ASSIGNMENT - 1

papergrid

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1) The open loop transfer function of a certain unity feedback system is given as $G(s) = \frac{K}{s(s+1)}$

$K_v = 10$ and phase margin is $60°$. Design a suitable phase lag compensator.

$$G(s) = \frac{K}{s(s+1)}$$

Step 1 : calculate gain K

given $K_v = 10$

also $K_v = \lim_{s \to 0} s\, G(s) H(s)$

$$10 = \lim_{s \to 0} s \frac{K}{s(s+1)}$$

$\therefore K = 10$ //.

Step 2 .

$$G(s) = \frac{10}{s(s+1)} = \frac{10}{s(1+s)}$$

Corner frequency, ω_c = 1 rad/sec.

$$G(j\omega) = \frac{10}{j\omega(1+j\omega)}$$

$$|G(j\omega)| = \frac{10}{\omega\sqrt{1+\omega^2}}$$

ω_{cl} = 0.1 rad/sec

ω_{ch} = 10 rad/sec

Magnitude plot

ω	10 $20\log 10$	$1/j\omega$ $-20\log j\omega$	$1/1+j\omega$ $-20\log \omega$ $\omega_c = 1$ rad/sec	total magnitude
0.1	20	20	0	40
1	20	0	0	20
10	20	-20	-20	-20

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$\phi = \angle G(j\omega) = -90 - \tan^{-1}\omega$

phase plot

ω	0.1	0.5	1	2	5	10	100
ϕ	-96	-117	-135	-153	-169	-174	-179

Step - 3

phase margin, $\gamma = 180 + \phi gc$

$\gamma = 180 - 164$

$= 16°$ //

Step - 4

phase margin of compensated system

$\gamma_n = \gamma_d + \varepsilon$

$= 60+5 = 65°$ //

$65 = 180 + \phi gcn$

$\phi gcn = 65 - 180 = -115°$ //

Step - 5

$\omega gcn = 0.44$ rad/sec

Step - 6

$A_{gcn} = 20 \log \beta$

$A_{gcn} = 27 db$.

$\beta = 10^{\frac{A}{20}} = 10^{\frac{27}{20}} = 22.39$ //

Step - 7

Zero of lag compensator, $Z_c = \frac{1}{T} = \frac{\omega gcn}{10} = \frac{0.44}{10}$

$T = 22.72$

Pole of compensator, $P_c = \frac{1}{\beta T} = \frac{1}{22.39 \times 22.72} = \frac{1}{508.7}$

Transfer function of lag compensator

$G(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} = \frac{\beta(1+sT)}{(1+s\beta T)} = \frac{22.39(1+22.72s)}{(1+508.7s)}$

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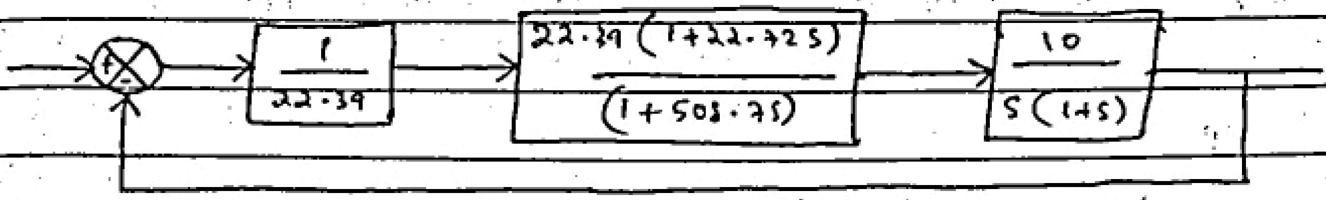
Step -8

To find the open loop transfer function of compensated system



$$G_o(s) = \frac{10\,(1+22.72s)}{s(1+s)(1+508.7s)}$$

step - 9

$$G_o(j\omega) = \frac{10\,(1+22.72\,j\omega)}{j\omega\,(1+j\omega)(1+503.7j\omega)}$$

$$\phi_o = \tan^{-1} 22.72\,\omega - 90 - \tan^{-1}\omega - \tan^{-1} 508.7\,\omega.$$

put $\omega = \omega_{gcn} = 0.44$ rad/sec.

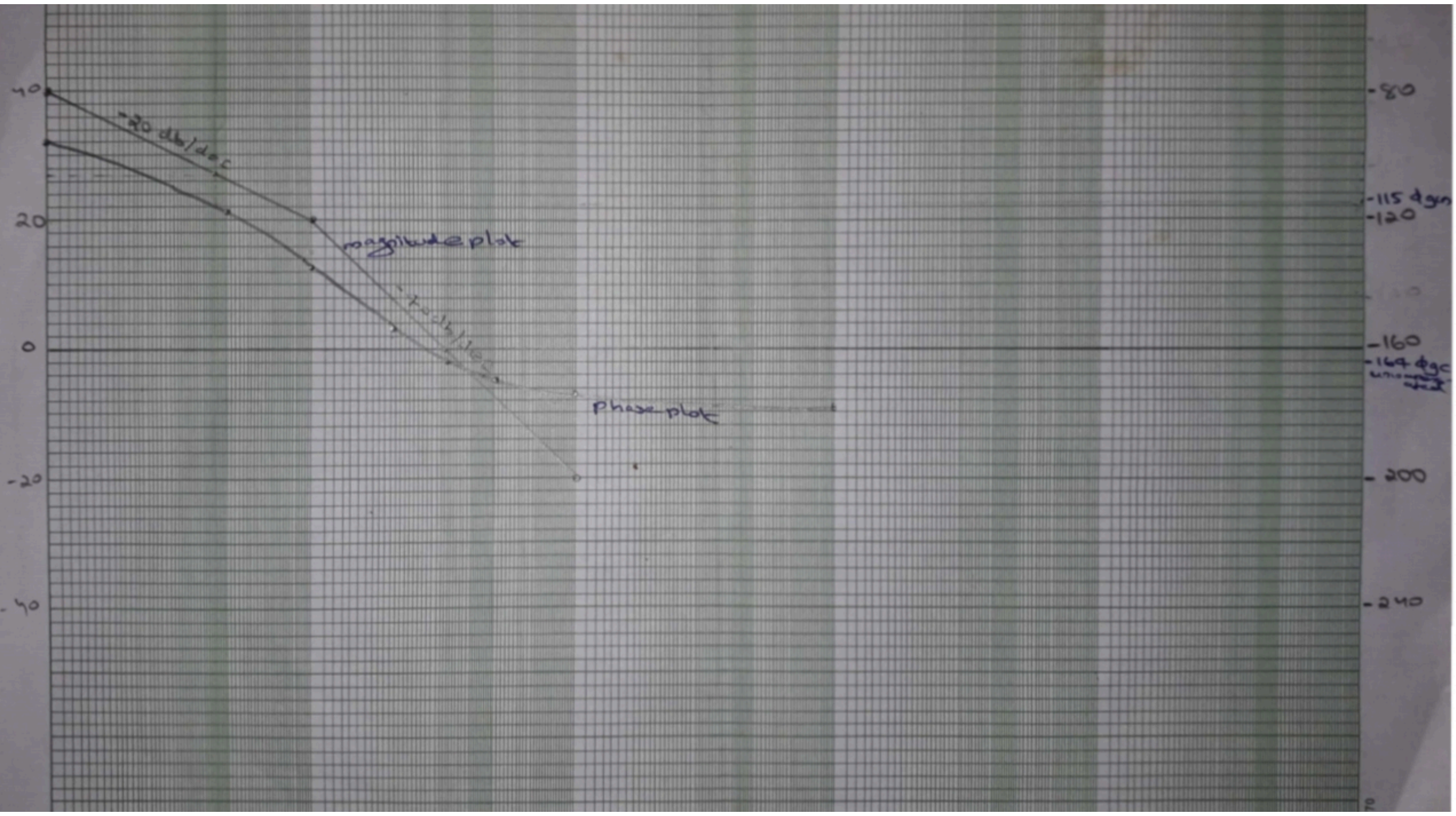
$$\phi_o = -120^\circ$$

Compensated system, $\gamma_o = 180 + \phi_{gcn}$

$$= 180 - 120 = 60^\circ$$

Transfer function of lag compensator $= G_c(s) = \dfrac{22.39(1+22.72s)}{(1+503.7s)}$

Transfer function of compensated system $G_o(s) = \dfrac{10\,(1+22.72s)}{s(1+s)(1+508.7s)}$


40
20
0
-20
-40
-20 db/dec
magnitude plot
-40db/dec
phase plot
-80
-115 dge
-120
-160
-164 dge
200
-240

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2.) The open loop transfer function of a certain unity feedback system is given as $G(s) = K/s(s+4)(s+5)$. It is desired that Kv should be atleast 5 and the damping ratio should be 0.707. Design a suitable lag compensator to meet these requirements.

$$G(s) = \frac{K}{s(s+4)(s+5)}$$

Step-1 to find root locus

poles of open loop system are $s = 0, -4, -5$

zeroes = no zeroes

Asymptote = $\frac{\pm 180 (2q+1)}{p-z}$

for $q=0$ = ± 60

for $q=1$ = ± 180

for $q=2$ = ± 300

Centroid

$$\frac{\Sigma P - \Sigma Z}{P-Z} = \frac{-4-5}{3} = -\frac{9}{3} = -3$$

Breakaway point

closed transfer function = $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

$$= \frac{K}{s(s+4)(s+5)} \div 1 + \frac{K}{s(s+4)(s+5)}$$

$$= \frac{K}{(s^2+4s)(s+5)+K}$$

$$= \frac{K}{s^3+5s^2+4s^2+20s+K}$$

$$= \frac{K}{s^3+9s^2+20s+K}$$

Characteristic eqn = $s^3+9s^2+20s+K = 0$

$$K = -(s^3+9s^2+20s)$$

$$\frac{dK}{ds} = -(3s^2+18s+20) = 0$$

$$3s^2 + 18s + 20 = 0$$

$$s = -1.47, \; -4.53$$

K is +ve and real for $s = -1.47$

Breakaway point is $s = -1.47$.

Crossing point on imaginary axis

put $s = j\omega$ in characteristic eqn :

$$(j\omega)^3 + 9(j\omega)^2 + 20j\omega + K = 0$$

$$-j\omega^3 - 9j\omega^2 + 20j\omega + K = 0$$

equate real and imaginary parts to zero.

$$-\omega^3 + 20\omega = 0 \qquad -9\omega^2 + K = 0$$

$$-\omega^3 = -20\omega \qquad -9\omega^2 = K$$

$$\omega = \sqrt{20} = \pm 4.47 \text{ rad/sec}$$

Step-2

$$\xi = 0.707$$

$$\cos^{-1}(\xi) = \cos^{-1}(0.707) = 45^\circ$$

$$S_d = -1.2 \pm j1.2$$

$$K = \frac{l_1 l_2 l_3}{1} = \frac{1.6 \times 3.2 \times 4.1}{1} = 20.992$$

Step-3

$$\beta = 1.2A$$

$$A = \frac{K_{vd}}{K_{vu}}$$

$$K_{vd} = 5, \quad K_{vu} = \lim_{s\to 0} s\,G(s)H(s) = \lim_{s\to 0} s \frac{20.992}{s(s+4)(s+5)} = 1.05$$

$$A = \frac{5}{1.05} = 4.76$$

$$\beta = 1.24 \times 4.76 = 5.712$$

Step-4

$$Z_c = -\frac{1}{T} = 0.1 \times \text{second pole of } G(s).$$

$$= 0.1 \times -4$$

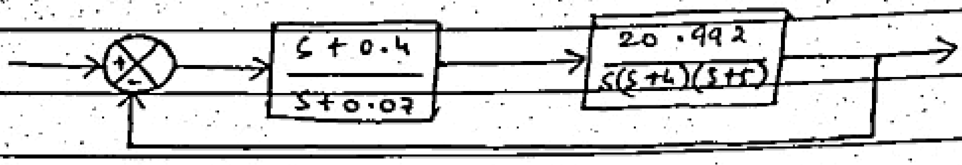
$$T = \frac{1}{0.1 \times 4} = 2.5$$

$$Z_c = \frac{-1}{2.5} = -0.4$$

$$P_c = \frac{-1}{\beta T} = \frac{-1}{5.712 \times 2.5} = -0.07 //.$$

$$G_c(s) = \frac{s+0.4}{s+0.07}$$

Step - 5



$$G_o(s) = \frac{20.992\,(s+0.4)}{s(s+4)(s+5)(s+0.07)}$$

Step - 6

$$K_{vc} = \lim_{s \to 0} s \cdot G_o(s)$$

$$= \lim_{s \to 0} s \cdot \frac{20.992\,(s+0.4)}{s\,(s+4)(s+5)(s+0.07)}$$

$$= \frac{20.992 \times 0.4}{4 \times 5 \times 0.07}$$

$$= 5.99 //.$$

$K_{vc} > 5$. $\therefore$ system gives desired o/p since the requirement was K_{vc} should be atleast 5, i.e $K_{vc} \geq 5$.

<u>Scale</u>

X-axis = 1unit = 1

Y-axis = 1unit = 1 jω