ARJUNA NEET BATCH

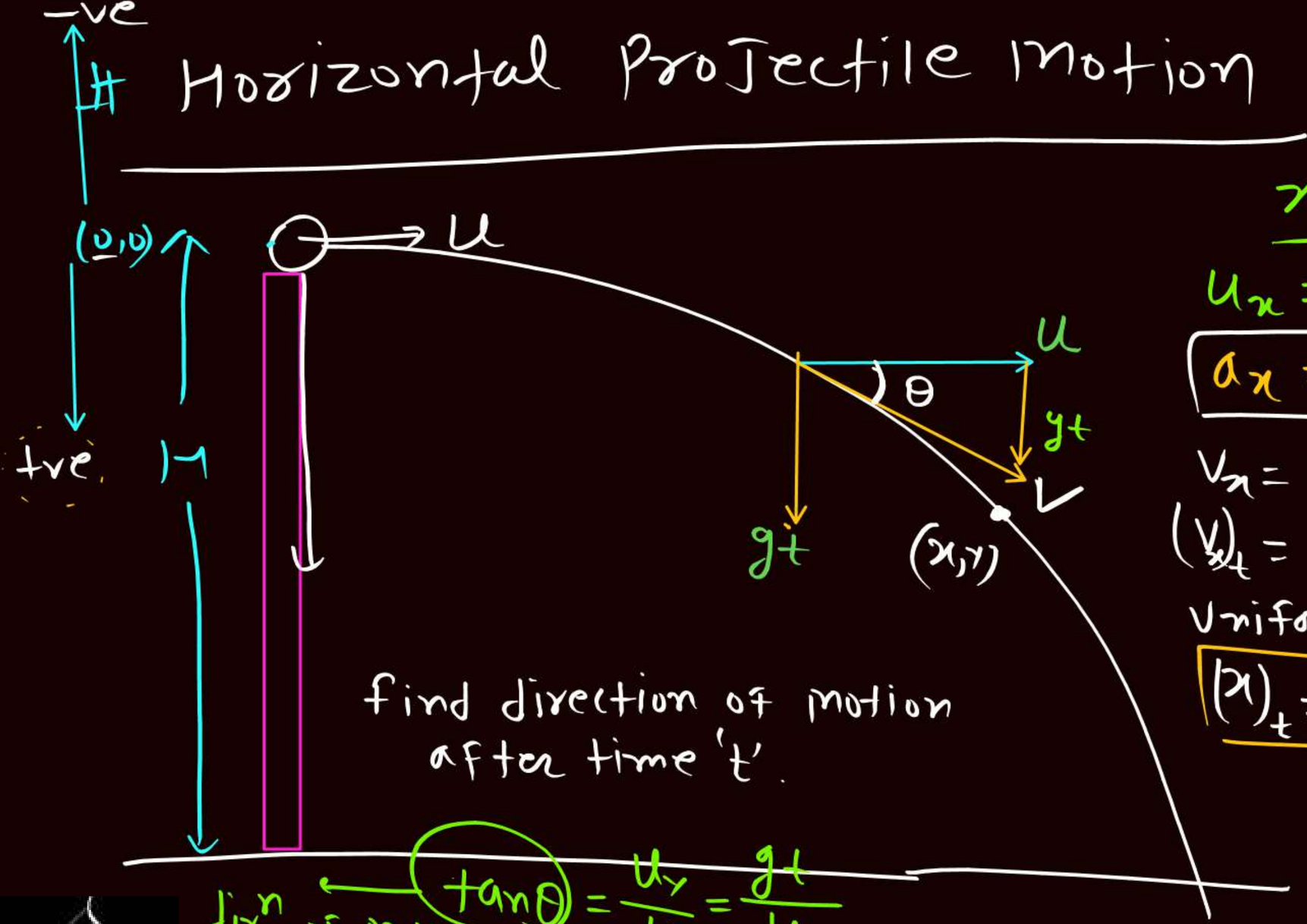
MOTION IN A PLANE

LECTURE - 04

Todays Goal

Horizontal Projectile Motion.

Horizontal Projectile Motion



x-axis	y-axis
$u_x = u$	$u_y = 0$
$a_x = 0$	$a_y = g\hat{j}$
$v_x = \text{constant}$	$(v_y)_t = u + at$
$(v_x)_t = u$	$v_y = 0 + gt\hat{j}$
Uniform motion	$v_y = gt\hat{j}$
$(x)_t = ut$	$y = \frac{1}{2}gt^2$
dx	dt

Net velocity at time 't'

$$\vec{V} = v_x \hat{i} + v_y \hat{j}$$

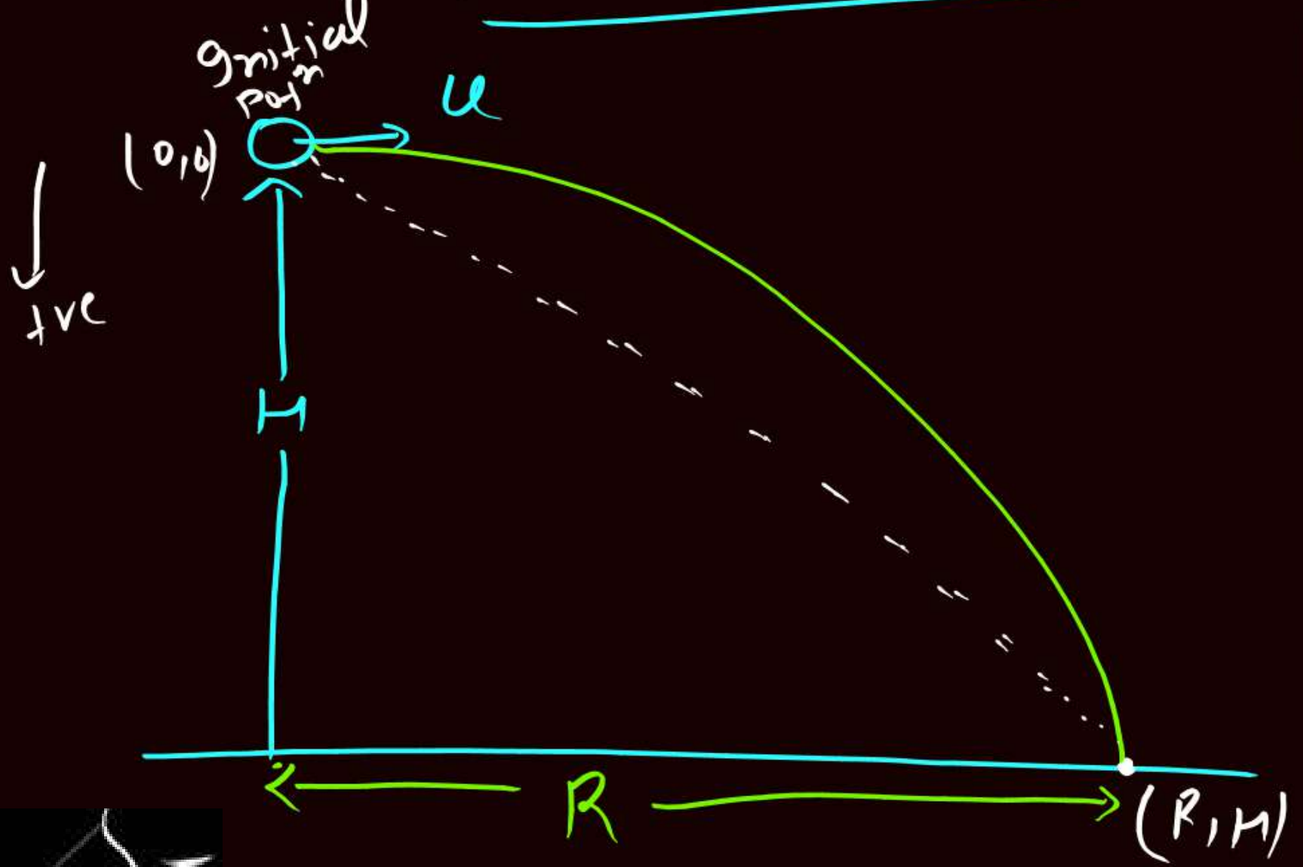
$$\vec{v} = u \hat{i} + gt \hat{j}$$

$$\text{Speed} = \sqrt{u^2 + (gt)^2}$$

Time of Flight :-

Consider motion along y-axis

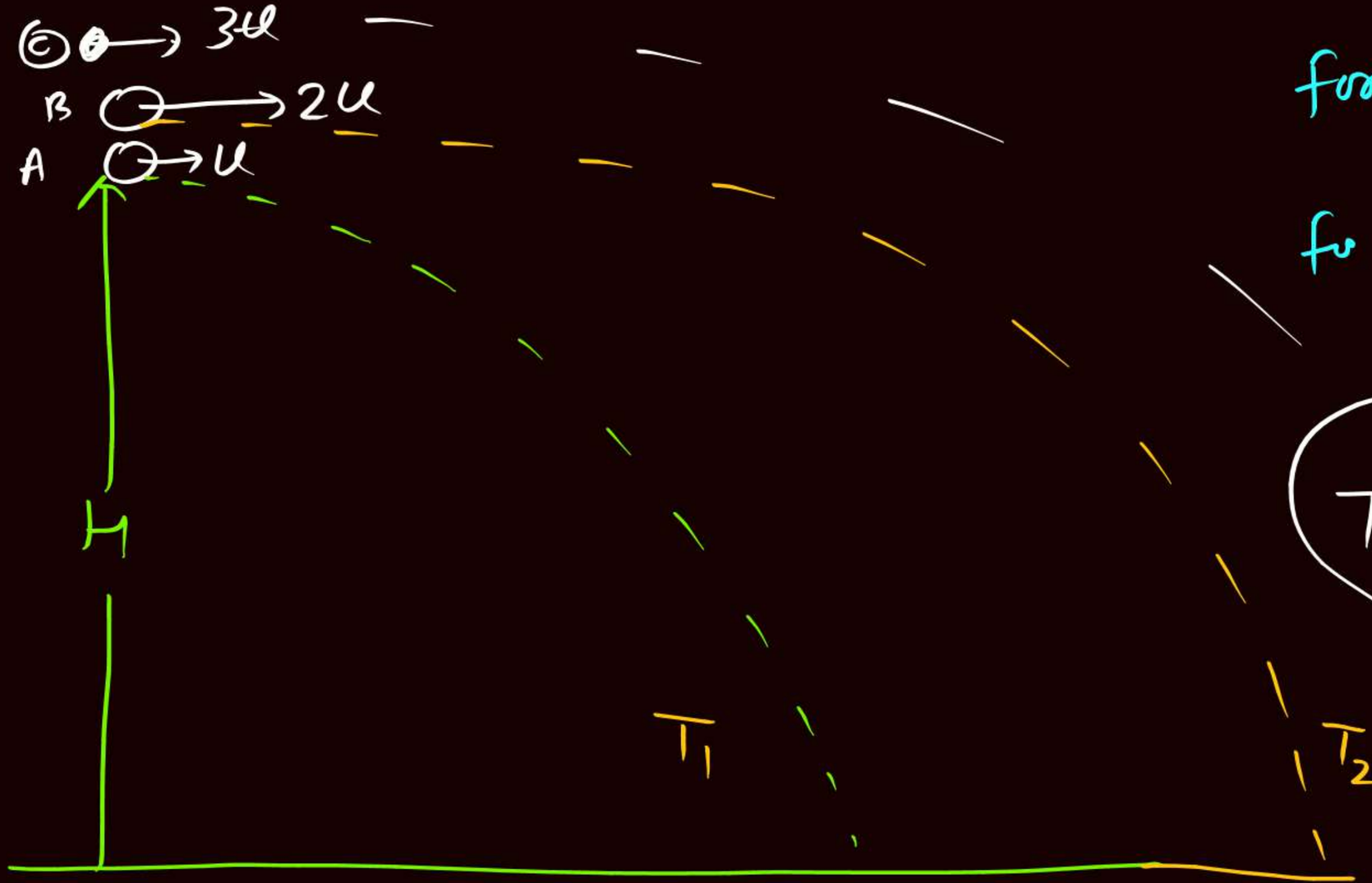
$$\left[s = u_y t^{0} + \frac{1}{2} a t^2 \right]_{y\text{th direction}}$$



$$H = 0 + \frac{1}{2} g t^2$$

$$t_f = \sqrt{\frac{2H}{g}}$$

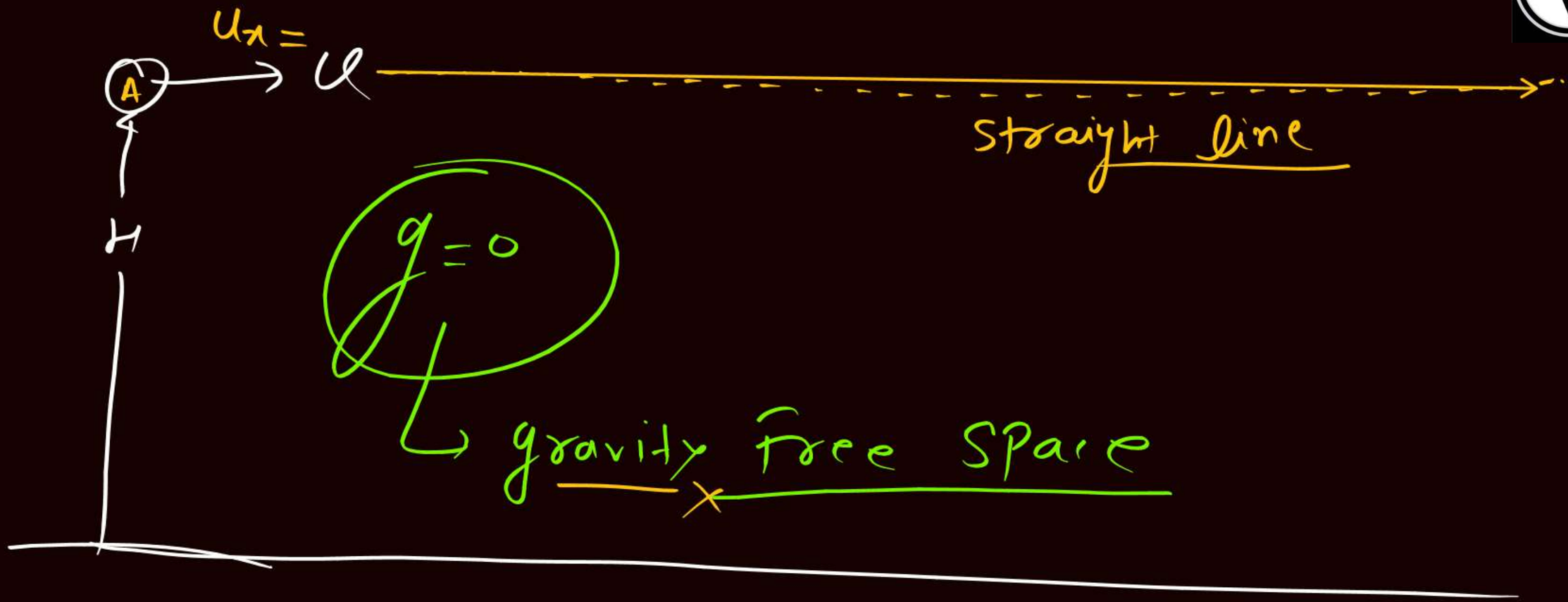
Time of flight does not depends upon velocity along x-axis.



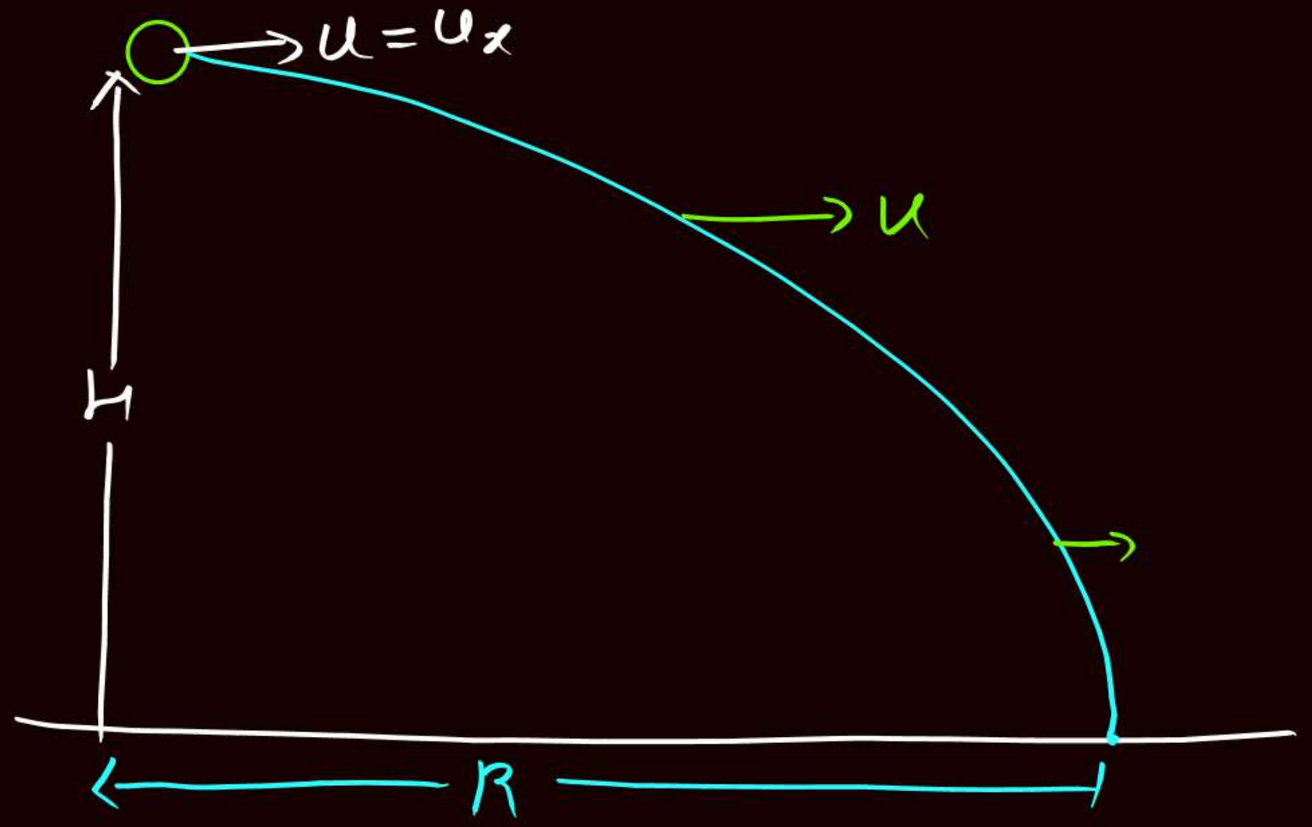
for (A) $(u_y)_{t=0} = 0$

for (B) $(u_y)_{t=0} = 0$

$$T_1 = T_2 = \sqrt{\frac{2H}{g}}$$


$u_x = u$
A
H
straight line
g = 0
gravity Free Space

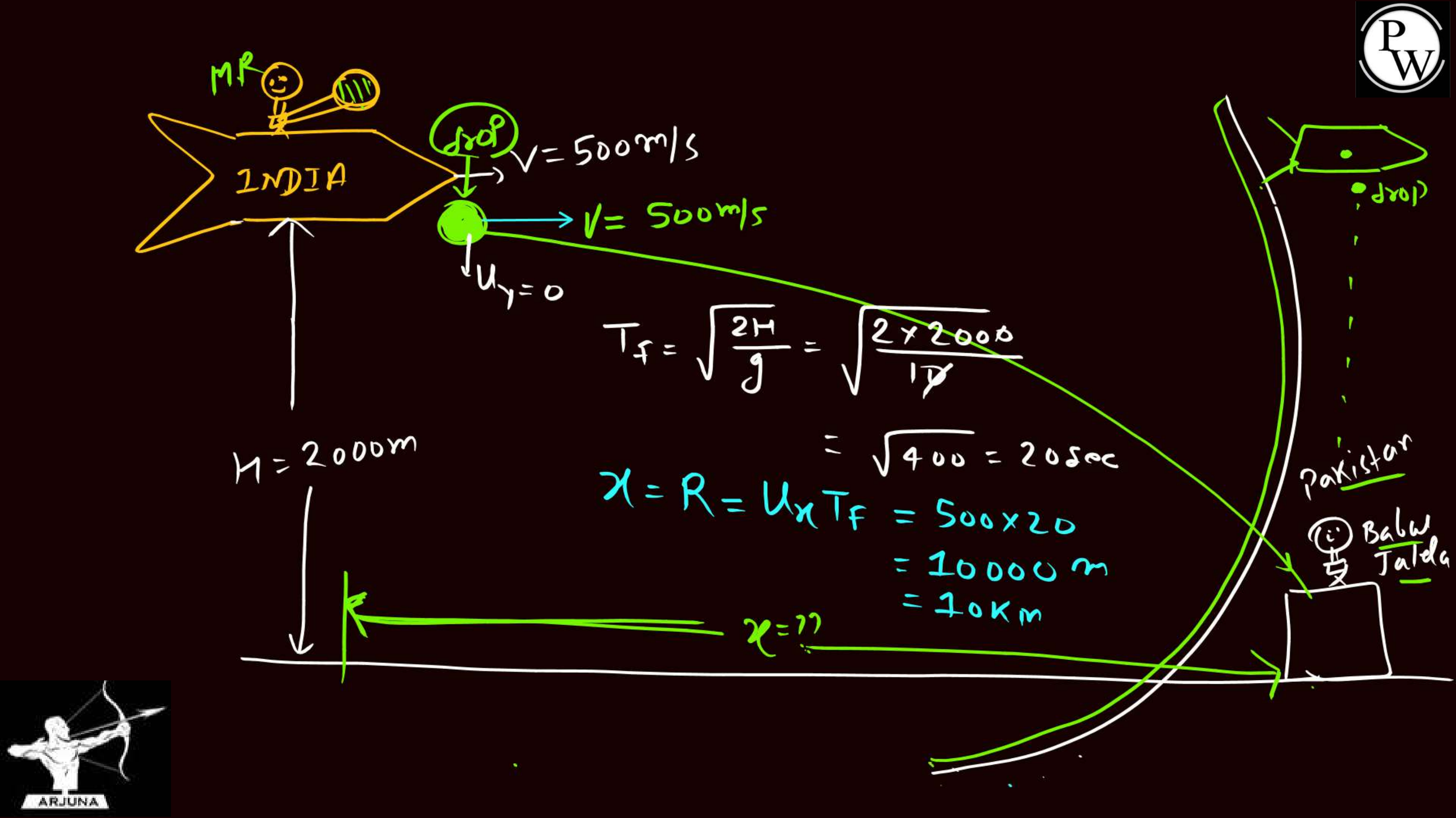
Horizontal Range

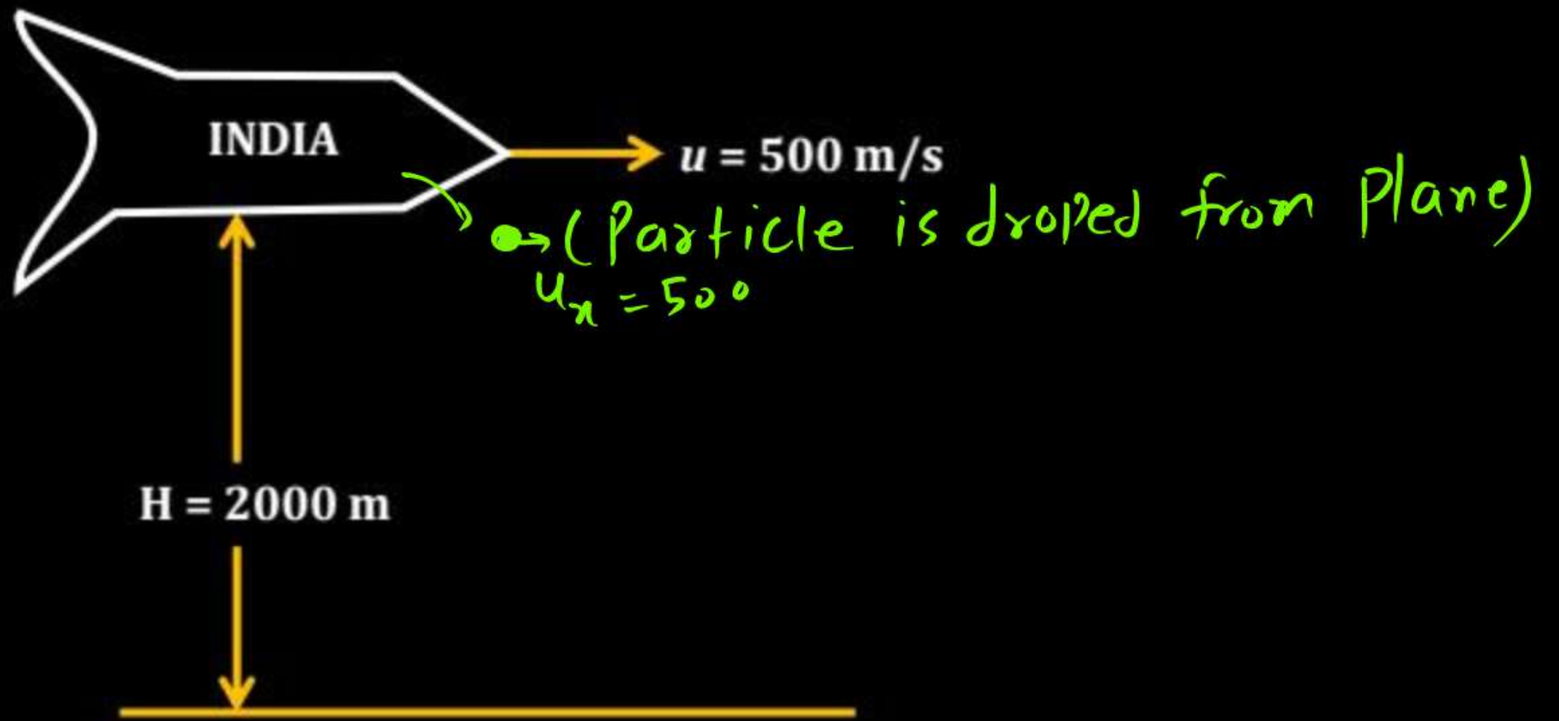


Consider motn along x-axis :—

$$x = u_x T_f$$

$$R = u \times \sqrt{\frac{2H}{g}}$$


MR
INDIA
drop
V = 500 m/s
V = 500 m/s
$u_y = 0$
H = 2000m
$T_f = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 2000}{10}}$
$= \sqrt{400} = 20 sec$
$x = R = u_x T_f = 500 \times 20$
$= 10000 m$
$= 10 km$
x = ??
drop
Pakistan
Balw Jatda


INDIA
$u = 500$ m/s
(Particle is droped from Plane)
$u_x = 500$
H = 2000 m

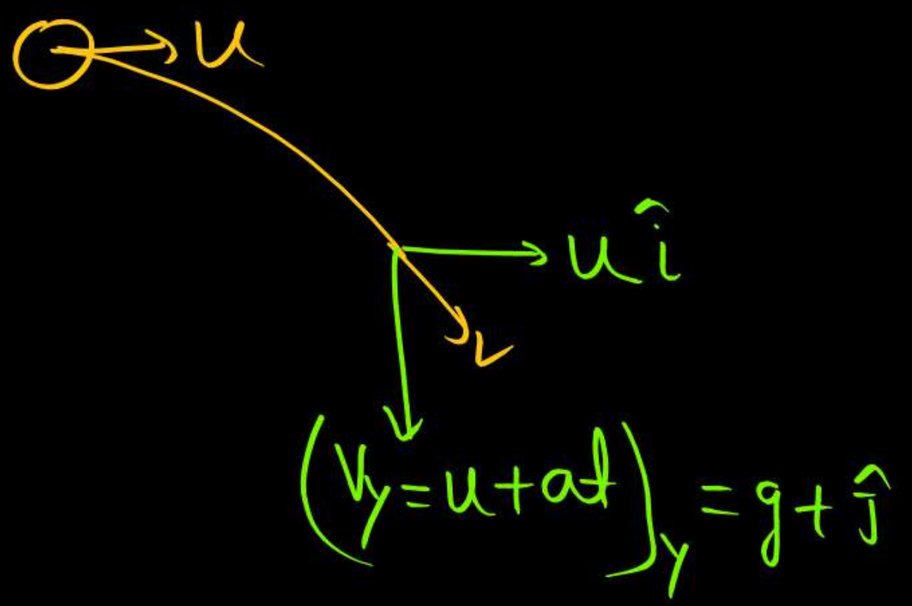
When a particle is thrown horizontally, with initial velocity 'u' the resultant velocity of the projectile at any time t is given by :

(a) gt

(b) $\frac{1}{2}gt^2$

(c) $\sqrt{u^2 + g^2t^2}$

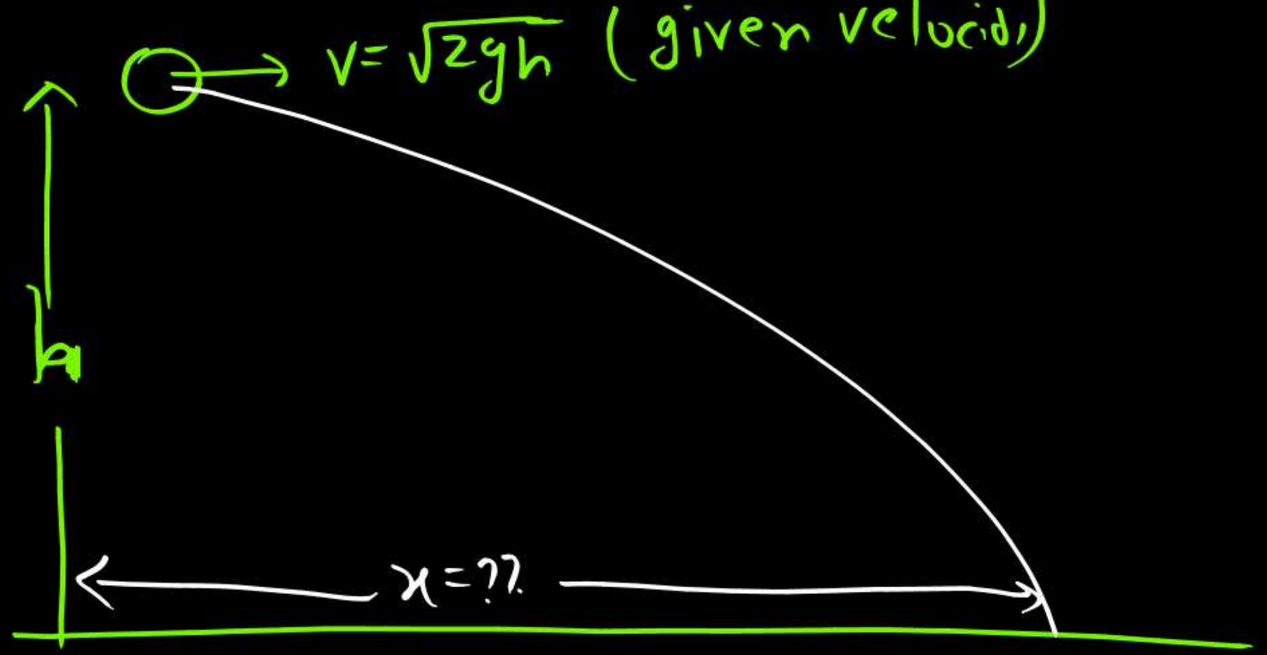
(d) $\sqrt{u^2 - g^2t^2}$



A body is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of the tower at a distance x from the tower. The value of x is :

(a) h (b) $h/2$

(c) $2h$ (d) $2h/3$



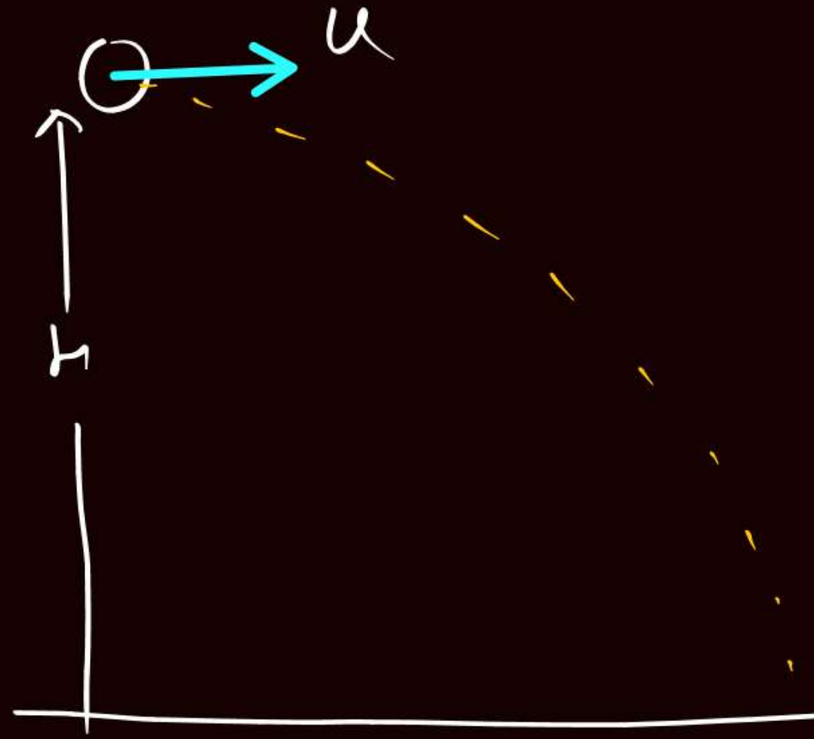
$$T_f = \sqrt{\frac{2h}{g}}$$

$$x = u_x T_f = \sqrt{2gh} \times \sqrt{\frac{2h}{g}}$$

$$x = \sqrt{2h} \times \sqrt{2h} = 2h$$

⑥ Particle is Projected in horizontal direction with velocity u then find eqn of Trajectory

Sol^n



$x = ut$ — (i)

$t = \frac{x}{u}$

$y = \frac{1}{2} g t^2$ — (ii)

$y = \frac{1}{2} g \left(\frac{x}{u}\right)^2$

$$y = \frac{1}{2} g \frac{x^2}{u^2}$$

$y \propto x^2$ → Parabolic

ARJUNA

A particle starts from the origin of coordinates time $t = 0$ and moves in the xy plane with a constant acceleration α in the y-direction. Its equation of motion is $y = \beta x^2$. Its velocity component in the x-direction :

IIT

(a) Variable

(b) $\sqrt{\frac{2\alpha}{\beta}}$

(c) $\frac{\alpha}{2\beta}$

(d) $\sqrt{\frac{\alpha}{2\beta}}$

$y = \frac{1}{2} g \frac{x^2}{u^2}$

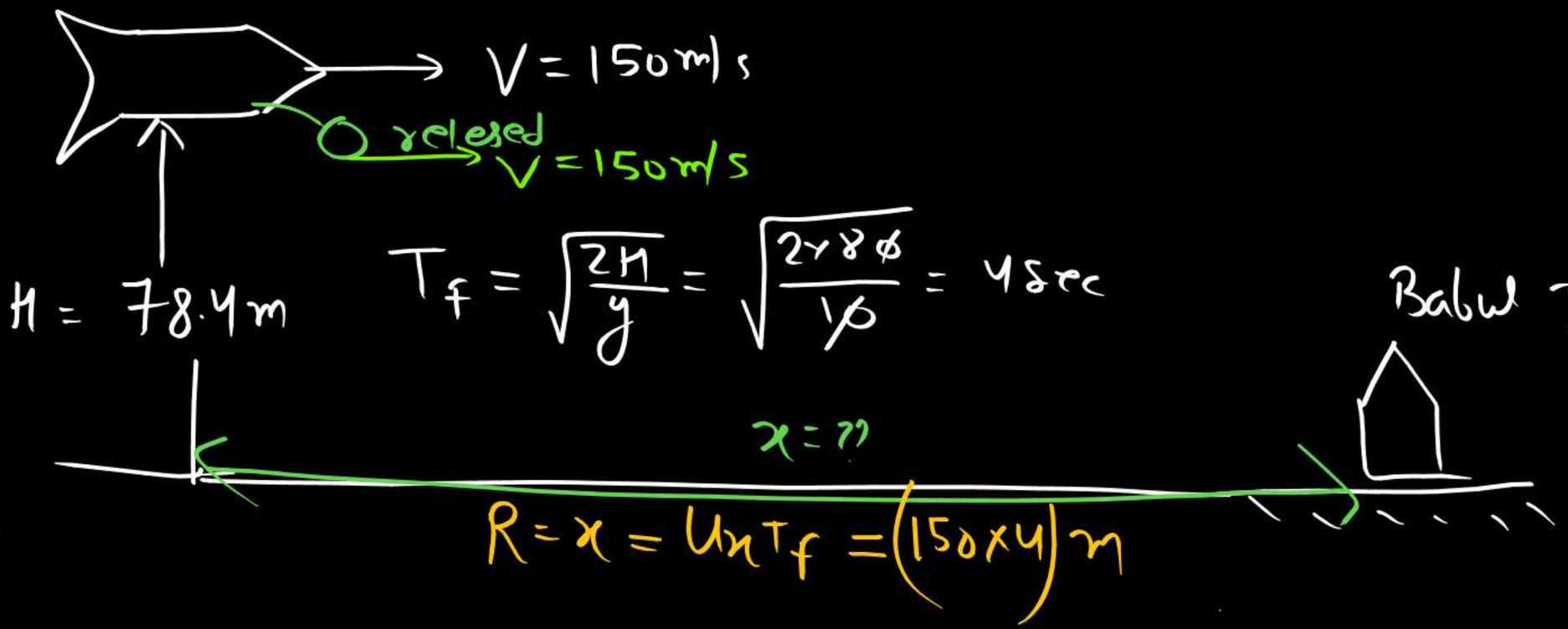
$\beta x^2 = \frac{1}{2} g \frac{x^2}{u^2}$

$u = \sqrt{\frac{g}{2\beta}}$ x

ARJUNA

A bomber is flying horizontally with a constant speed of 150 m/s at a height of 78.4 m. The pilot has to drop a bomb at the enemy target. At what horizontal distance from the target should he release the bomb:

(a) zero
(b) 300 m
(c) 600 m
(d) 750 m



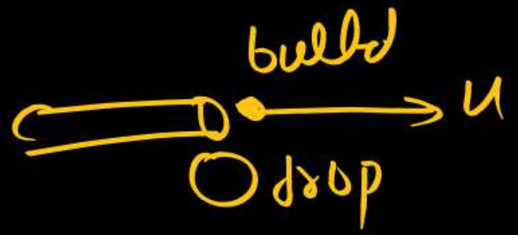
A bullet is fired in a horizontal direction from a tower while a stone is simultaneously dropped from the same point then :

(a) The bullet and the stone will reach the ground simultaneously

(b) The stone will reach earlier

(c) The bullet will reach earlier

(d) Nothing can be predicted



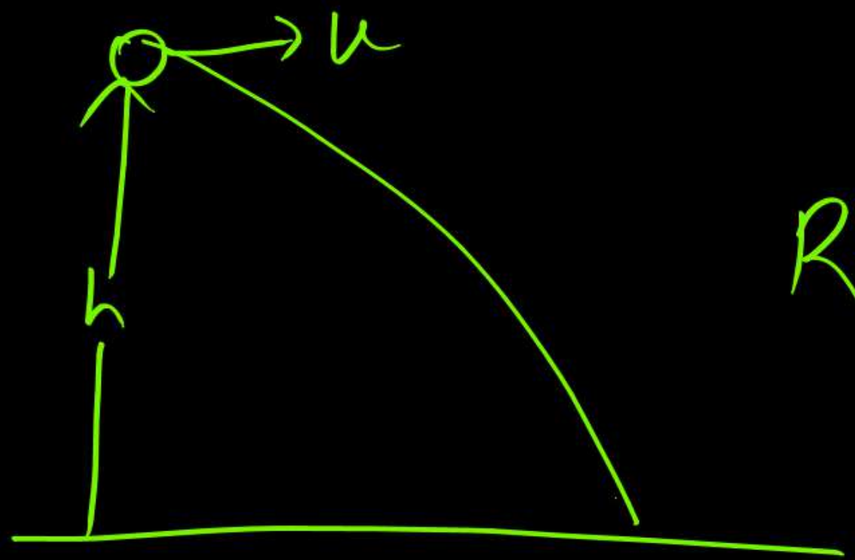
True / False :

A body *x* is dropped from the top of a tower. At the same time, another body *y* is thrown horizontally from the same position with a velocity U. Both bodies will reach the ground at same time

→ True ✓

A particle is projected from height h with horizontal velocity u, then range of particle is $u\sqrt{\frac{2H}{g}}$.

↳ True



$R = u_x T_f$

$= u\sqrt{\frac{2H}{g}}$ ✓

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At the highest point of the path of projectile speed is zero.

→ False

At the highest Point of the Path of projectile Speed in vertical direction is zero (True) ✓

Horizontal velocity in projectile motion is zero at the highest point.

→ false

$u_x = u\cos\theta$

*

A bomber plane moves horizontally with a speed of 500 m/s and a bomb released from it, strikes the ground in 10 sec. Angle at which it strikes the ground will be (g = 10 m/s^2)

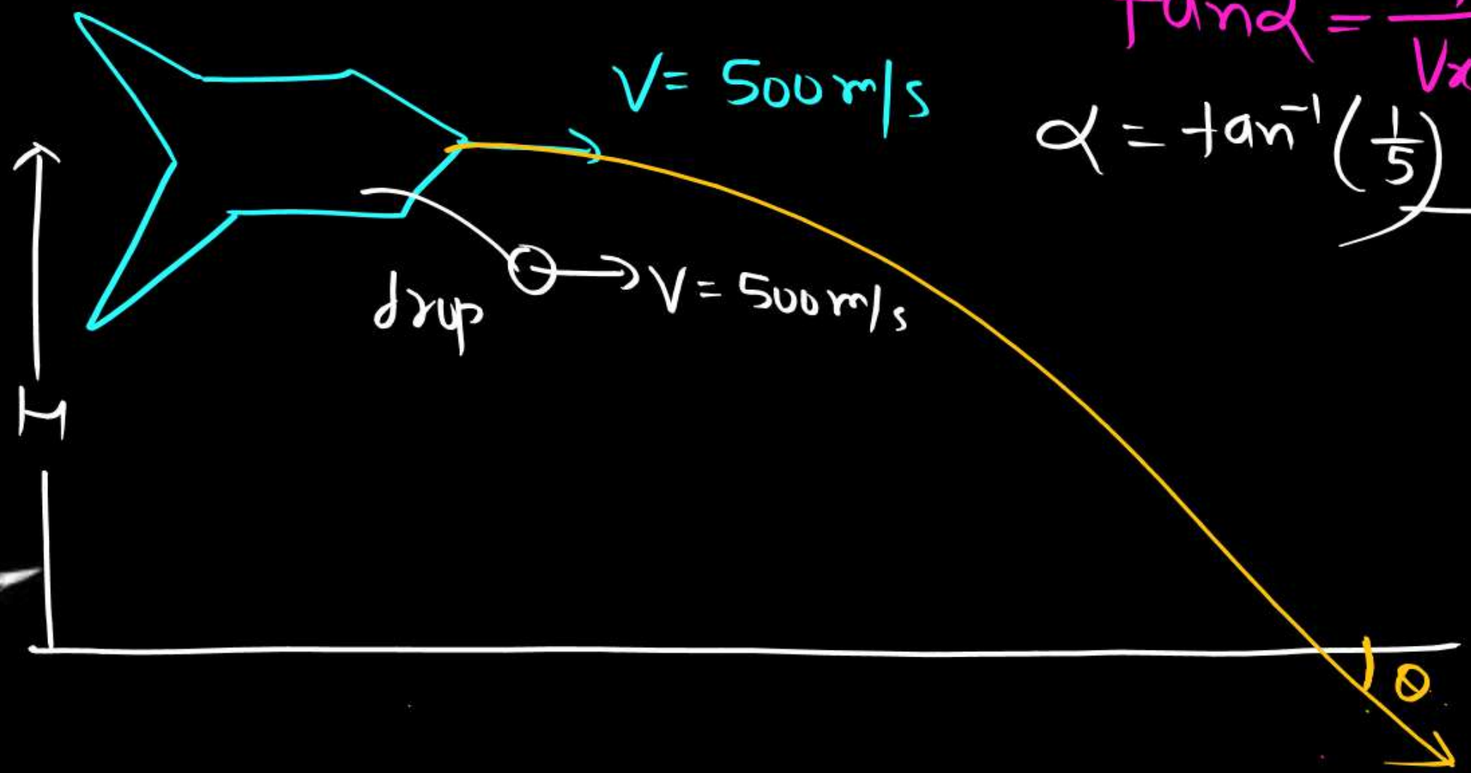
[MH CET 2003]

(a) $\tan^{-1}\left(\frac{1}{5}\right)$

(b) $\tan\left(\frac{1}{5}\right)$

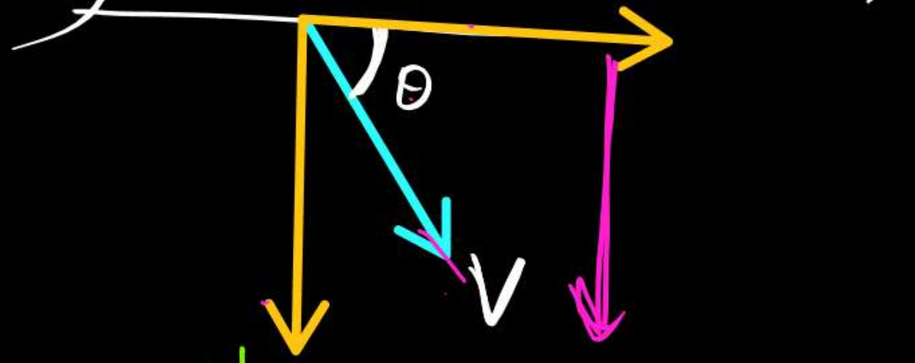
(c) $\tan^{-1}(1)$

(d) $\tan^{-1}(5)$



$\tan\alpha = \frac{V_y}{V_x} = \frac{100}{500}$

$\alpha = \tan^{-1}\left(\frac{1}{5}\right)$



$V_y = gt = 10 \times 10 = 100$ m/s

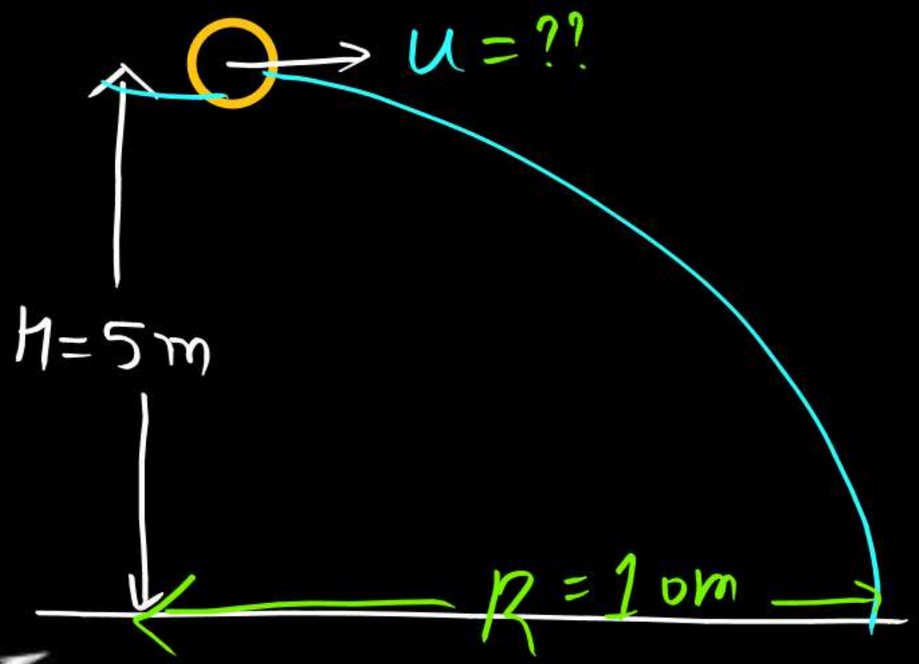
A body is thrown horizontally from the top of a tower of height 5 *m*. It touches the ground at a distance of 10 *m* from the foot of the tower. The initial velocity of the body is (g = 10 ms)

(a) 2.5 ms

(b) 5 ms

(c) 10 ms

(d) 20 ms



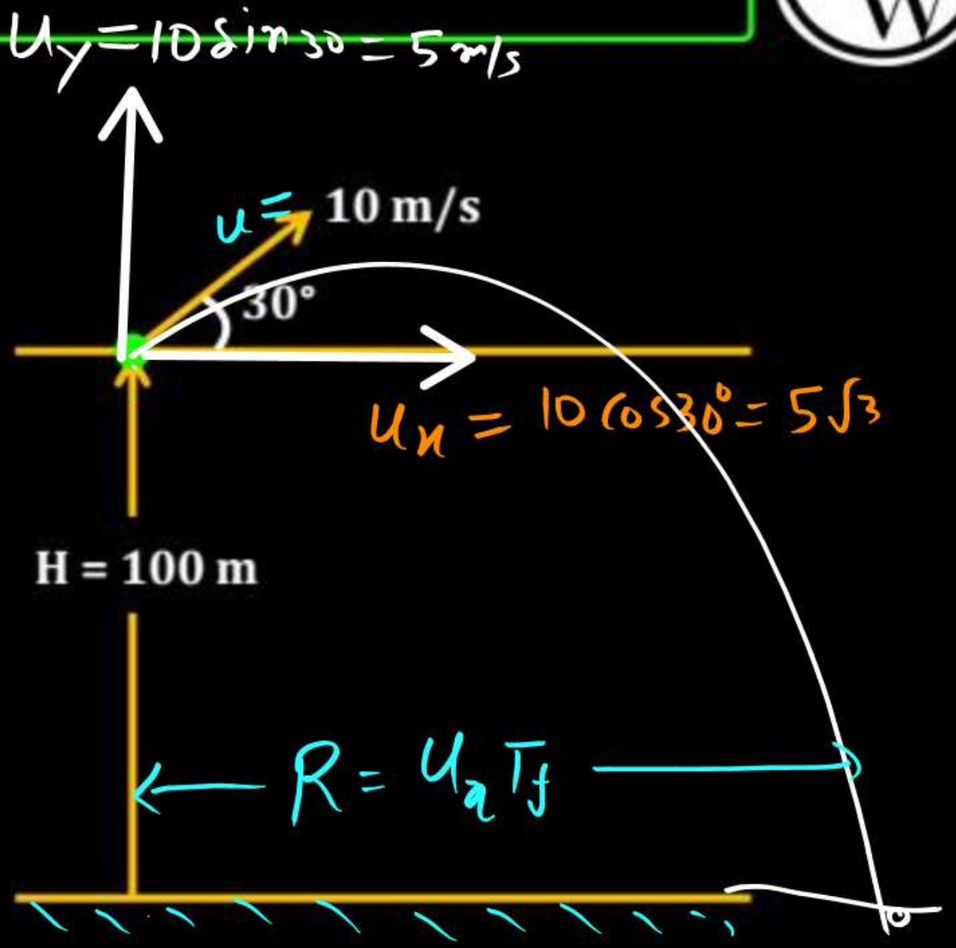
$$R = u_x T_f$$

$$10 = u \sqrt{\frac{2H}{g}}$$

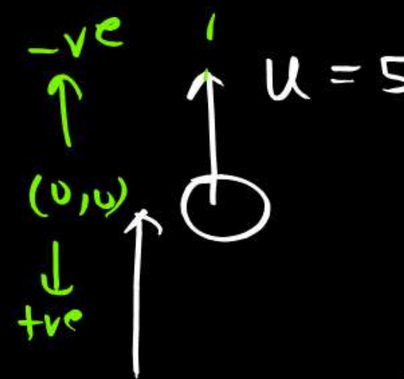
$$10 = u \sqrt{\frac{2 \times 5}{10}}$$

$$u = 10 \text{ m/s}$$

Find time of flight and range.



Consider motion along y-axis



$S = ut + \frac{1}{2}at^2$

$100 = -5t + \frac{1}{2}(10)t^2$

$5t^2 - 5t - 100 = 0$

$t^2 - t - 20 = 0$

$t^2 - 5t + 4t - 20 = 0$

$t(t-5) + 4(t-5) = 0$

$t = 5$ sec ✓ $\quad t = -4$ ✗

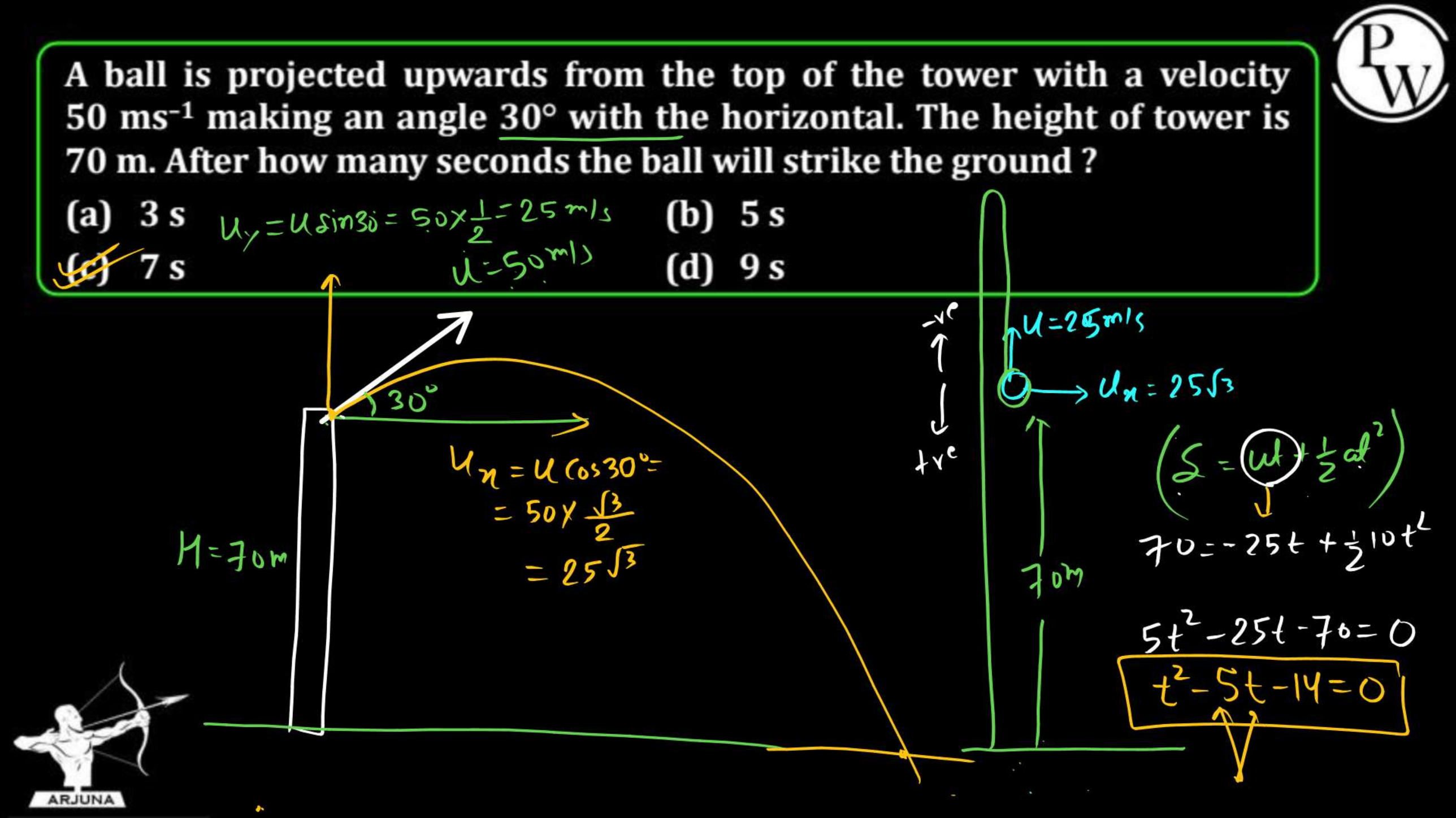
$R = 5\sqrt{3} \times 5$

$= 25\sqrt{3}$

A ball is projected upwards from the top of the tower with a velocity 50 ms^{-1} making an angle 30° with the horizontal. The height of tower is 70 m. After how many seconds the ball will strike the ground ?

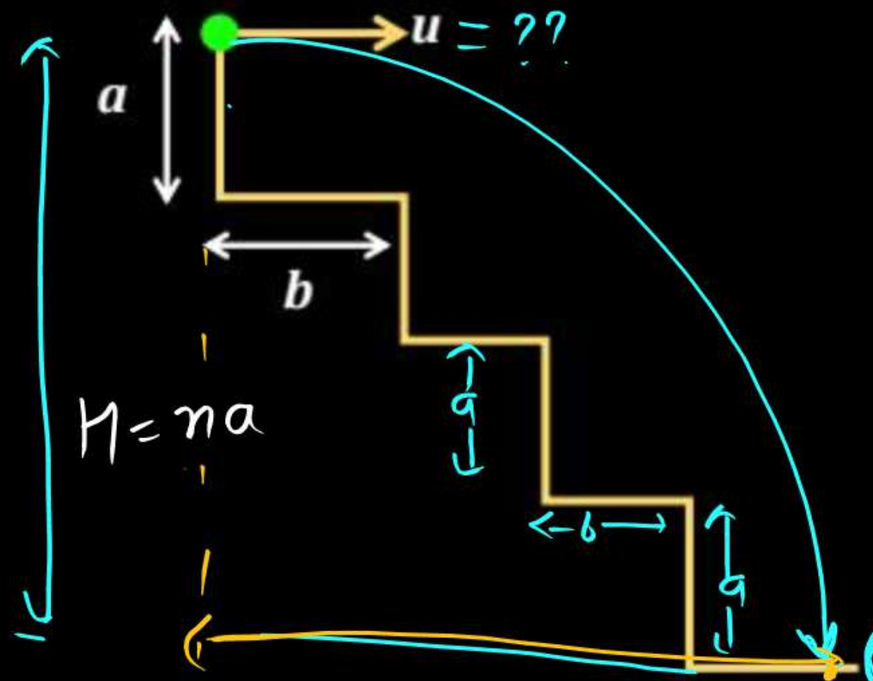
(a) 3 s (b) 5 s

(c) 7 s (d) 9 s



Find velocity so that ball will fall on *n*th.

HCV



$$T_f = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2na}{g}}$$

$$R = u \times T_f$$

$$\Downarrow$$

$$nb = u\sqrt{\frac{2na}{g}}$$

$R = nb$

$m R^x$ $\quad$ $\dim^n$

$$u = \sqrt{\frac{gb^2n}{2a}}$$

$\sqrt{g}$ dimension $= LT^{-1}$

$$u = nb\sqrt{\frac{g}{2na}} = \sqrt{\frac{g n^2 b^2 n}{2na}}$$

$$\boxed{u = \sqrt{\frac{gb^2n}{2a}}}$$

ARJUNA

(A) → Two bodies of different masses are projected horizontally with different speeds, they reach the ground simultaneously.

(B) → For both bodies, the vertical component of initial velocity is zero.

A → True

B → True.

IIT

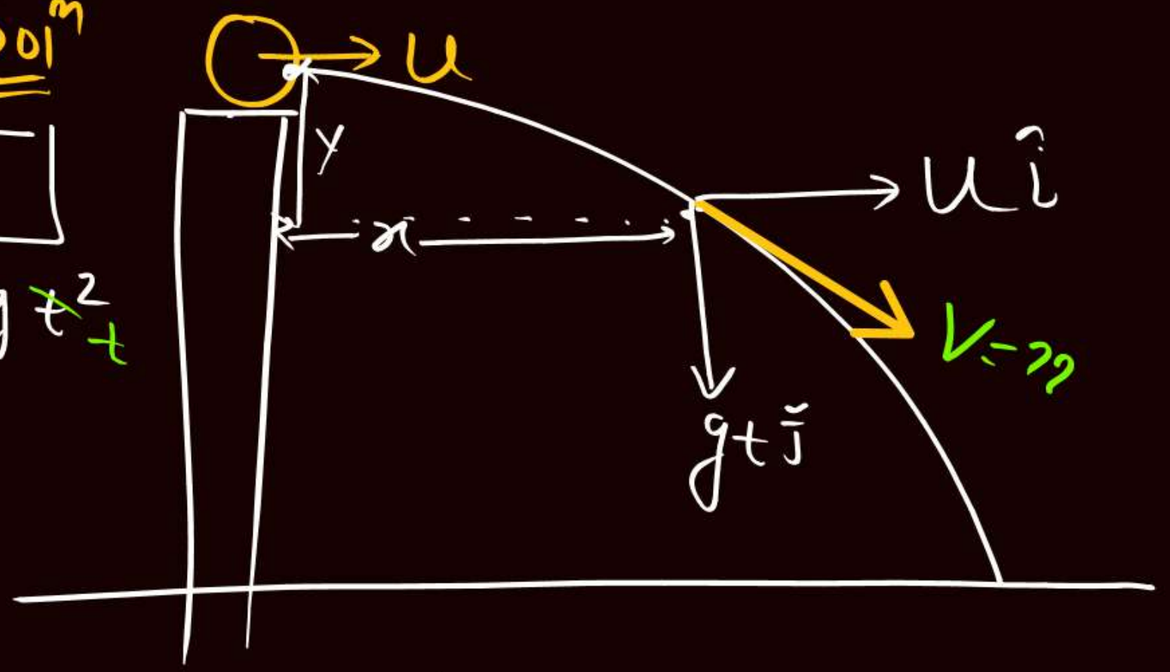
(Q). ball is Projected in Horizontal direction with velocity u; then find its speed when horizontal & vertical displacement is same.

Cond^n given in qu. ↓ Soln

$x = y$

$ut = \frac{1}{2} g t^2$

$t = \frac{2u}{g}$



$\vec{v} = u\hat{i} + gt\hat{j}$

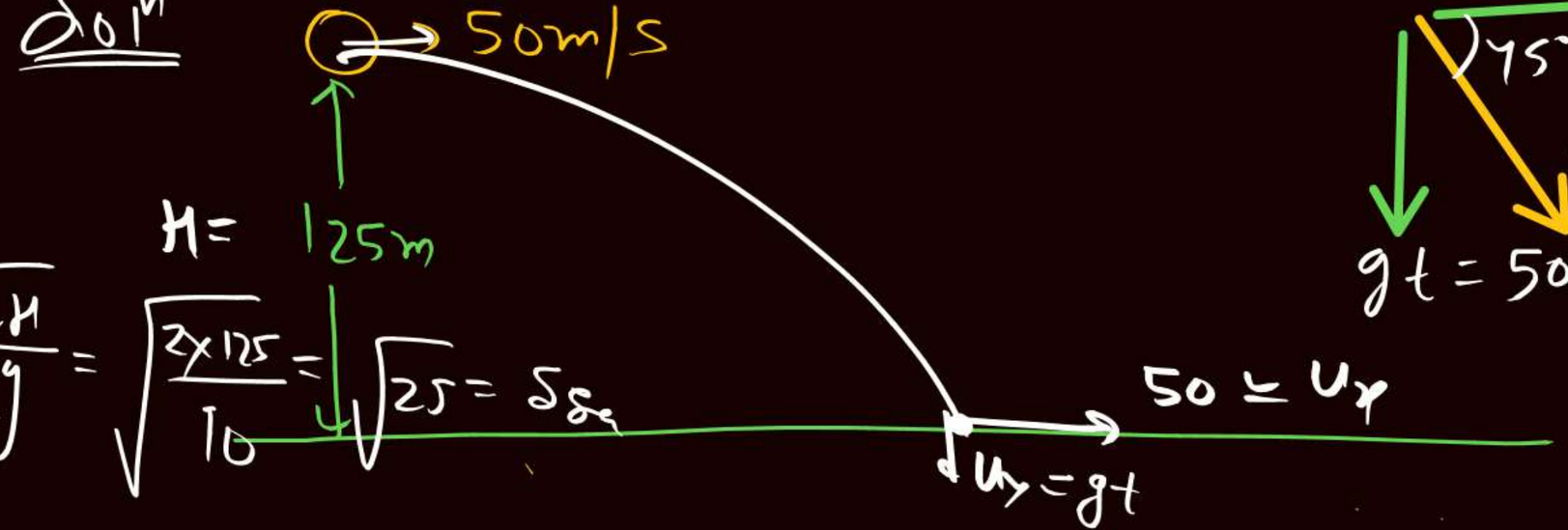
$\vec{v} = u\hat{i} + g\frac{2u}{g}\hat{j}$

$\vec{v} = u\hat{i} + 2u\hat{j}$

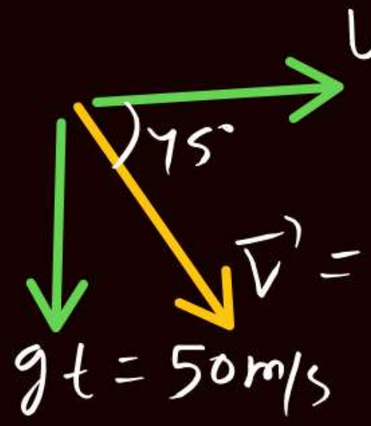
$|\vec{v}| = \sqrt{u^2 + 4u^2} = \sqrt{5}u$

Q. Ball is Projected with Speed 50 m/s in horizontal direction from 125 m Height then find speed by which it will strike on the ground and also find angle at which it will collide on Ground.

Soln



$t_f = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times125}{10}} = \sqrt{25} = 5s$



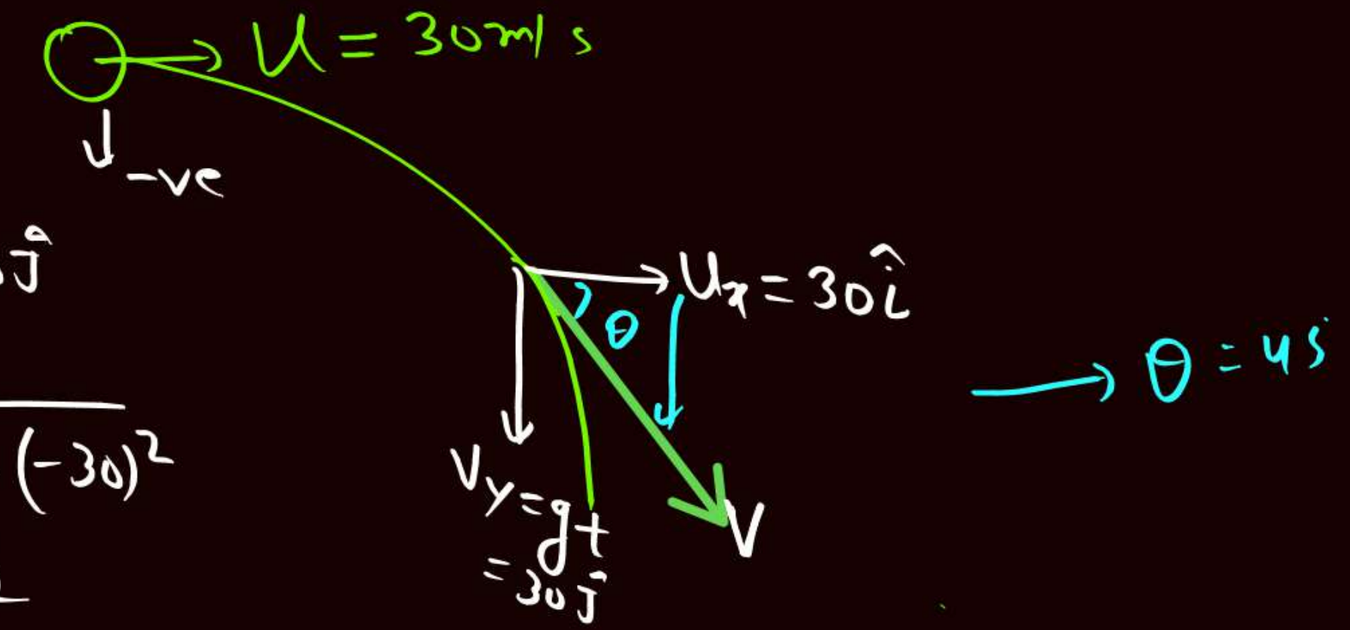
$\vec{V} = 50\hat{i} + 50\hat{j}$

$|\vec{V}| = \sqrt{(50)^2 + (50)^2}$

$= 50\sqrt{2}$ m/s

⑥ Ball is project in horizontal direction with 30 m/s; find direction of motion of this ball after $t = 3$ sec and speed at $t = 3$ sec.

Sol^{n}



$\vec{v} = 30\hat{i} - 30\hat{j}$

$|\vec{v}| = \sqrt{(30)^2 + (-30)^2}$

$= 30\sqrt{2}$

$\rightarrow \theta = 45^{\circ}$

THANK YOU ☺

ARJUNA