Machine Learning

Homework 5 solution - Shyam Arumugaswamy

1.

For points in convex hull of $\{xi\}_{i=1}{}^{N}$, the linear discriminant is

$z(x) = w^{\wedge T} x^{n} + \omega_0$

substituting the given eq in the above we get

$z(x) = w^{\wedge T} (\sum_N \alpha_N x^N) + \omega_0$

$z(x) = \sum_N \alpha_N (w^{\wedge T} + \omega_0$

we coNsider $\sum_N \alpha_N = 1$, so
$z(x) = \sum_N \alpha_N (w^{\wedge T} x^N + \omega_0)$ ---→ 1

Similarly linear discriminant for points on convex hull $\{yi\}_{i=1}{}^{M}$

$z(y) = \sum_M \beta_M (w^{\wedge T} y^M + \omega_0)$ ---→ 2

where $\beta_M > 0$ and $\sum_M \beta_M = 1$

if the convex hulls intersect, there must be at least one common point i.e xy between {x} and {y} as xy belongs to both convex hulls, there must be set of $\{\alpha_N\}$ and $\{\beta_M\}$ that give rise to xy. The linear discriminant for xy can be written in two separate but equivalent ways. From the two eqs, we get
$z(xy) = \sum_N \alpha_N (w^{\wedge T} x^N + \omega_0) = \sum_M \beta_M (w^{\wedge T} y^M + \omega_0)$ ----→ 3

for linear separability, we must have

$z(x^N) = w^{\wedge T} x^N + \omega_0 > 0$
and $z(y^M) = w^{\wedge T} y^M + \omega_0 < 0$ ---→ 4

for the non-negativity and simple constraints on α and β, 3 & 4, we have contradiction. The linear discriminant z(xy) has to be simultaneously greater than and less than zero which is impossible.

2.

The posterior class probabilities for each training point is ensured to be greater than 0.5 by finding a separate hyperplane which puts all training points on correct side.
each training point is assigned a posterior class probability of 1 by $\omega \rightarrow \infty$ and due to shape of sigmoid function

3. $\Phi(x_1,x_2) = x_1 x_2$

4. General form of linear classifier model is

$\omega_0 + \omega_1 x_1 + \omega_2 x_2 = 0$

$\omega_0 + 2\omega_1 = 0$

$\omega_1 = - \omega_0/2$

$\omega_0 + 5\omega_2 = 0$

$\omega_2 = - \omega_0/5$

if $\omega_0 = -1$, then $\omega_1 = ½$, $\omega_2 = 1/5$