HW #2

1 a) This is because this is a filter that allows only frequencies within a certain range to go through - it removes frequencies too low or too high.

b) 

$\pi \cdot \frac{1}{\pi} \longleftrightarrow \frac{\sin Wt}{\pi t} \cdot \cos(\omega_p t) \cdot \frac{1}{\pi} \cdot 2\pi$

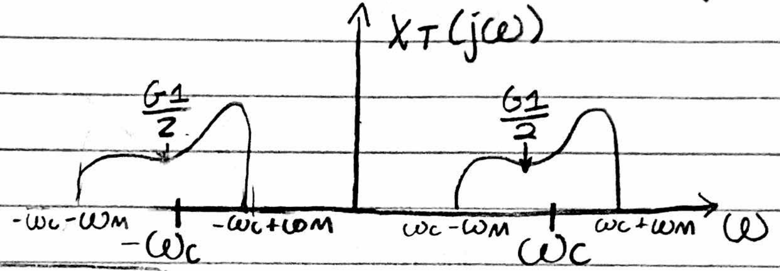
In frequency domain — In time domain

$$\boxed{2\left(\frac{\sin Wt}{\pi t}\right)\cos(\omega_p t)}$$

2. a) $X_T(t) = x(t) \times \cos(\omega_c t) \times G1$

Fourier Transform of $\cos(\omega_0 t)$ is $\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$

$$X_T(j\omega) = \frac{G1}{2\cancel{\pi}}\left(X(j\omega) * (\cancel{\pi}\delta(\omega-\omega_c) + \cancel{\pi}\delta(\omega+\omega_c))\right)$$



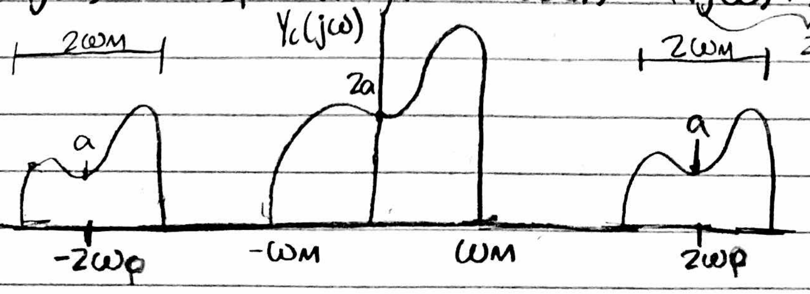
b) $\boxed{\omega_c = \omega_p}$ signals need to be aligned w/ bandpass filter

c) $Y_R(j\omega) = X_T(j\omega) \cdot H(j\omega)$

$$= \frac{10^{-6}G_1}{2}\left(X(j\omega) * (\delta(\omega-\omega_c) + \delta(\omega+\omega_c))\right)$$

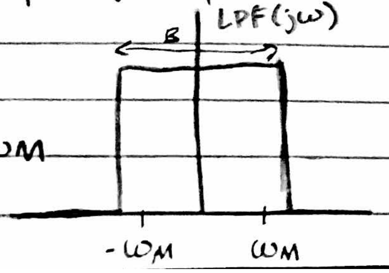
$$= \frac{10^{-6}G_1}{2}\left(X(j(\omega-\omega_c)) + X(j(\omega+\omega_c))\right)$$

$$Y_c(j\omega) = \frac{10^{-6}G_1G_2}{4}\left(X(j(\omega-2\omega_c)) + \underbrace{X(j\omega)+X(j\omega)}_{2} + X(j(\omega+2\omega_c))\right)$$



$a = \frac{10^{-6}G_1G_2}{4}$

d) Frequency Response LPF



$B > 2\omega_M$

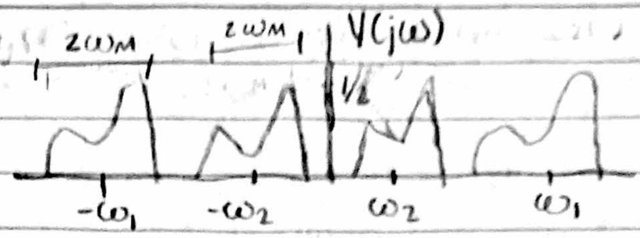
$2a = 1$

$10^{-6}G_1G_2 = 2$

$G_1G_2 = 2\times10^6$

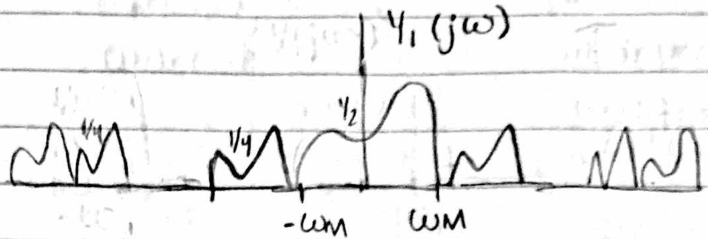
$$\boxed{\begin{array}{l}G_1 = 2{,}000\\ G_2 = 1{,}000\end{array}}$$

3 a)



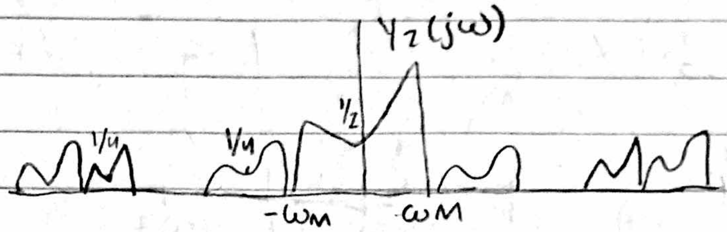
$|\omega_1-\omega_2| > 2\omega_M$

b)

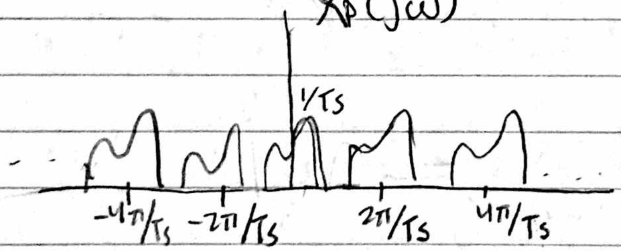


In both cases, you can recover $x_1(t)/x_2(t)$ from $y_1(t)/y_2(t)$ by multiplying by 2 and applying a low pass filter banded for frequencies $-\omega_M$ to ω_M.

c)



4. a)



b)

$2\omega_M < \frac{2\pi}{T_s}$ $A = T_s$

$\omega_a > \omega_M$

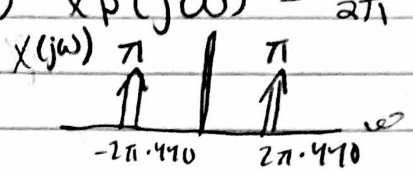
$\omega_b < \frac{2\pi}{T_s} - \omega_M$

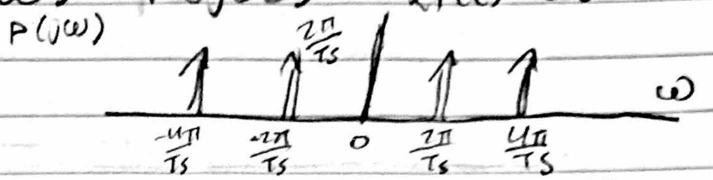
5. $x_p(t) = x(t)\,p(t) \qquad x_1(t) = \cos(\omega_1 t),\ x_2(t) = \cos(\omega_2 t)$

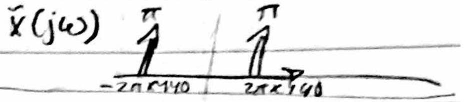
$$\omega_s = \frac{2\pi}{T_s}$$

a) $\omega_1 = 2\pi \times 440$ rad/s, $\omega_2 = 2560$ rad/s and $\omega_s = 2\pi \times 3000$ rad/s

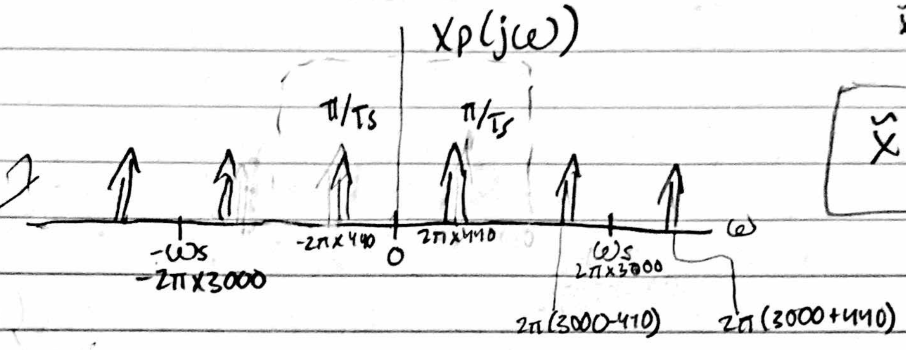
ai) $X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega) \qquad x_1(t) = \cos(2\pi \cdot 440 \frac{rad}{s}$





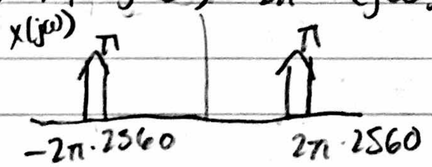


$$\boxed{\hat{x}(t) = \cos(2\pi \cdot 440 \tfrac{rad}{s} t) = x_1(t)}$$



same picture

aii) $X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega) \qquad x_2(t) = \cos(2\pi \cdot 2560 \frac{rad}{s})$



$$\boxed{\hat{x}(t) = \cos(2\pi \cdot 440 \tfrac{rad}{s} t) \neq x_2(t)}$$