ARJUNA NEET BATCH

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VECTOR SUBTRACTION AND DOT PRODUCT

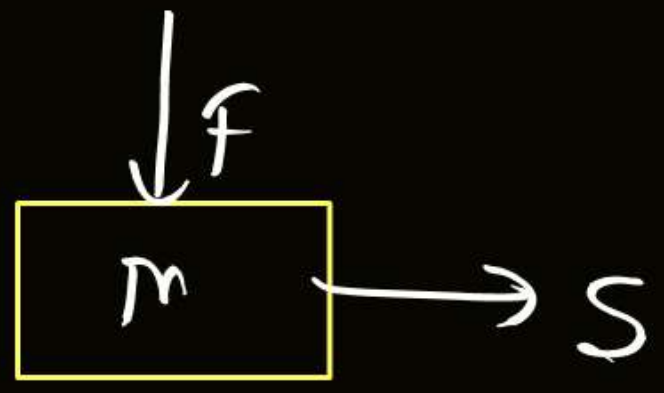
LECTURE

NEET

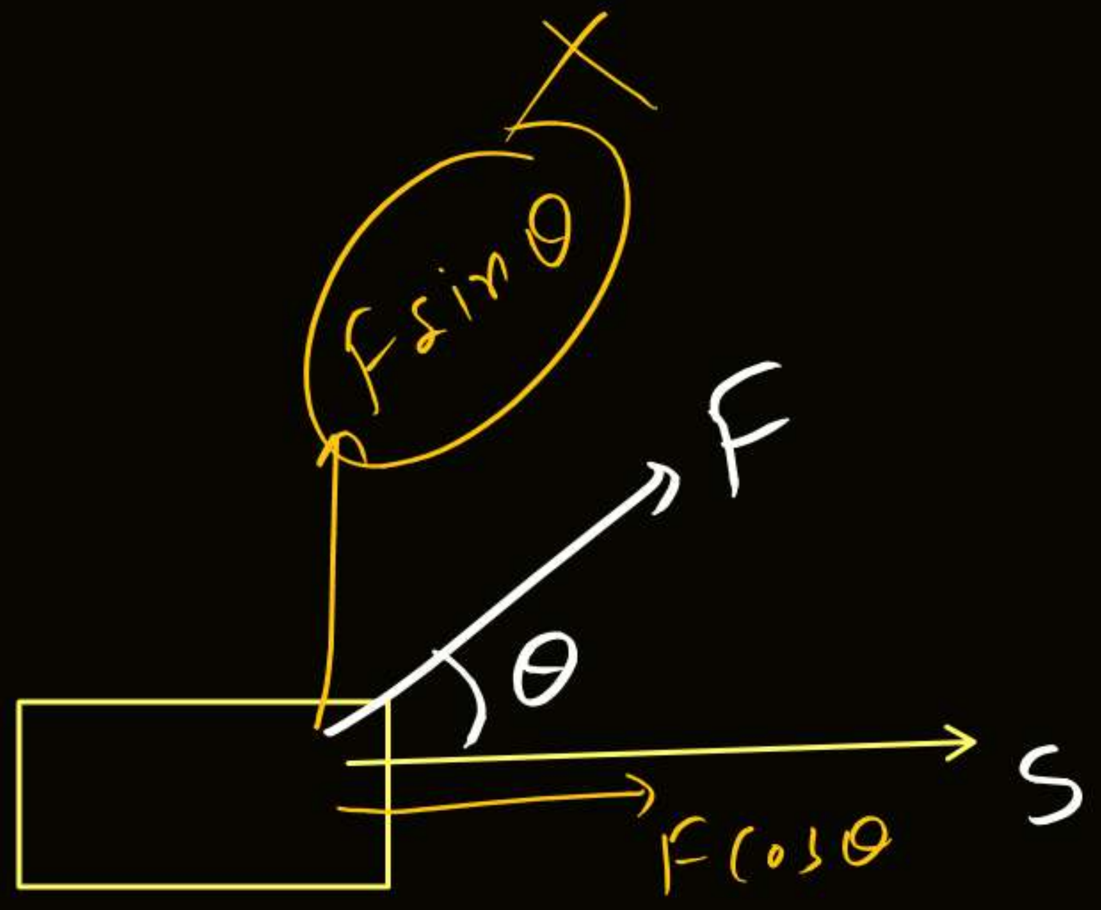
Todays goal

- Dot Product (scalor Product)
- Cross Product (vector Product)

$$W = F \cdot S$$



$$W = 0$$



$$W = \underbrace{(F\cos\theta) \cdot S}$$

$\downarrow$

scalar

SCALAR PRODUCT

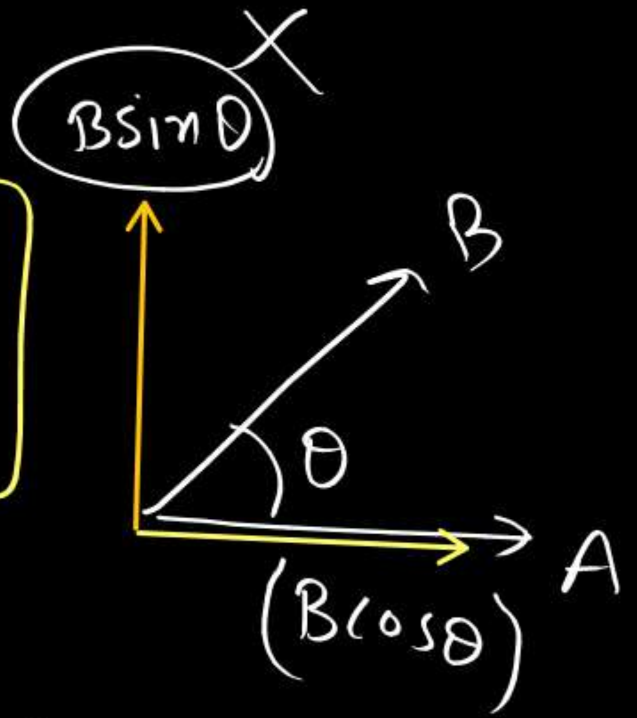
[Dot. Product of Vector]

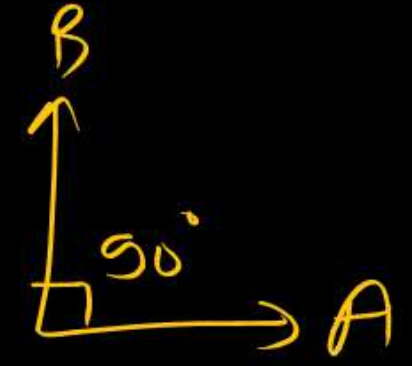
#

$$\vec{A} \bullet \vec{B} = (\text{Component of A along B}) \times |\vec{B}|$$
$$= (\text{Component of B along A}) \times A$$

$$\vec{A} \bullet \vec{B} = AB\cos\theta$$

θ = Angle between $\vec{A}$ and $\vec{B}$

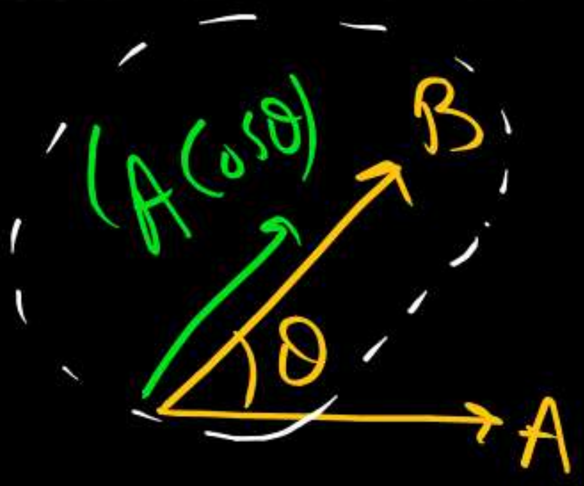




$$\vec{A}\cdot\vec{B} = 0$$

$$\vec{A}\cdot\vec{B} = A\,(B\cos\theta)$$

$$\vec{A}\cdot\vec{B} = A\,(\text{Component of B along A})$$



$$\vec{A}\cdot\vec{B} = B\,(A\cos\theta)$$

$$= B\,(\text{component of A along B})$$

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = C$ (scalar)

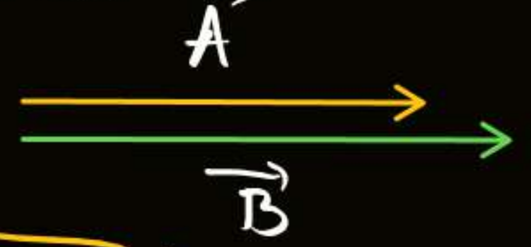
$C = \boxed{\vec{A} \cdot \vec{B}}$

Result of Dot Producto Of two vector is scalar

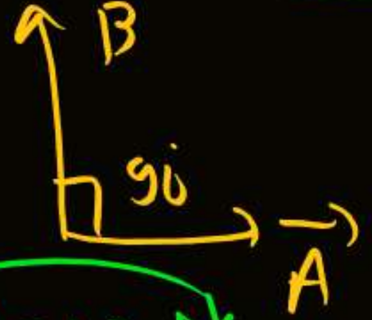
$\vec{A} \cdot \vec{B} = AB \cos\theta$

$\boxed{\text{If } \theta = 0^\circ}$

$(\vec{A} \cdot \vec{B})_{max} = AB \cos 0^\circ = AB$

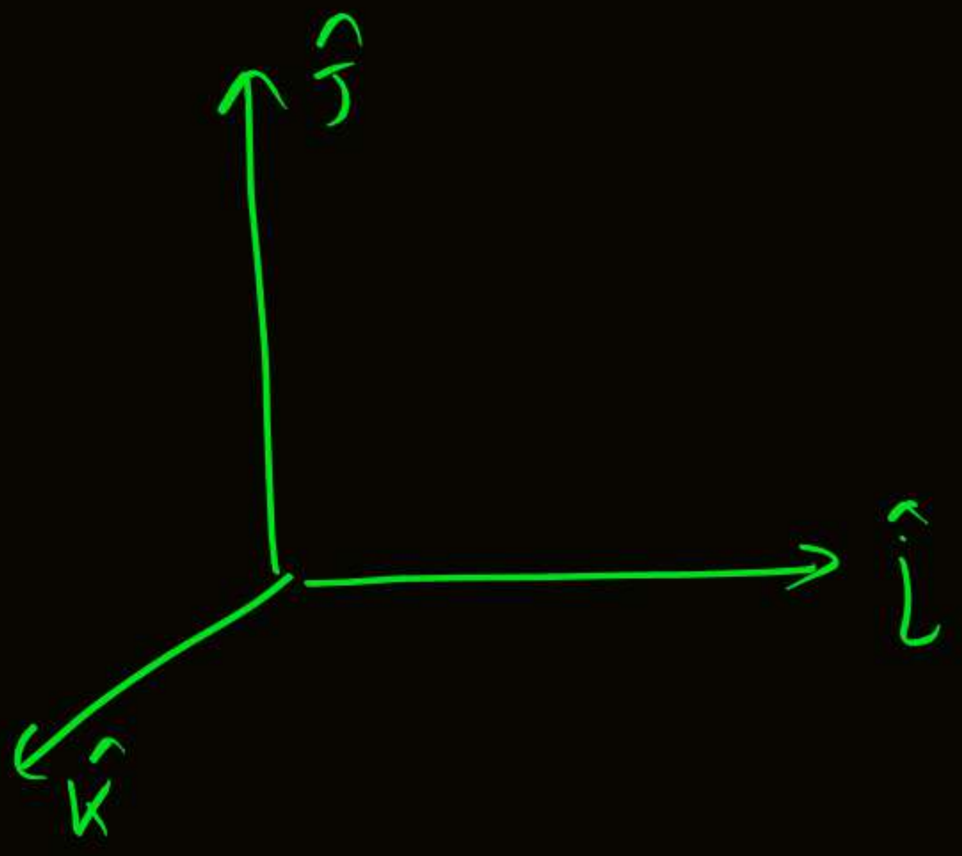


$\boxed{\text{If } \theta = 90^\circ}$



$\boxed{\vec{A} \cdot \vec{B} = 0}$

$\theta = 180^\circ$ $(\vec{A} \cdot \vec{B})_{min} = -AB$



$\hat{i}$, $\hat{j}$ & $\hat{k}$ are the unit vector of x-axis, y axis & z-axis

#

$$\boxed{\begin{aligned} \hat{i}\cdot\hat{i} &= 1 \times 1 \cos 0^\circ = 1 \\ \hat{j}\cdot\hat{j} &= 1 \\ \hat{k}\cdot\hat{k} &= 1 \end{aligned}}$$

imp.

$$\hat{i}\cdot\hat{j} = 0$$

$$i\cdot\hat{k} = 0$$

$$\hat{k}\cdot\hat{i} = 0$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

A_x = Component of $\vec{A}$ along x

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

MR^* $\Rightarrow$

$\vec{A} = 3\hat{i} + 4\hat{j} - 2\hat{k}$

, $\vec{B} = 2\hat{i} - 3\hat{j} + 5\hat{k}$

find $\vec{A} \cdot \vec{B} = ??$

$\underline{\text{Scalar}} = \underbrace{\vec{A} \cdot \vec{B}} = 6 - 12 - 10$

$= -16$ //

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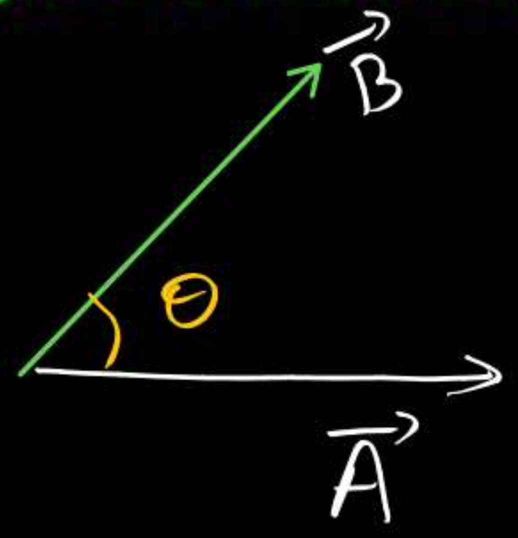
APPLICATION OF DOT PRODUCT

(i) Angle between vectors :



$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

Angle b/w $\vec{A}$ & $\vec{B}$

If $|\vec{A}| = 2$ and $|\vec{B}| = 4$, $\theta = 60°$ then find $\vec{A} \cdot \vec{B}$.

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$= 2 \times 4 \times \cos 60°$$

$$= 4$$

If $\vec{A} = 2\hat{\imath} + 2\hat{\jmath}$ and $\vec{B} = -2\hat{\imath} + 2\hat{\jmath}$ then find angle between $\vec{A}$ and $\vec{B}$.

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$(2\hat{i} + 2\hat{j}) \cdot (-2\hat{i} + 2\hat{j}) = \sqrt{(2)^2 + (2)^2}\sqrt{(-2)^2 + (2)^2}\cos\theta$$

$$4 - 4 = (2\sqrt{2})(2\sqrt{2})\cos\theta$$

$$\cos\theta = 0$$

$$\boxed{\theta = 90^\circ}$$

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Q $\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$ & $\vec{B} = 2\hat{i} + \hat{j}$

then find Angle b/w $\vec{A}$ & $\vec{B}$

MR $\Rightarrow$

$\vec{A} \cdot \vec{B} = AB \cos\theta$

$2 + 2 = \sqrt{6}\,\sqrt{5}\,\cos\theta$

$4 = \sqrt{30}\,\cos\theta$

$\cos\theta = \left(\frac{4}{\sqrt{30}}\right)$

$\theta = \cos^{-1}\left(\frac{4}{\sqrt{30}}\right)$

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(ii) To check unit vector :

$$\vec{A} \cdot \vec{A} = 1 \times 1 \times \cos 0^\circ$$

$$\boxed{\vec{A} \cdot \vec{A} = 1}$$

If $\vec{A} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$ then prove that $\vec{A}$ is a unit vector.

$$\vec{A} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\vec{A}\cdot\vec{A} = (\sin\theta\,\hat{i} + \cos\theta\,\hat{j})\cdot(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1$$

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If $\vec{A} = 0.5\hat{i} + 0.4\hat{j} - \alpha\hat{k}$ then find α if $\vec{A}$ is unit vector.

$$\vec{A} = 0.5\hat{i} + 0.4\hat{j} - \alpha\hat{k}$$

A is a unit

$$\vec{A}\cdot\vec{A} = 1$$

$$0.25 + 0.16 + \alpha^2 = 1$$

$$\alpha^2 = 1 - (.41)$$

$$\alpha^2 = 0.59$$

$$\boxed{\alpha = \sqrt{0.59}}$$

(iv) To check perpendicular $\vec{A}$ and $\vec{B}$:

$$\vec{A} \cdot \vec{B} = AB\cos 90^\circ$$

(If $\vec{A}$ & $\vec{B}$ is Perpendicular to each other)

$$\boxed{\vec{A} \cdot \vec{B} = 0}$$

If two vector $\vec{A}$ & $\vec{B}$ is Perpendicular to each other then $\boxed{\vec{A} \cdot \vec{B} = 0}$

If $\vec{A} = 2\hat{i} + 3\hat{j} - \alpha\hat{k}$ and $\vec{B} = \hat{i} - 2\hat{j} + 4\hat{k}$ find α. If $\vec{A}$ is perpendicular to $\vec{B}$.

Sol^n

$$\vec{A} \cdot \vec{B} = 0$$

$$2 - 6 - 4\alpha = 0$$

$$4\alpha = -4$$

$$\alpha = -1$$

Ans

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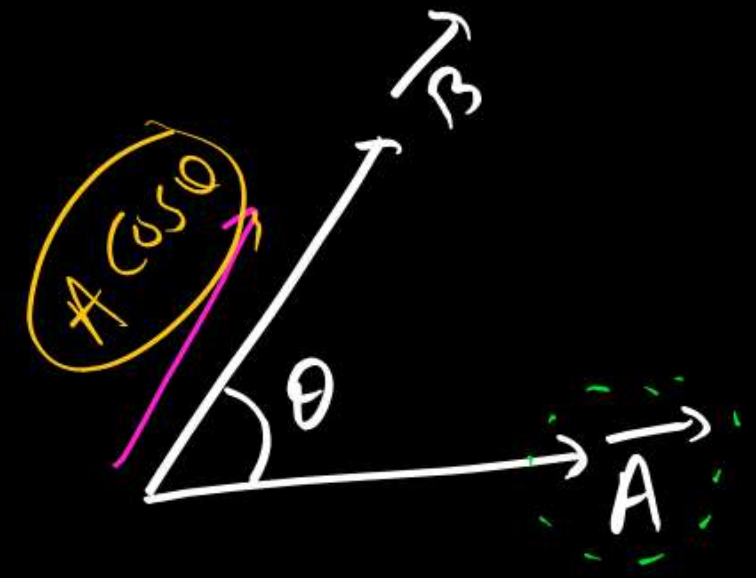
(iii) Projection (Component of $\vec{A}$ along $\vec{B}$) :



$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

Component of A along B $(A\cos\theta)$ $= \dfrac{\vec{A}\cdot\vec{B}}{B}$

$$\vec{A}\cdot\vec{B} = A\,B\cos\theta$$

$$B\cos\theta = \frac{\vec{A}\cdot\vec{B}}{A}$$

Component of B along A

If $\vec{A} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ and $\vec{B} = 4\hat{\imath}$. Find angle between $\vec{A}$ and $\vec{B}$.

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{8+0+0}{\sqrt{4+36+9}\,(4)} = \frac{8}{28} = \frac{4}{14}$$

$$\theta = \cos^{-1}\left(\frac{2}{7}\right)$$

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If a unit vector is represented by $0.5\hat{i} - 0.8\hat{j} + c\hat{k}$ then the value of c is

(A) $\sqrt{0.01}$

(B) $\sqrt{0.11}$

(C) 1

(D) $\sqrt{0.39}$

AIPMT

$$\left(0.5\hat{i} - 0.8\hat{j} + c\hat{k}\right)\cdot\left(0.5\hat{i} - 0.8\hat{j} + c\hat{k}\right) = 1$$

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If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is

(A) 45°
(B) 180°
(C) 0°
(D) 90°

AIPMT

m12

The vectors $\bar{A}$ and $\bar{B}$ are such that $|\bar{A}+\bar{B}| = |\bar{A}-\bar{B}|$. The angle between the two vectors is

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(A) 45°

(B) 90°

(C) 60°

(D) 75°

The magnitude of vectors $\overline{A}$, $\overline{B}$ and $\overline{C}$ are 3, 4 and 5 units respectively. If $\overline{A} + \overline{B} = \overline{C}$, the angle between $\overline{A}$ and $\overline{B}$ is

(A) $\pi/2$ (B) $\cos^{-1}(0.6)$

(C) $\tan^{-1}(7/5)$ (D) $\pi/4$

$$\vec{C} = \vec{A} + \vec{B}$$

$$C^2 = A^2 + B^2 + 2AB\cos\theta$$

$$(5)^2 = (3)^2 + (4)^2 + 2 \times 3 \times 4 \cos\theta$$

$$0 = \cos\theta \qquad \theta = 90^\circ$$

CROSS PRODUCT

[Vector - Product]

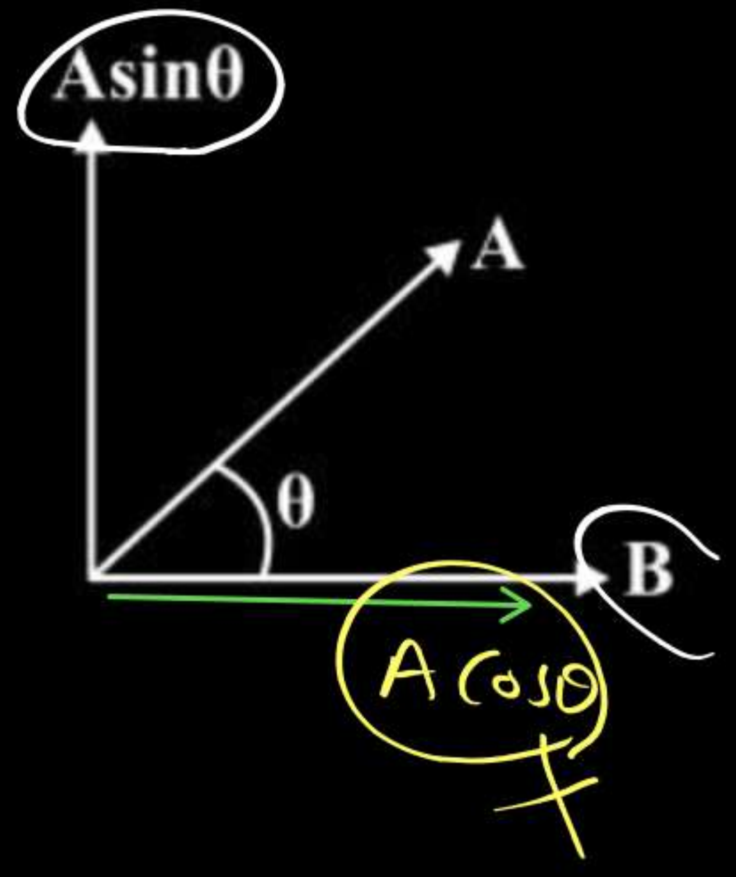
(Vector) × (Vector) = Vector

$\Rightarrow \quad \vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$

$= $ (Component of A perpendicular to B) B

$\Rightarrow \quad \vec{A} \times \vec{B} = B \, (A \sin\theta) \, \hat{n}$

$\hat{n}$ ↑ direction of $(\vec{A} \times \vec{B})$ – always perpendicular to $\vec{A}$ & $\vec{B}$



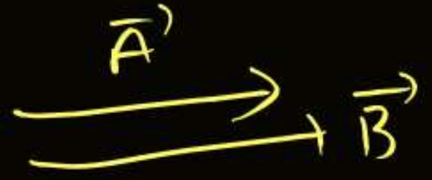
$\vec{C} = \vec{A} \times \vec{B} = AB \sin\theta$

$\hookrightarrow$ angle b/w $\vec{A}$ & $\vec{B}$

If $\theta = 90^\circ$

$\vec{C} = |\vec{A} \times \vec{B}|_{max} = AB$

If $\theta = 0^\circ$



$\vec{A} \times \vec{B} = 0$

$\vec{C}$ is always perpendicular to $\vec{A}$ & $\vec{B}$

Direction of $\vec{A} \times \vec{B}$ always perpendicular to the $\vec{A}$ and $\vec{B}$.

$(\vec{A} \times \vec{B}) \perp$ to $\vec{A}$

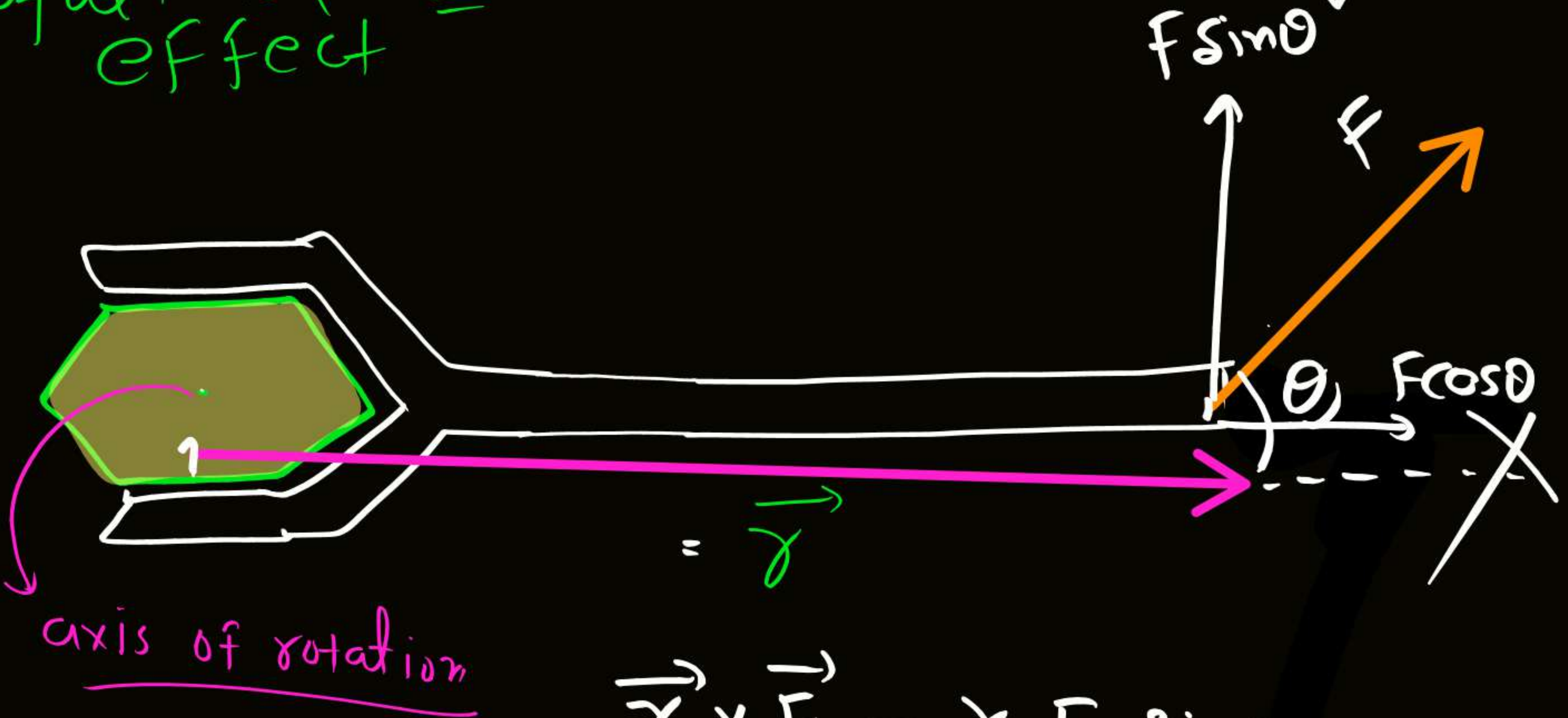
$(\vec{A} \times \vec{B}) \bullet \vec{A} = 0$

$$\underbrace{(\vec{A} \times \vec{B})} \cdot \vec{B} = ??$$

$$\vec{C} \cdot \vec{B} = 0$$

which is $\perp$r to $\vec{A}$ & $\vec{B}$

rotational effect =



$$\vec{r} \times \vec{F} = r\,F\,\sin\theta$$

Cross Product (vector Product)

$\hat{i}\times\hat{i} = 1\times 1\times \sin 0^\circ = 0$

$\hat{j}\times\hat{j} = 0$

$\hat{k}\times\hat{k} = 0$

$\hat{i}\cdot\hat{i} = 1$

$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

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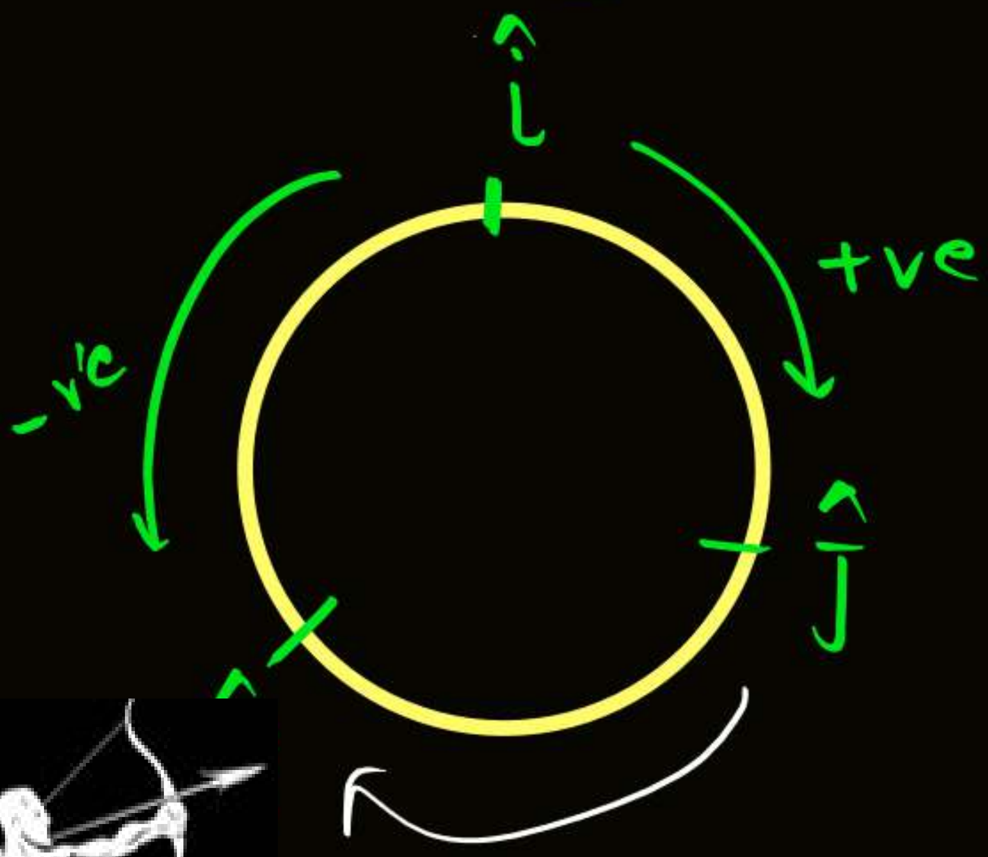


$\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{k} = \hat{i}$

$\hat{k}\times\hat{i} = \hat{j}$

$\hat{j}\times\hat{i} = -\hat{k}$

$\hat{i}\times\hat{k} = -\hat{j}$

$\hat{k}\times\hat{j} = -\hat{i}$

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

Director of $\vec{A} \times \vec{B}$:

#1

Right hand

$$\vec{R} = \vec{A} \times \vec{B}$$

Result — 1st vector — 2nd vector

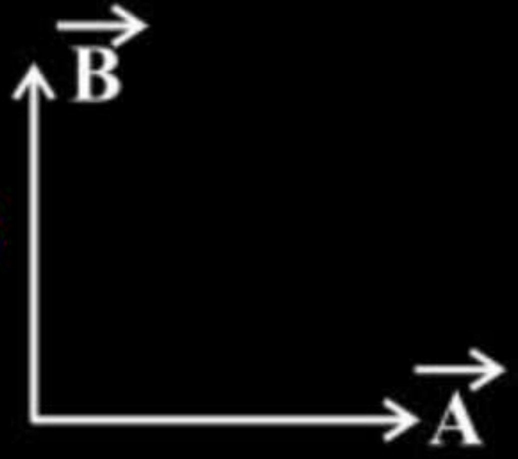
$\vec{R} \cdot \vec{A} = 0$

$\vec{R} \cdot \vec{B} = 0$

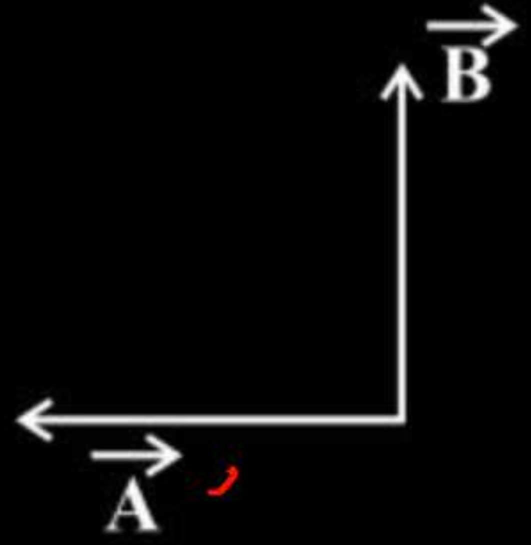
four

Place ~~Fair~~ finger in the director of 1^{st} vector and then turn it in the direction of $\vec{B}$ (2^{nd} vector) then thumb represent direction of $(\vec{A} \times \vec{B})$

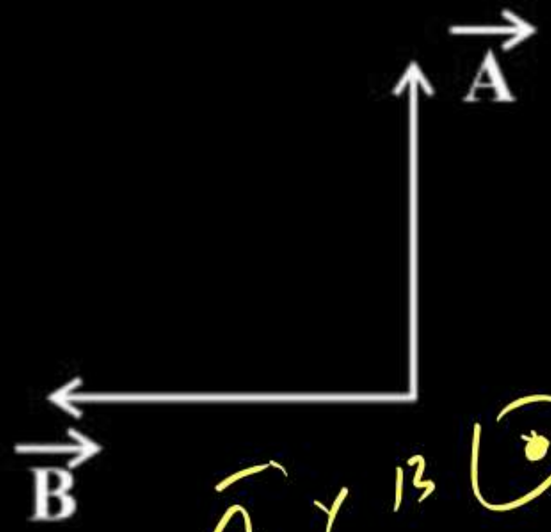
$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$



$\vec{A} \times \vec{B} =$ outwar $\odot$



$\vec{A} \times \vec{B} =$ Inward $\otimes$



$\vec{A} \times B$ $\odot$ = outw.

If $\vec{A} \times \vec{B} = \sqrt{3}\,(\vec{A} \cdot \vec{B})$ then find Angle between $\vec{A}$ and $\vec{B}$.

$$\vec{A} \times \vec{B} = \sqrt{3}\,(\vec{A} \cdot \vec{B})$$

$$\cancel{AB} \sin\theta = \sqrt{3}\,\cancel{AB} \cos\theta$$

$$\tan\theta = \sqrt{3}$$

$$\boxed{\theta = 60^\circ}$$

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If $\vec{A} = 2\hat{i} - 2\hat{j}$ and $\vec{B} = 5\hat{k}$ then find $\vec{A} \times \vec{B}$

$$\vec{A} \times \vec{B} = (2\hat{i} - 2\hat{j}) \times 5\hat{k}$$

$$= 10(\hat{i} \times \hat{k}) - 10(\hat{j} \times \hat{k})$$

$$= 10(-\hat{j}) - 10[\hat{i}]$$

$$= -10\hat{j} - 10\hat{i} = 10[-\hat{j} - \hat{i}]$$

If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ & $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

formula $\longrightarrow$ not useful for Physics

$$= \hat{i}\left[A_y B_z - A_z B_y\right] - \hat{j}\left[A_x B_z - A_z B_x\right] + \hat{k}\left[A_x B_y - A_y B_x\right]$$

If $\vec{A} = 2\hat{i} - 4\hat{j} + \hat{k}$ & $\vec{B} = 2\hat{i} + 3\hat{j}$

then find $\vec{A} \times \vec{B} = ??$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 1 \\ 2 & 3 & 0 \end{vmatrix}$$

$$= \hat{i}\,|0 - 3| - \hat{j}\,|0 - 2| + \hat{k}(6 - (-8))$$

$$= -3\hat{i} + 2\hat{j} + 14\hat{k}$$

THANK YOU ☺