<u>Homework</u>

<u>Question 5:</u>

Use the definition of Θ in order to show the following:

a. $5n^3 + 2n^2 + 3n = \Theta(n^3)$

<u>*Solution*</u>:

In order to prove that *f* =Θ(g), we must first prove that *f* = *O*(g) and *f* =Ω(g) :

Proof of *f* = *O*(g):

$$f(n) = 5n^3 + 2n^2 + 3n$$
$$g(n) = n^3$$

First, we will use c=10 and n_0=1.
We will show that for any n ≥1, $f(n) \leq c*g(n)$.
For n≥1, it is clear that $n \leq n^2 \leq n^3$:

$$f(n) = 5n^3 + 2n^2 + 3n \leq f(n) = 5n^3 + 2n^3 + 3n^3$$
$$f(n) = 5n^3 + 2n^2 + 3n^3 \leq f(n) = 10n^3 = 10 * g(n)$$

The result after combining the inequalities is:

$$f(n) = 5n^3 + 2n^2 + 3n \leq 10n^3 = 10 * g(n)$$

We have come to the conclusion that $f \leq 10 * g(n)$ and therefore *f* = *O*(g).

Proof of *f* = Ω(g):

$$f(n) = 5n^3 + 2n^2 + 3n$$
$$g(n) = n^3$$

First, we will use c=5 and n_0=1.
We will show that for any n ≥1, $f(n) \geq c*g(n)$.
For n≥1, it is clear that $n \geq 1, 2n^2 + 3n \geq 0$:

When adding $5n^3$ to both sides, we get the following:

$$5n^3 + 2n^2 + 3n \geq 5n^3 = 5n^3 + 2n^2 + 3n \geq 5 * g(n)$$

We have come to the conclusion that $f \geq 5 * g(n)$ and therefore *f* =Ω(g).

Both *f* = *O*(g) and *f* =Ω(g) have been proven and, as a result, we have proved that *f* =Θ(g). ■

b. $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

<u>*Solution*</u>:

In order to prove that *f* =Θ(g), we must first prove that *f* = *O*(g) and *f* =Ω(g) :

Proof of *f* = *O*(g):

$$f(\text{n}) = \sqrt{7n^2 + 2n - 8}$$

First, we will use c=10 and n_0=1.

We will show that for any n ≥1, *f*(n)≤c*g(n).

Since *f* (n) is included, we know the following :

$$7n^2 + 2n - 8 \le 7n^2 + 2n^2 = 7n^2 + 2n - 8 \le 9n^2$$

In order to get our *f* (n) , we can take the square root of both sides:

We solve for c*g(n) as $\sqrt{9n^2}$ = 3n.

It has also been shown that $7n^2 + 2n - 8 \le 9n^2$

And, $\sqrt{7n^2 + 2n - 8} \le$ 3n

In other words, *f*(n)≤3 * n.

Therefore, it has been proven that *f* = *O*(g).

Proof of *f* = Ω(g):

$$f(\text{n}) = \sqrt{7n^2 + 2n - 8}$$

$$\text{g(n)} = \sqrt{7n^2}$$

First, we will use c = 2 and n_0= 4.

We will show that for any n≥4, *f* (n) ≥ c*g(n).

Because *f* (n) must be greater than g(n), we can go ahead and begin to eliminate expressions:

In *f* (n) = $\sqrt{7n^2 + 2n - 8}$, it is understood that $7n^2 \ge 7n^2$, 2n≥2n and 0≥− 8.

We can go ahead and add that g(n) = $\sqrt{7n^2}$

Because 2n− 8≥0, it is understood that n≥4.

The bound must be less than $\sqrt{7}$and, consequently, we are left with c = 2.

Therefore, it has been proven that *f* = Ω(g).

Both *f* = *O*(g) and *f* =Ω(g) have been proven and, as a result, we have proved that *f* =Θ(g). ■