Machine Learning Worksheet 06

Optimization

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1 Convexity

Problem 1: Prove or disprove whether the following functions are convex on the given set D:

i) $f(x, y, z) = 3x + e^{y+z} - min\{-x^2, log(y)\}$ and $D = (-100, 100) \times (1, 50) \times (10, 20)$

ii) $f(x, y) = y \cdot x^3 - 2 \cdot y \cdot x^2 + y + 4$ and $D = (-10, 10) \times (-10, 10)$

iii) $f(x) = log(x) + x^3$ and $D = (1, \infty)$

iv) $f(x) = -\min(2\log(2x), -x^2 + 4x - 32)$ and $D = \mathbb{R}^+$

Problem 2: Prove the following statement: Let $f_1 : \mathbb{R}^d \to \mathbb{R}$ and $f_2 : \mathbb{R}^d \to \mathbb{R}$ be convex functions, then $h(x) := f_1(x) + f_2(x)$ is a also convex function.

Problem 3: Given two convex functions $f_1 : \mathbb{R} \to \mathbb{R}$ and $f_2 : \mathbb{R} \to \mathbb{R}$, prove or disprove that the function $g(x) = f_1(x) \cdot f_2(x)$ is also convex.

2 Minimization of convex functions

Problem 4: Prove that for convex functions each local minimum is a global minimum. More specifically, given a convex function $f : \mathbb{R}^N \to \mathbb{R}$, prove that if $\nabla f(\theta^*) = 0$ then θ^* is a global minimum.

3 Gradient Descent

Problem 5: Load the notebook `06_hw_optimization_logistic_regression.ipynb` from Piazza. Fill in the missing code and run the notebook. Convert the evaluated notebook to pdf and add it to the printout of your homework.

Upload a single PDF file with your solution to Moodle by 04.12.2017, 9:59am CET. The solution must be typeset (using Latex, Word, etc.). Handwritten solutions are not accepted.