

WORK POWER & ENERGY

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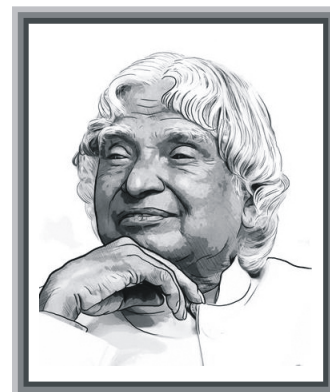
NEET SYLLABUS

Work done by a constant force and variable force; kinetic energy, work-energy theorem, power.

Notion of potential energy, potential energy of a spring, conservative forces; conservation of mechanical energy (kinetic and potential energies); non-conservative forces.

DR. APJ ABDUL KALAM AZAD

He was born in a Tamil Muslim family on 15th October 1931. He was a mediocre student at school and he had average grades, though he used to be highly appreciated for his will to study. He moved in Madras in 1955 to study aerospace engineering in Madras Institute of Technology. His contributions are immense. He was responsible for the development of India's first satellite launch vehicle, the SLV-3, development and operation of strategic missiles and building indigenous capability in critical technologies. He was appointed as the chairman of Technology information, Forecasting and Assessment council (TIFAC). He guided a number of technology projects and missions to take India into the 21st century. APJ Abdul Kalam was elected as the 11th



President of India in 2002 and stayed in his position for 5 years. Abdul Kalam held various positions under ISRO and DRDO and became Principal Scientific Adviser to the Government of India holding the rank of a Cabinet Minister. He has received honorary doctorates from 30 universities and the country's three highest civilian honours – Padma Bhushan (1981), Padma Vibhushan (1990) and Bharat Ratna (1997). This great personality had his last breath while giving a lecture at IIM, Meghalaya on 27th July 2015.

HOMI JAHANGIR BHABHA

He was born on October 30, 1909 in Mumbai. He was a nuclear physicist who laid the foundation for nuclear research in India. Often hailed as the “Father of Indian nuclear programme”, Bhabha was not only a scientist, but also a visionary and an institution builder. From childhood, he was an intelligent and hard working student and his parents dreamed of him becoming a mechanical engineer. However, young Bhabha's interest laid in studying physics and not in becoming an engineer. His parents too respected their son's true interest and supported him in his pursuit of scientific research. Bhabha studied in Europe where he became acquainted with many great physicists of his time, and was determined to contribute to India's scientific research upon his return.



He had very ambitious plans and upon returning to his home country, he set about establishing the Cosmic Ray Research Unit. He helped to formulate India's strategy in the field of nuclear power for which he is fondly remembered as the father of Indian nuclear power. The great scientist's brilliant career was cut short by a plane crash which claimed his life.

WORK, ENERGY & POWER

1. WORK

Whenever a force acting on a body, displaces it, work is said to be done by the force.

Work done by a constant force is equal to the scalar product of the force \vec{F} applied and the displacement \vec{d} of the point of application, $W = \vec{F} \cdot \vec{d}$

Work is a scalar quantity.

1.1 Work done by a constant force

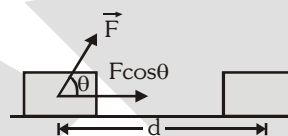
If the direction and magnitude of a force applied on a body is constant then the force is said to be constant.

Work done by a constant force, $W = \text{Force} \times \text{component of displacement along the force}$

$= \text{displacement} \times \text{component of force along the displacement.}$

i.e., work done will, be $W = (F \cos \theta) d$
 $= F (d \cos \theta)$

In vector form, $W = \vec{F} \cdot \vec{d}$



Note : The force of gravity (within small altitudes) is an example of constant force; consequently work done by it is an example of work done by a constant force.

1.2 Work done by a variable force

If the force applied on a body is changing in direction or magnitude or both, the force is said to be variable.

Suppose a variable force causes displacement of a body from position P_1 to position P_2 . To calculate the work done by the force, the path from P_1 to P_2 can be divided into infinitesimal elements; each element is so small that during the displacement of the body through it, the force is supposed to be constant. If $d\vec{r}$ be the small displacement of point of application and \vec{F} be the force acting on the body, the work done by force is $dW = \vec{F} \cdot d\vec{r}$

The total work done in displacing the body from P_1 to P_2 is given by $\int dW = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} \Rightarrow W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$

If \vec{r}_1 and \vec{r}_2 be the position vectors of the points P_1 and P_2 respectively then the total work done $W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$

Cartesian Form

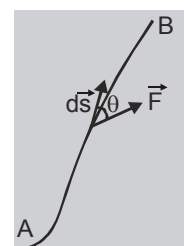
When magnitude and direction of the force varies with position, work done by the force for infinitesimal displacement $d\vec{s}$ is $dW = \vec{F} \cdot d\vec{s}$

The total work done for the displacement from position A to B is $W_{AB} = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B (F \cos \theta) ds$

In terms of rectangular components

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}, \quad d\vec{s} = dx \hat{i} + dy \hat{j} + dz \hat{k}$$

$$W_{AB} = \int_A^B (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) \cdot (dx \hat{i} + dy \hat{j} + dz \hat{k}) = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$



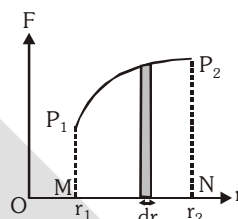
1.3 Calculating work form graph

Suppose a body, whose initial position is \vec{r}_1 , is acted upon by a variable force \vec{F} and consequently the body acquires its final position \vec{r}_2 . From position \vec{r} to $\vec{r} + d\vec{r}$ or for small displacement $d\vec{r}$, the work done will be $\vec{F} \cdot d\vec{r}$ whose value will be the area of the shaded strip of width $d\vec{r}$. The work done on the body in displacing it from position \vec{r}_1 to \vec{r}_2 will be equal to the sum of areas of all such strips

$$\text{Thus, total work done, } W = \sum_{r_1}^{r_2} dW = \sum_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \text{Area of } P_1P_2NM$$

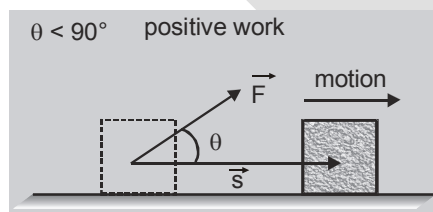
The area of the graph between curve and displacement axis is equal to the work done.

Note : To calculate the work done by graphical method, for the sake of simplicity, here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between $F \cos\theta$ and r .



1.4 Nature of Work Done

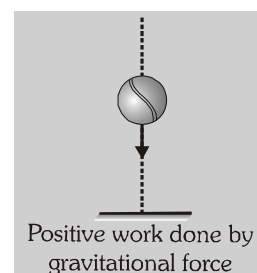
(i) Positive work



$$W = Fs \cos\theta$$

If the angle θ is acute ($\theta < 90^\circ$) then the work is said to be positive. Positive work signifies that the external force favours the motion of the body.

- When a body falls freely under the action of gravity ($\theta = 0^\circ$), the work done by gravity is positive.
- When a spring is stretched, stretching force and the displacement both are in the same direction. So work done by stretching force is positive.



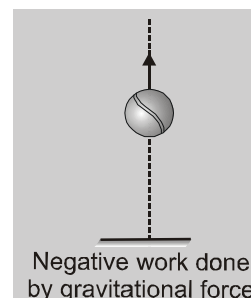
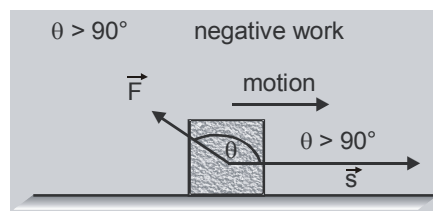
(ii) Negative work

$$W = Fs \cos\theta$$

If the angle θ is obtuse ($\theta > 90^\circ$).

Then the work is said to be negative.

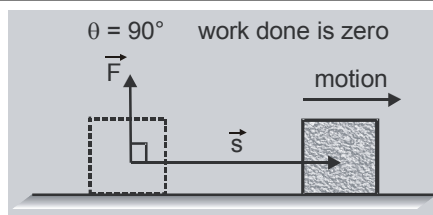
It signifies that the direction of force is such that it opposes the motion of the body.



- Work done by frictional force is negative when it opposes the motion.
- Work done by braking force on the car is negative.

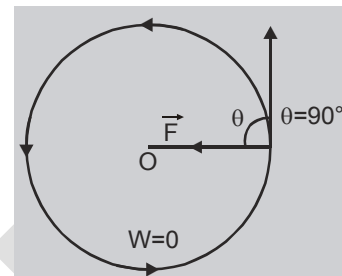
(iii) Zero work

$$W = Fs \cos \theta$$



Work done will be zero if ($F = 0$ or $s = 0$ or $\theta = 90^\circ$)

- body moving with uniform velocity.
- net force on the particle is zero.
- we push the wall and it remains at rest.
- a pendulum is oscillating, work done by tension.
- electron is moving round the nucleus.
- satellite is moving around the earth.
- coolie "Sahayak" is carrying baggage on a horizontal platform (W.D. by gravitational force is zero)



1.5 Dependency on Frame of Reference

A force does not depend on frame of reference and assumes same value in all frame of references, but displacement depends on frame of reference and may assume different values relative to different reference frames. Therefore, work of a force depends on choice of reference frame. For example, consider a man with a suitcase standing in a lift that is moving up. In the reference frame attached to the lift, the man applies a force equal to the weight of the bag but the displacement of the bag is zero, therefore work due to this force on the bag is zero. However, in a reference frame attached to the ground the bag has displacement equal to that of the lift and the force applied by the man does a non-zero work.

1.6 Unit & Dimensions

Unit :

SI Unit : joule (J).

joule : One joule of work is said to be done when a force of one newton displaces a body by one meter in the direction of force.

$$1 \text{ joule} = 1 \text{ newton} \times 1 \text{ meter} = 1 \text{ kg-m}^2/\text{s}^2$$

erg : One erg of work is said to be done when a force of one dyne displaces a body by one centimeter in the direction of force.

$$1 \text{ erg} = 1 \text{ dyne} \times 1 \text{ cm} = 1 \text{ g-cm}^2/\text{s}^2$$

Other Units :

(a) $1 \text{ joule} = 10^7 \text{ ergs}$

(b) $1 \text{ erg} = 10^{-7} \text{ joules}$

(c) $1 \text{ eV} = 1.6 \times 10^{-19} \text{ joules}$

(d) $1 \text{ joule} = 6.25 \times 10^{18} \text{ eV}$

(e) $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$

(f) $1 \text{ J} = 6.25 \times 10^{12} \text{ MeV}$

(g) $1 \text{ kilowatthour (kWh)} = 3.6 \times 10^6 \text{ joules}$

- **Dimensions :** $[\text{Work}] = [\text{Force}] [\text{Displacement}] = [\text{MLT}^{-2}][\text{L}] = [\text{ML}^2\text{T}^{-2}]$.

GOLDEN KEY POINTS

- Work is defined for an interval or displacement; there is no term like instantaneous work similar to instantaneous velocity.
- For a particular displacement, work is independent of time.
- When several forces act, work done by a force, for a particular displacement, is independent of other forces.
- Displacement depends on reference frame so work done by a force is reference frame dependent. Work done by a force can be different in different reference frames.
- Work is done by an energy source or agent that applies the force.
- When $\theta = 0^\circ$, force does maximum positive work.
- When $\theta = 180^\circ$, force does maximum negative work.

Illustrations

Illustration 1.

A position dependent force $F=7-2x+3x^2$ acts on a small body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. Calculate the work done in joules.

Solution

$$W = \int_{x_1}^{x_2} F dx = \int_0^5 (7 - 2x + 3x^2) dx = \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3} \right]_0^5 = 135 \text{ J.}$$

Illustration 2.

Calculate the amount of work done in raising a glass of water weighing 0.5 kg through a height of 20 cm. ($g = 10 \text{ m/s}^2$)

Solution

$$\text{Work done } W = \text{Force} \times \text{displacement} = mgh = 0.5 \times 10 \times 0.2 = 1 \text{ J.}$$

Illustration 3.

A body of mass 20 kg is at rest. A force of 5 N is applied on it. Calculate the work done in the first second.

Solution

$$\text{Displacement in the direction of force is } d = \frac{1}{2} at^2 = \frac{1}{2} \frac{F}{m} t^2 \quad (\because s = ut + \frac{1}{2} at^2)$$

$$\text{so, work done is } W = \frac{1}{2} \frac{F^2}{m} t^2 = \frac{1}{2} \times \frac{5^2}{20} \times 1^2 = \frac{5}{8} \text{ J.}$$

Illustration 4.

Corresponding to the force-displacement diagram shown in adjoining diagram, calculate the work done by the force in displacing the body

from $x=1$ cm to $x=5$ cm.

Solution

$$\begin{aligned} \text{Work} &= \text{Area between the curve and displacement axis} \\ &= 10 + 20 - 20 + 10 = 20 \text{ ergs.} \end{aligned}$$

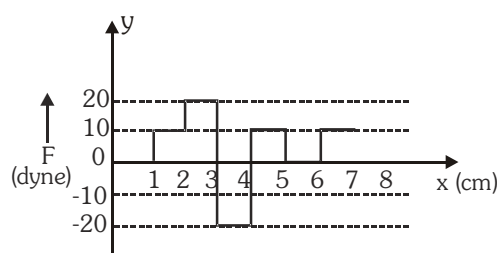


Illustration 5.

Calculate the work done to move a body of mass 10 kg along a smooth inclined plane ($\theta = 30^\circ$) with constant velocity through a distance of 10 m.

Solution

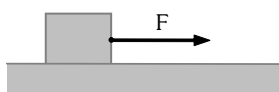
Here the motion is not accelerated, the resultant force parallel to the plane must be zero. So

$$F - Mg \sin 30^\circ = 0 \Rightarrow F = Mg \sin 30^\circ \text{ \& } d = 10 \text{ m}$$

$$W = Fd \cos \theta = (Mg \sin 30^\circ) d \cos 0^\circ = 10 \times 10 \times \frac{1}{2} \times 10 \times 1 = 500 \text{ J.}$$

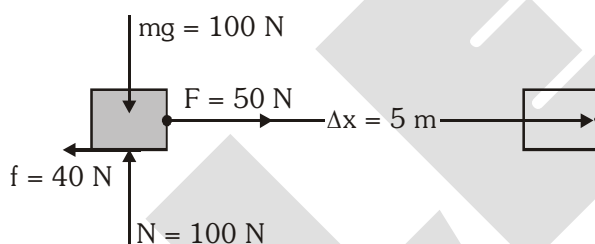
Illustration 6.

A 10 kg block placed on a rough horizontal floor is being pulled by a constant force of 50 N. Coefficient of kinetic friction between the block and the floor is 0.4. Find the work done by each individual force acting on the block over a displacement of 5 m.



Solution.

Forces acting on the block are (i) its weight ($mg = 100 \text{ N}$),



(ii) normal reaction ($N = 100 \text{ N}$) by the ground, (iii) force of kinetic friction ($f = 40 \text{ N}$) and (iv) the applied force ($F = 50 \text{ N}$). All these force and the displacement of the block are shown in the figure.

All these forces are constant forces, therefore we use equation $W_{i \rightarrow f} = \vec{F} \cdot \Delta \vec{r}$.

Work done W_g by the gravity i.e. weight of the block $W_g = 0 \quad (\because m\vec{g} \perp \Delta \vec{x})$

Work done W_N by the normal reaction $W_N = 0 \quad (\because \vec{N} \perp \Delta \vec{x})$

Work done W_F by the applied force $W_F = 250 \text{ J} \quad (\because \vec{F} \parallel \Delta \vec{x})$

Work done W_f by the force of kinetic friction $W_f = -200 \text{ J.} \quad (\because \vec{f} \uparrow \downarrow \Delta \vec{x})$

Illustration 7.

Calculate the work done by the force $\vec{F} = (3\hat{i} - 2\hat{j} + 4\hat{k}) \text{ N}$ in carrying a particle from point $(-2 \text{ m}, 1 \text{ m}, 3 \text{ m})$ to point $(3 \text{ m}, 6 \text{ m}, -2 \text{ m})$.

Solution.

The force \vec{F} is a constant force, therefore we can use equation $W_{i \rightarrow f} = \vec{F} \cdot \Delta \vec{r}$.

$$W = \vec{F} \cdot \Delta \vec{r} = (3\hat{i} - 2\hat{j} + 4\hat{k}) \cdot (5\hat{i} + 5\hat{j} - 5\hat{k}) = -15 \text{ J.}$$

Illustration 8.

A particle is shifted from point $(0,0,1\text{ m})$ to point $(1\text{ m}, 1\text{ m}, 2\text{ m})$, under the simultaneous action of several forces. Two of them are $\vec{F}_1 = (2\hat{i} + 3\hat{j} - \hat{k})\text{ N}$ and $\vec{F}_2 = (\hat{i} - 2\hat{j} + 2\hat{k})\text{ N}$. Find the work done by the resultant of these two forces.

Solution.

Work done by a constant force equals to dot product of the force and displacement vectors.

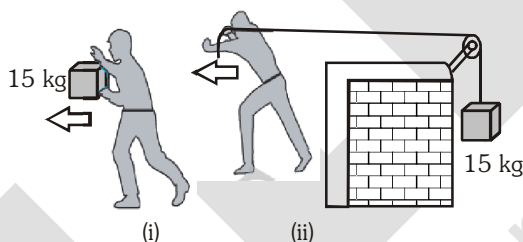
$$W = \vec{F} \cdot \Delta\vec{r} \Rightarrow W = (\vec{F}_1 + \vec{F}_2) \cdot \Delta\vec{r}$$

Substituting the given values, we have

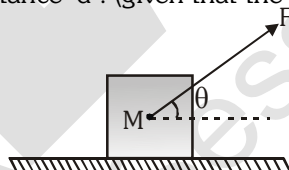
$$W = (3\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 3 + 1 + 1 = 5\text{ J.}$$

BEGINNER'S BOX-1

1. A body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done is 25 joules, find the angle which the force makes with the direction of motion of the body.
2. In Fig. (i), the man walks 2 m carrying a mass of 15 kg on his hand. In Fig. (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done by gravity is greater?

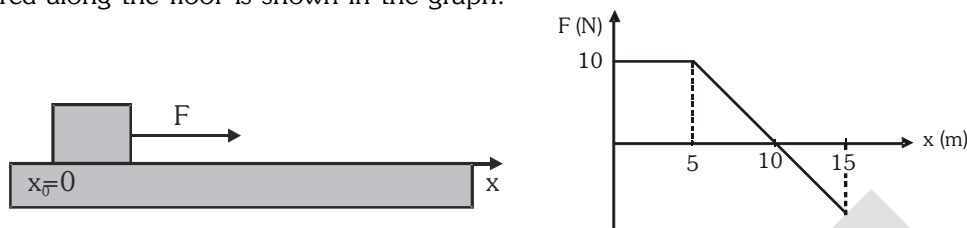


3. Calculate the work done in pulling an object with a constant force F as shown in figure, so that the block moves with a constant velocity through a distance 'd'. (given that the ground is rough with coefficient of friction μ)



4. A particle moves along the x-axis from $x = x_1$ to $x = x_2$ under the influence of a force given by $F = 2x$. Find the work done in the process.
5. A force $F = (10 + 0.5x)\text{ N}$ acts on a particle in x direction, where x is in metres. Find the work done by this force during a displacement from $x = 0$ to $x = 2$.
6. A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\text{ m}^{-1/2}\text{s}^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2\text{ m}$?
7. A particle moves from position $\vec{r}_1 = (3\hat{i} + 2\hat{j} - 6\hat{k})\text{ m}$ to position $\vec{r}_2 = (14\hat{i} + 13\hat{j} + 9\hat{k})\text{ m}$ under the action of force $(4\hat{i} + \hat{j} + 3\hat{k})\text{ N}$. Find the work done.
8. $\vec{F} = K(x\hat{i} + y\hat{j})$ is working on a particle which moves from $(0, 0)$ to $(a, 0)$ along x axis and $(a, 0)$ to (a, a) along y axis. Find the work done by the force.

- 9.** Three forces $3(\hat{i} + 3\hat{j} + \hat{k})$, $\frac{5}{7}(-2\hat{i} + 9\hat{k})$ and $11(2\hat{i} + \hat{j} + 6\hat{k})$ are acting on a particle. Calculate the work done in displacing the particle from point (4, -1, 1) to point (11, 6, 8).
- 10.** A horizontal force F is used to pull a box placed on a floor. Variation in the force with position coordinate x measured along the floor is shown in the graph.



- Calculate the work done by the force in moving the box from $x=0$ to $x=10$ m.
- Calculate the work done by the force in moving the box from $x=10$ m to $x=15$ m.
- Calculate the work done by the force in moving the box from $x=0$ to $x=15$ m.

2. ENERGY

Energy is defined as the internal capacity to do work.

When we say that a body has energy it means that it can do work.

2.1 Different forms of energy

Mechanical energy, electrical energy, optical (light) energy, acoustical (sound), molecular, atomic and nuclear energy etc, are various forms of energy.

These forms of energy can change from one form to the other.

2.2 Mass energy relation

According to Einstein's mass-energy equivalence principle, mass and energy are inter convertible i.e. they can be changed into each other.

Equivalent energy corresponding to mass m is $E = mc^2$

where, m : mass of the particle

c : speed of light

- Energy is a scalar quantity
- Dimensions : $[M^1L^2T^{-2}]$
- SI Unit : joule

Other units $1 \text{ erg} = 10^{-7} \text{ J}$

$$1\text{kWh} = 36 \times 10^5 \text{ J}$$
$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$
$$1 \text{ cal} = 4.2 \text{ J}$$

In mechanics we are only concerned with the mechanical energy, which is of two types.

- (a) Kinetic energy (b) Potential energy

2.3 Kinetic Energy

Kinetic energy is the internal capacity of doing work of an object by virtue of its motion.

or

K.E. of a body can be calculated by the amount of work done in stopping the moving body or by the amount of work done in imparting the present velocity to the body from the state of rest.

If a particle of mass m is moving with velocity ' v ' much less than the velocity of light, then the kinetic energy

K.E. is given by $\text{K.E.} = \frac{1}{2}mv^2$

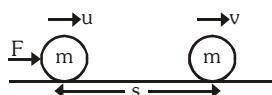
2.4 Work-Kinetic Energy Theorem

Work done by all the forces (conservative or non conservative, external or internal) acting on a particle or an object is equal to the change in its kinetic energy. So work done by all the forces = change in kinetic energy

$$W = \Delta KE = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$$

Proof :

- (i) For constant force :



$$v^2 = u^2 + 2as$$

$$\Rightarrow v^2 - u^2 = 2\left(\frac{F}{m}\right)s$$

$$\Rightarrow s = \frac{(v^2 - u^2)m}{2F} = \frac{\Delta KE}{F}$$

$$\Rightarrow Fs = \Delta KE$$

$$\Rightarrow W = \Delta KE$$

- (ii) For variable force :

$$W = \int F dx = \int m a dx = \int m \left(\frac{v dv}{dx} \right) dx = \int_u^v m v dv = m \left[\frac{v^2}{2} \right]_u^v = \frac{m}{2} [v^2 - u^2]$$

$$\Rightarrow W = \Delta KE$$

2.5 How to apply work-kinetic energy theorem

The work-kinetic energy theorem is deduced here for a single body moving relative to an inertial frame, therefore it is recommended at present, to use it for a single body in inertial frame. To use work-kinetic energy theorem the following steps should be followed :

- Identify the initial and final positions as position 1 and 2 and write expressions for kinetic energies, whether known or unknown.
- Draw the free body diagram of the body at any intermediate stage between positions 1 and 2. The forces shown will help in deciding their work. Calculate work by each force and add them to obtain the total work done $W_{1 \rightarrow 2}$ by all the forces.
- Use the work obtained in step 2 and kinetic energies obtained in step 1 in the equation $W_{1 \rightarrow 2} = K_2 - K_1$.

GOLDEN KEY POINTS

- If K.E. of the body decreases then work done is negative & vice-versa.
- In the above discussion, we have assumed that the work done by the force is effective only in changing the kinetic energy of the body. It should however be remembered that the work done on a body may also be stored in the system in the form of potential energy.
- As mass m and v^2 or $\vec{v} \cdot \vec{v}$ are always positive so K.E. can never be negative.
- The kinetic energy depends on the frame of reference.
- The expression $KE = \frac{1}{2}mv^2$ holds even when the force applied varies in magnitude or in direction or in both.

Illustrations

Illustration 9.

A body of mass 0.8 kg has initial velocity $(3\hat{i} - 4\hat{j})$ m/s and final velocity $(-6\hat{j} + 2\hat{k})$ m/s. Find the change in kinetic energy of the body ?

[AIPMT (Mains) 2006]

Solution

Change in kinetic energy

$$\begin{aligned}\Delta KE &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \text{ where } v_f = \sqrt{6^2 + 2^2} = \sqrt{40} \text{ and } v_i = \sqrt{3^2 + 4^2} = \sqrt{25} \\ &= \frac{1}{2} \times 0.8 \left((\sqrt{40})^2 - (\sqrt{25})^2 \right) = 0.4 [40 - 25] = 0.4 (15) = 6 \text{ joules.}\end{aligned}$$

Illustration 10.

A 5 kg ball when falls through a height of 20 m acquires a speed of 10 m/s. Find the work done by air resistance.

Solution.

The ball starts falling from position 1, where its speed is zero; hence, kinetic energy is also zero.

$$K_1 = 0 \text{ J} \quad \dots(i)$$

During the downward motion of the ball, constant gravitational force mg acts downwards and air resistance R of unknown magnitude acts upwards as shown in the free body diagram. The ball reaches position 20 m below the position-1 with a speed $v = 10$ m/s, so the kinetic energy of the ball at position 2 is

$$K_2 = \frac{1}{2}mv^2 = 250 \text{ J} \quad \dots(ii)$$

Work done by gravity

$$W_{g,1 \rightarrow 2} = mgh = 1000 \text{ J} \quad \dots(iii)$$

Denoting the work done by the air resistance as $W_{R,1 \rightarrow 2}$ and making use of eq. (i), (ii) and (iii) in work-kinetic energy theorem, we have $W_{1 \rightarrow 2} = K_2 - K_1 \Rightarrow W_{g,1 \rightarrow 2} + W_{R,1 \rightarrow 2} = K_2 - K_1 \Rightarrow W_{R,1 \rightarrow 2} = -750 \text{ J}.$

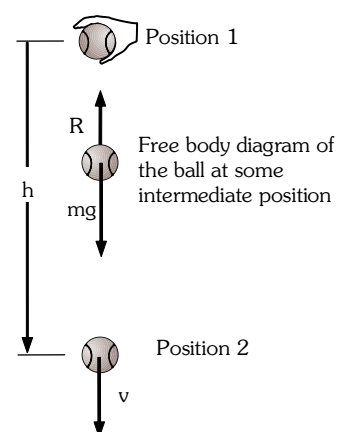
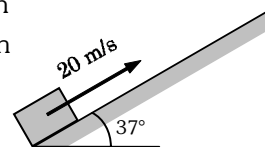
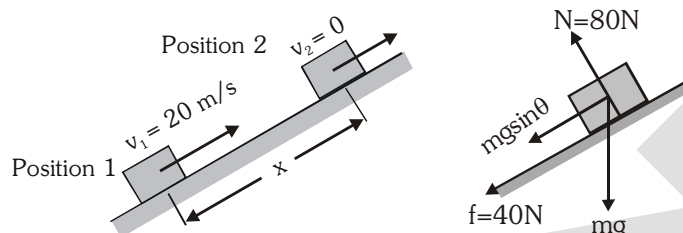


Illustration 11.

A box of mass $m = 10 \text{ kg}$ is projected up an inclined plane from its foot with a speed of 20 m/s as shown in the figure. The coefficient of friction μ between the box and the plane is 0.5 . Find the distance travelled by the box on the plane before it stops for the first time.

**Solution.**

The box starts from position 1 with speed $v_1 = 20 \text{ m/s}$ and stops at position 2.



Kinetic energy at position 1: $K_1 = \frac{1}{2}mv_1^2 = 2000 \text{ J}$

Kinetic energy at position 2: $K_2 = 0$

Work done by external forces as the box moves from position 1 to position 2 is,

$$W_{1 \rightarrow 2} = W_{g,1 \rightarrow 2} + W_{f,1 \rightarrow 2} = -60x - 40x = -100x \text{ J}$$

Applying work energy theorem for the motion of the box from position 1 to position 2, we have

$$W_{1 \rightarrow 2} = K_2 - K_1 \Rightarrow -100x = 0 - 2000 \Rightarrow x = 20 \text{ m.}$$

Illustration 12.

Kinetic energy of a particle is increased by 300%. Find the percentage increase in its momentum.

Solution

Kinetic energy $E = \frac{1}{2}mv^2$ momentum $p = mv$

When E is increased by 300%, $E' = E + 3E = 4E = 4\left(\frac{1}{2}mv^2\right) = 2mv^2$

If v' is the new velocity of the body, then $\frac{1}{2}mv'^2 = 2mv^2 \Rightarrow v' = 2v$ So $p' = mv' = 2mv$

Hence, percentage change in momentum $= \frac{2mv - mv}{mv} \times 100 = 100\%$.

Illustration 13.

A bullet weighing 10 g is fired with a velocity 800 m/s . Its velocity decreases to 100 m/s after passing through a 1 m thick mud wall. Find the average resistance offered by the mud wall.

Solution

Work done by the average resistance (offered by the wall) = change in K.E. of the bullet.

$$F_s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \Rightarrow F = \frac{m(v^2 - u^2)}{2s} = \frac{0.01(800^2 - 100^2)}{2 \times 1} = -3150 \text{ N}$$

$$\Rightarrow \text{Resistance offered} = 3150 \text{ N.}$$

Illustration 14.

In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with a speed of 200 m/s on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

Solution

$$\text{Initial kinetic energy, } K_i = \frac{1}{2} \times \frac{50}{1000} \times 200 \times 200 = 1000 \text{ J}$$

$$\text{Final kinetic energy, } K_f = \frac{10}{100} \times 1000 = 100 \text{ J}$$

If v_f is emergent speed of the bullet, then

$$\frac{1}{2} \times \frac{50}{1000} \times v_f^2 = 100 \Rightarrow v_f^2 = 4000 \Rightarrow v_f = 63.2 \text{ m/s.}$$

Note that the speed is reduced by approximately 68% and not 90%.

Illustration 15.

A particle of mass m moves with velocity $v = a\sqrt{x}$ where a is a constant. Find the total work done by all the forces during a displacement from $x = 0$ to $x = d$.

Solution

$$\text{Work done by all forces} = W = \Delta KE = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$$

$$\text{Here } v_1 = a\sqrt{0} = 0, v_2 = a\sqrt{d}, \text{ So } W = \frac{1}{2} ma^2d - 0 = \frac{1}{2} ma^2d.$$

BEGINNER'S BOX-2

1. If the linear momentum of a body is increased by 50%, then by what percentage will its kinetic energy increase?
2. The kinetic energy of a body is numerically equal to thrice the momentum of the body. Find the velocity of the body.
3. A body of mass 10 kg is released from the top of a tower which acquires a velocity of 10 m/s after falling through the distance of 20 m. Calculate the work done by the drag force of the air on the body? (take $g = 10 \text{ m/s}^2$)
[AIPMT(Mains) 2008]
4. The displacement x of a body of mass 1 kg on smooth horizontal surface as a function of time t is given by $x = \frac{t^3}{3}$ (where x is in metres and t is in seconds). Find the work done by the external agent for the first one second.

3. CONSERVATIVE, NON-CONSERVATIVE & CENTRAL FORCE

3.1 Conservative force

- A force is said to be conservative if the work done by the force is independent of the path and depends only on initial and final positions.
- It does not depend on the nature of the path followed between the initial and the final positions.

Examples of Conservative Force

All central forces are conservative like Gravitational, Electrostatic, Elastic force, Restoring force due to spring, Intermolecular force etc.

3.2 Central Force

A force whose line of action always passes through a fixed point (which is known as centre of force) and whose magnitude depends only on the distance from this point is known as **central force**.

$$\vec{F} = F(r)\hat{r}$$

All forces following inverse square law are called central forces.

$$\vec{F} = \frac{k}{r^2}\hat{r} \text{ is a central force such as Gravitational force and Coulomb force.}$$

- All central forces are conservative forces.
- Central forces are functions of position only.

3.3 Non Conservative Force

A force is said to be non-conservative if work done by the force in moving a body depends upon the path between the initial and final positions.

Work done in a closed path is not zero in a non-conservative force field.

Frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which the body is moved. It does not depend on the initial and final positions. The work done by frictional force in a round trip is not zero.

Examples of non-conservative force

All velocity-dependent forces such as air resistance, viscous force are non-conservative forces.

3.4 Difference Between Conservative forces & Non-conservative Forces

Conservative Forces	Non-Conservative Forces
<ul style="list-style-type: none"> • Work done does not depend upon path. • Work done in a round trip is zero e.g. gravitational force • Central forces, spring forces etc. are conservative forces • When a conservative force acts within a system, the kinetic energy and potential energy can change. However, their sum, the mechanical energy of the system, does not change. • Work done is completely recoverable. 	<ul style="list-style-type: none"> • Work done depends upon path. • Work done in a round trip is not zero e.g. friction • Forces which are velocity-dependent in nature e.g. dragging force, viscous force, etc • Work done against a non-conservative force may be dissipated as heat energy. • Work done is not completely recoverable.

4. POTENTIAL ENERGY

4.1 Potential Energy

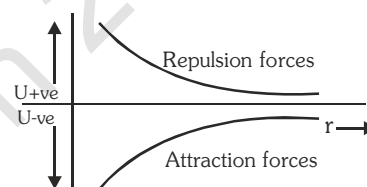
- The energy possessed by a body by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Potential energy of a body at any position in a conservative force field is defined as the external work done against the action of conservative force in order to shift it from a certain reference point (PE = 0) to the present position.
- Potential energy of a body in a conservative force field is equal to the work done by the conservative force in moving the body from its present position to reference position.
- At a certain reference position, the potential energy of the body is assumed to be zero.
- Relationship between conservative force field and potential energy :

$$\vec{F} = -\nabla U = -\text{grad}(U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

- If force varies with only one dimension (say along x-axis) then $F = -\frac{dU}{dx} \Rightarrow dU = -Fdx$

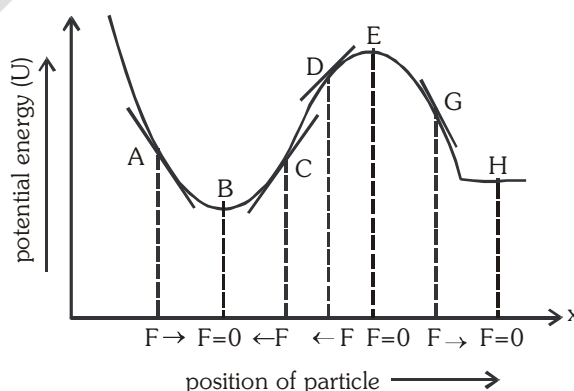
$$\Rightarrow \int_{U_1}^{U_2} dU = -\int_{x_1}^{x_2} Fdx \Rightarrow \boxed{\Delta U = -W_C}$$

- Potential energy may be positive or negative or even zero
 - i) Potential energy is positive, if force field is repulsive in nature
 - ii) Potential energy is negative, if force field is attractive in nature
- If $r \uparrow$ (separation between body and force centre), $U \uparrow$, force field is attractive or vice-versa.
- If $r \uparrow$, $U \downarrow$, force field is repulsive in nature.



4.2 Potential energy curve and equilibrium

It is a curve which shows the change in potential energy with the position of a particle.



- **Stable Equilibrium :**

After a particle is slightly displaced from its equilibrium position if it tends to come back towards equilibrium then it is said to be in stable equilibrium.

At point **A** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **C** : slope $\frac{dU}{dx}$ is positive so F is negative

At equilibrium $F = -\frac{dU}{dx} = 0$

At point **B** : it is the point of stable equilibrium.

At point **B** : $U = U_{\min}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} = \text{positive}$

- **Unstable equilibrium :**

After a particle is slightly displaced from its equilibrium position, if it tends to move away from equilibrium position then it is said to be in unstable equilibrium.

At point **D** : slope $\frac{dU}{dx}$ is positive so F is negative ; At point **G** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **E** : it is the point of unstable equilibrium ; At point **E** $U = U_{\max}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} = \text{negative}$

- **Neutral equilibrium :**

After a particle is slightly displaced from its equilibrium position if no force acts on it then the equilibrium is said to be neutral equilibrium.

Point **H** corresponds to neutral equilibrium $\Rightarrow U = \text{constant}$; $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = 0$.

5. LAW OF CONSERVATION OF ENERGY

Energy can neither be created nor be destroyed, it only can be converted from one form to another.

5.1 Conservation of mechanical energy

The total potential energy of a system and the total kinetic energy of all the constituent bodies together is known as the mechanical energy of the system. If E , K and U respectively denote the total mechanical energy, total kinetic energy, and the total potential energy of a system in any configuration then we have

$$E = K + U$$

Consider a system on which no external force acts and all the internal forces are conservative. If we apply work-kinetic energy ($W_{1 \rightarrow 2} = K_2 - K_1$) theorem, the work $W_{1 \rightarrow 2}$ will be the work done by internal conservative forces, negative of which equals the change in potential energy. Rearranging the kinetic energy and potential energy terms, we have

$$E = K_1 + U_1 = K_2 + U_2$$

The above equation takes the following form

$$E = K + U = \text{constant}$$

$$\Delta E = 0 \Rightarrow \Delta K + \Delta U = 0.$$

Above equations, express the principle of conservation of mechanical energy.

If there is no net work done by any external force or any internal non-conservative force, then the total mechanical energy of a system is conserved.

The principle of conservation of mechanical energy is developed from the work-kinetic energy principle for systems where change in configuration takes place under internal conservative forces only. Therefore, in physical situations, where external forces or non-conservative internal forces are involved, the use of work-kinetic energy principle should be preferred.

In systems, where external forces or internal nonconservative forces do work, the net work done by these forces becomes equal to the change in the mechanical energy of the system.

5.2 Application to gravitational field

Let a ball of mass m be dropped from position (3) (as shown in figure)

At point 1

$$PE = 0; \quad KE = \frac{1}{2}mv^2 \quad \because v = \sqrt{2gh} \quad \text{so} \quad KE = mgh$$

At point 2

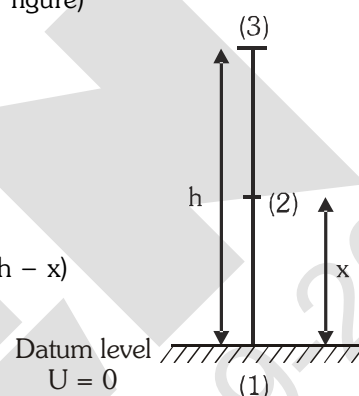
$$PE = mgx; \quad KE = \frac{1}{2}mv^2 \quad \because v = \sqrt{2g(h-x)} \quad \text{so} \quad KE = mg(h-x)$$

At point 3

$$PE = mgh; \quad KE = 0$$

so during the motion of the ball at any position

$$\boxed{ME_{(1)} = ME_{(2)} = ME_{(3)}} \quad \text{and} \quad \boxed{PE = mgx} ; \quad \boxed{KE = mg(h-x)}$$



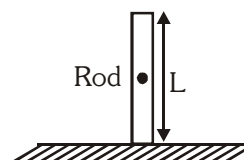
GOLDEN KEY POINTS

- In presence of conservative forces mechanical energy remains conserved.
- Work done along a closed path is zero. i.e. $\oint \vec{F} \cdot d\vec{r} = 0$
- Potential energy depends on the frame of reference but change in potential energy is independent of reference frame.
- Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.
- It is a function of position and does not depend on the path.
- Whenever work is done by the conservative forces the potential energy decreases and whenever work is done against the conservative forces, potential energy increases.
- For regularly shaped uniform bodies, the potential energy change can be calculated by considering their mass to be centered at the geometrical centre point.

For Example :

For a uniform vertical rod of length L

$$PE = mg \frac{L}{2}$$



Illustrations

Illustration 16.

Force between the atoms of a diatomic molecule has its origin in the interaction between the electrons and the nuclei present in each atom. This force is conservative and associated potential energy $U(r)$ is, to a good approximation, represented by the Lennard – Jones potential function.

$$U(r) = U_0 \left\{ \left(\frac{a}{r} \right)^{12} - \left(\frac{a}{r} \right)^6 \right\}$$

Here r is the distance between the two atoms and U_0 and a are positive constants. Develop expression for the associated force and find the equilibrium separation between the atoms.

Solution. Using equation $F = -\frac{dU}{dr}$, we obtain the expression for the force

$$F = \frac{6U_0}{a} \left\{ 2 \left(\frac{a}{r} \right)^{13} - \left(\frac{a}{r} \right)^7 \right\}$$

At equilibrium, force must be zero. Therefore the equilibrium separation is $r_0 = 2^{\frac{1}{5}}a$

Illustration 17.

The potential energy for a conservative force system is given by $U = ax^2 - bx$, where a and b are constants. Find out the (a) expression for force, (b) equilibrium position and (c) potential energy at equilibrium.

Solution (a) For conservative force $F = -\frac{dU}{dx} = -(2ax - b) = -2ax + b$

(b) At equilibrium $F = 0 \Rightarrow -2ax + b = 0 \Rightarrow x = \frac{b}{2a}$

(c) $U = a \left(\frac{b}{2a} \right)^2 - b \left(\frac{b}{2a} \right) = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$

Illustration 18.

The potential energy of a particle of mass 1 kg free to move along the x -axis is given by $U(x) = \left(\frac{x^2}{2} - x \right)$ joules.

If total mechanical energy of the particle is 2 J, then find its maximum speed.

Solution Potential energy $U = \left(\frac{x^2}{2} - x \right) = \frac{x^2}{2} - x$

For minimum U , $\frac{dU}{dx} = \frac{2x}{2} - 1 = 0$ and $\frac{d^2U}{dx^2} = 1 = \text{positive}$

so at $x = 1$, U is minimum. Hence $U_{\min} = -\frac{1}{2}$ J.

Total mechanical energy = Max KE + Min PE

$$\Rightarrow \text{Max KE} = \frac{1}{2}mv_{\max}^2 = 2 - \left(-\frac{1}{2} \right) = \frac{5}{2} \Rightarrow v_{\max} = \sqrt{\frac{2}{1} \times \frac{5}{2}} = \sqrt{5} \text{ ms}^{-1}$$

Illustration 19.

A body is dropped from height 8 m. After striking the surface it rises to 6 m, what is the fractional loss in kinetic energy during impact? Assuming the frictional resistance to be negligible.

(A) 1/2

(B) 1/4

(C) 1/6

(D) 1/8

Solution

Ans. (B)

$$\begin{aligned} \text{Fractional loss in kinetic energy} &= \frac{\text{loss in kinetic energy}}{\text{initial kinetic energy}} \\ &= \frac{\text{loss in potential energy}}{\text{initial potential energy}} \\ &= \frac{mg(8-6)}{mg(8)} = \frac{2}{8} = \frac{1}{4} \end{aligned}$$

Illustration 20.

A chain of mass m and length L is held on a frictionless table in such a way that $\frac{1}{n}$ th part is hanging below the edge of table. Calculate the work done to pull the hanging part of the chain. [AIPMT (Mains) 2008]

Solution

Required work done = change in potential energy of chain

Now, let Potential energy (U) = 0 at table level

so potential energies of chain initially and finally are respectively

$$U_i = -mg\left(\frac{L}{2n}\right) = -\left(\frac{M}{L}\right)\frac{L}{n}g\left(\frac{L}{2n}\right) = -\frac{MgL}{2n^2}, U_f = 0$$

$$\therefore \text{required work done} = U_f - U_i = \frac{MgL}{2n^2}$$

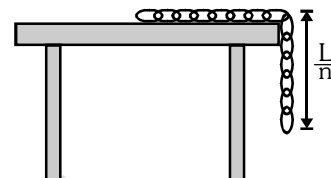
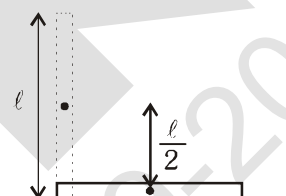


Illustration 21.

A rod of mass ' m ' and length ' ℓ ' is placed in a horizontal position. Find the work done by the external force against gravity to alter its configuration from horizontal position to vertical position.



Solution.

Displacement between the positions of center of mass of rod is $\frac{\ell}{2}$ (in a direction parallel to the force of gravity)

$$W = -\text{change in PE of the rod} = mg\frac{\ell}{2}.$$

BEGINNER'S BOX-3

1. Match the following items -

- | | |
|----------------------------|-------------------------------------|
| (A) Conservative force | (p) Mechanical energy = constant |
| (B) Non conservative force | (q) Kinetic energy = constant |
| | (r) Work done in a closed path = 0 |
| | (s) $F = -\frac{dU}{dx}$ |
| | (t) Potential energy is not defined |

2. The kinetic energy of a particle increases continuously with time. Then select correct alternative.

- (A) the magnitude of its linear momentum is increasing continuously.
 (B) its height above the ground must continuously decrease.
 (C) the work done by all forces acting on the particle must be positive.
 (D) the resultant force on the particle must be parallel to the velocity at all times.

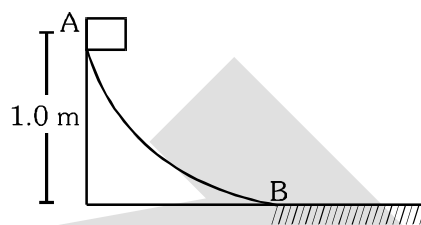
3. A particle moves in one dimensional field with total mechanical energy E . If its potential energy is $U(x)$, then

- (A) particle has zero speed where $U(x) = 0$ (B) particle has zero acceleration where $U(x) = E$
 (C) particle has zero velocity where $\frac{dU(x)}{dx} = 0$ (D) particle has zero acceleration where $\frac{dU(x)}{dx} = 0$

4. Match the following items –

Column-I		Column-II	
A	Work done by all the forces	p	Change in mechanical energy
B	Work done by internal conservative forces	q	Change in kinetic energy
C	Work done by external forces	r	Negative of change in potential energy

5. A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface. The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, then it would move a distance S on the rough surface. Calculate the value of S [$g = 10 \text{ m/s}^2$]



6. SPRING POTENTIAL ENERGY AND SPRING - BLOCK SYSTEM

6.1 Work done by Spring force

$$W = \int_{x_1}^{x_2} F \cos \theta dx$$

- Work required to stretch a spring by a length x

$$x_1 = 0 \quad x_2 = x \quad \text{and} \quad \theta = 0^\circ$$

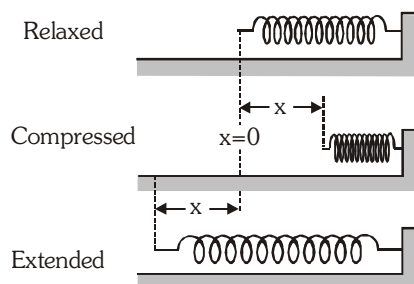
$$W = \int_0^x F dx \quad \because F = kx \quad W = \int_0^x kx dx \quad \boxed{W = \frac{1}{2} kx^2}$$

- Work required to stretch a spring from a length x_1 to x_2 :

$$\text{Then } x_1 = x_1 \quad x_2 = x_2 \quad \text{and} \quad \theta = 0^\circ$$

$$W = \int_{x_1}^{x_2} F dx \quad \because F = kx \quad W = \int_{x_1}^{x_2} kx dx \quad \boxed{W = \frac{1}{2} k(x_2^2 - x_1^2)}$$

6.2 Potential energy associated with spring force



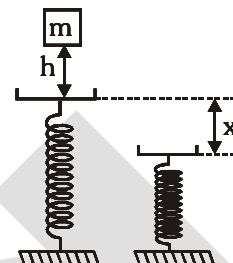
The potential energy of an ideal spring associated with a spring force when compressed or elongated by a distance x from its natural length is given by the following expression $U = \frac{1}{2} kx^2$

GOLDEN KEY POINTS

- A particle of mass m is freely released from a height h on a spring; if the spring gets compressed by x then :

$$mgh + 0 \text{ (Elastic potential energy)} + 0 \text{ (Kinetic energy)} = -mgx + \frac{1}{2}kx^2 + 0 \text{ (Kinetic energy)}$$

$$mg(h+x) = \frac{1}{2}kx^2$$



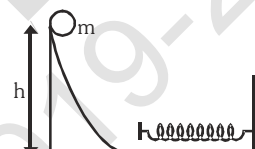
- If a body of mass m moving with speed v collides with a spring. The spring gets compressed by x_{\max} , then

$$\frac{1}{2}mv^2 = \frac{1}{2}kx_{\max}^2 \Rightarrow \sqrt{\frac{m}{k}}v = x_{\max}$$



- If a body is released from a height h and collides with a spring.

$$mgh = \frac{1}{2}kx_{\max}^2 \Rightarrow \sqrt{\frac{2mgh}{k}} = x_{\max}$$



- If a block is attached to the lower end of a spring hanging from a fixed support

- Equilibrium condition:

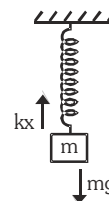
$$F_{\text{net}} = 0$$

$$mg - kx = 0 \Rightarrow \frac{mg}{k} = x$$

- Maximum extension:**

Decrease in G.P.E. of the block = increase in P.E. of the spring

$$mgx_{\max} = \frac{1}{2}kx_{\max}^2 \Rightarrow \frac{2mg}{k} = x_{\max}$$

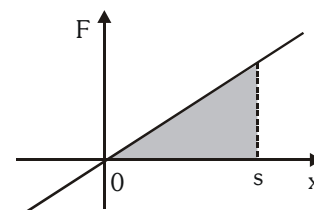


- When a spring is compressed or elongated from its natural length by the same amount x then the W.D. by spring force = $-\frac{kx^2}{2}$.

- When spring is elongated or compressed from its natural length by the same amount x the W.D. by external force = $\frac{+kx^2}{2}$.

- The variation in F with extension x in the spring is linear therefore area under the force-extension graph can easily be calculated. This area equals to the work done by the applied force. The graph showing variation in F with x is shown in the adjoining figure.

$$W_F = \int_0^s F dx = \text{Area of the shaded portion} = \frac{1}{2}ks^2$$



Illustrations

Illustration 22.

A spring of force constant 100 N/m is stretched by 5 cm. Find the work done by applied force.

Solution

Spring is stretched by a length $\ell = 5 \text{ cm} = 0.05 \text{ m}$

Work done for small displacement dx is $dW = Fdx = kx dx$

$$\text{Total work done is } W = \int_0^\ell Fdx = \int_0^\ell kx dx = \frac{1}{2}k\ell^2 = \frac{1}{2}100(.05)^2 = 0.125 \text{ J}$$

Illustration 23.

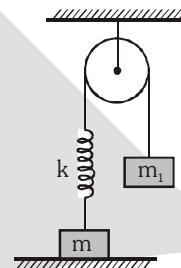
For what minimum value of m_1 will the block of mass m just leave contact with the surface?

Solution

Let extension in the spring be x_0 due to m_1

$$\text{then } m_1 g x_0 = \frac{1}{2} k x_0^2 \Rightarrow k x_0 = 2 m_1 g$$

$$\text{but } k x_0 \geq mg \text{ so } 2 m_1 g \geq mg \Rightarrow m_1 \geq \frac{m}{2}; \text{ therefore minimum value of } m_1 = \frac{m}{2}.$$

**Illustration 24.**

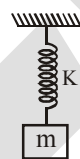
A block of mass m is lowered slowly from the natural length position of a massless spring by an external agent to equilibrium position. The extension produced in the spring was asked to two students.

$$\text{Student A : } \frac{1}{2} k x^2 = mgx \quad \therefore x = \frac{2mg}{k}$$

$$\text{Student B : } mg = kx \quad \therefore x = \frac{mg}{k}$$

(A) Student A is incorrect, Student B is correct.

(C) Both are correct



(B) Student A is correct, Student B is incorrect.

(D) Both are incorrect

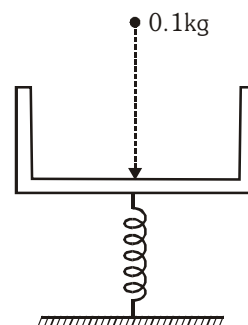
Solution

In equilibrium position, $mg = kx \Rightarrow x = \frac{mg}{k}$ so student A is incorrect.

Ans. (A)

Illustration 25.

A massless platform is kept on a light elastic spring as shown in Fig. When a sand particle of 0.1 kg mass is dropped on the pan from a height of 0.24 m, the particle strikes the pan (in a perfectly inelastic manner) and the spring compresses by 0.01 m. From what height should the particle be dropped so as to generate a compression of 0.04 m?

**Solution**

$$mg(h + x) = \frac{1}{2} k x^2 \dots\dots(1) \quad \& \quad mg(h' + x') = \frac{1}{2} k x'^2 \dots\dots(2)$$

Divide (1) by (2)

$$\frac{h+x}{h'+x'} = \frac{x^2}{x'^2} \Rightarrow \frac{0.24+0.01}{h'+0.04} = \left(\frac{0.01}{0.04}\right)^2 \Rightarrow \frac{0.25}{h'+0.04} = \frac{1}{16} \Rightarrow h' + 0.04 = 4 \Rightarrow h' = 4 - 0.04 = 3.96 \text{ m.}$$

Illustration 26.

A block strikes the free end of a horizontal spring with the other end fixed, placed on a smooth surface with a speed v . After compressing the spring by x , the speed of the block reduces to half. Calculate the maximum compression of the spring.

Solution

Let maximum compression be x_{\max} then by COME

$$\frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{2}\right)^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}kx^2}{\frac{1}{2}kx_{\max}^2} \Rightarrow \frac{3}{4} = \frac{x^2}{x_{\max}^2} \Rightarrow x_{\max} = \frac{2}{\sqrt{3}}x.$$

Illustration 27.

If in the above question KE of the block reduces to half, then find the out maximum compression of the spring.

Solution

$$KE - \frac{KE}{2} = \frac{1}{2}kx^2 \Rightarrow \frac{KE}{2} = \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}kx_{\max}^2 \Rightarrow kx^2 = \frac{1}{2}kx_{\max}^2 \Rightarrow x_{\max} = \sqrt{2}x.$$

Illustration 28.

A spring is initially compressed by x and then, it is further compressed by y . Find out the work done during the latter compression. (spring constant is k .)

Solution

$$W_1 = \frac{1}{2}kx^2; \quad W_2 = \frac{1}{2}k(x+y)^2$$

$$W.D. = W_2 - W_1 = \frac{1}{2}k(x^2 + y^2 + 2xy) - \frac{1}{2}kx^2$$

$$\frac{1}{2}k(y^2 + 2xy) \Rightarrow \frac{1}{2}ky(y + 2x)$$

7. POWER

7.1 Average Power & Instantaneous Power

- When we purchase a car or jeep we are interested in the horsepower of its engine. We know that, usually an engine with large horsepower is most effective in accelerating the automobile.

In many cases, it is useful to know not just the total amount of work being done, but also how fast the work is being done. We define power as the rate at which work is being done.

$$\text{Average Power} = \frac{\text{Total Work done}}{\text{Time taken to do work}} = \frac{\text{Total change in kinetic energy}}{\text{Total change in time}}$$

If ΔW is the amount of work done in a time interval Δt , then $P = \frac{\Delta W}{\Delta t} = \frac{W_2 - W_1}{t_2 - t_1}$

- Instantaneous power is the scalar product of force and velocity at any instant.

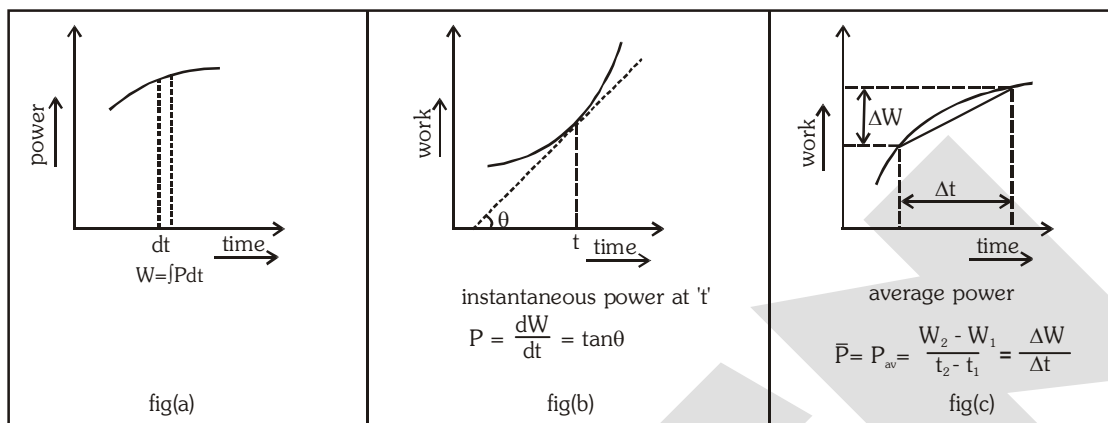
When work is measured in joules and t is in seconds, the unit for power is joules per second, which is called watt. For motors and engines, power is usually measured in horsepower, where $1 \text{ hp} = 746 \text{ W}$. The definition of power is applicable to all types of energies also like mechanical, electrical, thermal.

$$\text{Instantaneous power } P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

where v is the instantaneous velocity of the particle. Here dot product is used because only that component of force will contribute to power which is acting in the direction of instantaneous velocity.

- For a system of varying mass $F = \frac{d}{dt}(mv) = m \frac{dv}{dt} + v \frac{dm}{dt}$ If $v = \text{constant}$ then $F = v \frac{dm}{dt}$ then $P = \vec{F} \cdot \vec{v} = v^2 \frac{dm}{dt}$
- Power is a scalar quantity with dimensions $M^1L^2T^{-3}$
- SI unit of power is J/s or watt.
- 1 horsepower = 746 watts = 550 ft-lb/s.

7.2 Graphical Analysis



- Area under power-time graph gives the work done. $W = \int P dt$ (See Fig. a)
- The slope of tangent at a point on work-time graph, gives instantaneous power (See Fig. b)
- The slope of a straight line joining two points on work-time graph gives average power between two points (See Fig. c)

7.3 Efficiency

Machines are designed to convert energy into useful work, however because of frictional effects and other dissipative forces work performed by the machine is always less than the energy supplied to the machine.

The efficiency of a machine is given by $\eta = \frac{\text{work done}}{\text{energy input}}$

Illustration 29.

A truck pulls a mass of 1200 kg at a constant speed of 10 m/s on a levelled road. The tension in the coupling is 1000 N. What is the power spent on the mass? Find the tension when the truck moves up a road of inclination 1 in 6.

Solution

Force applied by truck $f = 1000$ N

Power spent in pulling the mass $P = fv = 1000 \times 10 = 10^4$ W

Here $\sin \theta = 1/6$, the force required by the truck to move up is

$$F = f + Mg \sin \theta \Rightarrow F = 1000 \text{ N} + 1200 \times 9.8 \times \frac{1}{6} = 2960 \text{ N}$$

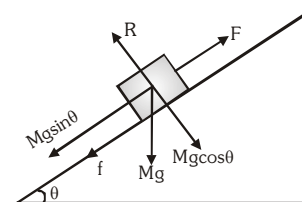


Illustration. 30

A body of mass m starting from rest from the origin moves along the x -axis with a constant power (P). Calculate the :

- relation between velocity and time.
- relation between distance and time.
- relation between velocity and distance.

$$(i) \quad P = Fv = mav = m \frac{dv}{dt} v \Rightarrow \int_0^t \frac{P}{m} dt = \int_0^v v dv \Rightarrow \frac{P}{m} t = \frac{v^2}{2}$$

$$\Rightarrow v = \sqrt{\frac{2P}{m} t} \Rightarrow \boxed{v \propto t^{1/2}} \quad \dots(1)$$

$$(ii) \quad \frac{dx}{dt} = \sqrt{\frac{2P}{m} t^{1/2}} \Rightarrow \int_0^x dx = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2} dt$$

$$\Rightarrow x = \sqrt{\frac{2P}{m}} \frac{2}{3} t^{3/2} \Rightarrow \boxed{x \propto t^{3/2}} \quad \dots(2)$$

$$(iii) \quad \text{From (1) \& (2), } x \propto (t^{1/2})^3 \Rightarrow \boxed{x \propto v^3} \quad \dots(3)$$

Illustration 31.

A pump is used to deliver water at a certain rate from a given pipe. By what amount should velocity of water, Force on the water and power of motor be increased to obtain n times water from the same pipe in the same time ?

Solution

Amount of water flowing per unit time $\frac{dm}{dt} = Av\rho$

v = velocity of flow, A is area of cross-section, ρ = density of liquid

To get n times water in the same time, $\left(\frac{dm}{dt}\right)' = n \frac{dm}{dt} \Rightarrow A v' \rho = n Av \rho \Rightarrow v' = nv$

$$F = \frac{v dm}{dt} \Rightarrow F' = v' \frac{dm'}{dt} = n^2 v \frac{dm}{dt} = n^2 F$$

To get n times water, force must be increased n^2 times.

$$\therefore P = v^2 \frac{dm}{dt} \quad \text{So } \frac{P'}{P} = \frac{v'^2 (dm/dt)'}{v^2 (dm/dt)} = \frac{n^2 v^2 n (dm/dt)}{v^2 (dm/dt)} = n^3 \Rightarrow P' = n^3 P$$

Thus to get n times water, the power must be increased n^3 times.

Illustration 32.

The force required to tow a boat at constant velocity is proportional to the speed. If a speed of 4.0 km/h requires 7.5 kW, how much power does a speed of 12 km/h require ?

Solution

Let the force be $F = \alpha v$, where v is speed and α is a constant of proportionality. the power required is

$$P = Fv = \alpha v^2$$

Let P_1 be the power required for speed v_1 and P_2 be the power required for speed v_2 .

$$P_1 = 7.5 \text{ kW and } v_2 = 3v_1, \quad P_2 = \left(\frac{v_2}{v_1}\right)^2 P_1 \Rightarrow P_2 = (3)^2 \times 7.5 \text{ kW} = 67.5 \text{ kW}.$$

Illustration 33.

A 30000 kg air-plane takes off at a speed of 50 m/s and 5 min. later it is at an elevation of 3 km with a speed of 100 m/s. What average power is required during this 5 min. if 40 percent of the power is used in overcoming dissipative forces ?

Solution

Energy supplied to the plane in 300 s (5 min.) is

$$(PE + KE)_{\text{at altitude}} - KE_{\text{ground}} = (30000 \times 9.8 \times 3000) + \frac{1}{2} (30000) (100)^2 - \frac{1}{2} (30000) (50)^2$$

$$= 882 \times 10^6 + 150 \times 10^6 - 37.5 \times 10^6 = 994.5 \text{ MJ}$$

For Average Power we have $0.6 P = \frac{994.5 \text{ MJ}}{300 \text{ s}}$ so, $P = 5.525 \text{ MW}$

Illustration 34.

In unloading grains from the hold of a ship, an elevator lifts the grains through a distance of 12 m. Grains are discharged at the top of the elevator at a rate of 2.0 kg per second and the discharge speed of each grain particle is 3.0 m/s. Find the minimum horsepower of the motor that can elevate grains in this way.

Solution

Work done by the motor each second, i.e. power = $mgh + \frac{1}{2} mv^2$

here the mass of grain discharged (and lifted) in one second $m = 2.0 \text{ kg}$, $v = 3.0 \text{ m/s}$ and $h = 12 \text{ m}$

\therefore power = $249 \text{ W} = 0.33 \text{ H.P.}$

The motor must have an output of at least 0.33 H.P.

Illustration 35.

A pump can take out 7200 kg of water per hour from a 100 m deep well. Calculate the power of the pump, assuming that its efficiency is 50%. ($g = 10 \text{ m/s}^2$)

Solution

Output power = $\frac{mgh}{t} = \frac{7200 \times 10 \times 100}{3600} = 2000 \text{ W}$

Efficiency $\eta = \frac{\text{output power}}{\text{input power}}$

Input power = $\frac{\text{output power}}{\eta} = \frac{2000 \times 100}{50} = 4 \text{ kW}$

Illustration 36.

A truck of mass 10,000 kg moves up an inclined plane rising 1 in 50 with a speed of 36 km/h. Find the power of the engine ($g = 10 \text{ m/s}^2$).

Solution

Force against which work is done $F = mg \sin \theta = 10,000 \times 10 \times \frac{1}{50} = 2000 \text{ N}$

speed $v = \frac{36 \times 5}{18} = 10 \text{ m/s}$ so $P = 2000 \times 10 = 20 \text{ kW.}$

Illustration 37.

An engine pumps water of density ρ , through a hose pipe. Water leaves the hose pipe with a velocity v . Find the

- rate at which kinetic energy is imparted to water
- power of the engine.

Solution

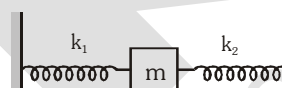
(i) Rate of change of kinetic energy = $\frac{dE_k}{dt} = \frac{d}{dt} \left(\frac{1}{2} mv^2 \right) = \frac{1}{2} v^2 \frac{dm}{dt} = \frac{1}{2} v^2 \frac{d(\rho Ax)}{dt}$

$$= \frac{1}{2} \rho A v^2 \frac{dx}{dt} = \frac{1}{2} \rho A v^3$$

(ii) Power = $Fv = \left(v \frac{dm}{dt} \right) v = v^2 \frac{dm}{dt} = v^2 (\rho Av) = \rho A v^3$

BEGINNER'S BOX-4

- Two springs have their respective force constants K_1 and K_2 . Both are stretched till their elastic potential energies are equal. If the stretching forces are F_1 and F_2 find $\frac{F_1}{F_2}$.
- A long spring is stretched by 1 cm. If work done in this process is W , then find out the work done in further stretching it by 1 cm.
- A 20 newtons stone falls accidentally from a height of 4 m on to a spring of stiffness constant 1960 N/m. Write down the equation to find out the maximum compression (x_m) of the spring.
- A block of mass m moving with speed v compresses a spring through distance x before its speed is halved. What is the value of spring constant ?
 (A) $\frac{3mv^2}{4x^2}$ (B) $\frac{mv^2}{4x^2}$ (C) $\frac{mv^2}{2x^2}$ (D) $\frac{2mv^2}{x^2}$
- A block of mass m is attached to two springs of spring constants k_1 and k_2 as shown in figure. The block is displaced by x towards right and released. The velocity of the block when it is at $x/2$ will be :
 (A) $\sqrt{\frac{(k_1 + k_2)x^2}{2m}}$ (B) $\sqrt{\frac{3(k_1 + k_2)x^2}{4m}}$ (C) $\sqrt{\frac{(k_1 + k_2)x^2}{m}}$ (D) $\sqrt{\frac{(k_1 + k_2)x^2}{4m}}$
- A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?
- An engine pumps up 100 kg of water through a height of 10 m in 5 s. If the efficiency of the engine is 60%, find out the power of the engine ($g = 10 \text{ m/s}^2$).



ANSWERS

BEGINNER'S BOX-1

- 60°
- In second case work has to be done against gravity.
- $\frac{\mu Mgd \cos \theta}{\cos \theta + \mu \sin \theta}$
- $x_2^2 - x_1^2$
- 21 J
- 50 J
- 100 J
- Ka^2
- 833 units
- (a) 75 J (b) -25 J (c) 50 J

BEGINNER'S BOX-2

- 125%
- 6 units
- 1500 J
- 0.5 J

BEGINNER'S BOX-3

- (A) - p, r, s, (B) - t
- (A, C)
- D
- (A) - q, (B) - r, (C) - p
- 5 m

BEGINNER'S BOX-4

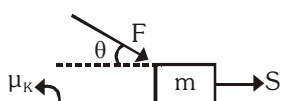
- $\sqrt{\frac{K_1}{K_2}}$
- 3W
- $49x_m^2 - x_m - 4 = 0$
- A
- B
- 43.6 kW
- 3.33 kW

EXERCISE-I (Conceptual Questions)

Build Up Your Understanding

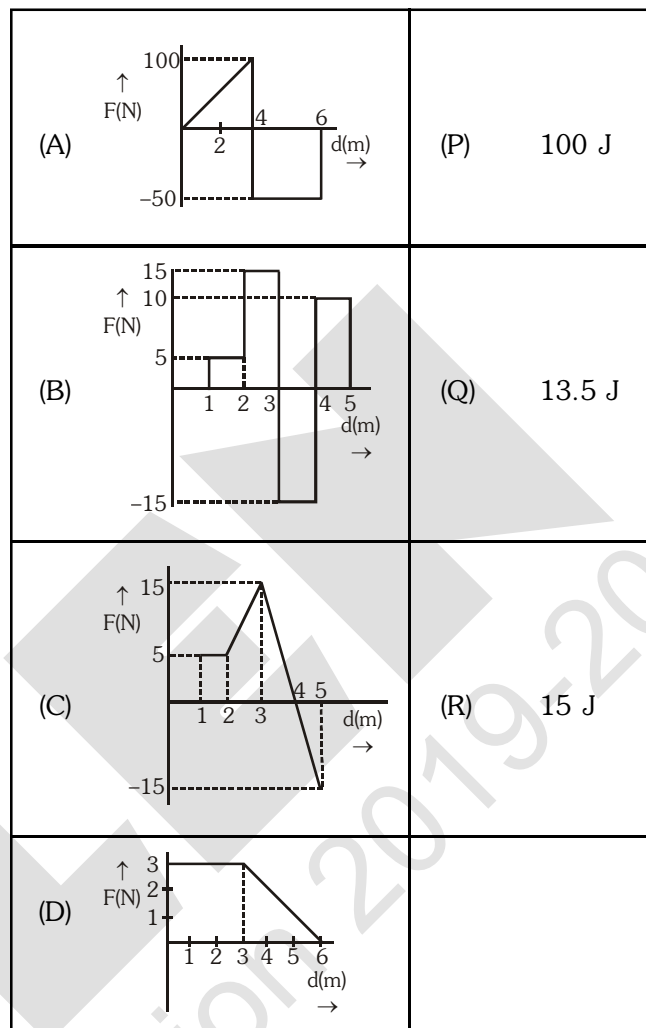
WORK

- A force $\vec{F} = (2\hat{i} - \hat{j} + 4\hat{k})$ N displaces a particle upto $\vec{d} = (3\hat{i} + 2\hat{j} - \hat{k})$ m, calculate work done.
(1) Zero (2) 8 J (3) 4 J (4) 12 J
- A body of mass m is displaced from point A(3, 1, 2) to point B(4, 3, 3) under the effect of a force $\vec{F} = (3\hat{i} + 2\hat{j} + 4\hat{k})$ N, calculate W.D. by the force.
(1) 57 J (2) 11 J (3) 0 (4) 22 J
- A force $\vec{F} = (3x^2 + 2x - 7)$ N acts on a 2 kg body as a result of which the body gets displaced from $x=0$ to $x=5$ m. The work done by the force will be—
(1) 35 J (2) 70 J (3) 115 J (4) 270 J
- A person of mass m is standing on one end of a plank of mass M and length L and floating in water. The person moves from one end to another and stops. Work done by normal force is —
(1) MgL (2) mgL (3) $\frac{mMgL}{M+m}$ (4) 0
- A 2 kg mass lying on a table is displaced in the horizontal direction through 50 cm. The work done by the normal reaction will be —
(1) 0 (2) 100 joule
(3) 100 erg (4) 10 joule
- A body of mass M tied to a string is lowered at a constant acceleration of $(g/4)$ through a vertical distance h . The work done by the string will be.....
(1) $\frac{3}{4}Mgh$ (2) $\frac{1}{4}Mgh$
(3) $\frac{-3}{4}Mgh$ (4) $\frac{-1}{4}Mgh$
- Find work done by friction for displacement 'S'?



- (1) $\mu_k(mg + F\sin\theta) \cdot S$ (2) $-\mu_k(mg + F\sin\theta) \cdot S$
- (3) $\mu_k(mg - F\sin\theta) \cdot S$ (4) $-\mu_k(mg - F\sin\theta) \cdot S$

- Calculate the work done for following F-d curves



- (1) (A - P); (B - Q); (C - Q); (D - R)
(2) (A - P); (B - R); (C - R); (D - Q)
(3) (A - P); (B - P); (C - Q); (D - R)
(4) (A - P); (B - P); (C - R); (D - Q)
- A force $F = Kx^2$ acts on a particle at an angle of 60° with the x-axis. the work done in displacing the particle from x_1 to x_2 will be —
(1) $\frac{kx^2}{2}$ (2) $\frac{k}{2}(x_2^2 - x_1^2)$
(3) $\frac{k}{6}(x_2^3 - x_1^3)$ (4) $\frac{k}{3}(x_2^3 - x_1^3)$
- The work done against gravity in taking 10 kg. mass at 1 m height in 1 s will be :—
(1) 49 J (2) 98 J
(3) 196 J (4) None of these

11. A force of 10N displaces an object by 10m. If work done is 50J then direction of force make an angle with direction of displacement :-

(1) 120° (2) 90°
(3) 60° (4) None of these

12. A body of mass 6 kg under a force which causes

displacement in it given ' $S = \frac{t^2}{4}$ ', metres where 't' is time. The work done by the force in 2 seconds is :-

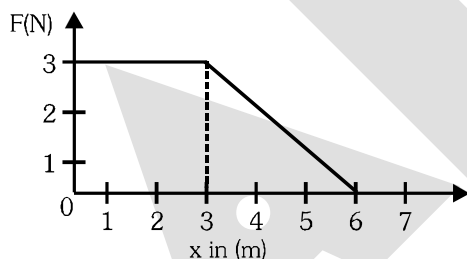
(1) 12J (2) 9J
(3) 6J (4) 3J

13. A force acts $\vec{F} = (5\hat{i} + 4\hat{j})\text{N}$ on a body and produced a displacement $\vec{S} = (6\hat{i} - 5\hat{j} + 3\hat{k})\text{m}$. The work done will be

(1) 30 J (2) 40 J
(3) 10 J (4) 20 J

14. A force F acting on an object varies with distance x as shown here.

The work done by the force in moving the object from $x = 0$ to $x = 6\text{m}$ is :-



(1) 18.0 J (2) 13.5 J
(3) 4.5 J (4) 9.0 J

15. A stone of mass m is tied to a string of length ℓ at one end and by holding second end it is whirled into a horizontal circle, then work done will be :-

(1) 0 (2) $\left(\frac{mv^2}{\ell}\right)2\pi\ell$
(3) $(mg) \cdot 2\pi\ell$ (4) $\left(\frac{mv^2}{\ell}\right)\ell$

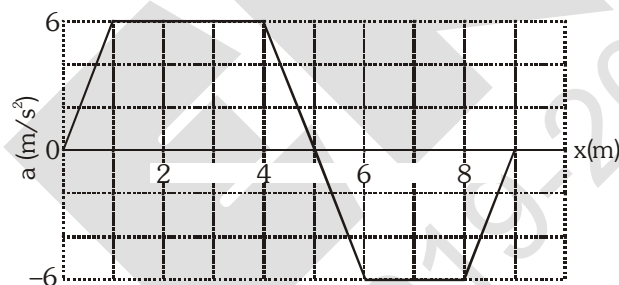
16. A particle is moved from a position $\vec{r}_1 = (3\hat{i} + 2\hat{j} - 6\hat{k})$ metre to a position $\vec{r}_2 = (14\hat{i} + 13\hat{j} - 9\hat{k})$ metre under the action of a force $\vec{F} = (4\hat{i} + \hat{j} + 3\hat{k})\text{N}$.

The work done is equal to :-

(1) 32 J (2) 64 J
(3) 96 J (4) 46 J

17. Figure gives the acceleration of a 2.0 kg body as it moves from rest along x axis while a variable force acts on it from $x = 0\text{m}$ to $x = 9\text{m}$. The work done by the force on the body when it reaches

(i) $x = 4\text{m}$ and
(ii) $x = 7\text{m}$ shall be as given below



(1) 21 J and 33 J respectively
(2) 21 J and 15 J respectively
(3) 42 J and 60 J respectively
(4) 42 J and 30 J respectively

KINETIC ENERGY & WORK ENERGY THEOREM

18. A force acts on a 30 g particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 second is :-

(1) 5.28 J (2) 450 mJ
(3) 490 mJ (4) 530 mJ

19. A constant force F is applied to a body of mass m moving with initial velocity u. If after the body undergoes a displacement S, its velocity becomes v, then the total work done is :-

(1) $m[v^2 + u^2]$ (2) $\frac{m}{2}[u^2 + v^2]$
(3) $\frac{m}{2}[v^2 - u^2]$ (4) $m[v^2 - u^2]$

20. If the momentum of a body is increased n times, its kinetic energy increases.

(1) n times (2) $2n$ times
(3) \sqrt{n} times (4) n^2 times

21. If K.E. increases by 3%. Then momentum will increase by :-

(1) 1.5% (2) 9%
(3) 3% (4) 2%

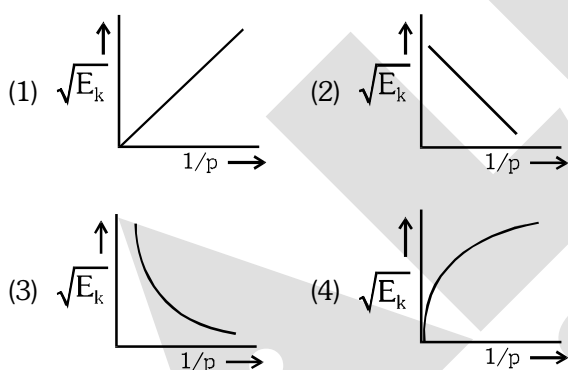
22. If K.E. body is increased by 100%. Then % change in 'P'.

(1) 50% (2) 41.4%
(3) 10% (4) 20%

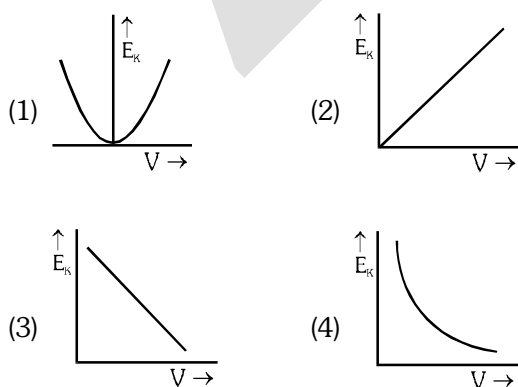
23. 2 particles of mass 1 Kg and 5 kg have same momentum, calculate ratio of their K.E.

(1) 5 : 1 (2) 25 : 1
(3) 1 : 1 (4) 10 : 1

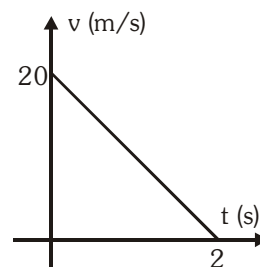
24. The graph between $\sqrt{E_k}$ and $\frac{1}{p}$ is
(E_k = kinetic energy and p = momentum) -



25. The graph between kinetic energy E_k and velocity V is -



26. Velocity-time graph of a particle of mass 2 kg moving in a straight line is as shown in figure. Work done by all the forces on the particle is :



(1) 400 J (2) -400 J
(3) -200 J (4) 200 J

27. The kinetic energy of a body becomes four times of its initial value. The new linear momentum will be :-

(1) Four times the initial value
(2) Thrice the initial value
(3) Twice the initial value
(4) Same as the initial value

28. A 0.5 kg ball is thrown up with an initial speed 14 m/s and reaches a maximum height of 8.0 m. How much energy is dissipated by air drag acting on the ball during the ascent ?

(1) 19.6 J (2) 4.9 J
(3) 10 J (4) 9.8 J

29. If the kinetic energy of a body is double of its initial kinetic energy, then the momentum of the body will be :-

(1) $2\sqrt{2}$ times (2) $\sqrt{2}$ times
(3) $\frac{1}{\sqrt{2}}$ times (4) none of these

30. A ball of mass 2 kg and another of mass 4 kg are dropped together from a 60 feet tall building. After a fall of 30 feet each towards earth, their respective kinetic energies will be in the ratio of :-

(1) 1 : 4 (2) 1 : 2
(3) $1 : \sqrt{2}$ (4) $\sqrt{2} : 1$

31. Two bodies of mass 1kg and 4kg have equal K.E. then the ratio of their momentum is :-

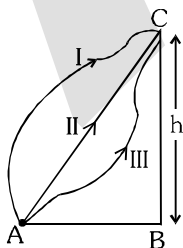
(1) 2 : 1 (2) 1 : 2
(3) 4 : 1 (4) 1 : 4

32. If the kinetic energy is increased by 300%, the momentum will increase by :-
 (1) 100% (2) 200% (3) 150% (4) 300%
33. If the kinetic energy of a body increases by 4% the momentum :
 (1) increases by 2% (2) increases by 4%
 (3) increases by 8% (4) increases by 16%

**CONSERVATIVE FORCE POTENTIAL ENERGY
& LAW OF CONSERVATION OF ENERGY**

34. Which of the following statements is true for work done by conservative forces :-
 (1) It does not depend on path
 (2) It is equal to the difference of initial and final potential energy function
 (3) It can be recovered completely
 (4) All of the above
35. Which of the following statement is incorrect for a conservative field ?
 (1) Work done in going from initial to final position is equal to change in kinetic energy of the particle.
 (2) Work done depends on path but not on initial and final positions.
 (3) Work done does not depend on path but depends only on initial and final positions
 (4) Work done on a particle in the field for a round trip is zero.

36. As shown in the diagram a particle is to be carried from point A to C via paths (I), (II) and (III) in gravitational field, then which of the following statements is correct :-



- (1) Work done is same for all the paths
 (2) Work done is minimum for path (II)
 (3) Work done is maximum for path (I)
 (4) None of the above

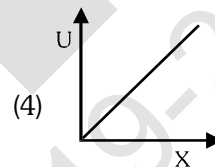
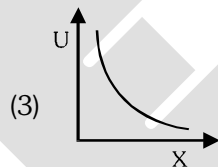
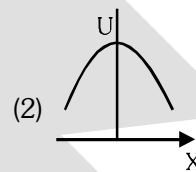
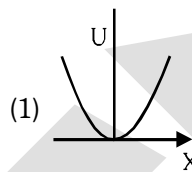
37. The relation between conservative force and potential energy U is given by :-

$$(1) \vec{F} = \frac{dU}{dx} \quad (2) \vec{F} = \int U dx$$

$$(3) \vec{F} = - \frac{dU}{dx} \quad (4) F = \frac{dU}{dx}$$

38. Which of the following is a non conservative force:-
 (1) Electric force (2) Gravitational force
 (3) Spring force (4) Viscous force

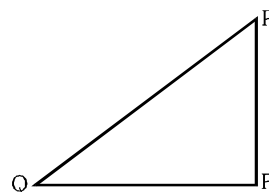
39. The graph between potential energy U and displacement X in the state of stable equilibrium will be-



40. A particle moves in a potential region given by $U = 8x^2 - 4x + 400$ J. Its state of equilibrium will be-

- (1) $x = 25$ m (2) $x = 0.25$ m
 (3) $x = 0.025$ m (4) $x = 2.5$ m

41. For the path PQR in a conservative force field the amounts work done in carrying a body from P to Q and from Q to R are 5 Joule and 2 Joule respectively. The work done in carrying the body from P to R will be -



- (1) 7 J (2) 3 J (3) $\sqrt{21}$ J (4) Zero

42. The mass of a bucket full of water is 15 kg. It is being pulled up from a 15m deep well. Due to a hole in the bucket 6 kg water flows out of the bucket. The work done in drawing the bucket out of the well will be ($g = 10\text{m/s}^2$)-

- (1) 900 J (2) 1500 J
 (3) 1800 J (4) 2100 J

43. A uniform chain of length L and mass M is lying on a smooth table and $\frac{2}{3}$ of its length is hanging down over the edge of the table. If g is the acceleration due to gravity, the work done to pull the hanging part on the table is :-

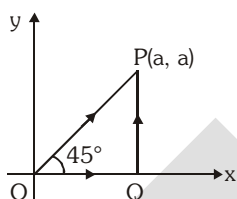
(1) MgL (2) $\frac{MgL}{3}$ (3) $\frac{MgL}{9}$ (4) $\frac{2MgL}{9}$

44. A 10 kg satellite completes one revolution around the earth at a height of 100 km in 108 minutes. The work done by the gravitational force of earth will be -

(1) $108 \times 100 \times 10 \text{ J}$ (2) $\frac{108 \times 10}{100} \text{ J}$

(3) 0 J (4) $\frac{100 \times 10}{108} \text{ J}$

45. A particle is moved from $(0, 0)$ to (a, a) under a force $\vec{F} = (3\hat{i} + 4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then :



(1) $W_1 = W_2$ (2) $W_1 = 2W_2$
(3) $W_2 = 2W_1$ (4) $W_2 = 4W_1$

46. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is :-

(1) 16 J (2) 8 J
(3) 32 J (4) 24 J

47. If a spring extends by x on loading then energy stored by the spring is :- (T is tension in the spring, K = spring const.)

(1) $\frac{T^2}{2x}$ (2) $\frac{T^2}{2k}$ (3) $\frac{2k}{T^2}$ (4) $\frac{2T^2}{k}$

48. A body of mass 2 kg falls from a height of 20 m. What is the loss in potential energy :-

(1) 400 J (2) 300 J
(3) 200 J (4) 100 J

49. In stretching a spring by 2 cm energy stored is given by U , then stretching by 10 cm energy stored will be :-

(1) U (2) $5U$
(3) $\frac{U}{25}$ (4) $25U$

50. If the potential energy of two molecules is given by,

$$U = \frac{A}{r^{12}} - \frac{B}{r^6}$$

then at equilibrium position, its potential energy is equal to :

(1) $\frac{A^2}{4B}$ (2) $-\frac{B^2}{4A}$ (3) $\frac{2B}{A}$ (4) $3A$

51. A 2 g ball of glass is released from the edge of a hemispherical cup whose radius is 20 cm. How much work is done on the ball by the gravitational force during the ball's motion to the bottom of the cup ?

(1) 1.96 mJ
(2) 3.92 mJ
(3) 4.90 mJ
(4) 5.88 mJ



52. 4 J of work is required to stretch a spring through 10 cm beyond its unstretched length. The extra work required to stretch it through additional 10 cm shall be

(1) 4 J (2) 8 J
(3) 12 J (4) 16 J

53. A particle in a certain conservative force field has a potential energy given by $U = \frac{20xy}{z}$. The force exerted on it is

(1) $\left(\frac{20y}{z}\right)\hat{i} + \left(\frac{20x}{z}\right)\hat{j} + \left(\frac{20xy}{z^2}\right)\hat{k}$

(2) $-\left(\frac{20y}{z}\right)\hat{i} - \left(\frac{20x}{z}\right)\hat{j} + \left(\frac{20xy}{z^2}\right)\hat{k}$

(3) $-\left(\frac{20y}{z}\right)\hat{i} - \left(\frac{20x}{z}\right)\hat{j} - \left(\frac{20xy}{z^2}\right)\hat{k}$

(4) $\left(\frac{20y}{z}\right)\hat{i} + \left(\frac{20x}{z}\right)\hat{j} - \left(\frac{20xy}{z^2}\right)\hat{k}$

CONSERVATION OF MECHANICAL ENERGY

54. A body is dropped from a height h . When loss in its potential energy is U then its velocity is v . The mass of the body is –

(1) $\frac{U^2}{2v}$ (2) $\frac{2v}{U}$ (3) $\frac{2v}{U^2}$ (4) $\frac{2U}{v^2}$

55. A ball of mass 1 kg is released from the tower of Pisa. The kinetic energy generated in it after falling through 10m will be –

(1) 10 J (2) 9.8 J
(3) 0.98 J (4) 98 J

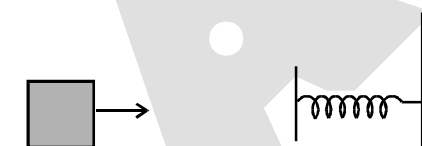
56. A stone is projected vertically up to reach maximum height ' h '. The ratio of its kinetic energy to potential energy, at a height $\frac{4h}{5}$ will be :-

(1) 5 : 4 (2) 4 : 5 (3) 1 : 4 (4) 4 : 1

57. A block of mass 16 kg is moving on a frictionless horizontal surface with velocity 4m/s and comes to rest after pressing a spring. If the force constant of the spring is 100 N/m then the compression in the spring will be –

(1) 3.2 m (2) 1.6 m (3) 0.6 m (4) 6.1 m

58. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant $k=50\text{N/m}$. The maximum compression of the spring would be :-



(1) 0.12 m (2) 1.5 m
(3) 0.5 m (4) 0.15 m

59. A projectile is fired at 30° with momentum p , neglecting friction, the change in kinetic energy, when it returns back to the ground, will be :-

(1) zero (2) 30 %
(3) 60 % (4) 100 %

POWER

60. An electric motor produces a tension of 4500N in a load lifting cable and rolls it at the rate of 2 m/s. The power of the motor is –

(1) 9 kW (2) 15 kW
(3) 225 kW (4) 9×10^3 hp

61. A motor of 100 hp is moving a car with a constant velocity of 72 km/hour. The forward force exerted by the engine on the car is –

(1) 3.73×10^3 N (2) 3.73×10^2 N
(3) 3.73×10^1 N (4) None of the above

62. A crane lifts 300 kg weight from earth's surface upto a height of 2m in 3 seconds. The average power generated by it will be –

(1) 1960 W (2) 2205 W
(3) 4410 W (4) 0 W

63. Two men with weights in the ratio 5 : 3 run up a staircase in times in the ratio 11 : 9. The ratio of power of first to that of second is –

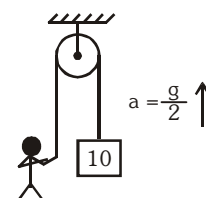
(1) $\frac{15}{11}$ (2) $\frac{11}{15}$ (3) $\frac{11}{9}$ (4) $\frac{9}{11}$

64. A car is moving with a speed of 40 Km/hr. If the car engine generates 7 kilowatt power, then the resistance in the path of motion of the car will be–

(1) 360 newton (2) 630 newton
(3) Zero (4) 280 newton

65. Calculate power generated by tension in the string in first 2 seconds of motion :-

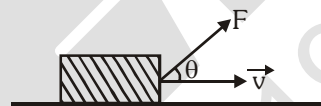
(1) 250 W
(2) 500 W
(3) 750 W
(4) 1000 W



66. A body of mass m starting from rest from origin moves along x -axis with constant power (P). Calculate relation between velocity/distance :-

(1) $x \propto v^{1/2}$ (2) $x \propto v^2$
(3) $x \propto v$ (4) $x \propto v^3$

- 67.** A pump is used to deliver water at a certain rate from a given pipe. To obtain n times water from the same pipe in the same time, by what factor, the force of the motor should be increased?
- (1) n times (2) n^2 times
(3) n^3 times (4) $\frac{1}{n}$ times
- 68.** A body of mass 4 kg is moving up an inclined plane rising 1 in 40 with velocity 40 m/sec if efficiency is 50% the calculate power required.
- (1) 38.4 W (2) 55 W
(3) 78.4 W (4) 108 W
- 69.** A 1.0 hp motor pumps out water from a well of depth 20 m and fills a water tank of volume 2238 liters at a height of 10 m from the ground. The running time of the motor to fill the empty water tank is ($g = 10\text{m/s}^2$)
- (1) 5 minutes (2) 10 minutes
(3) 15 minutes (4) 20 minutes
- 70.** Water is falling on the blades of a turbine at a rate of 100 kg/s from a certain spring. If the height of the spring be 100 metres, the power transferred to the turbine will be :-
- (1) 100 kW (2) 10 kW
(3) 1 kW (4) 1000 kW
- 71.** If the force applied is F and the velocity gained is v , then the average power developed is :-
- (1) $\frac{F}{v}$ (2) $\frac{v}{F}$
(3) $\frac{Fv}{2}$ (4) Fv^2
- 72.** What average horsepower is developed by an 80kg man while climbing in 10 s flight of stairs that rises 6 m vertically ?
- (1) 0.63 hp (2) 1.26 hp
(3) 1.8 hp (4) 2.1 hp
- 73.** A constant force \vec{F} is acting on a body of mass m with constant velocity \vec{v} as shown in the figure. The power P exerted is



- (1) $Fv \cos \theta$ (2) $\frac{F \cos \theta}{mg}$
(3) $\frac{Fmg \cos \theta}{v}$ (4) $\frac{mg \sin \theta}{F}$

EXERCISE-I (Conceptual Questions)

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	2	3	4	1	3	2	2	3	2	3	4	3	2	1
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	4	4	1	3	4	1	2	1	3	1	2	3	4	2	2
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	2	1	1	4	2	1	3	4	1	2	1	3	4	3	1
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	2	1	4	2	2	3	2	4	4	3	2	4	1	1
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73		
Ans.	1	1	1	2	3	4	2	3	3	1	3	1	1		

EXERCISE-II (Previous Year Questions)

AIPMT/NEET & AIIMS (2006-2018)

AIPMT 2006

- 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10m. The work done against friction is :-
(take $g = 10 \text{ m/s}^2$)
(1) zero (2) 100 J (3) 200 J (4) 300 J
- A body of mass 3 kg is under a constant force which causes a displacement s in metres in it, given by the relation $s = \frac{1}{3} t^2$, where t is in seconds. Work done by the force in 2 seconds is :-
(1) $\frac{5}{19} \text{ J}$ (2) $\frac{3}{8} \text{ J}$
(3) $\frac{8}{3} \text{ J}$ (4) $\frac{19}{5} \text{ J}$

AIIMS 2008

- A vertical spring with force constant k is fixed on a table. A ball of mass m at a height h above the free upper end of the spring falls vertically on the spring, so that the spring is compressed by a distance d . The net work done in the process is:
(1) $mg(h+d) + \frac{1}{2}kd^2$ (2) $mg(h+d) - \frac{1}{2}kd^2$
(3) $mg(h-d) - \frac{1}{2}kd^2$ (4) $mg(h-d) + \frac{1}{2}kd^2$

AIPMT 2009

- A block of mass M is attached to the lower end of a vertical spring. The spring is hung from a ceiling and has force constant value k . The mass is released from rest with the spring initially unstretched. the maximum extension produced in the length of the spring will be :-
(1) $Mg/2k$ (2) Mg/k
(3) $2 Mg/k$ (4) $4 Mg/k$
- An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water jet. What is the rate at which kinetic energy is imparted to water :-
(1) $\frac{1}{2} m^2 v^2$ (2) $\frac{1}{2} m v^3$
(3) $m v^3$ (4) $\frac{1}{2} m v^2$

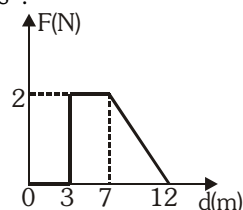
- A body of mass 1 kg is thrown upwards with a velocity 20 m/s. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction ? ($g = 10 \text{ m/s}^2$) :-
(1) 10 J (2) 20 J
(3) 30 J (4) 40 J

AIPMT 2010

- An engine pumps water through a hose pipe. Water passes through the pipe and leaves it with a velocity of 2 m/s. The mass per unit length of water in the pipe is 100 kg/m. What is the power of the engine?
(1) 800 W (2) 400 W
(3) 200 W (4) 100 W

AIPMT 2011

- The potential energy of a system increases if work is done :-
(1) Upon the system by a nonconservative force
(2) By the system against a conservative force
(3) By the system against a nonconservative force
(4) Upon the system by a conservative force
- A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest :-
(1) At the highest position of the body
(2) At the instant just before the body hits the earth
(3) It remains constant all through
(4) At the instant just after the body is projected
- Force F on a particle moving in a straight line varies with distance d as shown in the figure. The work done on the particle during its displacement of 12 m is :



- (1) 18 J (2) 21 J
(3) 26 J (4) 13 J

AIPMT (Pre) 2012

11. The potential energy of a particle in a force field

$$\text{is : } U = \frac{A}{r^2} - \frac{B}{r}$$

where A and B are positive constants and r is the distance of particle from the centre of the field. For stable equilibrium, the distance of the particle is :

- (1) A/B (2) B/A
(3) B/2A (4) 2A/B

AIPMT (Mains) 2012

12. A car of mass m starts from rest and accelerates so that the instantaneous power delivered to the car has a constant magnitude P_0 . The instantaneous velocity of this car is proportional to :-

- (1) $t^{-1/2}$ (2) t/\sqrt{m}
(3) $t^2 P_0$ (4) $t^{1/2}$

NEET (UG) 2013

13. A uniform force of $(3\hat{i} + \hat{j})$ newton acts on a particle of mass 2kg. Hence the particle is displaced from position $(2\hat{i} + \hat{k})$ meter to position $(4\hat{i} + 3\hat{j} - \hat{k})$ meter. The work done by the force on the particle is :-

- (1) 15 J (2) 9 J
(3) 6 J (4) 13 J

AIPMT 2015

14. A block of mass 10 kg, moving in x direction with a constant speed of 10 m/s, is subjected to a retarding force $F = 0.1x$ J/m during its travel from $x = 20$ m to 30 m. Its final KE will be :

- (1) 450 J (2) 275 J
(3) 250 J (4) 475 J

15. A particle of mass m is driven by a machine that delivers a constant power k watts. If the particle starts from rest the force on the particle at time t is :-

- (1) $\sqrt{mk} t^{-1/2}$ (2) $\sqrt{2mk} t^{-1/2}$
(3) $\frac{1}{2}\sqrt{mk} t^{-1/2}$ (4) $\sqrt{\frac{mk}{2}} t^{-1/2}$

Re-AIPMT 2015

16. The heart of a man pumps 5 litres of blood through the arteries per minute at a pressure of 150 mm of mercury. If the density of mercury be 13.6×10^3 kg/m³ and $g = 10$ m/s² then the power of heart in watt is:

- (1) 1.50 (2) 1.70
(3) 2.35 (4) 3.0

AIIMS 2015

17. A boy of mass 50 kg jumps to a height of 0.8 m from the ground then momentum transferred by the ground to boy is

- (1) 400 kg m/s (2) 200 kg m/s
(3) 800 kg m/s (4) 500 kg m/s

NEET-I 2016

18. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (2t\hat{i} + 3t^2\hat{j})$ N,

where \hat{i} and \hat{j} are unit vectors along x and y axis.

What power will be developed by the force at the time t ?

- (1) $(2t^2 + 3t^3)$ W
(2) $(2t^2 + 4t^4)$ W
(3) $(2t^3 + 3t^4)$ W
(4) $(2t^3 + 3t^5)$ W

NEET-II 2016

19. A particle moves from a point $(-2\hat{i} + 5\hat{j})$ to $(4\hat{j} + 3\hat{k})$ when a force of $(4\hat{i} + 3\hat{j})$ N is applied.

How much work has been done by the force ?

- (1) 5 J (2) 2 J
(3) 8 J (4) 11 J

NEET(UG) 2017

20. Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take 'g' constant with a value 10 m/s². The work done by the (i) gravitational force and the (ii) resistive force of air is :-

- (1) (i) 1.25 J (ii) - 8.25 J
(2) (i) 100 J (ii) 8.75 J
(3) (i) 10 J (ii) - 8.75 J
(4) (i) - 10 J (ii) - 8.25 J

AIIMS 2017

21. A block of mass 20 kg is moved with constant velocity along an inclined plane of inclination 37° with help of a force of constant power 50 W. If the coefficient of kinetic friction between block and surface is 0.25, then what fraction of power is used against gravity?

- (1) $\frac{3}{4}$ (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) $\frac{1}{8}$

22. Velocity of a particle of mass 1 kg moving rectilinearly is given by $v = 25 - 2t + t^2$. Find the average power of the force acting on the particle between time interval $t = 0$ to $t = 1$ sec.

- (1) 49 W (2) 24.5 W
(3) -49 W (4) -24.5 W

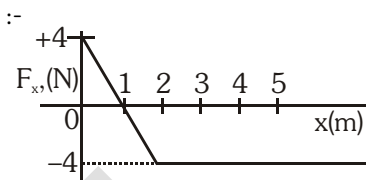
EXERCISE-II (Previous Years Questions)

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	3	2	3	2	2	1	2	2	4	4	4	2	4	4
Que.	16	17	18	19	20	21	22								
Ans.	2	2	4	1	3	1	4								

EXERCISE-III (Analytical Questions)

Check Your Understanding

1. A body of mass 10 kg is displaced from point A(2, 1, 3) to point B(3, 3, 4) under the effect of a force of magnitude 20 N in the direction of $6\hat{i} + 8\hat{j}$. Calculate W.D. by the force :-
 (1) 22 J (2) $20\sqrt{6}$ J (3) 44 J (4) Zero
2. Work done in time t on a body of mass m which is accelerated from rest to a speed v in time t_1 as a function of time t is given by :-
 (1) $\frac{1}{2}m\frac{v}{t_1}t^2$ (2) $m\frac{v}{t_1}t^2$
 (3) $\frac{1}{2}\left(\frac{mv}{t_1}t\right)^2$ (4) $\frac{1}{2}m\frac{v^2}{t_1^2}t^2$
3. A particle moves on a rough horizontal ground with some initial velocity say v_0 . If $\frac{3}{4}$ of its kinetic energy is lost due to friction in time t_0 then coefficient of friction between the particle and the ground is :-
 (1) $\frac{v_0}{2gt_0}$ (2) $\frac{v_0}{4gt_0}$ (3) $\frac{3v_0}{4gt_0}$ (4) $\frac{v_0}{gt_0}$
4. A body of mass m accelerates uniformly from rest to v_1 in time t_1 . The instantaneous power delivered to the body as a function of time t is-
 (1) $\frac{mv_1t}{t_1}$ (2) $\frac{mv_1^2t}{t_1^2}$ (3) $\frac{mv_1t^2}{t_1}$ (4) $\frac{mv_1^2t}{t_1}$
5. The only force F_x acting on a 2.0 kg body as it moves along the x-axis varies as shown in the figure. The velocity of the body along positive x-axis at $x = 0$ is 4 m/s. The kinetic energy of the body at $x = 3.0$ m is :-
 (1) 4 J (2) 8 J (3) 12 J (4) 16 J
- 
6. A machine which is 75% efficient, uses 12 J of energy in lifting 1 kg mass through a certain distance. The mass is then allowed to fall through the same distance, the velocity at the end of its fall is :-
 (1) $\sqrt{12}$ m/s (2) $\sqrt{18}$ m/s
 (3) $\sqrt{24}$ m/s (4) $\sqrt{32}$ m/s
7. A variable force, given by the 2-dimensional vector $\vec{F} = (3x^2\hat{i} + 4\hat{j})$, acts on a particle. The force is in newton and x is in metre. What is the change in the kinetic energy of the particle as it moves from the point with coordinates (2,3) to (3,0) ? (The coordinates are in meters)
 (1) -7 J (2) Zero
 (3) +7J (4) +19 J

EXERCISE-III (Analytical Questions)

ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	3	4	1	2	3	2	3

EXERCISE-IV (Assertion & Reason)

Target AIIMS

Directions for Assertion & Reason questions

These questions consist of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses.

- (A) If both Assertion & Reason are True & the Reason is a correct explanation of the Assertion.
 (B) If both Assertion & Reason are True but Reason is not a correct explanation of the Assertion.
 (C) If Assertion is True but the Reason is False.
 (D) If both Assertion & Reason are false.

1. **Assertion :** No work is done by a force on an object when the point of application of the force does not move.

Reason : A small block of mass m is kept on a rough inclined surface of inclination α fixed in an elevator. The elevator goes up with a uniform velocity u and the block does not slide on the wedge. Then the work done by the force of friction on the block in time t will be $mgut\sin^2\alpha$.

- (1) A (2) B (3) C (4) D

2. **Assertion :** Kinetic energy of a body becomes four times, when its linear momentum is doubled.

Reason : This is because, $K.E. = p^2/2m$.

- (1) A (2) B (3) C (4) D

3. **Assertion :** A spring has potential energy, when it is compressed (or stretched).

Reason : In compressing or stretching, work is done on the spring against the restoring force which is stored as potential energy.

- (1) A (2) B (3) C (4) D

4. **Assertion :** Work done by the force or against the force does not depend on path, then force is called conservative force.

Reason : All central forces which follows the inverse square law are conservative forces.

- (1) A (2) B (3) C (4) D

5. **Assertion :** Potential energy is possible only in conservative force field.

Reason : Potential energy is a relative quantity but K.E. is an absolute quantity.

- (1) A (2) B (3) C (4) D

6. **Assertion :** A body can have energy without having momentum.

Reason : A body can have momentum without having mechanical energy.

- (1) A (2) B (3) C (4) D

7. **Assertion :** A truck and a car moving with the same kinetic energy are brought to rest by the application of breaks which provide equal retarding forces. Both come to rest in equal distance.

Reason : It is possible that the speed of a body is zero but velocity is not zero.

- (1) A (2) B (3) C (4) D

8. **Assertion :** If work done by the force depends on path then force is said to be conservative.

Reason : All the conservative forces are central force.

- (1) A (2) B (3) C (4) D

9. **Assertion :** Frictional forces are conservative forces. [AIIMS 2005]

Reason : Potential energy can be associated with frictional forces.

- (1) A (2) B (3) C (4) D

10. **Assertion :** The change in kinetic energy of a particle is equal to the work done on it by the net force. [AIIMS 2009]

Reason : Change in kinetic energy of particle is equal to the work done only in case of system of one particle.

- (1) A (2) B (3) C (4) D

- 11. Assertion :** When a force of 20 N displaces a body by 5 m then work done by this force must be 100J for every observer watching that body.
Reason : Work done on a body does not depend on the observer that is observing the body.
 (1) A (2) B (3) C (4) D
- 12. Assertion :** Work done by or against gravitational force in moving a body from one point to another is independent of the actual path followed between the two points.
Reason : Gravitational forces are conservative forces.
 (1) A (2) B (3) C (4) D
- 13. Assertion :** When a spring is elongated work done by spring is negative but when it compressed work done by spring is positive.
Reason : Work done by spring is path dependent.
 (1) A (2) B (3) C (4) D
- 14. Assertion :** Work done by force of friction in moving a body in any round trip is zero.
Reason : Frictional force is a conservative force.
 (1) A (2) B (3) C (4) D
- 15. Assertion :-** A particle moving one dimensionally in conservative field may have more than one turning points. [AIIMS 2017]
Reason :- Turning point depends on initial velocity.
 (1) A (2) B (3) C (4) D
- 16. Assertion :-** For an isolated system, mechanical energy is not conserved. [AIIMS 2017]
Reason :- For an isolated system internal energy is constant.
 (1) A (2) B (3) C (4) D
- 17. Assertion :-** For an isolated system mechanical energy is conserved. [AIIMS 2018]
Reason :- Kinetic frictional force is absent for isolated system.
 (1) A (2) B (3) C (4) D

EXERCISE-IV (Assertion & Reason)

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	1	1	2	3	2	3	4	4	3	4	1	4	4	1
Que.	16	17													
Ans.	4	4													