Particle-Techniques

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1 Assignment 1 (Computer Issues)

1.1 Revised Method

Using the the hits from the drift chamber and the fact the geometry of the magnet is a cube, a revised method for calculating the momentum was done. (see the code Tracker.py for details).

- First, the direction vector of the particle exiting drift chamber 1 and entering drift chamber 2 are found by using the hits in the drift chamber to trace a line through the points, and then the direction vector is found by normalising the vector.
- The entry and exit points of the particle is found by extrapolating the direction vector of the hits of drift chambers 1 and 2 (1 for the entry point and 2 for the exit point), using the real distance between the drift chambers and the magnet. As a result the direction vector of the entry and exit points are the same as the direction vectors of the tracks.
- The deflection angle θ is found by seeing how much the direction vector of the particle changes as it exits the magnetic field. This is done by calculating the angle of the direction vectors of the particle entry and exit points and taking the difference.
- The distance between the entry and exit point of the magnet d is found using $d^2 = h^2 + l^2$, where l is the length of the magnet along z and h is the difference in the x position for the particle entry and exit points of the magnet.
- The bending radius r is found using the equation $d/\arcsin(\theta)$.
- Calculate the momentum using $p = qBr$ (need to switch to relativistic equation).

1.2 Part 1:

Beam Momentum is 100 GeV and the magnetic field is set to 0.5T. The beam angle (also all other parts) is zero. 1000 events are generated (same for subsequent parts). x precision is 10^{-4}m and y precision is 10^{-2}m.

Momentum is estimated using the deflection of the charged particle as it travels through the magnetic field. the equation for the momentum of a charged particle travelling through a magnetic field is

$$p = qBr, \tag{1}$$

where q is the charge, B is the Field strength and r is the radius of deflection. Following the diagrams on figure 6 the momentum can be expressed as

$$p = qB\sqrt{\frac{l^2 z}{4x}}, \tag{2}$$

where z is the distance between the drift chambers, x is the amount the particle moves along the x axis (using the hit points), and l is the distance between the entry and exit point of the particle as it passes through the field.

The resolution (I think) is dependant on how well the deflection radius can be calculated, so its relative uncertainty scales proportionally:

$$\frac{\sigma_r}{r} = \frac{\sigma_p}{p}. \tag{3}$$

To determine σ_r, propagation of uncertainties is used (see figure 7) so the momentum resolution is:

$$\sigma_p = \sigma_x \frac{z}{x} \left(2(l^2 - 9.6^2) + \frac{l^2}{16x^2} \right). \tag{4}$$

Here, σ_x is the uncertainty in x which I take as the precision in x i.e. 10^{-4}m. Note it is the only uncertainty in the equation because all other parameters are dependant on the x precision (z is assumed to be perfectly known as it is part of the detector geometry).

These methods were implemented in python to generate the momentum distribution, the average momentum and the momentum resolution.

Average momentum (GeV)	35.47 ± 0.03
Resolution (GeV)	3.12 ± 0.03

Table 1: Average momentum and momentum resolution for part 1. Uncertainties are statistical

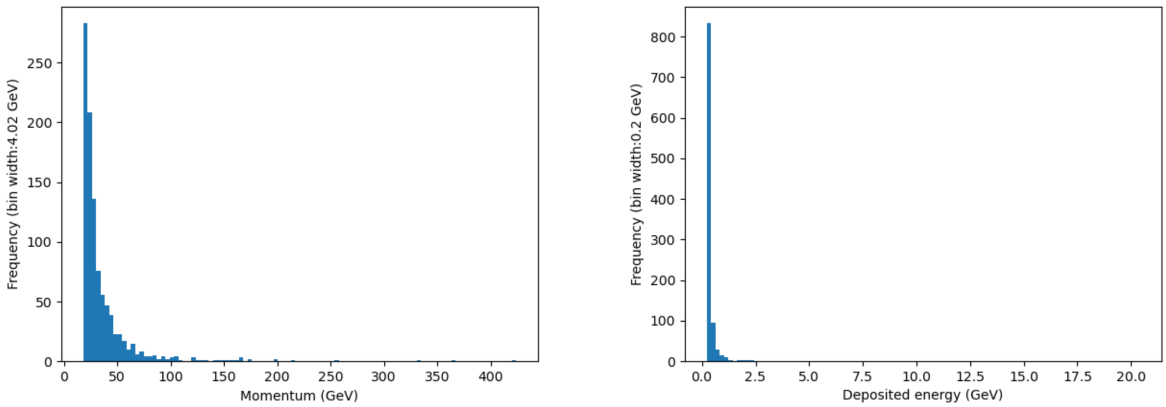

Figure 1: Plot of the momentum distribution and the total deposited energy (summed hadron and EM calorimeter data).

So, from figure 1 the momentum distribution as calculated using equation 2 follows the deposited energy distribution, which may be expected given the energy - mass relation. The average momentum calculated shown in table 1 is reasonable, but it is worth noting the momentum distribution spans a larger range than the deposited energy which was not expected. The reason there are very large values in the momentum distribution is likely due to the poor resolution for the events in that range.

It is very likely the calculation is inaccurate and could use much more thought to improve the estimation of the deflection.

1.3 Part 2:

Average momentum (GeV)	18.51 ± 0.02
Resolution (GeV)	2.15 ± 0.03

Table 2: Average momentum and momentum resolution for part 2a. Uncertainties are statistical. For Beam momentum of 100GeV and B field of 0.25T.

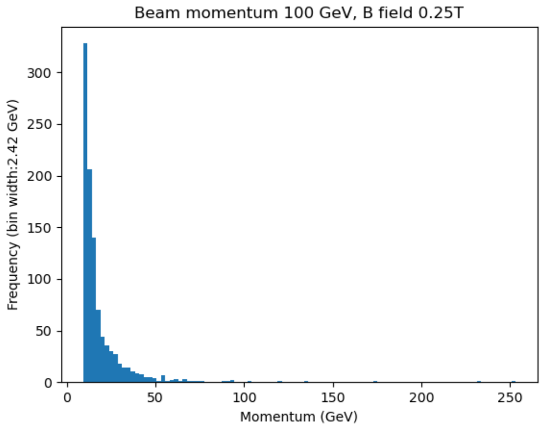

Figure 2: Plot of the momentum distribution and the total deposited energy (summed hadron and EM calorimeter data).

Average momentum (GeV)	74.94 ± 0.08
Resolution (GeV)	17.0 ± 0.2

Table 3: Average momentum and momentum resolution for part 2b. Uncertainties are statistical. For Beam momentum of 100GeV and B field of 1T.

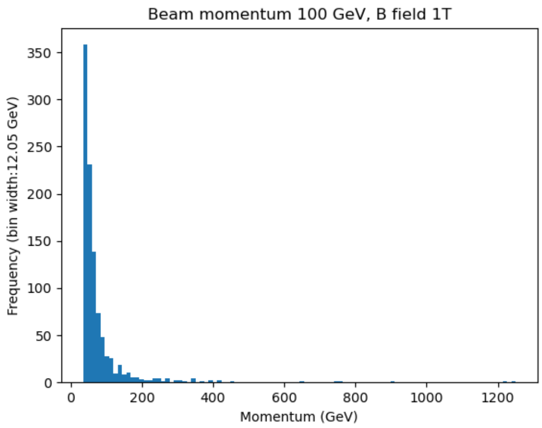

Figure 3: Plot of the momentum distribution and the total deposited energy (summed hadron and EM calorimeter data).

From the results it seems that increasing the field strength will tend the average momentum closer to the beam momentum value (100GeV) but more testing would be needed to confirm this. The resolution increases with the field strength as well so increasing the field too much will reduce how well σ_r and hence σ_p is resolved.

1.4 Part 3:

Average momentum (GeV)	36.25 ± 0.03
Resolution (GeV)	2.82 ± 0.02

Table 4: Average momentum and momentum resolution for part 3. Uncertainties are statistical. For Beam momentum of 50GeV and B field of 0.5T.

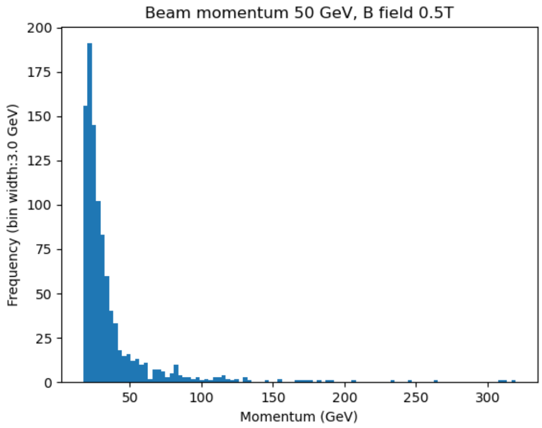

Figure 4: Plot of the momentum distribution and the total deposited energy (summed hadron and EM calorimeter data).

Average momentum (GeV)	37.29 ± 0.06
Resolution (GeV)	31.3 ± 0.8

Table 5: Average momentum and momentum resolution for part 3. Uncertainties are statistical. For Beam momentum of 200GeV and B field of 0.5T.

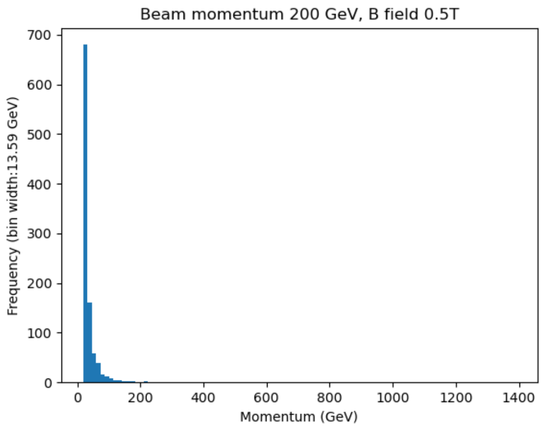

Figure 5: Plot of the momentum distribution and the total deposited energy (summed hadron and EM calorimeter data).

From the results of part 3, a lower beam energy compared to a higher beam energy is favourable as a beam energy that is too high results in poor momentum resolution. This and the results from part 2 can be explained by the fact the momentum resolution depends on the bending radius resolution. So more curved paths in the field will improve the momentum resolution as bending radius can be more precisely resolved (up to a limit).

1.5 Part 4

If the trajectory is perfectly known, σ_x will be zero, so following equation 4, σ_p will be zero so you can perfectly determine the resolution (not sure?).

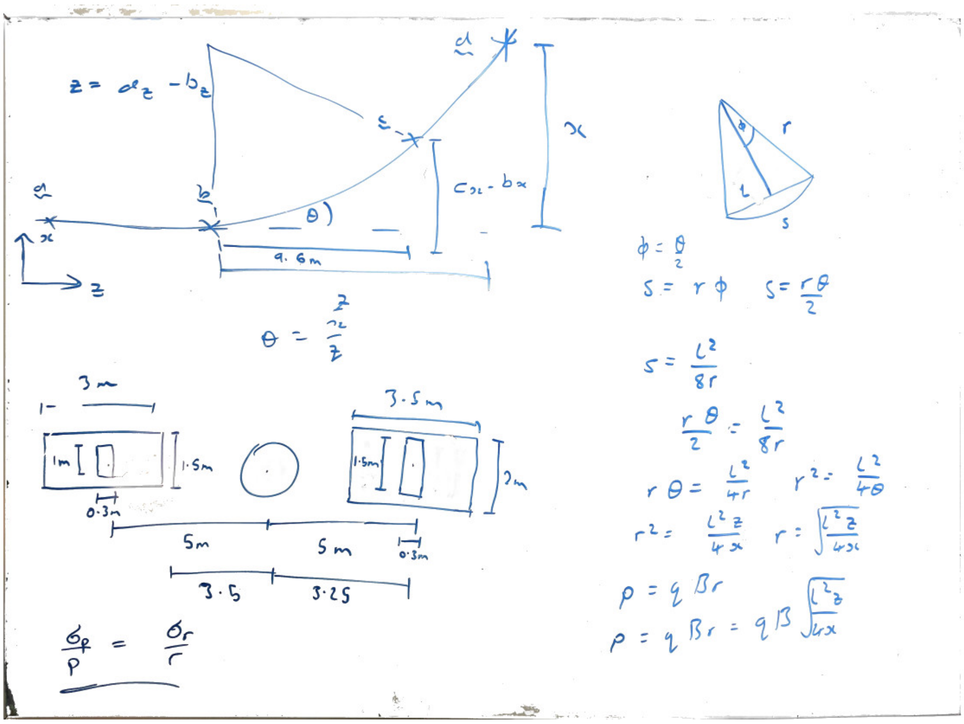

Figure 6: Rough working for finding the momentum via the deflection radius due to the magnetic field.

1.6 Whiteboard Work

$$\frac{\sigma_p}{p} = \frac{\sigma_r}{r} \qquad r = \sqrt{\frac{l^2 z}{4x}} \qquad l^2 = (c_x - b_x)^2 + (9.6)^2$$

$$\partial_x y = \frac{\partial y}{\partial x} \qquad \sigma_z = 0$$

$$\sigma_r^2 = (\sigma_l \, \partial_l r)^2 + (\sigma_z \, \partial_z r)^2 + (\sigma_x \, \partial_x r)^2$$

$$\sigma_l^2 = (\sigma_{c_x} \partial_{c_x} l)^2 + (\sigma_{b_x} \partial_{b_x} l)^2$$

$$\partial_l r = \frac{1}{2}\sqrt{\frac{z}{x}} \qquad \partial_z r = \frac{l}{4\sqrt{xz}} \qquad \partial_x r = -\frac{l z}{4 x^2}\sqrt{\frac{x}{z}} = -\frac{l}{4x}\sqrt{\frac{z}{x}}$$

$$\partial_{c_x} l = 2(c_x - b_x) \qquad \partial_{b_x} l = -2(c-b) = -\partial_{c_x} l$$

$$\sigma_l^2 = 4(c_x - b_x)^2(\sigma_{c_x}^2 + \sigma_{b_x}^2)$$

$$\sigma_r^2 = \frac{\sigma_l^2}{4}\frac{z}{x} + \sigma_x^2 \frac{l^2 z}{16 x^3} \qquad \sigma_x = 10^{-4}\,m$$

$$\sigma_r^2 = [l^2 - (9.6)^2](\sigma_{c_x}^2 + \sigma_{b_x}^2)\frac{z}{x} + \sigma_x^2 \frac{l^2 z}{16 x^3}$$

$$\sigma_x = \sigma_{c_x} = \sigma_{b_x} \qquad \sigma_r^2 = \sigma_x^2 \frac{z}{x}\left[2(l^2 - (9.6)^2) + \frac{l^2}{16 x^2}\right]$$

Figure 7: Rough working for finding the momentum resolution via the deflection radius due to the magnetic field (propagated uncertainty).