# **Homework**

## **Question 7:**

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.1.1, sections a-g

Use the definitions for the sets given below to determine whether each statement is true or false:

$A = \{ x \in Z$: x is an integer multiple of 3 $\}$
$B = \{ x \in Z$: x is a perfect square $\}$
$C = \{ 4, 5, 9, 10 \}$
$D = \{ 2, 4, 11, 14 \}$
$E = \{ 3, 6, 9 \}$
$F = \{ 4, 6, 16 \}$

a) $27 \in A$ = True
To be an element of A, 27 must be a multiple of 3, which it is. Therefore, 27 is an element of A.

b) $27 \in B$ = False
B implies that 27 must be a perfect square, which it isn't. Therefore, 27 is not an element of B.

c) $100 \in B$ = True
B implies that 100 must be a perfect square, which it is. $10^2 = 100$. Therefore, 100 is an element of B.

d) $E \subseteq C$ or $C \subseteq E$ = False.
Not every element in E is an element of C. Therefore, it is false.

e) $E \subseteq A$ = True.
To be a subset of A, E={3,6,9} must be an integer multiple of 3, which it is. Therefore E is a subset of A.

f) $A \subset E$ = False
To be a proper subset of E, All of A must be in E. A implies that an integer must be multiple of 3 for Z (all integers). There are more integers of 3 aside from E={3,6,9}. E.g. 12 is not an element of E

g) $E \in A$ = False
A set or a subset cannot be made an element.Therefore, E is not an element of A

b) Exercise 3.1.2, sections a-e

Use the definitions for the sets given below to determine whether each statement is true or false:

$A = \{ x \in Z\text{: x is an integer multiple of 3} \}$
$B = \{ x \in Z\text{: x is a perfect square} \}$
$C = \{ 4, 5, 9, 10 \}$
$D = \{ 2, 4, 11, 14 \}$
$E = \{ 3, 6, 9 \}$
$F = \{ 4, 6, 16 \}$

a) $15 \subset A =$ False
To be a proper subset of A, 15 must be in A, which it isn't. Therefore, 15 is <u>not</u> a proper subset of A.

b) $\{15\} \subset A =$ True
To be a proper subset of A, {15} must be in A, which it is. Therefore, {15} is a proper subset of A.

c) $\emptyset \subset A =$ True
The Ø (empty set) is a proper subset of every set. It is true.

d) $A \subseteq A =$ True
$A \subseteq A$ implies that all the element in A are a subset of A. All the same elements are in both A's.

e) $\emptyset \in B =$ False
Ø implies that there are no elements in B, which is false. B is made up of perfect square integers.

c) Exercise 3.1.5, sections b, d

Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

b) { 3,6,9,12, ....}

*Solution*: { 3,6,9,12, ....} = { x $\in$ Z: x is an integer multiple of 3}.

The set is infinite.

d) { 0,10,20,30,...., 1000}

*Solution*: { x $\in$ **N**: x $\geq$ 100, x = 10x }
[ x= ( 10 $\cdot$ 100) = 1000]

The set is finite. The cardinality is 101.

d) Exercise 3.2.1, sections a-k

Let X = {1, {1}, {1, 2}, 2, {3}, 4 }. Which statements are true?

a) 2 $\in$ X = True 2 is an element of X

b) {2} $\subseteq$ X = True {2} is a subset of X

c) {2} $\in$ X = False {2} is not an element of X

d) 3 $\in$ X = False 3 is not an element of X

e) {1, 2} $\in$ X = True {1,2} is an element of X

f) {1, 2} $\subseteq$ X = True {1,2} is a subset of X

g) {2, 4} $\subseteq$ X = True {2,4} is a subset of X

h) {2, 4} $\in$ X = False {2,4} is not an element of X

i) {2, 3} $\subseteq$ X = False {2,3} is not a subset of X

j) {2, 3} $\in$ X = False {2,3} is not an element of X

k) |X| = 7 = False The cardinality of |X| is 6.

**Question 8:**

Solve exercises 3.2.4, section b from the Discrete Math zyBook.

b) Let A = {1, 2, 3}. What is $\{X \in P(A): 2 \in X\}$?

*Solution*:

$$A = \{1, 2, 3\}$$

$$P(A) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$$

$$\{X \in P(A): 2 \in X \} = \{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}$$

**Question 9:**

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.3.1, sections c-e

Define the sets A, B, C, and D as follows:

$$A = \{-3, 0, 1, 4, 17\}$$
$$B = \{-12, -5, 1, 4, 6\}$$
$$C = \{x \in Z: \text{x is odd}\}$$
$$D = \{x \in Z: \text{x is positive}\}$$

For each of the following set expressions, if the corresponding set is finite, express the set using roster notation. Otherwise, indicate that the set is infinite.

c) $A \cap C =$

*Solution*:

$A \cap C = \{-3,1,17\}$ ← They are all <u>odd</u> numbers

d) $A \cup ( B \cap C ) =$

*Solution*:

$(B \cap C)$ = B union C = $\{-5, 1\}$

$A \cup ( B \cap C )$ = A intersects ( B union C) = $\{-5, -3, 0, 1, 4, 17\}$

e) $A \cap B \cap C =$

*Solution*:

$A = \{-3, 0, 1, 4, 17\}$

$B = \{-12, -5, 1, 4, 6\}$

$C = \{x \in Z: \text{x is odd}\}$

$A \cap B \cap C = \{ 1, 4 \}$ ← But 4 is <u>not</u> odd, only 1 is.

Therefore: $A \cap B \cap C = \{ 1 \}$

b) Exercise 3.3.3 , sections a, b, e, f

Use the following definitions to express each union or intersection given. You can use roster or set builder notation in your responses, but no set operations.

$A_i = \{ i^0, i^1, i^2 \}$ (Recall that for any number $x$, $x^0 = 1$.)

$B_i = \{ x \in \mathbf{R} : -i \leq x \leq 1/i \}$

$C_i = \{ x \in \mathbf{R} : -1/i \leq x \leq 1/i \}$

a) $\bigcap_{i=2}^{5} A_i$

*Solution*:

$A_2 = \{1, 2, 4\} \cap A_3 = \{1, 3, 9\} \cap A_4 = \{1, 4, 16\} \cap A_5 = \{1, 5, 25\}$

$= \{ 1 \}$

b) $\bigcup_{i=2}^{5} A_i$

*Solution*:

$A_2 = \{1, 2, 4\} \cup A_3 = \{1, 3, 9\} \cup A_4 = \{1, 4, 16\} \cup A_5 = \{1, 5, 25\}$

$= \{ 1, 2, 3, 4, 5, 9, 16, 25 \}$

e) $\bigcap_{i=1}^{100} C_i$

*Solution*:

$\bigcap_{i=1}^{100} C_i = C_{100}$

$= \{ x \in \mathbf{R} : -\frac{1}{100} \leq x \leq \frac{1}{100} \}$

f) $\bigcup_{i=1}^{100} C_i$

*Solution*:

$$= \{ x \in \mathbf{R} : -\frac{1}{i} \leq x \leq \frac{1}{i} \}$$
$$= \{ x \in \mathbf{R} : -\frac{1}{1} \leq x \leq \frac{1}{1} \}$$
$$= \{ x \in \mathbf{R} : -1 \leq x \leq 1 \}$$

c) Exercise 3.3.4, sections b, d

Use the set definitions A = {a, b} and B = {b, c} to express each set below. Use roster notation in your solutions.

b) P ( A ∪ B )

*Solution*:

P ( A ∪ B ) = { Ø , {a}, {b}, {c}, {a,b}, {b,c}, {a,c}, {a,b,c} }

d) P ( A ) ∪ P ( B)

*Solution*:

P ( A ) ∪ P ( B) = { Ø , {a}, {b}, {c}, {a,b}, {b,c} }

**Question 10:**

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.5.1, sections b,c

The sets A, B, and C are defined as follows:

A = {tall, grande, venti}
B = {foam, no-foam}
C = {non-fat, whole}

Use the definitions for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

b) Write an element from the set B x A x C

*Solution:* B x A x C = { no-foam,grande,non-fat }

c) Write the set B x C using roster notation

*Solution:* B x C = { (foam, non-fat), (foam, whole),(no-foam, non-fat), (no-fam, whole) }

b) Exercise 3.5.3, sections b, c, e

Indicate which of the following statements are true.

b) $Z^2 \subseteq R^2$

*Solution*: True

Let us say (c,d) $\in Z^2$, therefore both c and d are elements of $Z$. If $Z \subseteq R$, then c and d are also elements of $R$. With that being said, (c,d) $\in R^2$.

c) $Z^2 \cap Z^3 = \emptyset$

*Solution*: True

Set $Z^2$ and set $Z^3$ have no elements that are equally the same.

e) For any three sets, A, B, and C, if A ⊆ B, then A × C ⊆ B × C.

*Solution*: True

Let us say (c,d) ∈ A × C, therefore c ∈ A and d ∈ C. If it is stated that A ⊆ B, then c is ∈ of B. With that being said, (c,d) ∈ B × C.

c) Exercise 3.5.6, sections d, e

Express the following sets using the roster method. Express the elements as strings, not n-tuples.

d) { xy: where x ∈ {0} ∪ {0}2 and y ∈ {1} ∪ {1}2 }

*Solution*:

x = {0,00}
y = {1,11}
= { 01, 011, 001, 0011}

e) { xy: x ∈ {aa, ab} and y ∈ {a} ∪ {a}2 }

*Solution*:

x = { aa,ab }
y = { a,aa }
= { aaa,aaaa,aba,abaa }

d) Exercise 3.5.7, section c, f, g

Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as strings.

- A = {a}
- B = {b, c}
- C = {a, b, d}

c) (A × B) ∪ (A × C)

*Solution*:

= (A × B) ∪ (A × C)
= { (a,b),(a,c)} ∪ {(a,a),(a,b),(a,d) }
= { (a,a),(a,b),(a,c),(a,d) }

f) P(A × B)

*Solution*:

A = { a }
B = { b,c }
A x B = { (a,b),(a,c) }
P(A × B) = { Ø, {(a,b)}, {(a,c)}, {(a,b),(a,c)} }

g) P(A) × P(B). Use ordered pair notation for elements of the Cartesian product.

*Solution*:

P(A) = { {Ø}, {a} }

P(B) = { {Ø}, {b,c}, {b}, {c} }

P(A) × P(B) =

{({Ø}, {Ø}), ({Ø}, {b,c}), ({Ø}, {b}), ({Ø}, {c}),({a}, {Ø}), ({a}, {b,c}), ({a}, {b}), ({a}, {c})}

**Question 11:**

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.6.2, sections b, c

Use the set identities given in the table to prove the following new identities. Label each step in your proof with the set identity used to establish that step.

b) $(B \cup A) \cap (\overline{B} \cup A) = A$

| | |
|---|---|
| $( B \cup A ) \cap ( \overline{B} \cup A )$ | |
| $( B \cup A ) \cap ( B \cup A )$ | Double Complement Law |
| $A \cap ( B \cup A )$ | Distributive Law |
| $A \cap ( A \cup B )$ | Commutative Law |
| A | Absorption Law |

c) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

| | |
|---|---|
| $\overline{A \cap \overline{B}}$ | |
| $\overline{A} \cup \overline{B}$ | De Morgan's Law |
| $\overline{A} \cup B$ | Double Complement Law |

b) Exercise 3.6.3, sections b, d

A set equation is not an identity if there are examples for the variables denoting the sets that cause the equation to be false. For example $A \cup B = A \cap B$ is not an identity because if $A = \{1, 2\}$ and $B = \{1\}$, then $A \cup B = \{1, 2\}$ and $A \cap B = \{1\}$, which means that $A \cup B \neq A \cap B$.

Show that each set equation given below is not a set identity.

b) $A - (B \cap A) = A$ = False

*Solution:*

$$A = \{a,b,c\}$$
$$B = \{a\}$$

Then $B \cap A = \{a\}$

$$A - (B \cap A)$$
$$= \{a,b,c\} - \{a\}$$
$$= \{b,c\}$$

$A - (B \cap A)$ <u>does not</u> equal to A. It equals to $\{b,c\}$. Therefore, it is False.

d) $(B - A) \cup A = A$ = False

*Solution:*

$$A = \{a,b,c\}$$
$$B = \{d\}$$

Then $(B - A) = \{d\}$

$$(B - A) \cup A$$
$$= \{d\} \cup \{a,b,c\}$$
$$= \{a,b,c,d\}$$

$(B - A) \cup A$ <u>does not</u> equal to A. It equals to $\{a,b,c,d\}$. Therefore, it is False.

c) Exercise 3.6.4, sections b, c

The set subtraction law states that $A - B = A \cap B$. Use the set subtraction law as well as the other set identities given in the table to prove each of the following new identities. Label each step in your proof with the set identity used to establish that step.

b) $A \cap (B - A) = \varnothing$

| | |
|---|---|
| $A \cap (B \cap \overline{A})$ | Subtraction Law |
| $(A \cap \overline{A}) \cap B$ | Associative Law |
| $\varnothing \cap B$ | Complement Law |
| $\varnothing$ | Domination Law |

c) $A \cup (B - A) = A \cup B$

| | |
|---|---|
| $A \cup (B \cap \overline{A})$ | Subtraction Law |
| $(A \cup B) \cap (A \cup \overline{A})$ | Distributive Law |
| $(A \cup B) \cap \cup$ | Complement Law |
| $(A \cup B)$ | Identity Law |