: [AIPMT 2000]

(a) Time of flight

(b) Horizontal range of projectile

(c) Max height acquired

(d) All of them

For angles of projection of a projectile at angles $(45° - \theta)$ and $(45° + \theta)$, the horizontal ranges described by the projectile are in the ratio of :-

speed of projection is same

[A1PMT 2006]

(a) 1 : 1

(b) 2 : 3

(c) 1 : 2

(d) 2 : 1

$$\alpha = 45° - \theta$$

$$\beta = 45° + \theta$$

If $\alpha + \beta = 90°$ then $R_1 = R_2$

A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection, is :

[AIPMT 2011]

(a) 60°

(b) $\tan^{-1}\frac{1}{2}$

(c) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

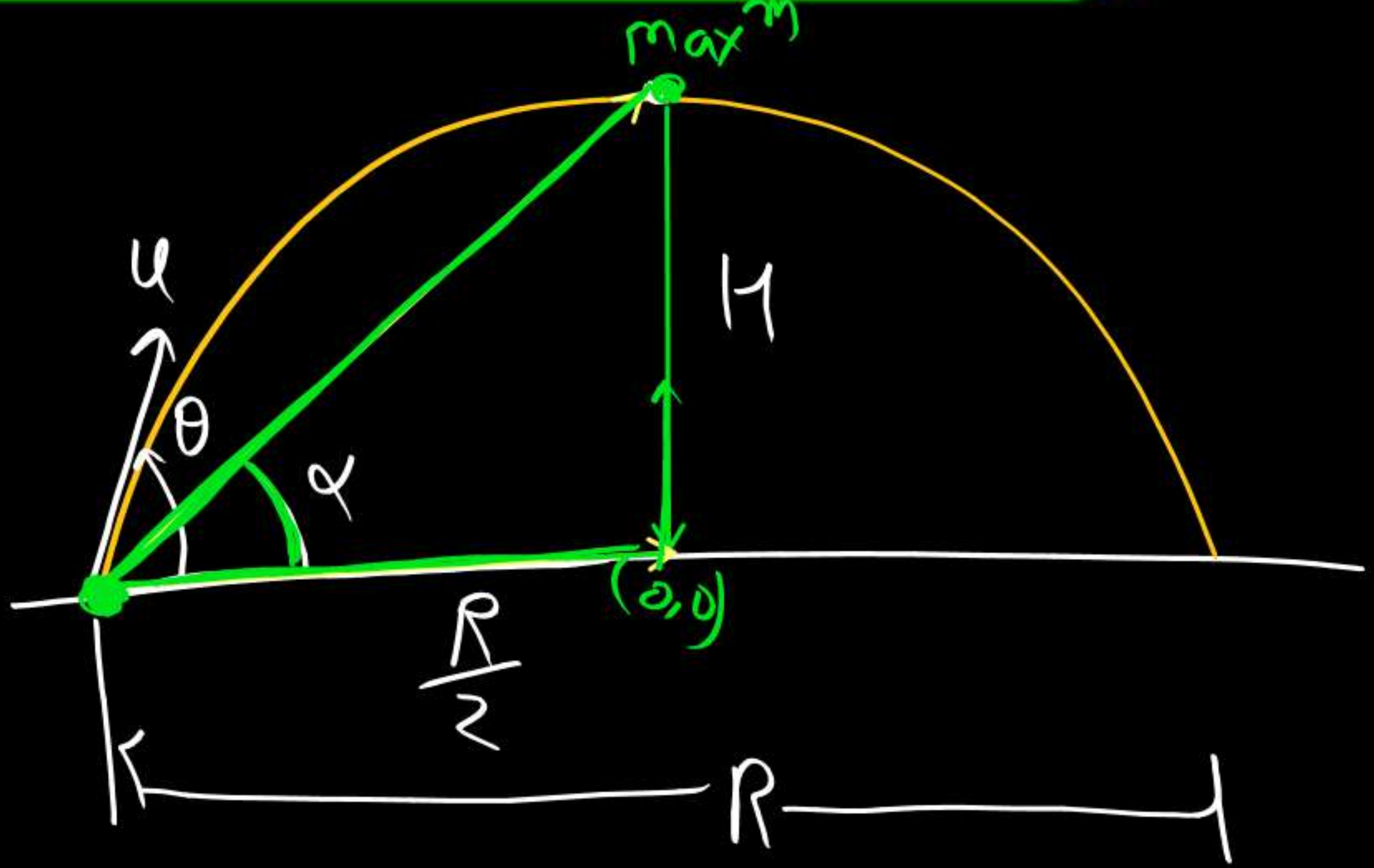
(d) 45°

$$\tan\alpha = \frac{2H}{R}$$

$$= \frac{\cancel{2}\,\cancel{R}\tan\theta}{\cancel{R}\,\cancel{4}\,2}$$

$$\tan\alpha = \frac{\tan 45^\circ}{2}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{1}{2}\right)}$$



A projectile is fired from the surface of the earth with a velocity of $5\ ms^{-1}$ and angle θ with the horizontal. Another projectile fired from another planet with a velocity of $3\ ms^{-1}$ at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is: (given $g = 9.8\ m/s^2$) [AIPMT 2014]

(a) 3.5
(b) 5.9
(c) 16.3
(d) 110.8

$$(H_{max})_{earth} = (H_{max})_P$$

$$\frac{(5)^2 \sin^2\theta}{\cancel{2} g} = \frac{(3)^2 \sin^2\theta}{\cancel{2} g'}$$

$$g' = \frac{9}{25} \times 9.81$$

$$= 3.5$$

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Eq^n of Trajectory of Projected Particle

→ relation b/w x - y^{th} co-ord

$x = u\cos\theta\, t$ — ①

$t = \frac{x}{u\cos\theta}$

$y = (u\sin\theta)t - \frac{1}{2}gt^2$ — ⑪

$$y = \cancel{u}\sin\theta\left[\frac{x}{\cancel{u}\cos\theta}\right] - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

a (const)

$\cos t^n$ (b)

$$\boxed{y = ax - bx^2}$$

↳ eq^n of Parabola

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \times \frac{\tan\theta}{\tan\theta}$$

$$y = x\tan\theta\left[1 - \frac{gx}{2u^2\cos^2\theta\,\tan\theta}\right]$$

$$\tan\theta \rightarrow \frac{\sin\theta}{\cos\theta}$$

$$y = x\tan\theta\left[1 - \frac{gx}{2u^2\sin\theta\cos\theta}\right]$$

$$\boxed{y = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

The equation of projectile is $y = \sqrt{3}\,x - \frac{gx^2}{2}$ the angle of projection is :

(a) 30°

~~(b)~~ 60°

NEET

(c) 45°

(d) none

$$y = \sqrt{3}x - \frac{gx^2}{2}$$

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$y = \boxed{\sqrt{3}x}\left[1 - \frac{gx}{2\sqrt{3}}\right]$$

$$\sqrt{3}\cancel{x} = \cancel{x}\tan\theta$$

$$\boxed{\tan\theta = \sqrt{3}}$$

$$\frac{1}{R} = \frac{g}{2\sqrt{3}}$$

$$R = \frac{2\sqrt{3}}{10} = \frac{\sqrt{3}}{5}\ R$$

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The equation of projectile is $y = 16x - \frac{x^2}{4}$ the horizontal range is :

(a) 16 m

(b) 8 m

(c) 64 m

(d) 12.8 m

$$\Rightarrow y = 16x - \frac{x^2}{4}$$

$$\Rightarrow y = 16x\left[1 - \frac{x}{4\times 16}\right]$$

$$\Rightarrow y = 16x\left[1 - \frac{x}{64}\right]$$

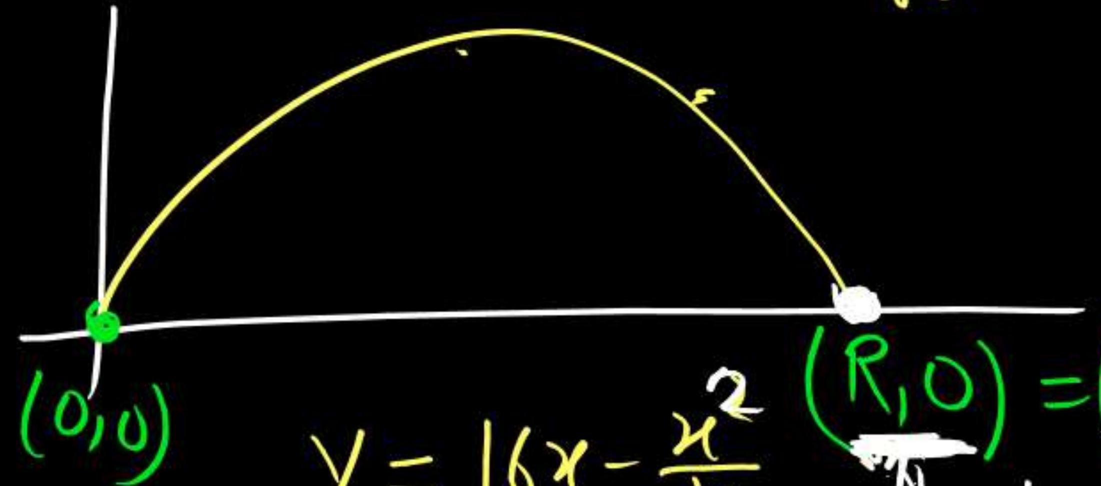
$$\Rightarrow y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

MOKSHA With MR*

eqn of Trajectory

$\Downarrow$

reln b/w x & y



$$y = 16x - \frac{x^2}{4}$$

If $y = 0$ $(x = R)$

$$16R = \frac{R^2}{4}$$

$R = 64$ m

THANK YOU ☺