Homework

Question 3:

Solve the following questions from the Discrete Math zyBook:

a) Exercise 4.1.3, sections b, c

Which of the following are functions from R to R? If f is a function, give its range.

(b) $f(x) = 1/(x^2 - 4)$

Solution: $f(x) = 1/(x^2 - 4)$ is not a function because it would not apply for x = 2 and x =−2.

(c) $f(x) = \sqrt{x^2}$

Solution: $f(x) = \sqrt{x^2}$ is a function for <u>all</u> values in R . When taking the square root of any number, it will <u>never</u> be negative. Therefore, the range is all positive real numbers (R^+) and 0.

b) Exercise 4.1.5, sections b, d, h, i, l

Express the range of each function using roster notation

(b) Let A = {2, 3, 4, 5}.
f: A → **Z**

Solution: The range is {4, 9, 16, 25}

(d) f: $\{0, 1\}^5$ → **Z**. For x ∈ $\{0, 1\}^5$, f (x) is the number of 1's that occur in x. For example f (01101) = 3, because there are three 1's in the string "01101".

Solution: The range is {0, 1, 2, 3, 4, 5}

(h) Let A = {1, 2, 3}.
f: A x A → **Z** x **Z**, where f (x,y) = (y, x).

Solution: The range is {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}

(i) Let A = {1, 2, 3}.
f: A x A $\longrightarrow$ **Z** x **Z**, where f (x,y) = (x, y + 1).

Solution: The range is {(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)}

(l) Let A = {1, 2, 3}.
f: P(A) $\longrightarrow$ P(A). For X $\subseteq$ A, f(x) = X - {1}

Solution: The range is {$\emptyset$, {2}, {3}, {2,3}}

Question 4:

I. Solve the following question from the Discrete Math zyBook:

a. Exercise 4.2.2, sections c, g, k

For each of the functions below, indicate whether the function is onto, one-to-one, neither or both. If the function is not onto or not one-to-one, give an example showing why.

(c) h: $\mathbf{Z} \to \mathbf{Z}$. h(x) = x^3

Solution: The function is one-to-one but not onto. h(x) is <u>not</u> onto because if h(x) = 2 which also equals to x^3 then x will equal to $\sqrt[3]{2}$ which is <u>not</u> in Z.

(g) f: **Z** x **Z** $\to$ **Z** x **Z**, $f(x, y) = (x + 1, 2y)$

Solution: The function is one-to-one but not onto. For y, it will <u>always</u> be even. 2y will always come out to even numbers.

(k) f: $\mathbf{Z}^+ \times \mathbf{Z}^+ \to \mathbf{Z}^+$, f(x,y) = 2^x + y.

Solution: The function is not one-to-one and not onto. For example: (2,1) and (1,3), if evaluated, they both result in 5. If you were to solve for (1,1), you would get 3 as a result and there is no value below that since Z^+(set of all integers) is positive numbers <u>only</u>. And because 1 is the lowest positive number available for evaluation there is no pair x,y that will be equivalent to f(x,y) =1.

b. Exercise 4.2.4, sections b, c, d, g

For each of the functions below, indicate whether the function is onto, one-to-one, neither or both. If the function is not onto or not one-to-one, give an example showing why.

(b) f:$\{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, f(001)=101 and f(110) = 110.

Solution: Not one-to-one and not onto. Not one-to-one because f(000) will equal to f(100) which will equal to 100. And it can't be onto because there is no item that would map to 000.

(c) f:$\{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example f(011) = 110.

Solution: One-to-one and onto

(d) $f:\{0,1\}^3 \to \{0,1\}^4$. The output of f is obtained by taking the input string and adding an extra copy of the first bit to the end of the string. For example, f(100) = 1001.

Solution: One-to-one and not onto. It can't be onto because they are not the same size. If the target is bigger than the domain, it cannot be onto.

(g) Let A be defined to be the set {1, 2, 3, 4, 5, 6, 7, 8} and let B = {1}.
f: $P(A) \to P(A)$. For $X \subseteq A$, $h(X) = X \cup \{a\}$.

Solution: Not one-to-one and not onto. It is not one-to-one because if x1 = {1,2} and x2= {2} then the function of the two will be equal to {2} (the set of 2). And not onto because there will never be a {1} (a set of 1).

II. Give an example of a function from the set of integers to the set of positive integers that is:

a. one-to-one, but not onto.

Solution: $f(x) = 2x$ for $x \geq 0$ and $2|x|+1$ for $x < 0$

b. onto, but not one-to-one.

Solution: $f(x) = |x| + 1$

c. one-to-one and onto.

Solution: $f(x) = 2x$ for $x \geq 0$ and $2|x| - 1$ for $x < 0$

d. neither one-to-one nor onto

Solution: $f(x) = 1$

Question 5:

Solve the following question from the Discrete Math zyBook:

a) Exercise 4.3.2, sections c, d, g, i

For each of the following functions, indicate whether the function has a well-defined inverse. If the inverse is well-defined, give the input/output relationship of f^{-1}.

(c) f: **R** ⟶ **R**. f(x) = 2x + 3

Solution: It is well-defined. The inverse function of x equals to $\frac{x-3}{2}$.

(d) Let A be defined to be the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$.
f: P(A) ⟶ $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$. For X ⊆ A, f(X) = |X|. Recall that for a finite set A, P(A) denotes the power set of A which is the set of all subsets of A.

Solution: It is not one-to-one and therefore not well-defined.

(g) f:$\{0, 1\}^3$ ⟶ $\{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example, f(011) = 110.

Solution: It is well-defined. The inverse of f is f itself.

(i) f: **Z** x **Z** ⟶ **Z** x **Z**, f(x,y) = (x+5, y-2)

Solution: It is well-defined. The inverse of f (x,y)= (x-5, y+2).

b) Exercise 4.4.8, sections c, d

The domain and target set of functions f, g, and h are Z. The functions are defined as:

$$f(x) = 2x + 3$$
$$g(x) = 5x + 7$$
$$h(x) = x^2 + 1$$

Give an explicit formula for each function given below.

(c) f o h

Solution: f o h(x)= $2x^2 + 5$

(d) h o f

Solution: h o f(x) = $4x^2 + 12x + 10$

c) Exercise 4.4.2, sections b-d

Consider three functions f, g, and h, whose domain and target are **Z.** Let

$$f(x) = x^2 \qquad g(x) = 2^x \qquad h(x) = \left\lceil \frac{x}{5} \right\rceil$$

(b) Evaluate f o h(52)

Solution: f o h (52) $= \left(\left\lceil \frac{52}{5} \right\rceil\right)^2$

$= 11^2$

$= 121$

(c) Evaluate g o h o f(4)

Solution: g o h o f(4) = g o h (16)

= g(x) = 2^x

= g(4) = $2^4 = 16$

(d) Give a mathematical expression for h o f.

Solution: h o f

$$= h(x) = \left\lceil \frac{x}{5} \right\rceil \;\&\; f(x) = x^2$$

$$= \left\lceil \frac{x^2}{5} \right\rceil$$

d) Exercise 4.4.6, sections c-e

Define the following functions f, g, and h:

f:$\{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, f(001) = 101 and f(110) = 110.

g:$\{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of g is obtained by taking the input string and reversing the bits. For example, g(011) = 110.

h:$\{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of h is obtained by taking the input string x, and replacing the last bit with a copy of the first bit. For example, h(011) = 010.

(c) What is h o f(010)?

Solution: h o f(010) = 111

(d) What is the range of h o f?

Solution: The range of h o f is {101, 111}.

(e) What is the range of g o f?

Solution: The range g o f {001, 011, 101, 111}.

e) **Extra Credit**: Exercise 4.4.4, sections c, d

Let f: X ⟶ Y and g: Y ⟶ Z be two functions.

(c) Is it possible that f is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g.

Solution: No. If g o f is one to one then f is one to one. If x1 and x2 were elements in X they will not be equivalent to each other and since g o f is one-to-one, the function of both will not be equivalent either.

(d) Is it possible that g is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g.

Solution: Yes. It is possible that g is not one-to-one and g o f is one-to-one.