UNIT-1

7/8/2023
monday

MATHEMATICAL LOGIC

Propositional calculus:

statement and notations, connectivities, well formed formulas, truth tables, Tautologies, Equivalence and of formulas, Duality law, Tautological, Implications, normal forms, Theory of Inference for statement calculas.

Predicate calculus:

Predicate logic, statement functions, variables and quantifiers, free and bound variables. Influence theory for predicate calculas.

Mathematical logic:

* It is the discipline that deals with methods of reasoning.
* It provides rules and techniques for determining whether a given argument or mathematical proof or conclusion in a scientific is valid or not.

Logic has two parts or components:

1. preposition logic
2. Predicate logic

Preposition logic:

It deals with statements with values true and false and is concened with analysis of prepositions.

Predicate logic:

It deals with the predicates which are preposition containing variables.

Prepositions:

A preposition or statement is a declarative sentence that is either true or false.

Example:

→ five plus eight equals thirteen.

→ Sun rises in the west

Ans) This is a sentence which is false but it is a declarative sentence so it is a statement or preposition.

→ $2+4=6$

Ans: True and declarative sentence. So, it is a preposition.

→ $\{5,6\} \subset \{7,6,5\}$

Ans: True and declarative sentence. So, it is preposition.

→ Do you know Hindi?

Since it is a question not a declarative sentence. So, it is not a preposition.

→ $4-x=8$

Here, we don't know the value of x. So, it is not a preposition.

→ close the door.

It is a command so, not a preposition.

Truth Values

The truth or falsy of a statement is called its truth value.

If the value is true which is denoted by (T) and if the value is false(F)

Example:

1. p; "Three is a prime number"

A. Truth value of 'p' is 1 or T.

2. q; "Every rectangle is a square"

A. The truth value of q is '0' or F

Types of Declarative sentences;

Declarative sentences are two types.

1. Atomic statement
2. compound statement.

1) Atomic statement:

The statement which do not contain any of the connective is called an atomic or primary or primitive or simple statement.

These will be denoted by alphabetical capital letters A, B.., P, Q

2) Compound statement:

A preposition obtained from the combination of two or more prepositions by means of logical operators or connectivities is known as compound or molecular or composite statement

Connectives:

The words and phrases (or symbols) are used to form compound prepositions are called connectives.

Types of connectives;

There are 5 basic connectives

1. Negation ($\neg$: NOT)
2. conjuction ($\wedge$: AND)
3. Disjunction ($\vee$: OR)
4. Condition ($\rightarrow$; Implication).
5. Bi-conditional ($\Leftrightarrow$) - Biimplication or double implication (Implies or implies that)

I. Negation : ($\neg$, $\neg$, $\sim$)

The negation of the statement is introducing the word 'NOT'

Let 'p' is a proposition which is having the word "NOT" before 'p' is called negation of 'p' denoted by $\sim p$.

Truth table for negation;

p	$\sim p$
T	F
F	T

Example;

If p: It is raining outside

Then, Negation of p is

$\sim p$: It is not raining outside.

II conjuction;

Two proposition "p" and "q" are joined by the word 'and' then it is called conjunction of 'p' and 'q'

Note; 'p' and 'q' has truth value 'T' whenever both 'p' and 'q' have the truth value 'T' otherwise it has the truth value 'F'

Truth table for conjunction:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example:

1) If p: It is raining outside } both are
then q: It is cooling outside } True(T)
~~x. Then, negation of 'p' is~~
~~x ~p: It is not raining outside.~~

2) If p : 2+4 = 6 (T) } F
then q : 1>2 (F) }

III Disjunction;

Two statements 'p' and 'q' are joined by the word 'or' then it is called disjunction of 'p' & 'q' and is denoted by $p \vee q$ and is read as "p" or "q".

Rule: The statement $p \vee q$ has the truth value 'F' whenever both 'p' & 'q' have value 'F' otherwise it has the truth value 'F' otherwise it has the truth value 'T'.

Truth table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Examples:1

If p and q are two propositions

- p : $2+4=6$ (T)
- q : It is raining outside (T) or (F)

Example: 2 :-

If 'p' and q are two propositions

- p : It is raining outside
- q : It is cooling outside } (T)

4. **Conditional** :- or **implication statements**:

Two statements 'p' and 'q' can be written as " if p then q" is called the conditional statement and is denoted by $p \to q$ and is read as if 'p' then 'q'.

Rule: The statement $(p \to q)$ has the truth value 'F' when 'p' is true and 'q' is false otherwise it has truth-value "T".

Note:- If $p \to q$; the statement 'p' is called "Ascend" (True) and 'q' is false otherwise it has truth value 'T'.

Truth value table for conditional statement

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Example:-
- If $a=b$ & $b=c$ then $a=c$
- If it is raining outside then it will stop after some time.

5. Bi-conditional or bi-implication statements:
Two statements 'p' & 'q' are of the form 'p' and only if 'q' then it is called bi-conditional statements and is denoted by 'p' iff 'q' or $p \leftrightarrow q$.

The biconditional statement is the conjunction of conditionals $p \to q$ and $q \to p$ i.e., $(p \to q) \wedge (q \to p)$

Rule: The statement 'p iff q' is true whenever both 'p' and 'q' have identical truth values otherwise it is false.

Truth table for Bi-conditional:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example:
If p: It is book (T)
then q: It has papers (T) } T

Example;
If p: $2-3=1$ (F)
then q: $1>2$ (F) } T

Example: Find the truth table of

1) $(\sim p \vee q)$

Sol:

p	q	$\sim p$	$\sim p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$(\sim p \vee q) \leftrightarrow (p \to q)$ ($\leftrightarrow$) → biconditional or biimplication

$(\sim p \vee q) \leftrightarrow (p \to q)$
T
T
T
T

→ (+ | -)

2) $(p \to q) \wedge (q \to p)$ (marriage)

Sol:

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

3. $(p \to q) \wedge (q \to r)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Tautology:

A statement formula which is true regardless of the truth Values of the statements which replace the Variables it is called universally Valid formula or a tautology or a logical truth. ie. if each entry in the final column of the truth table of a statement formula is always T then it is called Tautology.

contradiction:

A statement formula which is false regardless of the truth Values of the the statements which replace the Variables in it is said to be contradiction. ie: if each entry in the final column of the truth table of a statement formula is always F then it is called contradiction.

Note:

Negation of contradiction is the Tautology and same as Vice Versa

contingency:

A proposition that is neither a 'tautology' or 'contradiction' is called the contingency.

Show that $(\neg P \wedge (\neg q \wedge r)) \vee (q \wedge r)$ $\vee (P \wedge r)$ are logically equivalent to r. 7-8-2023 monday

(or)

show that $(\neg P \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (P \wedge r) \equiv$

* Example: prove that $\sim p \wedge (p \vee q) \rightarrow q$ is a tautology.

Ans

p	q	$\sim p$	$p \vee q$	$\sim p \wedge (p \vee q)$	$\sim p \wedge (p \vee q) \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

In the final column of T.T values are 'T' only. So, it is tautology.

* Example: prove that $(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$ is a contradiction.

Sol:-

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T	T	F	F	F	F
T	F	F	F	T	T	F
F	T	F	T	F	T	F
F	F	F	T	T	T	F

In the final column of the truth-table the truth values are 'F' only. So, it is a contradiction.

* Prove that proposition (statement) "$(p \rightarrow q) \rightarrow (p \wedge q)$" is a contigency

Sol)

P	q	P→q	p∧q	(P→q)→(p∧q)
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

In the final column of the T.T, the truth values are neither 'T' only nor 'F' only so it is contigency.

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Q Show that $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r)$ are logically equivalent to r

(or)

Show that $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

Sol)

p	q	r	¬p	¬q	¬q∧r	[A] ¬p∧(¬q∧r)
T	T	T	F	F	F	F
T	T	F	F	F	F	F
T	F	T	F	T	T	F
T	F	F	F	T	F	F
F	T	T	T	F	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

[B] q∧r	[C] p∧r	[D] A∨B	[D] (A∨B)∨(C)	(D∨C)↔r
T	T	T	T	T
F	F	F	F	T
F	T	F	T	T
F	F	F	F	T
T	F	T	T	T
F	F	F	F	T

$q \wedge r$ (B)	$p \wedge r$ (C)	$A \vee B$	$(A \vee B) \vee (C)$ (D)	$D \vee$ ... $\leftrightarrow$
F	F	T	T	T
F	F	F	F	T

i) $(D \vee C) \equiv r$

Both are having same truth value

ii) $(D \vee C) \leftrightarrow r$ is a tautology

The given statement is logically equivalent.

2) Show that $(p \rightarrow q) \wedge (q \rightarrow r)$ & $p \rightarrow (q \wedge r)$ are logically equivalent.

quest & answer

Sol:

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

$q \wedge r$	$p \rightarrow (q \wedge r)$
T	T
F	F
F	F
F	F
T	T
F	T
F	T
F	T

∴ Both are not equal

∴ The $(p \rightarrow q) \wedge (q \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically not equivalent.

Logical equivalence of formulas

1. **Idempotent law:**

a) $P \vee P \Leftrightarrow P$ b) $P \wedge P \Leftrightarrow P$

2. Associative law:

a) $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$

b) $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$

3. commutative law:

a) $P \vee Q \Leftrightarrow Q \vee P$

b) $P \wedge Q \Leftrightarrow Q \wedge P$

4. Distributive law

a) $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

b) $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

5 **Identity laws:**

a) i) $P \vee F \Leftrightarrow P$ ii) $P \vee T \Leftrightarrow T$

b) i) $P \wedge T \Leftrightarrow P$ ii) $P \wedge F \Leftrightarrow F$

6 **component laws:**

a) i) $P \vee \neg P \Leftrightarrow T$ ii) $P \wedge \neg P \Leftrightarrow F$

b) i) $\neg\neg P \Leftrightarrow P$ ii) $\neg T \Leftrightarrow F$, $\neg F \Leftrightarrow T$

7 **Absorption laws:-**

a) $P \vee (P \wedge Q) \Leftrightarrow P$

b) $P \wedge (P \vee Q) \Leftrightarrow P$

8) Demorgan's laws:-

a) $\neg (P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

b) $\neg (P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

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show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically Equivalent, without using truth tables.

$\neg(A \vee B) \Leftrightarrow \neg A \wedge \neg B$

Sol:- $\neg(p \vee (\neg p \wedge q))$

$\neg p \wedge \neg(\neg p \wedge q)$

$\neg p \wedge (\neg\neg p \vee \neg q)$

$\neg p \wedge (p \vee \neg q)$

$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$

false $F \vee (\neg p \wedge \neg q)$

$(\neg p \wedge \neg q) \vee F \Rightarrow (\neg p \wedge \neg q)$ by law of disjunction.

2) show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology, without using truth tables.

$\underset{A}{(p \wedge q)} \rightarrow \underset{B}{(p \vee q)}$ implication

$\Leftrightarrow \neg(p \wedge q) \vee (p \vee q)$, by defn of implication

$\Leftrightarrow (\neg p \vee \neg q) \vee (p \vee q)$, Demorgan's law

$\Leftrightarrow \neg p \vee (\neg q \vee p) \vee q$, by associative law

$\Leftrightarrow \neg p \vee (p \vee \neg q) \vee q$, by commutative law

$\Leftrightarrow (\neg p \vee p) \vee (\neg q \vee q)$

$\Leftrightarrow T \vee T \Leftrightarrow T$, a tautology.

3) S.T $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

Sol:- $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r)$

$\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee (r \wedge q) \vee (r \wedge p)$

$\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee (r \wedge (q \vee p))$

$\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee (r \wedge (p \vee q))$

$\Leftrightarrow ((\neg p \wedge \neg q) \wedge r) \vee (r \wedge (p \vee q))$

$\Leftrightarrow \neg(p \vee q) \wedge r) \vee (r \wedge (p \vee q))$

$\Leftrightarrow (r \wedge \neg(p \vee q)) \vee (r \wedge (p \vee q))$

$(a \wedge b) \vee (a \wedge c)$

$\Leftrightarrow r \wedge (\neg(p \vee q) \vee (p \vee q))$

$\Leftrightarrow r \wedge T$

12-8-20
Saturday

Normal forms:

1) Conjuctive normal form.
2) Desjuctive normal form
3) Principle conjuctive normal form
4) Principle desjuctive normal form

1) conjuctive normal form: ($\wedge$)

A formula which is equivalent to a given formula which consists of product of elementary sums is called a conjuctive normal form (CNF).

Ex: $\underset{\text{E-sum}}{(\vee)} \underset{\text{product}}{\wedge} \underset{\text{E-sum}}{(\vee)} \wedge (\vee) \wedge (\vee) \ldots$

2) Desjuctive normal form: ($\vee$) sum

A formula which is equivalent to a given formula which consists of sum of elementary products is called a desjuctive normal form.

Ex: $\underset{\text{E-produ}}{(\wedge)} \underset{\text{E-sum}}{\vee} \underset{\text{E-produ}}{(\wedge)} \vee (\wedge) \vee (\wedge) \ldots$

Some formula

1) $\rightarrow$

$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \vee (Q \rightarrow P)$

2) $\neg (P \wedge Q)$

$\neg P \vee \neg Q$

Problems:-

1) Find C.N.F of $P \wedge (P \to Q)$.

Sol:- $P \wedge (P \to Q) \Leftrightarrow P \wedge (\neg P \vee Q)$ | $P \to Q \Leftrightarrow \neg P \vee Q$

$\Leftrightarrow (P \vee P) \wedge (\neg P \vee Q)$ — using idempotent law $P \vee P \Leftrightarrow P$

$\therefore$ It is a C.N.F

2) Obtain C.N.F of $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$ (or) $\leftrightarrow$

Sol: $\neg(P \vee Q) \rightleftarrows (P \wedge Q) \Leftrightarrow$

$[\neg(P \vee Q) \to (P \wedge Q)] \wedge$

$[(P \wedge Q) \to \neg(P \vee Q)]$

$P \to Q \Leftrightarrow \neg P \vee Q$

$\Leftrightarrow [(\neg(\neg(P \vee Q))) \vee (P \wedge Q)] \wedge$

$[\neg(P \wedge Q) \vee \neg(P \vee Q)]$

→ demorgan law

$\Leftrightarrow [(P \vee Q) \vee (P \wedge Q)] \wedge [(\neg P \vee \neg Q) \vee (\neg P \wedge \neg Q)]$

$a \vee (b \wedge c)$ $\qquad$ $a \vee (b \wedge c)$

→ Applying distributive law. $(a \vee b) \wedge (a \vee c)$

$\Leftrightarrow [(P \vee Q) \vee P] \wedge [(P \vee Q) \vee Q] \wedge$

$[(\neg P \vee \neg Q) \vee \neg P] \wedge (\neg P \vee \neg Q) \vee \neg Q)$

$\Leftrightarrow (P \vee Q \vee P) \wedge (P \vee Q \vee Q) \wedge (\neg P \vee \neg Q \vee \neg P)$

$\wedge (\neg P \vee \neg Q \vee \neg Q)$

$(P \vee Q) \vee (P \vee Q) \wedge (\neg P \vee \neg Q) \wedge (\neg P \vee \neg Q)$

$(P \vee Q) \wedge (\neg P \vee \neg Q)$ // C.N.F

$\boxed{P \vee P = P}$

$\neg P \vee \neg P = \neg P$

3) Find D.N.F of $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$

↳ $(\wedge) \vee (\wedge) \vee (\wedge)$.

Sol: $P \rightarrow Q$

$P \rightarrow [(P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P)]$

$P \rightarrow Q \Leftrightarrow \neg P \vee Q$

$\Leftrightarrow \neg P \vee ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P)]$

$\Leftrightarrow \neg P \vee [(\neg P \vee Q) \wedge \neg(\neg Q \vee \neg P)]$

$\neg(\neg P \vee \neg q)$

$= \neg\neg P \wedge \neg\neg q$

$\Leftrightarrow \neg P \vee [\underbrace{(\neg P \vee Q)}_{b \vee c} \wedge \underbrace{(Q \wedge P)}_{a}]$

interchanging these two Applying commutative law

$\Leftrightarrow \neg P \vee [(Q \wedge P) \wedge (\neg P \vee Q)]$

$\Leftrightarrow \neg P \vee [((Q \wedge P) \wedge \neg P) \vee ((Q \wedge P) \wedge Q)]$

removing brackets.

$\Leftrightarrow \neg P \vee [(Q \wedge P \wedge \neg P) \vee (Q \wedge P \wedge Q)]$

$\Leftrightarrow (\neg P \wedge \neg P) \vee (Q \wedge P \wedge \neg P) \vee (P \wedge Q)$

→ DNF to get product idempotent law

4) Find D.N.F of $P \wedge (P \rightarrow Q)$

Sol $P \wedge (P \rightarrow Q) \Leftrightarrow P \wedge (\neg P \vee Q)$

$\Leftrightarrow (P \wedge \neg P) \vee P \wedge Q$

It is in D.N.F

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5) Obtain P.N.F of $(p \to (q \wedge r)) \wedge (\neg p \to (\neg q \wedge \neg r))$ by using Truth table.

p	q	r	$q \wedge r$	$p \to (q \wedge r)$ (A)	$\neg p$	$\neg q$	$\neg r$
T	T	T	T	T	F	F	F
T	T	F	F	F	F	F	T
T	F	T	F	F	F	T	F
T	F	F	F	F	F	T	T
F	T	T	T	T	T	F	F
F	T	F	F	T	T	F	T
F	F	T	F	T	T	T	F
F	F	F	F	T	T	T	T

$\neg q \wedge \neg r$	$\neg p \to (\neg q \wedge r)$ (B)	$A \wedge B$	minterms
F	T	T	$p \wedge q \wedge r$
F	T	F	$p \wedge q \wedge \neg r$
F	T	F	$p \wedge \neg q \wedge r$
T	T	F	~~$p \wedge \neg q \wedge \neg r$~~ $p \wedge \neg q \wedge \neg r$
F	F	F	$\neg p \wedge q \wedge r$
F	F	F	$\neg p \wedge q \wedge \neg r$
F	F	F	$\neg p \wedge \neg q \wedge r$
T	T	T	$\neg p \wedge \neg q \wedge \neg r$

minterms having truth table is
$(p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r) \to$ PDNF

Note: max terms.

$\neg(p \wedge q \wedge \neg r) \wedge \neg(p \wedge \neg q \wedge r) \wedge \neg(p \wedge \neg q \wedge \neg r)$
$\wedge \neg(\neg p \wedge q \wedge r) \wedge \neg(\neg p \wedge q \wedge \neg r) \wedge$
$\neg(\neg p \wedge \neg q \wedge r)$.

$\Rightarrow$ PCNF $= (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge$
$(\neg p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge$
$(p \vee \neg q \vee r) \wedge (p \vee q \vee \neg r)$

6Q) Find PDNF of $(\neg p \vee \neg q) \rightarrow (p \leftrightarrow \neg q)$ by truth table

p	q	$\neg p$	$\neg q$	A: $\neg p \vee \neg q$	B: $p \leftrightarrow \neg q$	$A \rightarrow B$	min ter…
T	T	F	F	F	F	T	$p\wedge$…
T	F	F	T	T	T	T	$p\wedge$…
F	T	T	F	T	T	T	$\neg p$…
F	F	T	T	T	F	F	$\neg p$…

PCNF is

$\neg(\neg p \wedge \neg q) \Rightarrow p \vee q.$

PDNF is

$(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)$

Min Terms:

* For a given number of variables, the min terms consists of conjuctions in which each statement variable & its negation, but not both appears only once.

* Ex: If P and Q are any two statements, then $P \wedge Q$, $\sim P \wedge Q$, $P \wedge \sim Q$, $\sim P \wedge \sim Q$ are min terms

* Ex: If P, Q and R are any 3 statements; then $P \wedge Q \wedge R$, $\sim P \wedge Q \wedge R$, $P \wedge \sim Q \wedge R$, $P \wedge Q \wedge \sim R$, $\sim P \wedge \sim Q \wedge R$, $\sim P \wedge Q \wedge \sim R$, $P \wedge \sim Q \wedge \sim R$, $\sim P \wedge \sim Q \wedge \sim R$ are the min terms.

MAXTERMS:

The dual of a minterm is called maxterm. For a given number of variables, the maxterm consists of disjunctions in which each variable or its negation, but not both appears only once. Each of the maxterms has the truth value F for exactly one combination of the truth values of the variables.

* If P and Q are any two statements, then they are $2^{2} = 4$ maxterms.

ie: $P \vee Q$, $\sim P \vee Q$, $P \vee \sim Q$, $\sim P \vee \sim Q$

* If P and Q and R are any 3 statements, then there are $2^3 = 8$ maxterms

ie $P \vee Q \vee R$, $\sim P \vee Q \vee R$, $P \vee \sim Q \vee R$, $P \vee Q \vee \sim R$, $\sim P \vee \sim Q \vee R$, $\sim P \vee Q \vee \sim R$, $P \vee \sim Q \vee \sim R$, $\sim P \vee \sim Q \vee \sim R$

Principal Disjuctive Normal form:

Definition: For a given formula, an equivalent formula consisting of disjunctions of minterms only is called the principle disjuctive normal form of the formula.

The principle disjuctive normal formula is also called the sum-of-products canonical form

PDNF ≡ (min Ter) ∨ (min Ter) ∨ (min Ter)

Methods to obtain PDNF of a given formula:

a) By Truth table:

i) construct a truth table of given formula.

~~ii) for every truth value~~

ii) Select the min term with Truth value T.

iii) Take the disjunction of above minterms

1) Find the PDNF of $P \to Q$

Sol:

P	Q	$P \to Q$	min. Terms
T	T	T	$P \wedge Q$
T	F	F	$P \wedge \neg Q$
F	T	T	$\neg P \wedge Q$
F	F	T	$\neg P \wedge \neg Q$

P.D.N.F = $(P \wedge Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$

2) Obtain the PDNF for $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

Sol:

P	Q	R	min. Term	$P \wedge Q$	$\neg P \wedge R$	$Q \wedge R$	$(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$
T	T	T	$P \wedge Q \wedge R$	T	F	T	T
T	T	F	$P \wedge Q \wedge \neg R$	T	F	F	T
T	F	T	$P \wedge \neg Q \wedge R$	F	F	F	F
T	F	F	$P \wedge \neg Q \wedge \neg R$	F	F	F	F
F	T	T	$\neg P \wedge Q \wedge R$	F	T	T	T

F	T	F	$\neg P \wedge Q \wedge \neg R$	F	F	F	F
F	F	T	$\neg P \wedge \neg Q \wedge R$	F	T	F	T
F	F	F	$\neg P \wedge \neg Q \wedge \neg R$	F	F	F	F

The PDNF of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

b) with out constructing Truth table:

i) First replace $\leftrightarrow$, by theire equivalent form containing only $\wedge, \rightarrow$

(or)

$$P \leftrightarrow Q = ((P \rightarrow Q) \wedge (Q \rightarrow P))$$

$$P \rightarrow Q = \neg P \vee Q$$

Replace $\rightarrow$ by their equivalent formula containing only $\wedge, \vee$ and $\neg$

2) Next negations are applied to the variables by De-morgan's laws followed by the application of distributive laws

$$\neg (P \wedge Q) = \neg P \vee \neg Q$$

$$\neg (P \wedge Q \wedge R) = \neg P \vee \neg Q \vee \neg R$$

$$\neg (\neg P) = P$$

3) Any elementary product which is a contradiction is dropped. Min terms are obtained in the disjunctions by introducing the missing factors. Identical minterms appearing in the disjunctions are deleted

19-8-23
Saturday.

1) Obtain the principal disjunctive normal form of $P \to ((P \to Q) \wedge \neg(\neg Q \vee \neg P))$.

Sol) Using $P \to Q \Leftrightarrow \neg P \vee Q$ & demorgan's law, we obtain

anything $\vee (F)$ = anything
$GA \wedge T = GB$
(anything) $\wedge T$ = anything

$\Rightarrow \to ((P \to Q) \wedge \neg(\neg Q \vee \neg P)) \Leftrightarrow \neg P$
$\vee ((\neg P \vee Q) \wedge (Q \wedge P))$

$\Leftrightarrow \neg P \vee ((\neg P \wedge Q \wedge P) \vee (Q \wedge Q \wedge P)$

$\Leftrightarrow \neg P \vee F \vee (P \wedge Q)$

$\Leftrightarrow \neg P \vee (P \wedge Q)$

$\Leftrightarrow (\neg P \wedge T) \vee (P \wedge Q)$

$\Leftrightarrow (\neg P \wedge (Q \vee \neg Q)) \vee (P \wedge Q)$

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q)$

$\Leftrightarrow$ P.D.N.F obtained

2) Obtain PDNF of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

$\Leftrightarrow [(P \wedge Q) \wedge T] \vee [(\neg P \wedge R) \wedge T] \vee [(Q \wedge R) \wedge T]$

$\Rightarrow [\underset{a}{(P \wedge Q)} \wedge \underset{b \vee c}{(R \vee \neg R)}] \vee [\underset{a}{(\neg P \wedge R)} \wedge \underset{b \vee c}{(Q \vee \neg Q)}$

$\vee [\underset{a}{(Q \wedge R)} \wedge \underset{b \vee c}{(P \vee \neg P)}$

$\Leftrightarrow [((P \wedge Q) \wedge R) \vee ((P \wedge Q) \wedge \neg R)]$

$\Leftrightarrow \vee [((\neg P \wedge R) \wedge Q) \vee (\neg P \wedge R) \wedge \neg Q)]$

$\Leftrightarrow \vee [(Q \wedge R) \wedge P) \vee (Q \wedge R) \wedge \neg P)]$

$\Leftrightarrow [(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R)]$

$\Leftrightarrow \vee [(\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)]$

$\vee [P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R)]$

P.D.N.F obtained

Find PCNF of the PDNF $(P \wedge Q) \vee (\neg P \vee Q) \vee (\neg P \wedge$

This involves only 2 variables P and Q and has $2^2 = 4$ minterms

This possible minterms are $P \wedge Q$, $P \wedge \neg Q$, $\neg P \wedge Q$, $\neg P \wedge \neg Q$.

This minterms which is not present in the PDNF is $P \wedge \neg Q$. Since there is only 1 term we don't disjunct it with anything.

Taking negation of that we have $\neg(P \wedge \neg Q) \equiv \neg P \vee Q$ and it is the required PCNF..

Q) Find the PCNF of the PDNF $(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

This involves only 3 variable P, Q & R has $2^3 = 8$ minterms

This possible minterms are $P \wedge Q \wedge R$, $\neg P \wedge Q \wedge R$, $P \wedge \neg Q \wedge R$, $P \wedge Q \wedge \neg R$, $\neg P \wedge \neg Q \wedge R$, $P \wedge \neg Q \wedge \neg R$, $\neg P \wedge Q \wedge \neg R$, $\neg P \wedge \neg Q \wedge \neg R$

This minterms which is not present in the PDNF is $\neg P \wedge Q \wedge R$, $P \wedge \neg Q \wedge R$, $P \wedge \neg Q \wedge \neg R$, $\neg P \wedge \neg Q \wedge \neg R$

Theory of ~~Inference~~ Inference — 28/08/23 monday

Absent

The procedure of verifying the argument (premisis, or statements or assumptions + conclusion) is valid or not by using Roots of inference is called Theory of Inference

1)

1) with truth table: we have to prepare the truth table for $(H_1 \wedge H_2 \wedge H_3 \cdots \wedge H_n) \to C$

→ where $H_1, H_2 \ldots H_n$ is called premisis (or) Assumptions or hypothesis & C is conclusion

Ex: Verify the C: q follows from the premises H_1: P H_2: $P \to Q$ by TT.

H_1 P	C Q	H_2 $P \to Q$	$(H_1 \wedge H_2)$	$(H_1 \wedge H_2) \to C$
T	T	T	T	T
T	F	F	F	T (Tautology)
F	T	T	F	T
F	F	T	F	T

2) H_1: $\neg P$ H_2: $P \vee Q$ C: Q;

P	Q	H_1 $\neg P$	H_2 $(P \vee Q)$	$H_1 \wedge H_2$	$(H_1 \wedge H_2) \to C$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

2) Without Truth tables:

We are verifying the argument is valid or not by using i) Rools of interface (or) implication and Equivalence laws

For Example:

Demonstrate that R is a valid statement from the premises $P \to Q$, $Q \to R$ and P without using truth table. C: R

$$\begin{array}{c} P \\ P \to Q \\ \hline Q \end{array} \qquad \begin{array}{c} Q \\ Q \to R \\ \hline R \end{array}$$

$P, P \to Q \Rightarrow Q$

{1} (1) P Rule P

{2} (2) $P \to Q$ Rule P

{1,2} {3} Q Rule T (1) (2) $P, P \to Q \Rightarrow Q$

{4} (4) $Q \to R$ Rule P

{1,2,4} (5) R Rule T (3) (4) $P, P \to Q \Rightarrow Q$

② $C \vee D$, $(C \vee D) \to \neg H$, $\neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to R \vee S$

① $$\begin{array}{c} C \vee D \\ (C \vee D) \to \neg H \\ \hline \neg H \end{array}$$ ② $$\begin{array}{c} \neg H \\ \neg H \to (A \wedge \neg B) \\ \hline A \wedge \neg B \end{array}$$ ③ $$\begin{array}{c} (A \wedge \neg B) \\ (A \wedge \neg B) \to (R \vee S) \\ \hline R \vee S \end{array}$$

{1} (1) $(C \vee D)$ Rule P

{2} (2) $(C \vee D) \to \neg H$ Rule P

{1,2} {3} $\neg H$ Rule T (1) (2) $P, P \to Q \Rightarrow Q$

{4} (4) $\neg H \to (A \wedge \neg B)$ Rule P

{1,2,4} (5) $(A \wedge \neg B)$ Rule T (3) (4) $P, P \to Q \Rightarrow Q$

{6} (6) $(A \wedge \neg B) \to (R \vee S)$ Rule P

{1,2,4,6} (7) $(R \vee S)$ Rule T (5,6) $P, P \to Q \Rightarrow Q$

3) Verify the conclusion $R \to S$ follows the premises $P \to (Q \to S)$, $\neg R \vee P$ and Q

$A \to B$

① R	② P	③ $Q \to S$
$\frac{\neg R \vee P}{R}$	$\frac{P \to (Q \to S)}{Q \to S}$	$\frac{Q}{S}$
$\frac{R \to P}{P}$		

{1} (1) R Rule CP $\boxed{P \to Q \Leftrightarrow \neg P \vee Q}$

{2} (2) $\neg R \vee P$ Rule P

{2} (3) $R \to P$ Rule T

{1,2} (4) P Rule T (1) (3) $P \to Q$, $P \Rightarrow Q$

{5} (5) $P \to (Q \to S)$ Rule P

{1,2,5} (6) $(Q \to S)$ Rule T (4,5) $P, P \to Q \Rightarrow Q$

{7} (7) Q Rule P

{1,2,5,7} (8) S Rule T (6),(7) $P, P \to Q \Rightarrow Q$

$\therefore$ By CP rule $R \to S$ is a valid statement

29-8/23

Absent:

Tuesday

Inference theory for predicate calculus

Predicate: Statement which describes the action of the subject

Ex: $\underset{\text{sub}}{\underline{x}}$ is $\underset{\text{pridicate}}{\underline{\text{greater than 3}}}$

→ The process of verifying the validity of the given predicate formulas by using some inference rules is known as theory of inference.

Rules of inference:

1) Rule US: universal specification $(\forall x)(p(x)) \Rightarrow p(x)$
2) Rule UG: universal Generalization $p(x) \Rightarrow (\forall x)p(x)$
3) Rule ES: Existential specification $(\exists x)(p(x)) \Rightarrow p(x)$
4) Rule EG: Existential Generalization $p(x) \Rightarrow (\exists x)(p(x))$

Implication:

I_{11} : $P, P \to Q \Rightarrow Q$ (modus ponens)

I_{10} : $\neg P, P \vee Q \Rightarrow Q$ (disjunctive syllogism)

I_9 : $P, Q \Rightarrow P \wedge Q$

I_{12} : $P \to Q, \neg Q \Rightarrow \neg P$ (modus tollens)

I_{13} : $P \to Q, Q \to R \Rightarrow P \to R$ (hypothetical syllogism)

I_{14} : $P \vee Q, P \to R, Q \to R \Rightarrow R$ (dillemma)

Equivalence Relations:-

$P \to Q \Rightarrow \neg P \vee Q$

$P \to Q \Leftrightarrow \neg Q \to \neg P$

1) PT the following argument is Valid

$\neg \exists x, p(x) \wedge Q(x)$

$P(x)$

conclusion is $\neg Q(x)$

$\neg (\exists x) \cdot (p(x) \wedge Q(x))$

$\neg (\exists (x)) \cdot \neg (p(x) \wedge Q(x))$

$$\frac{(\forall x)\ [\neg p(x) \vee \neg Q(x)]}{\neg (p(x)) \vee \neg Q(x)}$$

$$\frac{p(x)}{\neg Q(x)}$$

$\{1\}$ (1) $\neg(\exists x).(p(x) \wedge Q(x))$ Rule P

$\{1\}$ (2) $\neg(\exists x).\neg(p(x) \wedge Q(x))$ Rule T (1)

$\neg(PQ) = \neg P \neg Q$

$\{1\}$ (3) $(\forall x)(\neg p(x) \vee \neg Q(x))$ Rule (2)

$\neg(\exists x) \Rightarrow \forall(x)$

$\neg(p \wedge Q) = \neg P \vee \neg Q$

$\{1\}$ (4) $\neg p(x) \vee \neg Q(x)$ Rule US

(3) $(\forall x)\, P(x) \Rightarrow P(x)$

$\{5\}$ (5) $p(x)$ Rule P

$\{1,5\}$ (6) $\neg Q(x)$ Rule T $P \vee Q, P \rightarrow Q$ (4) (5)

2) Verify the Validity of the following argument

All men are mortal

Socrates is a man

therefore Socrates is mortal

Sol:) Rewrite the state: all men are mortal as for all x, if x is a man there x is a mortal.

let $M(x)$: x is a man

$L(x)$: x is mortal

$(\forall x)\,(m(x)) \rightarrow L(x)$

Socrates is a man $\Rightarrow M(x) \Rightarrow (x:s) \Rightarrow M(s)$

s: socrate

Socrate is a mortal $\Rightarrow L(x) \Rightarrow (x:s)$

$\Rightarrow L(s)$

$(\forall x)(M(x)) \rightarrow L(x))$

$M(s) \rightarrow L(s)$

$M(s)$

$L(s)$

UNIT-2

30-8-2 Wednes

Binary Relation and properties

Relation: Let 'R' be a relation defined from a set A and to set B is a subset of cross product of A and B $(A \times B)$

Ex: $A = \{1, 2\}$ $B = \{5, 6\}$

$$A \times B = \{(1,5), (1,6), (2,5), (2,6)\}$$

$$R = \{(1,5), (1,6)\} \quad R = \{(2,5)(2,6)\}$$

$$R = \{(1,5), (2,6)\}$$

is known as relation.

Properties of Relation:

1) Reflexive: If $aRa \ \forall a \in A$ then R is said to be Reflexive Relation.

2) Symmetric: If $aRb \rightarrow bRa \ \forall a, b \in A$ then R is said to be symmetric

3) Transitive: $aRb, bRc \Rightarrow aRc$ then R is said to be Transitive Relation.

Problems:

1. If R_1 is $= \{(1,1) (1,2) (2,2) (2,3) (3,3)\}$ be a Relation defined on $A = \{1,2,3\}$. Verify R_1 is i) Reflexive ii) Symmetric iii) Transitive

Sol: i) Reflexive: $(1,1) \in R_1 \Rightarrow 1R1 \ \forall \ 1 \in A$

$(2,2) \in R_1$ $2 \in A$

$(3,3) \in R_1$ $3 \in A$

i.e. $aR_1a \ \forall a \in A$

$\therefore R_1$ is Relative.

ii) Symmetric: $(1,1) \in R_1 \Rightarrow (1,1) \in R_1$
$(1,2) \in R_1 \Rightarrow (2,1) \notin R_1$
$(2,2) \in R_1 \Rightarrow (2,2) \in R_1$
$(2,3) \in R_1 \Rightarrow (3,2) \notin R_1$
$(3,3) \in R_1 \Rightarrow (3,3) \in R_1$

$(a,b) \in R \Rightarrow (b,a) \in R$ then
$\therefore$ R is said to be symmetric
$(1,2) \in R_1$, $(2,1) \notin R_1$
$(2,3) \in R_1$, $(3,2) \notin R_1$
$\therefore R_1$ is not symmetric

iii) Transitive:- $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$
$(1,1) \in R$ $(1,2) \in R \Rightarrow (1,2) \in R$
$(1,2) \in R$ $(2,2) \in R \Rightarrow (1,2) \in R$
$(1,2) \in R_1$ $(2,3) \in R_1 \Rightarrow (1,3) \notin R$
$\therefore$ R is not Transitive.

2) $R = \{(1,1)\ (1,2)\ (2,3)\ (3,3)\}$.
i) Reflexive: $(1,1) \in R \Rightarrow |R|\ \forall\ 1 \in A$
$(2,2) \notin R$
$3 \in A$
$(3,3) \in R$
$\therefore R_1$ is not reflexive

ii) Symmetric:
$(1,1) \in R \Rightarrow (1,1) \in R$
$(1,2) \in R \Rightarrow (2,1) \notin R$
$(2,3) \in R \Rightarrow$ $(3,2) \notin R$
$(3,3) \in R \Rightarrow (3,3) \in R$
$(a,b) \in R \Rightarrow (b,a) \in R$ then
$\therefore$ R is said to be symmetric
$(1,2) \in R$, $(2,1) \notin R$
$(2,3) \in R$, $(3,2) \notin R$

$\therefore$ R is not symmetric.

iii) Transitive: $(a,b) \in R, (b,c) \in R \rightarrow (a,c) \in R$

$(1,1) \in R \quad (1,2) \in R \Rightarrow (1,2) \in R$

$(1,2) \in R \quad (2,3) \in R \Rightarrow (1,3) \notin R$

$\therefore$ R is not Transitive.

Matrix Representation of a relation:-

It is denoted by $M_R = M_{ij} = \begin{cases} 1 & (a_i, b_j) \\ 0 & (a_i, b_j) \end{cases}$

*1) R is the relation on $A = \{1, 2, 3\}$ such that $(a,b) \in R \Leftrightarrow a+b =$ even. Find the relation matrix M_R.

Sol:- $A \times A = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$

$R = \{(1,1)\ (1,3)\ (2,2)\ (3,1)\ (3,3)\}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

	1	2	3
1	1	0	1
2	0	1	0
3	1	0	1

$$M_R^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Note: $M_R^T = M_R$ R is symmetric

diagonal elements are 1 $\therefore$ R is Reflexive