Homework

Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. $10011011_2 = 155_{10}$

1	0	0	1	1	0	1	1
128	64	32	16	8	4	2	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	0	0	16	8	0	2	1

$$128 + 0 + 0 + 16 + 8 + 0 + 2 + 1$$
$$= 155$$
$$10011011_2 = 155_{10}$$

2. $456_7 = 237_{10}$

$$(4 \cdot 7^2 + 5 \cdot 7^1 + 6 \cdot 7^0)$$
$$(4 \cdot 49 + 5 \cdot 7 + 6 \cdot 1)$$
$$(196 + 35 + 6)$$
$$= 237$$
$$456_7 = 237_{10}$$

3. $38A_{16} = 906_{10}$

$$(10 \cdot 16^0 + 8 \cdot 16^1 + 3 \cdot 16^2)$$
$$(10 \cdot 1 + 8 \cdot 16 + 3 \cdot 256)$$
$$(10 + 128 + 768)$$
$$= 906$$
$$38A_{16} = 906_{10}$$

4. $2214_5 = 309_{10}$

$$(4 \cdot 5^0 + 1 \cdot 5^1 + 2 \cdot 5^2 + 2 \cdot 5^3)$$
$$(4 \cdot 1 + 1 \cdot 5 + 2 \cdot 25 + 2 \cdot 125)$$
$$(4 + 5 + 50 + 250)$$
$$= 309$$
$$2214_5 = 309_{10}$$

B. Convert the following numbers to their binary representation:

1. 69_{10} = 1000101

1	0	0	0	1	0	1
64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

64 + 0 + 0 + 0 + 4 + 0 + 1
= 69

2. 485_{10} = 111100101

1	1	1	1	0	0	1	0	1
256	128	64	32	16	8	4	2	1
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

256 + 128 + 64 + 32 + 0 + 0 + 4 + 0 + 1
= 485

3. $6D1A_{16}$ = 0110 1101 0001 1010

Hexadecimal digits	4-bit binary representation
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

$6D1A_{16}$ = $\underline{0110}$ $\underline{1101}$ $\underline{0001}$ $\underline{1010}$

6 D 1 A

$6D1A_{16}$ = 0110110100011010

C. Convert the following numbers to their hexadecimal representation:

1. $1101011_2 = (6B)_{16}$

1101 011

$$
\begin{array}{cccccccc}
1 & 1 & 0 & 1 & & 0 & 1 & 1 \\
\downarrow & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow
\end{array}
$$

$$= \quad 1\cdot2^0 + 1\cdot2^1 + 0\cdot2^2 + 1\cdot2^3 \quad + \quad 0\cdot 2^4 + 1\cdot 2^5 + 1\cdot2^6$$

$$= (1\cdot2^0 + 1\cdot2^1 + 0\cdot2^2 + 1\cdot2^3)\, 2^0 \quad + \quad (0\cdot2^0 + 1\cdot 2^1 + 1\cdot2^2)2^4$$

$$= (1\cdot 1 + 1\cdot 2 + 0\cdot4 + 1\cdot8)\, 2^0 \quad + \quad (0\cdot 1 + 1\cdot 2 + 1\cdot4)\, 2^4$$

$$= (1 + 2 + 0 + 8)\, 2^0 \quad + \quad (0 + 2 + 4)\, 2^4$$

$$= (11)\, 2^0 \quad + \quad (6)\, 2^4$$

$$= (11)\, 16^0 \quad + \quad (6)\, 16^1 \qquad 2^0 = 16^0 \quad 2^4 = 16^1$$

$$= (6) \quad + \quad (11)$$

$$\downarrow \qquad\qquad \downarrow$$

$$= 6 \quad + \quad B$$

Hexadecimal Digits
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9,
A(10), B(11), C (12), D(13)}

$$= (6B)_{16}$$

2. $895_{10} = (37F)_{16}$

$895 \div 16 = 55.9375 = 55$ R 15	R= $16 \cdot .9375 = 15$
$55 \div 16 = 3.4375 = 3$ R 7	R= $16 \cdot .4375 = 7$
$3 \div 16 = 0.1875 = 0$ R 3	R= $16 \cdot .1875 = 3$

We convert the R (Remainder) numbers into hexadecimal numbers:

15 = F (least significant number)
7 = 7
3 = 3 (most significant number)

Hexadecimal Digits
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9,
A(10), B(11), C (12), D(13),
E(14), F(15)}

We write the answer in the order from the most significant number (3) to the least significant number (F).

$$37F_{16}$$

$$895_{10} = (37F)_{16}$$

<u>Question 2:</u>

Solve the following, do all calculations in the given base. Show your work.

1. $7566_8 + 4515_8 = 14303_8$

Octal Number System = {0, 1, 2, 3, 4, 5, 6, 7}

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ 1\ \ } \\ 7\,5\,6\,6 \\ +\ \ \underline{\ 4\,5\,1\,5} \\ 1\ 4\,3\,0\,3 \end{array}$$

6+5=11(decimal) → 11÷8 = 1 R 3 (octal)
6+1+1=8(decimal) → 8÷ 8 = 1 R 0 (octal)
5+5+1=11(decimal) → 11÷8 = 1 R 3 (octal)
7+4+1=12(decimal) → 12 ÷8 = 1 R 4 (octal)
Since there is no more addition to be done,
The carry over one moves down giving the final result 14303.

$7566_8 + 4515_8 = 14303_8$

2. $10110011_2 + 1101_2 = 11000000_2$

Binary System = {0,1}

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ \ \ } \\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 1 \\ +\ \ \underline{0\ 0\ 0\ 0\ 1\ 1\ 0\ 1} \\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0 \end{array}$$

← We add 0 0 0 0 before 1101

$10110011_2 + 1101_2 = 11000000_2$

3. $7A66_{16} + 45C5_{16} = C02B_{16}$

Hexadecimal Number System = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A(10), B(11), C (12), D(13), E(14), F(15)}

$$\begin{array}{cccccc} & & {\scriptstyle 1} & {\scriptstyle 1} & & \\ & & 7 & A & 6 & 6 \\ + & & 4 & 5 & C & 5 \\ \hline & & C & 0 & 2 & B \end{array}$$

6 + 5 =11 = B

6 + C =16 + 12= 18

18=16+2 (1 carry over & 2 write down)

A + 5 + 1 =10 + 5+ 1 = 16

16=16+0 (1 carry over & 0 write down)

7 + 4 + 1= 12 = C

$7A66_{16} + 45C5_{16} = C02B_{16}$

4. $3022_5 - 2433_5 = (34)_5$

Base 5 = {0, 1, 2, 3, 4}

$$\begin{array}{ccccc} & {\scriptstyle 2} & {\scriptstyle 9} & {\scriptstyle 1} & \\ & \cancel{3} & \cancel{0} & \cancel{2} & 2 \\ - & 2 & 4 & 3 & 3 \\ \hline & 0 & 0 & 3 & 4 \end{array}$$

$3022_5 - 2433_5 = (34)_5$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} = (01111100)_{8-bit\ 2's\ complement}$

Zero is added to the left → to make it an 8-bit

0	1	1	1	1	1	0	0
128	64	32	16	8	4	2	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$$= 0 + 64 + 32 + 16 + 8 + 4 + 0 + 0$$

$$= 124$$

$$124_{10} = (01111100)_{8-bit\ 2's\ complement}$$

2. $-124_{10} = (10000100)_{8-bit\ 2's\ complement}$

$$\begin{array}{rccccccccccl} & 1 & 1 & 1 & 1 & 1 & 1 & & & & \\ & & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & \leftarrow & 124_{10} \\ + & & \underline{1} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{1} & \underline{0} & \underline{0} & \leftarrow & -124_{10} \\ & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \leftarrow & 2^8 \end{array}$$

$$-124_{10} = (10000100)_{8-bit\ 2's\ complement}$$

3. $109_{10} = (01101101)_{8-bit\ 2's\ complement}$

Zero is added to the left → to make it an 8-bit

0	1	1	0	1	1	0	1
128	64	32	16	8	4	2	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$$= 0 + 64 + 32 + 0 + 8 + 4 + 0 + 1$$

$$= 109$$

$$109_{10} = (01101101)_{8-bit\ 2's\ complement}$$

4. $-79_{10} = (10110011)_{8-bit\ 2's\ complement}$

79 ÷ 2 = 38 R 1 (top)
38 ÷ 2 = 19 R 0
19 ÷ 2 = 9 R 1
9 ÷ 2 = 4 R 1
4 ÷ 2 = 2 R 0
2 ÷ 2 = 1 R 0
1 ÷ 2 = 0 R 1 (bottom)

We arrange the numbers, from the bottom to the top: 1 0 0 1 1 0 1 and proceed to the next step, addition.

$$
\begin{array}{rccccccccccl}
 & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & & \\
 & & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & & \leftarrow\ 79_{10} \\
+ & & \underline{1} & \underline{0} & \underline{1} & \underline{1} & \underline{0} & \underline{0} & \underline{1} & \underline{1} & & \leftarrow\ -79_{10} \\
 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \leftarrow\ 2^8
\end{array}
$$

$-79_{10} = (10110011)_{8-bit\ 2's\ complement}$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8-bit\ 2's\ complement} = 30_{10}$

$$
\begin{array}{cccccccc}
\underline{0} & \underline{0} & \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} \\
128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\
2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0
\end{array}
$$

$= \ 0 + 0 + 0 + 16 + 8 + 4 + 2 + 0$

$= 30$

$00011110_{8-bit\ 2's\ complement} = 30_{10}$

2. $11100110_{8-bit\ 2's\ complement} = -26_{10}$

1	1	1	0	0	1	1	0
128	64	32	16	8	4	2	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$= 128 + 64 + 32 + 0 + 0 + 4 + 2 + 0$

$= 230$

$$\begin{array}{rccccccccl}
 & ^1 & ^1 & ^1 & ^1 & ^1 & ^1 & ^1 & & \\
 & & 0 & 0 & 0 & 1 & 1 & 0 & 1\ 0 & \leftarrow 26 \\
+ & & 1 & 1 & 1 & 0 & 0 & 1 & 1\ 0 & \leftarrow -26_{10} \\
\hline
 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\ 0 & \leftarrow 2^8
\end{array}$$

$11100110_{8-bit\ 2's\ complement} = -26_{10}$

3. $00101101_{8-bit\ 2's\ complement} = 45_{10}$

0	0	1	0	1	1	0	1
128	64	32	16	8	4	2	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$= 0 + 0 + 32 + 0 + 8 + 4 + 0 + 1$

$= 45$

$00101101_{8-bit\ 2's\ complement} = 45_{10}$

4. $10011110_{8-bit\ 2's\ complement} = -98_{10}$

$$\begin{array}{cccccccc} \underline{1} & \underline{0} & \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{0} \\ 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$$= 128 + 0 + 0 + 16 + 8 + 4 + 2 + 0$$

$$= 158$$

$$\begin{array}{rccccccccl} & & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & \leftarrow 98_{10} \\ + & & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & \leftarrow -98_{10} \\ \hline & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \leftarrow 2^8 \end{array}$$

$10011110_{8-bit\ 2's\ complement} = -98_{10}$

Question 4:
Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, sections b, c

Write a truth table for each expression

(b) $\neg (p \lor q)$

p	q	$p \lor q$	$\neg (p \lor q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

(c) $r \lor (p \land \neg q)$

p	q	r	$\neg q$	$(p \land \neg q)$	$r \lor (p \land \neg q)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

2. Exercise 1.3.4, section b, d

(b) $(p \to q) \to (q \to p)$

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$\neg q$	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7, sections b,c

Consider the following pieces of identification a person might have in order to apply for a credit card:

B: Applicant presents a birth certificate.
D: Applicant presents a driver's license.
M: Applicant presents a marriage license.

Write a logical expression for the requirements under the following conditions:

(b) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage's license.

$$(B \land (D \lor M)) \lor (D \land M)$$

(c) Applicant must present either a birth certificate or both a driver's license and a marriage license.

$$B \lor (D \land M)$$

2. Exercise 1.3.7, sections b - e

Define the following propositions:

s: a person is a senior
y: a person is at least 17 years of age
p: a person is allowed to park in the school parking lot

(b) A person can park in the school parking lot if they are a senior or at least seventeen years of age.

$$(s \lor y) \rightarrow p$$

(c) Being 17 years of age is a necessary condition for being able to park in the school parking lot.

$$p \rightarrow y$$

(d) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

$$p \leftrightarrow (s \land y)$$

(e) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

$$p \to (s \lor y)$$

3. 1.3.9, sections c, d

Use the definitions of the variables below to translate each English statement into an equivalent logical expression.

y: the application is at at least eighteen years old
p: the applicant has parental permission
c: the application can enroll in the course

(c) The applicant can enroll in the course only if the applicant has parental permission.

$$c \to p$$

(d) Having parental permission is a necessary condition for enrolling in the course.

$$c \to p$$

Question 6:
Solve the following questions from Discrete Math zyBook:

1. Exercise 1.3.6, sections b - d

Give an English sentence in the form "If...then.." that is equivalent to each sentence.

(b) Maintaining a B average is necessary for Joe to be eligible for the honors program.

If Joe maintains a B average, then he is available for the honors program.

(c) Rajiv can go on the roller coaster only if he is at least four feet tall.

If Rajiv can go to the roller coaster, then he is at least four feet tall.

(d) Rajiv can go on the roller coaster if he is at least four feet tall.

If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10, sections c - f

The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

(c) $(p \vee r) \leftrightarrow (q \wedge r)$

False. The derived expression will always be <u>true</u>, despite any value r may have.

(d) $(p \wedge r) \leftrightarrow (q \wedge r)$

Unknown. If r was to be <u>true</u> then the expression would be <u>false</u>. But if r was to be <u>false</u>, then the expression would be <u>true</u>.

(e) $p \rightarrow (r \vee q)$

Unknown. If r was to be <u>true</u> then the expression would be <u>false</u>. But if r was to be <u>false</u> then the expression would be <u>true</u>.

(f) $(p \wedge q) \rightarrow r$

True. The derived expression will always be <u>true</u>, despite any value r may have.

Question 7:
Solve exercise 1.4.5, sections b - d, from the Discrete Math zyBook:

Define the following propositions:

j: Sally got the job.
l: Sally was late for her interview
r: Sally updated her resume.

Express each pair of sentences using a logical expression. Then prove whether the two expressions are logically equivalent.

(b) If Sally did not get the job, then she was late for interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

$$\neg j \rightarrow l \lor \neg r \equiv r \land \neg l \rightarrow j$$

j	l	r	$\neg j$	$\neg l$	$\neg r$	$\neg j \rightarrow l \lor \neg r$	$r \land \neg l \rightarrow j$
T	T	T	F	F	F	T	T
T	T	F	F	F	T	T	T
T	F	T	F	T	F	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	F	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	F	F	F
F	F	F	T	T	T	T	T

As we can see, according to the truth table, $\neg j \rightarrow l \lor \neg r$ is <u>equivalent to</u> $r \land \neg l \rightarrow j$.

Their values <u>are</u> equal.

(c) If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

$$j \rightarrow \neg l \equiv \neg j \rightarrow l$$

j	l	¬j	¬l	j→¬l	¬j→l
T	T	F	F	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	F

As we can see, according to the truth table, $j \rightarrow \neg l$ is <u>not equivalent to</u> $\neg j \rightarrow l$.

Their values are <u>not</u> equal.

(d) If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

$$r \lor \neg l \rightarrow j \equiv j \rightarrow r \land \neg l$$

j	l	r	¬l	r ∨ ¬l → j	j → r ∧ ¬l
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	T	T
F	F	T	T	F	T
F	F	F	T	F	T

As we can see, according to the truth table, $r \lor \neg l \rightarrow j$ is <u>not logically equivalent</u> to $j \rightarrow r \land \neg l$.

Their values are <u>not</u> equal.

<u>Question 8:</u>

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2, sections c, f, i

Use the laws of propositional logic to prove the following:

(c) $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to q) \land (p \to r)$

	Laws of Propositional Logic
$(p \to q) \land (p \to r)$	
$(\neg p \lor q) \land (p \to r)$	Conditional Identity
$(\neg p \lor q) \land (\neg p \lor r)$	Conditional Identity
$\neg p \lor (q \land r)$	Distributive Law
$p \to (q \land r)$	Conditional Identify

Therefore, $(p \to q) \land (p \to r)$ is <u>logically equivalent</u> to $p \to (q \land r)$.

$$(p \to q) \land (p \to r) \equiv p \to (q \land r)$$

(f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q))$

	Law of Propositional Logic
$\neg(p \lor (\neg p \land q))$	
$\neg p \land \neg(\neg p \land q)$	De Morgan's Law
$\neg p \land (\neg\neg p \lor \neg q)$	De Morgan's Law
$\neg p \land (p \lor \neg q)$	Double Negation Law
$(\neg p \land p) \lor (\neg p \land \neg q)$	Distributive Law
$F \lor (\neg p \land \neg q)$	Complement Law
$\neg p \land \neg q$	Identity Law

Therefore, $\neg(p \lor (\neg p \land q))$ is <u>logically equivalent</u> to $\neg p \land \neg q$.

$$\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$$

(i) $(p \land q) \rightarrow r \equiv (p \land \neg r) \rightarrow \neg q$

$(p \land q) \rightarrow r$

	Law of Propositional Logic
$(p \land q) \rightarrow r$	
$\neg (p \land q) \lor r$	Conditional Identity
$(\neg p \lor \neg q) \lor r$	De Morgan's Law
$(\neg p \lor r) \lor \neg q$	Associative Law
$(\neg p \lor \neg\neg r) \lor \neg q$	Double Negation Law
$\neg (p \land \neg r) \lor \neg q$	De Morgan's Law
$(p \land \neg r) \rightarrow \neg q$	Conditional Identity

Therefore, $(p \land q) \rightarrow r$ is logically equivalent to $(p \land \neg r) \rightarrow \neg q$.

$$(p \land q) \rightarrow r \equiv (p \land \neg r) \rightarrow \neg q$$

2. Exercise 1.5.3, sections c, d

Use the laws of propositional logic to prove that each statement is a tautology.

(c) $\neg r \vee (\neg r \to p)$

Show that $\neg r \vee (\neg r \to p) \equiv T$

	Law of Propositional Logic
$\neg r \vee (\neg r \to p)$	
$\neg r \vee (\neg\neg r \vee p)$	Conditional Law
$\neg r \vee (r \vee p)$	Double Negation Law
$(\neg r \vee r) \vee p$	Associative Law
$(r \vee \neg r) \vee p$	Commutative Law
$T \vee p$	Complement Law
$p \vee T$	Commutative Law
T	Domination Law

$$\neg r \vee (\neg r \to p) \equiv T$$

(d) $\neg(p \to q) \to \neg q$

Show that $\neg(p \to q) \to \neg q \equiv T$

	Law of Propositional Logic
$\neg(p \to q) \to \neg q$	
$\neg(\neg p \lor q) \to \neg q$	Conditional Law
$(\neg\neg p \lor q) \to \neg q$	Distributive Law
$(p \lor \neg q) \to \neg q$	Double Negation Law
$\neg(p \lor \neg q) \lor \neg q$	Conditional Law
$(\neg p \lor \neg\neg q) \lor \neg q$	Distributive Law
$(\neg p \lor \neg\neg q) \lor \neg q$	Double Negation Law
$\neg p \lor (q \lor \neg q)$	Associative Law
$\neg p \lor T$	Complement Law
T	Domination Law

$$\neg(p \to q) \to \neg q \equiv T$$

Question 9:

Solve the following question from the Discrete Math zyBook:

1. Exercise 1.6.3, sections c, d

Consider the following statements in English.Write a logical expression with the same meaning. The domain of discourse is the set of all real numbers.

(c) There is a number that is equal to its square.

$$\exists x(x = x^2)$$

(d) Every number is less than or equal to its square.

$$\forall x(x \leq x^2)$$

2. Exercise 1.7.4, sections b - d

In the following question, the domain of discourse is aset of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:

S(x): x was sick yesterday
W(x): x went to work yesterday
V(x): x was on vacation yesterday

Translate the following English statements into a logical expression with the same meaning.

(b) Everyone was well and went to work yesterday.

$$\forall x(\neg S(x) \land W(x))$$

(c) Everyone who was sick yesterday did not go to work.

$$\forall x(S(x) \rightarrow \neg W(x))$$

(d) Yesterday someone was sick and went to work.

$$\exists x(S(x) \land W(x))$$

Question 10:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9, sections c - i

Using these values, determine whether each quantified expression evaluates to true or false.

(c) $\exists x((x=c) \rightarrow P(x))$

True. On the truth table, a, b, d & e are all True. When x=c is false and the values of P, Q and R are true.

(d) $\exists x(Q(x) \land R(x))$

True. If we substitute x=e in both $Q(x) \land R(x)$ we come up with $Q(e) \land R(e)$ and both of their values are True.

(e) $Q(a) \land P(d)$

True. Both, $Q(a) \land P(d)$ are both true. The expression is indeed true.

(f) $\forall x((x \neq b) \rightarrow Q(x)$

True. All but b evaluate to T (true).

(g) $\forall x(P(x) \lor R(x))$

False. When x=c, the expression turns out to be false, therefore, not all of the values are true.

(h) $\forall x(R(x) \rightarrow P(x))$

True. Whether x is equal to a, b, c, d or e, the expression is true.

x = a	F → T = T (True)
x = b	F → T = T (True)
x = c	F → F = T (True)
x = d	F → T = T (True)
x = e	T → T = T (True)

(i) $\exists x(Q(x) \lor R(x))$

True. The expression is true for at least one.

x = a	$T \lor F = T$
x = b	$F \lor F = F$
x = c	$T \lor F = T$
x = d	$T \lor F = T$
x = e	$T \lor T = T$

2. Exercise 1.9.2, sections b-i

Indicate whether each of the quantified statements is true or false.

(b) $\exists x \forall y\ Q(x, y)$

True. In the second row in Q all the values are True.

(c) $\exists x \forall y\ P(y, x)$

True. When x=1, all p (1, x), p(2, x) and p (3,x) are true.

(d) $\exists x \exists y\ S(x, y)$

False. Every value comes up to be false.

(e) $\forall x \exists y\ Q(x, y)$

False. When x=1, every single y is false.

(f) $\forall x \exists y\ P(x, y)$

True.

x = 1 & y = 1	T
x = 2 & y = 1	T
x = 3 & y = 1	T

(g) $\forall x \, \forall y \, P(x, y)$

False. $\forall x \, \forall y$ implies for all x and y to be true but not all x and y are true.

(h) $\exists x \, \exists y \, Q(x, y)$

True. There exists at least one x and one y, therefore, it is true.

(i) $\forall x \, \forall y \, \neg S(x, y)$

True. After applying negation, all x and y's result in false.

Question 11:

Solve the following question from the Discrete Math zyBook:

1. Exercise 1.10.4, sections c - g.

Translate each of the following English statements into logical expressions. The domain of discourse is the set of all real numbers.

(c) There are two numbers whose sum is equal to their product.

$$\exists x \ \exists y \ (x + y = xy)$$

(d) The ratio of every two positive numbers is also positive.

$$\forall x \ \forall y \ ((x > 0 \land y > 0) \rightarrow (x/y > 0))$$

(e) The reciprocal of every positive number less than one is greater than one.

$$\forall x \ (((x > 0 \land x < 1) \rightarrow ((1/x > 1))$$

(f) There is no smallest number.

$$\neg \exists x \ \forall y \ (x \leq y)$$

(g) Every number besides 0 has a multiplicative inverse.

$$\forall x \ \exists y \ ((x \neq 0) \rightarrow (xy=1))$$

2. Exercise 1.10.7, sections c - f.

The domain of discourse is a group working on a project at a company. One of the members of the group is named Sam. Define the following predicates.

P(x,y): x knows y's phone number. (A person may or may not know their own phone number)
D(x): x missed the deadline
N(x): x is a new employee

(c) There is at least one new employee who missed the deadline.

$$\exists x\ (N(x)\ \wedge\ D\ (x))$$

(d) Sam knows the phone number of everyone who missed the deadline.

$$\exists x\ \forall y\ (D(y) \rightarrow P(x=Sam,\ y))$$

(e) There is a new employee who knows everyone's phone number.

$$\exists x\ \forall y\ (N(x)\ \wedge\ P(x,y))$$

(f) Exactly one new employee missed the deadline.

$$\exists x\ \forall y\ (((N(x)\ \wedge\ D\ (x))\ \wedge\ (y \neq x) \wedge (N(y) \rightarrow \neg D(y)))$$

3. Exercise 1.10.10, sections c - f.

The domain for the first input variable to predicate T is a set of students at a university. The domain for the second input variable to predicate T is the set of Math classes offered at that university. The predicate T(x, y) indicates that student x has taken class y. Sam is a student at the university and Math 101 is one of the courses offered at the university. Give a logical expression for each sentence.

(c) Every student has taken at least one class besides Math 101.

$$\forall x\ \exists y\ ((y \neq \text{Math } 101) \rightarrow T(x,y))$$

(d) There is a student who has taken every math besides Math 101.

$$\exists x\ \forall y\ ((y \neq \text{Math}101 \rightarrow T(x,y))$$

(e) Everyone besides Sam has taken at least two different math classes.

$$\forall x\ \exists y\ \exists z\ (((x \neq \text{Sam})) \rightarrow ((y \neq z) \land T(x,y) \land T(x,z)))$$

(f) Sam has taken exactly two math classes.

$$\exists x\ \exists y\ \forall z\ (x \neq y \land T(\text{Sam},y) \land T(\text{Sam},z) \land (w \neq y \land w \neq z)) \rightarrow \neg T(\text{Sam},w)$$

Question 12:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2, sections b - e

In the following question, the domain of discourse is a set of male patients in a clinical study. Define the following predicates:

P(x): x was given the placebo
D(x): x was given the medication
M(x): x had migraines

Translate each statement into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

(b) Every patient was given the medication or the placebo or both.

$$\forall x\ (D(x) \lor P(x))$$

After Negation Operation: $\neg \forall x\ (D(x) \lor P(x))$

After Applying De Morgan's Law: $\exists x\ (\neg D(x) \land \neg P(x))$

English: There is a patient who was not given the medication and not given the placebo.

(c) There is a patient who took the medication and had migraines.

$$\exists x\ (D(x) \land M(x))$$

After Negation Operation: $\neg \exists x\ (D(x) \land M(x))$

After Applying De Morgan's Law: $\forall x\ (\neg D(x) \lor \neg M(x))$

English: Every patient did not get the medication or did not have migraines or both.

(d) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \to q \equiv \neg p \lor q$.)

$$\forall x\ (P(x) \to M(x))$$

After Negation Operation: $\neg \forall x\ (P(x) \to M(x))$

After Applying De Morgan's Law: $\exists x\ (P(x) \land \neg M(x))$

English: Some patient took the placebo and did not have migraines.

(e) There is a patient who had migraines and was given the placebo.

$$\exists x\ (M(x) \land P(x))$$

After Negation Operation: $\neg \exists x\ (M(x) \land P(x))$

After applying De Morgan's Law: $\forall x\ (\neg M(x) \lor \neg P(x))$

English: Every patient either did not have migraine or was not given the placebo or both.

2. Exercise 1.9.4, sections c - e

Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

(c) $\exists x\ \forall y\ (P(x, y) \rightarrow Q(x, y))$

After Negation: $\forall x\ \exists y\ (P(x,y) \land \neg\ Q(x, y))$

(d) $\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x))$

$$\neg\ \exists x\ \forall y\ ((P(x,y) \rightarrow\ P(y,x)) \land\ P(y,x)\ \rightarrow P(x,y))))$$

$$\neg\ (\exists x\ \forall y\ ((\neg P(x,y) \lor\ P(y,x)) \land\ \neg P(y,x) \lor\ P(x,y))))$$

$$\forall x\ \exists y\ (\neg (\neg\ P(x,y)\ \ vP(y,x) \lor\ (\neg (\neg\ P(x,y)\ v\ P(x,y)))$$

After Negation: $\forall x\ \exists y\ (((P(x,y) \land\ \neg\ (P(y,x) \lor\ P(y,x) \land\ \neg\ (P(x,y)))$

(e) $\exists x\ \exists y\ P(x, y) \land\ \forall x\ \forall y\ Q(x, y)$

$$\neg\ (\ \exists x\ \exists y\ P(x, y) \land\ \forall x\ \forall y\ Q(x, y)$$

After Negation: $\forall x\ \forall y\ (\neg\ P(x,y)\ v\ \exists x\ \exists y\ \neg\ Q(x, y)$