ARJUNA NEET BATCH

KINEMATICS

LECTURE - 06

$$\underbrace{(x_f - x_i)}_{\text{dispm}} = \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v\,dt$$

$$\int_{u_i}^{v_f} dv = \int_{t_i}^{t_f} a\,dt$$

गली $\vec{x}$ (Position) — velocity — acc^n | नाली

$$* \quad \vec{V}_{inst} = \frac{d\vec{x}}{dt}$$

$$\vec{V}_{Avg} = \frac{\text{total disp}}{\text{total time}} = \frac{\vec{x}_f - \vec{x}_i}{\Delta t}$$

$$\vec{V}_{Avg} = \frac{\int v\,dt}{\Delta t}$$

$$\vec{a}_{inst} = \frac{d\vec{v}}{dt} = v\frac{dv}{dx}$$

$$\vec{a}_{Avg} = \frac{\vec{V}_f - \vec{V}_i}{t_f - t_i} = \frac{\int a\,dt}{\Delta t}$$

If the displacement of a particle varies with time as $\sqrt{x} = t + 7$, then

(a) Velocity of the particle is inversely proportional t

(b) Velocity of the particle is proportional to t^2

(c) Velocity of the particle is proportional to $\sqrt{t}$

(d) The particle moves with constant acceleration

(AIEEE)

MR $\sqrt{x} = t+7$

$x \propto (t+7)^2$

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$\sqrt{x} = (t+7)$

$x = (t+7)^2 = t^2 + 49 + 14t$

$\frac{dx}{dt} = v = 2t + 0 + 14$

$\boxed{v = 2t + 14}$

$a = \frac{dv}{dt} = 2\frac{dt}{dt} + 0$

$\boxed{a = 2\ m/s^2}$

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The position x of particle moving along x-axis varies with time t as $x = A \sin(\omega t)$ where A and w are the positive constants. The acceleration a of particle varies with its position (x) as

AIPMT-2014

(a) $a = Ax$

(b) $a = -\omega^2 x$

(c) $a = A\omega x$

(d) $a = \omega^2 x A$

$x = A \sin(\omega t)$

diff^n w.r.t. time.

$V = \frac{dx}{dt} = A\cos(\omega t) \times \frac{d(\omega t)}{dt}$

$V = A\omega \cos(\omega t)$

$\Rightarrow a = \frac{dv}{dt} = -(A\omega)\sin(\omega t) \times \omega$

$a = -A\omega^2 \sin(\omega t)$

$\boxed{a = -\omega^2 x}$

A body is moving with variable acceleration (a) along a straight line. The average acceleration of body in time interval t_1 to t_2 is

(a) $\frac{a[t_2 + t_1]}{2}$

(b) $\frac{a[t_2 - t_1]}{2}$

(c) $\frac{\int_{t_1}^{t_2} a\,dt}{t_2 + t_1}$

(d) $\frac{\int_{t_1}^{t_2} a\,dt}{t_2 - t_1}$

MR* last class

a = variable acc^n

$$\vec{a}_{Avg} = \frac{\vec{V_f} - \vec{V_i}}{\Delta t} = \frac{\int_{t_1}^{t_2} a\,dt}{t_2 - t_1}$$

$$\boxed{\Delta t = t_f - t_i}$$

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A particle moving along x-axis has acceleration f, at time t, given $f = f_0 \left(1 - \frac{t}{T}\right)$, Where f_0 and T are constants. The particle at $t = 0$ has zero velocity. At the instant when $t=T$, the particle's velocity is

(a) $\frac{1}{2} f_0 T$ (b) $f_0 T$

(c) $\frac{1}{2} f_0 T^2$ (d) $f_0 T^2$ [AIPMT (Prelims)-2007]

$$\boxed{f} = f_0 \left(1 - \frac{t}{T}\right)$$

$$\Downarrow$$

$$\frac{dv}{dt} = f_0 - \frac{f_0 t}{T}$$

$$dv = f_0 dt - \frac{f_0 t}{T} dt$$

$t=0 \rightarrow u=0$

$$\int_0^v dv = f_0 \int_0^T dt - \frac{f_0}{T} \int_0^T t\, dt$$

$$[v]_0^v = f_0 [t]_0^T - \frac{f_0}{T} \left(\frac{t^2}{2}\right)_0^T$$

$$v = f_0 T - \frac{1}{2} f_0 T$$

(Q) If acceleration $a = 2t$ then find velocity at 2 sec if initial velocity at $t=0$ is $10\,m/s$.

(Q) If acceleration $\boxed{a = 2t}$ then find velocity at $t = 2\,sec$ if initial velocity at $t=0$ is $10\,m/s$

Soln

$a = 2t$ (variable)

$$\frac{dv}{dt} = 2t$$

$$\int_{u=10\,m/s}^{v} dv = 2\int_{t=0}^{t=2} t\,dt$$

$$[v]_{10}^{v} = \cancel{2}\left(\frac{t^2}{\cancel{2}}\right)_0^2$$

$$[v - 10] = (t^2)_0^2 = (2)^2 - (0)$$

$$v - 10 = 4$$

$$\boxed{v = 14\,m/s}$$

The position x of a particle varies with time, (t) as $x = at^2 - bt^3$. The acceleration will be zero at time t equal to

(a) $\frac{a}{3b}$ (b) zero

(c) $\frac{2a}{3b}$ (d) $\frac{a}{b}$

$x = at^2 - bt^3$

diff^n w.r.t. time

$v = \frac{dx}{dt} = a(2t) - b(3t^2)$

$\boxed{V = 2at - 3bt^2}$

$a = \frac{dv}{dt}$

$a = 2a \times 1 - 3b(2t)$

$a = 2a - 6bt$

$0 = 2a - 6bt$

$\boxed{t = \frac{a}{3b}}$

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A particle moves along a straight line such that its displacement at any time t is given by $s = (t^3 - 6t^2 - 3t + 4)$ metres. The velocity when the acceleration is zero is

(a) 3 m/s (b) 42 m/s

(c) – 9 m/s (d) – 15 m/s

$$S = (t^3 - 6t^2 - 3t + 4)$$

$$\boxed{\vec{V} = \frac{ds}{dt} = 3t^2 - 12t - 3}$$

$$\vec{a} = \frac{dv}{dt} = 6t - 12$$

$$0 = 6t - 12$$

$$\boxed{t = \frac{\cancel{12}}{\cancel{6}} = 2\ \text{sec}}$$

$$V_{(t=2)} = 3(2)^2 - 12 \times 2 - 3$$

$$= 3 \times 4 - 24 - 3$$

$$= -15\ \text{m/s}$$

The initial velocity of a particle moving along x-axis is u (at $t = 0$ and $x = 0$) and its acceleration a is given by $a = kx$. Which of the following equation is correct between its velocity (v) and position (x)?

(a) $v^2 - u^2 = 2kx$

~~(b)~~ $v^2 = u^2 + 2kx^2$

~~(c)~~ $v^2 = u^2 + kx^2$

(d) $v^2 + u^2 = 2kx$

at $t = 0$
$x = 0$
$u_i = 0$

$a = kx$

$\frac{dv}{dt} = kx$ — (A)

3rd eqn motion

$v^2 - u^2 = 2ax$

$v^2 = u^2 + 2(kx^2)$

Wrong

$v\frac{dv}{dx} = kx$ — (B)

$\int_u^v v\,dv = \int_{x=0}^{x} kx\,dx$

$\left[\frac{v^2}{\cancel{2}}\right]_u^v = \left[\frac{kx^2}{\cancel{2}}\right]_0^x$

$\boxed{v^2 - u^2 = kx^2}$

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The velocity of a body depends on time according to the equation $v = \frac{t^2}{10} + 20$. The body is undergoing

(a) ~~Uniform acceleration~~

(b) ~~Uniform retardation~~

(c) Non-uniform acceleration

(d) ~~Zero acceleration~~

$$\boxed{V = \frac{t^2}{10} + 20}$$

$$a = \frac{dv}{dt} = \frac{2t}{10} + 0$$

$$\boxed{a = \frac{t}{5}}$$

↳ variable accelr (Non-uniform)

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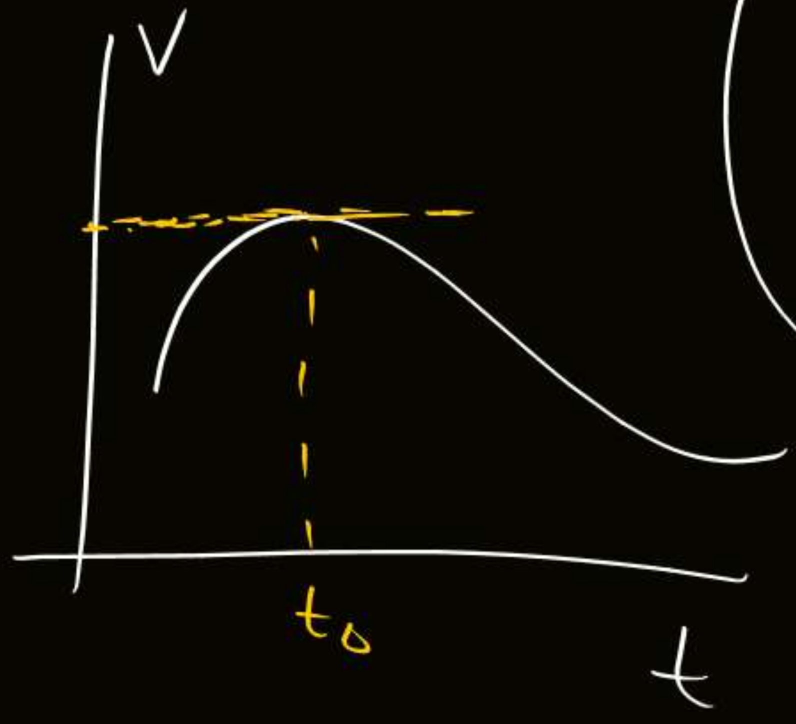
Q If velocity of object $V = 2t(3-t)$ then find time when velocity will be maximum.

$$V = 2t(3-t) = (6t - 2t^2)$$

$$\boxed{\text{'}v\text{' will be max}^n \text{ or min}^m \text{ when } \frac{dv}{dt} = 0}$$



$$\frac{dv}{dt} = (6) - 2 \times 2t$$

$$\frac{dv}{dt} = 6 - 4t = 0$$

$$t = \frac{3}{2} = 1.5\,sec$$

(Q) object is moving such that its velocity $\boxed{V \propto \sqrt{x}}$ then position depends upon time as

(1) $x \propto t$ (2) $x \propto \sqrt{t}$ ~~(3)~~ $\boxed{x \propto t^2}$ (4) $x \propto t^3$

IIT-2006

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$\boxed{V = K\sqrt{x}}$

$$\frac{dx}{dt} = K\sqrt{x}$$

$$\int \frac{dx}{\sqrt{x}} = \int K\,dt$$

$$\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = Kt$$

$$2x^{1/2} = Kt$$

$$\sqrt{x} = \frac{Kt}{2}$$

$$x = \left(\frac{K^2}{4}\right)t^2$$

$$\boxed{x \propto t^2}$$

MR*

MR* इअ

$V \propto \sqrt{x} \Rightarrow a = \text{const}^n$

$x \propto t^2$

object is moving such that its time related with position $\boxed{t = \alpha x + \beta x^2}$ then find acceleration in terms of velocity

$v = \frac{dx}{dt}$

(IIT-Adv)

(DPP-4)

वाली

• NEET में नहीं आएगा

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(i) $-2\alpha v^2$

(ii) $-2\alpha v^3$

(iii) $-2\alpha\beta v^3$

(iv) $\boxed{-2\beta v^3}$

mixed.

$= \frac{t}{x}\left(\frac{x}{t}\right)^2 = \frac{x}{t}$

$= \frac{t}{x}\left(\frac{x^3}{t^3}\right) = x^2$

$\beta v^3 = \left(\frac{t}{x^2}\right)\left(\frac{x}{t}\right)^3$

$= \frac{x}{t^2} = LT^{-2}$

"$t = \alpha x + \beta x^2$"

diff^n w.r.t (x)

$$\left(\frac{dt}{dx}\right) = \alpha \frac{dx}{dx} + \beta \frac{dx^2}{dx}$$

$$\frac{1}{v} = \alpha + 2\beta x$$

$$v = \frac{1}{\alpha + 2\beta x} \Longrightarrow \frac{dv}{dx} = \frac{0 - 1(2\beta)}{(\alpha + 2\beta x)^2}$$

$$a = v\frac{dv}{dx} = \left(\frac{1}{\alpha + 2\beta x}\right) \times \frac{-2\beta}{(\alpha + 2\beta x)^2}$$

$$a = \frac{-2\beta}{(\alpha+2\beta x)^3}$$

$$\boxed{v = \frac{1}{\alpha+2\beta x}}$$

$$a = -2\beta\left(\frac{1}{\alpha+2\beta x}\right)^3$$

$$\boxed{a = -2\beta v^3}$$

Motion with constant acceleration ($a = \text{const}^n$) uniform acc^n.

\# objects starts his motion with velocity $\boxed{u \text{ at } t=0}$ and $\boxed{\text{Constafant acceleration } a}$ then find its velocity and $disp^m$ after time 't' (Derive equation of motion)

$$a = \frac{dv}{dt}$$

$$\int_u^v dv = \int_{t=0}^{t} a \, dt$$

$$\int_u^v dv = a \int_{t=0}^{t} dt$$

$$[v]_u^v = a [t]_0^t$$

$$\boxed{v - u = at}$$

$$\boxed{v = u + at}$$

→ $a = \text{const}^n$

→ Not valid for variable acc^n

$'V_t' = u + at$

$\hookrightarrow$ velocity at time 't'

not speed

$$\frac{dx}{dt} = (u + at)$$

$$\int_{x_i}^{x_f} dx = \int_{t=0}^{t} u\,dt + \int_{t=0}^{t} at\,dt$$

$$\left[x\right]_{x_i}^{x_f} = u\left[t\right]_0^t + \left[a\frac{t^2}{2}\right]_0^t$$

$$\boxed{x_f - x_i = ut + \frac{1}{2}at^2}$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$ #

3^{rd} eq^n of motion

$$a = v\frac{dv}{dx}$$

$$\int_{x_i}^{x_f} a\,dx = \int_{u}^{v} v^1\,dv$$

$$\boxed{a = \text{const}^n}$$

$$a\left[x\right]_{x_i}^{x_f} = \left[\frac{v^2}{2}\right]_{u_i}^{v_f}$$

$$a(x_f - x_i) = \frac{v_f^2 - u_i^2}{2}$$

$$\boxed{2as = v_f^2 - u_i^2}$$

Motion with Constant Acceleration

$$\vec{V} = \vec{u} + \vec{a}\,t$$

$$V_{Avg} = \frac{\vec{u} + \vec{v}}{2}$$

Your text here

$$V^2 - u^2 = 2\,\vec{a}\cdot\vec{s}$$

$$\vec{s} = \vec{u}\,t + \frac{1}{2}at^2$$

$$\vec{s} = \left(\frac{\vec{u} + \vec{v}}{2}\right)t$$

$$s_{nth} = \vec{u} + \frac{a}{2}(2n-2)$$

1. Object starts his motion from rest and constant acceleration then find Ratio of distance in 1-sec, 2-sec, 3-sec.

2. Object starts his motion from rest and const^{n} acc^{n} then find disp^{ro} in 1st – 5sec: 10-sec : 15 sec.

3. Object starts his motion from rest and constant acceleration then find ratio of $disp^n$ in 1^{st} s, 2^{nd} s and 3^{rd} sec.

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THANK YOU ☺