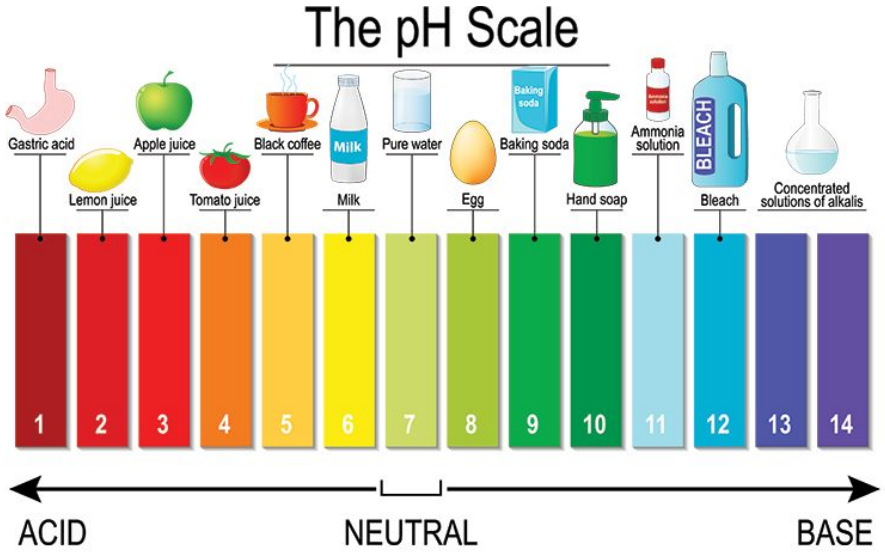
Chapter 3: Computing with Numbers Part 2

Jan 16, 2020



Today’s Outline

- Review: data types and Math library
- Factorials
- Working with large numbers
- NumPy library

Two Data Types in Python

INT: -2, -1, 0, 1, 2

FLOAT: 0.5, 0.23425, 30.2, -3.23, 1.0, 1030.0

/ (float division) vs. // (integer division)

ex:

10/3 = 3.3333333333333335

10//3 = 3

BEDMAS

- Python follows the typical ‘BEDMAS’ conventions
- The order of evaluating an expression is:
  - 1) Brackets
  - 2) Exponents
  - 3) Division and Multiplication
  - 4) Addition and Subtraction

- *Note: Python also evaluates values within function brackets first.
  Example:
  math.cos(math.pi)*3 = -1.0*3 = -3.0

Practice 1

What will be the result of evaluating each expression in Python?

Will the result be an INT or a FLOAT?

1. 5.0 / 10.0 + 3.5*2
2. 10 % 4 + 6//2
3. 3 * (10 // 3 + 10% 3)
4. abs (20 // 3 -4)**3

Math Library

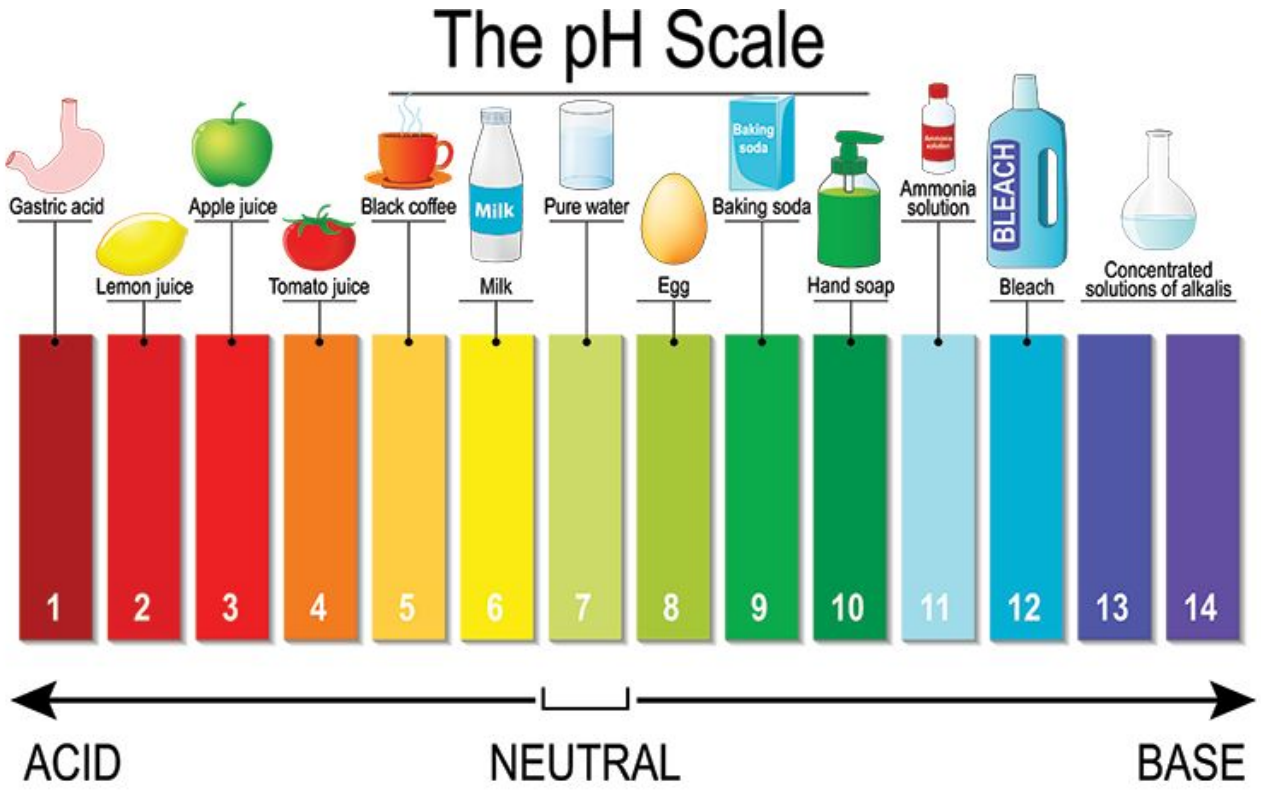
import math

Python	mathematics	English
`pi`	π	An approximation of pi.
`e`	e	An approximation of e.
`sqrt(x)`	$\sqrt{x}$	The square root of x.
`sin(x)`	$\sin x$	The sine of x.
`cos(x)`	$\cos x$	The cosine of x.
`tan(x)`	$\tan x$	The tangent of x.
`asin(x)`	$\arcsin x$	The inverse of sine x.
`acos(x)`	$\arccos x$	The inverse of cosine x.
`atan(x)`	$\arctan x$	The inverse of tangent x.
`log(x)`	$\ln x$	The natural (base e) logarithm of x.
`log10(x)`	$\log_{10} x$	The common (base 10) logarithm of x.
`exp(x)`	e^x	The exponential of x.
`ceil(x)`	$\lceil x \rceil$	The smallest whole number $>= x$.
`floor(x)`	$\lfloor x \rfloor$	The largest whole number $<= x$.

Table 3.2: Some math library functions

Practice 2

The pH scale is the decimal logarithm of the reciprocal of hydrogen ion (H^+) activity in a solution:

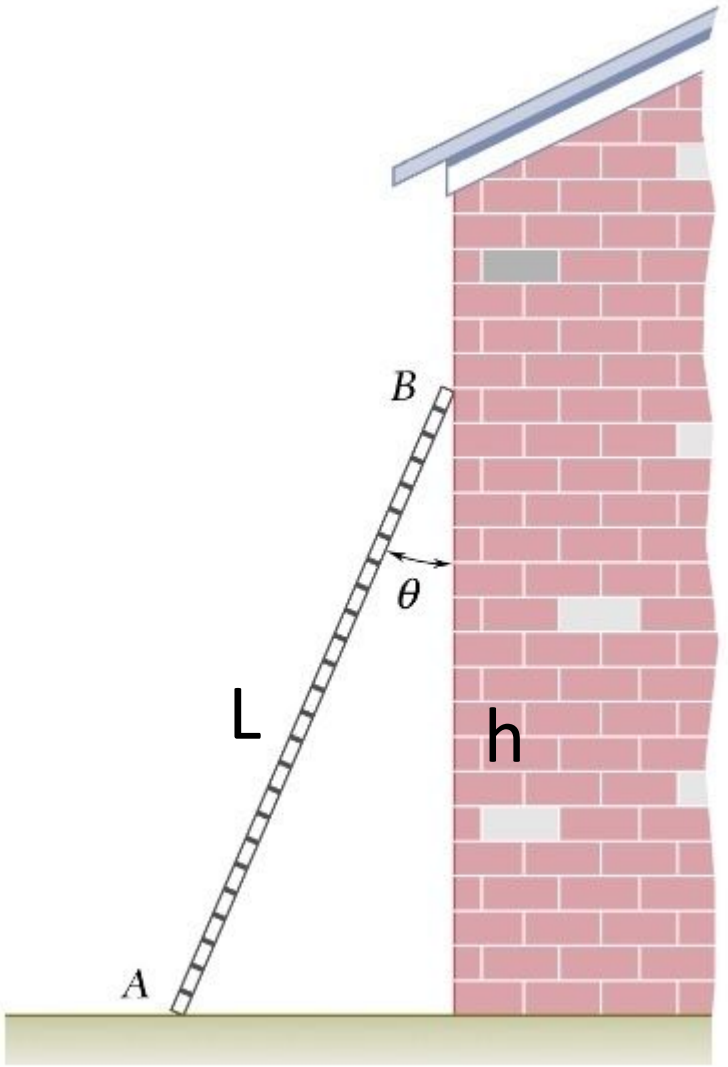


$$pH = - \log10(a_{H+})$$

Write a program that allows the user to input the hydrogen ion activity and outputs a single integer value of the pH scale.

Practice 3

Write a program to determine the length of a ladder required to reach a given height when leaned against a house. The height and angle of the ladder are given as inputs.



Hint:

sin(θ) = opposite / hypotenuse

cos(θ) = adjacent / hypotenuse

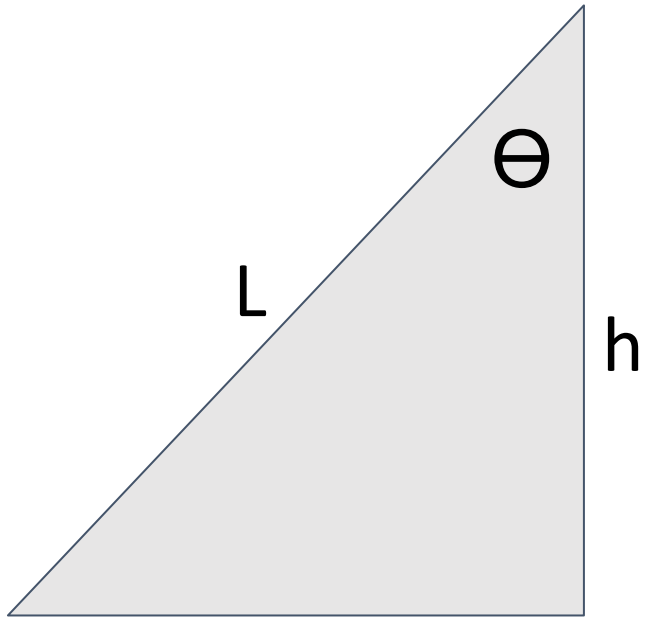
tan(θ) = opposite / adjacent

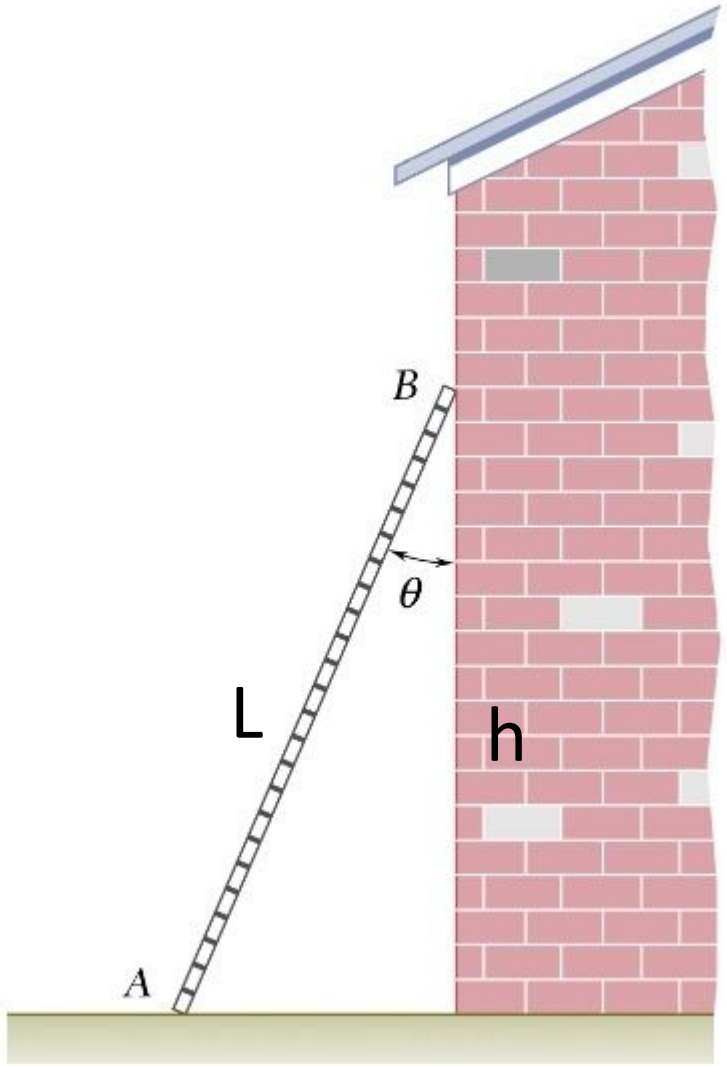
Practice 3

Write a program to determine the length of a ladder required to reach a given height when leaned against a house. The height and angle of the ladder are given as inputs.

$\cos(\Theta) = h/L$

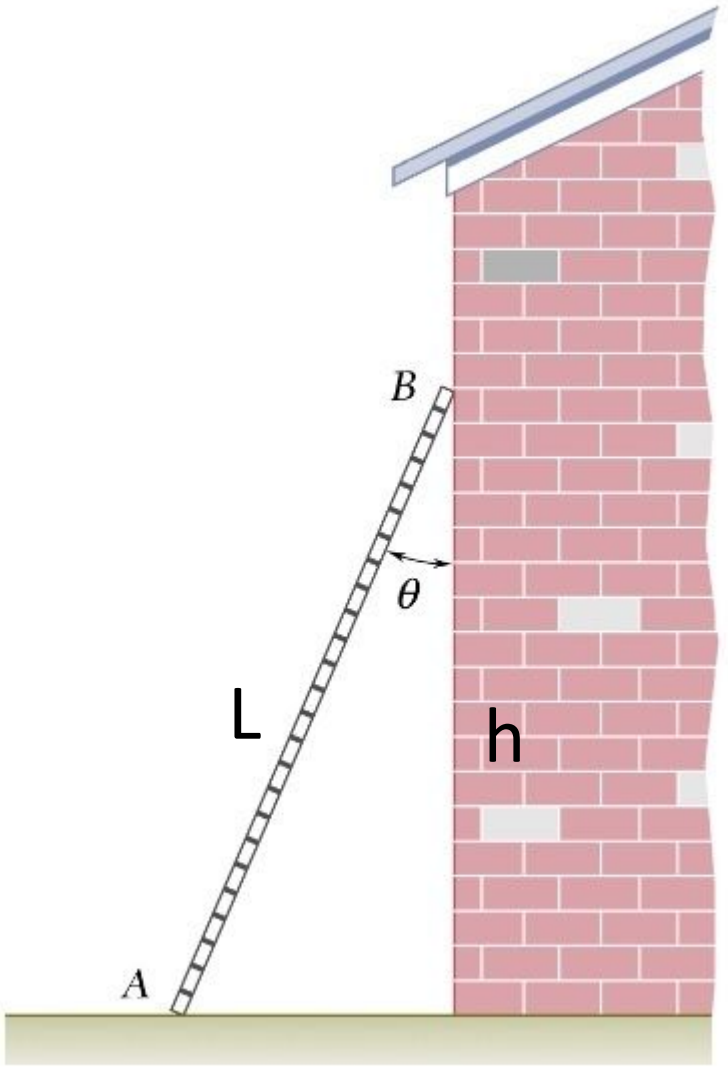
$L = h / \cos(\Theta)$





Practice 3

Write a program to determine the length of a ladder required to reach a given height when leaned against a house. The height and angle of the ladder are given as inputs.



Hint #2:

Trigonometric Python math functions use radians as the default.

$$radians = \frac{\pi}{180} degrees$$

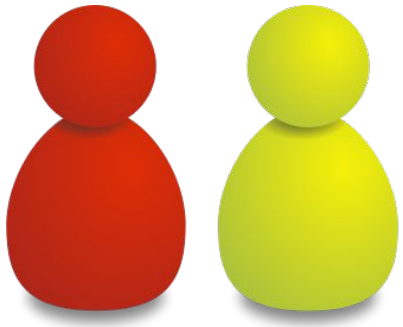
K-pop Problem

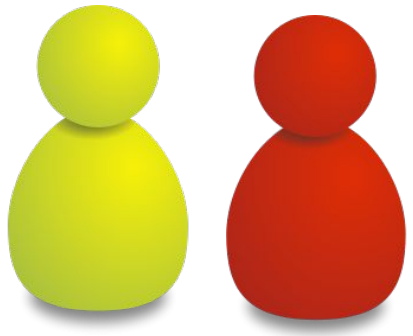
NCT is a group with 21 members. If they all want to line up for a photograph, how many possible ways can they assemble?

Start with easier problem

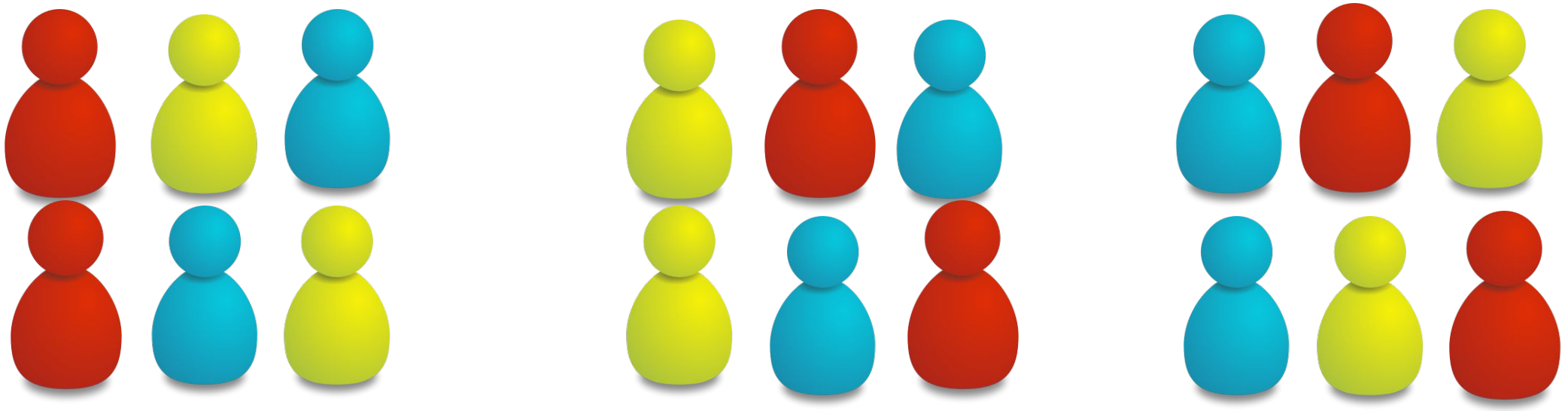
1 member: only one way to line up

2 members: two ways to line up

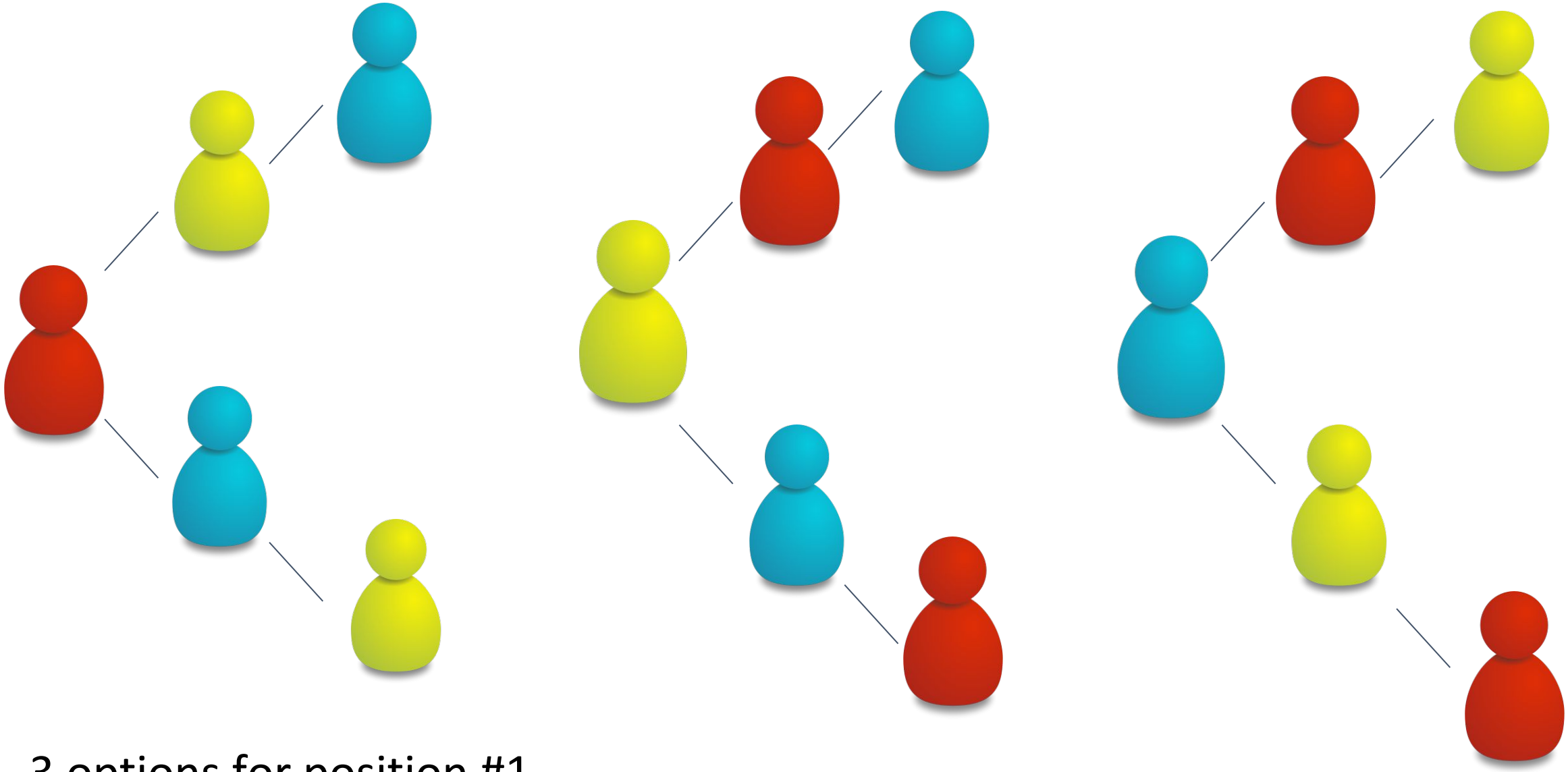




3 members:

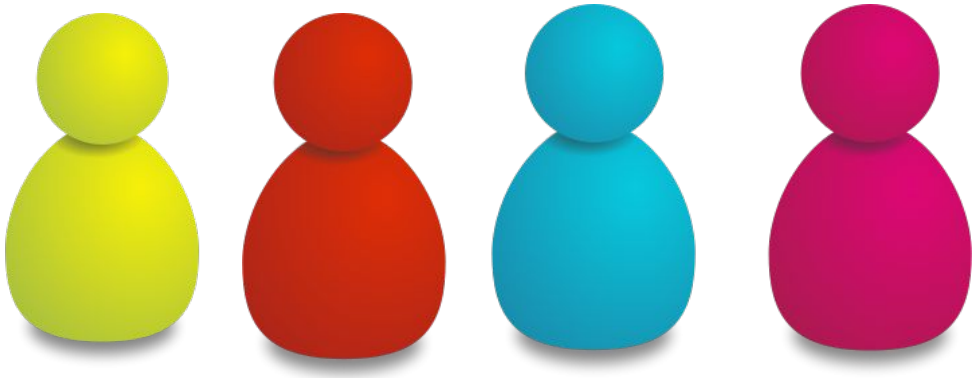


3 members: 6 ways to line up



3 options for position #1
Once position #1 is selected, have 2 options for position #2
Once #1 and #2 are selected, have 1 option for position #3

4 members:



4 options for position #1 (red, yellow, blue, pink)
3 options for position #2
2 options for position #3
1 option for position #4

4 * 3 * 2* 1 = 24

Factorials

- 4! = 4*3*2*1 = 24

- n! = n*(n-1)*(n-2)*(n-3)....*2*1

Factorial Problem

Write a function that can compute the factorial of a number entered by a user.

- n! = (n)*(n-1)*(n-2) … (1)

Factorial Problem

Write a program that can compute the factorial of a number entered by a user.

Input: an integer, n

Process: n! = (n)*(n-1)*(n-2) … (1)

Output: factorial, n!

Using factorials

- Number of ways of arranging objects in a sequence
- Permutations and combinations
- Important in calculus problems (ex. Taylor series)

K-pop Problem

NCT is a group with 21 members. If they all want to line up for a photograph, how many possible ways can they assemble?

K-pop Problem Part 2

NCT-U is a subunit of NCT that always contains 10 NCT members. How many possible variations of NCT-U are there, given that there are 21 total NCT members?

Combinations and Permutations

Combinations and permutations are ways of arranging members of a set into smaller subsets.

Combinations: order of items in the subset doesn’t matter

Permutations: order of items in the subset matters

Combinations and Permutations

Combination example: How many ways can I choose 4 textbooks out of my collection of 10 textbooks to bring to school?

Permutation example: How many ways can I choose 4 textbooks out of my collection of 10 textbooks and arrange them on my bookshelf?

Permutations

How many ways can I choose 4 textbooks out of my collection of 10 textbooks and arrange them on my bookshelf?

- 10 ways to choose book 1
- 9 ways to choose book 2
- 8 ways to choose book 3
- 7 ways to choose book 4

10*9*8*7 = 5040

Permutations (nPr)

How many ways can I choose 4 textbooks out of my collection of 10 textbooks and arrange them on my bookshelf?

10*9*8*7 = 5040

10*9*8*7 = (10*9*8*7*6*5*4*3*2*1)/(6*5*4*3*2*1) = 10!/6!

In general, if total number of items, n = 10, and number to select, r =4

nPr = n!/(n-r)! $_{10}P_4$ = 10!/6!

Combinations

How many ways can I choose 4 textbooks out of my collection of 10 textbooks to bring to school?

- 10 ways to choose book 1
- 9 ways to choose book 2
- 8 ways to choose book 3
- 7 ways to choose book 4
- 10*9*8*7 = 5040

*** BUT I don’t care what order the books are in

Combinations

How many ways can I choose 4 textbooks out of my collection of 10 textbooks to bring to school?

10*9*8*7 = 5040

*** BUT I don’t care what order the books are in

Need to divide 5040 by the 4! different ways to arrange the books in order.

5040/4! = 210

nCr

The formula for combinations is as follows:

$nCr = n!/ [r!(n-r)!]$

K-pop Problem Part 2

NCT-U is a subunit of NCT that always contains 10 NCT members. How many possible variations of NCT-U are there, given that there are 21 total NCT members?

Develop a function that can compute the required calculation given any n and any r.

Factorials and Combinations Part 2

I am too lazy to do all of these factorial calculations myself…

Modify our programs using libraries:

Computing with Big Numbers

Use Python to calculate 100!

Bits

Bit 0 or 1

1 bit: can represent 2 numbers (0, 1)

2 bits: can represent 4 numbers (00, 01, 10, 11)

3 bits: can represent 8 numbers (000, 001, 010, 011, 100, 101, 110, 111)

n bits: can represent 2^n numbers

Number of INT values

How many INT values are possible for a 64 bit computer?

Number of INT values

How many INT values are possible for a 64 bit computer?

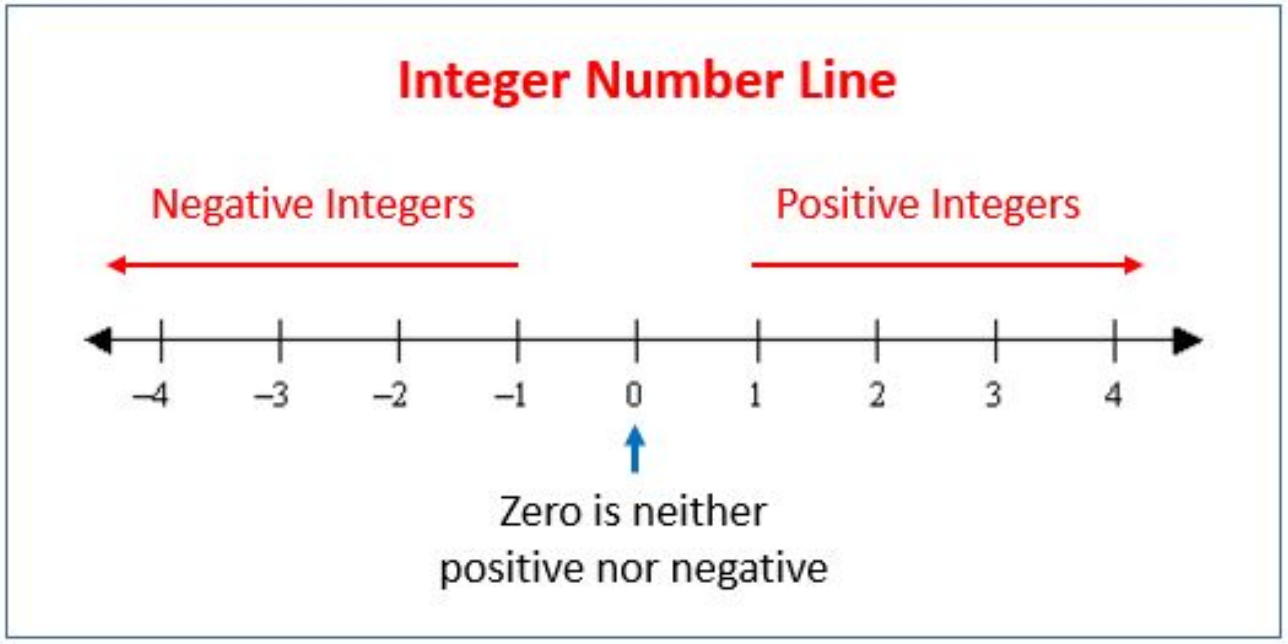
2^64 = 1.8446744e+19

This is the number of integers a computer can represent with 64 bits.

Positive and negative integers

2^64 = 1.8446744e+19 must include both negative and positive integers

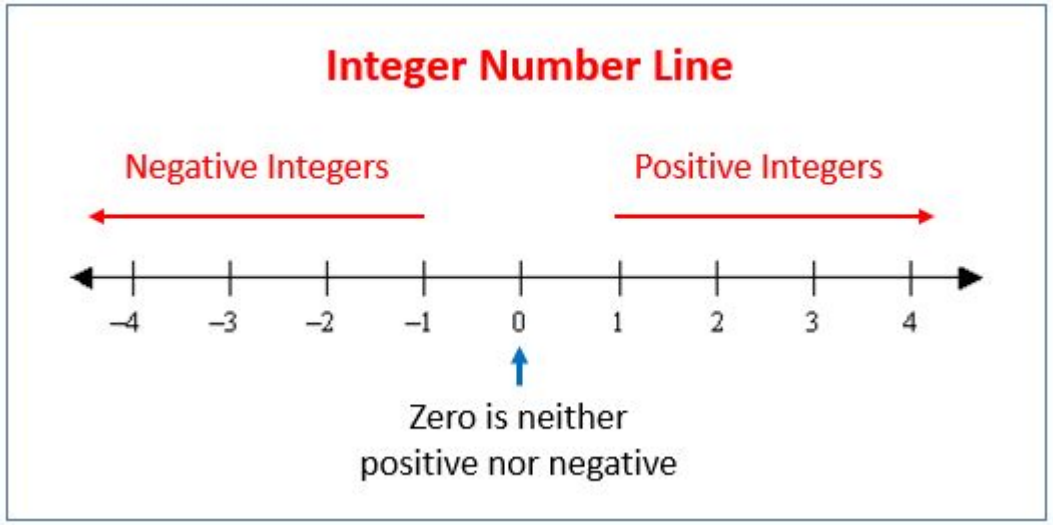
How many of each type of integer are represented?



Positive and negative integers

2^64/2 = 2^63 negative integers and 2^63 positive integers

0 is considered to be a positive integer by the computer, so the highest INT value would be 2^63-1 and the lowest INT value would be -2^63.



Big integers in Python

We calculated that the largest INT a computer should be able to represent is 9.2e18, but Python was correctly able to calculate 100! = 9.33e157.

How?

Big integers in Python

Python integers are not fixed sizes.

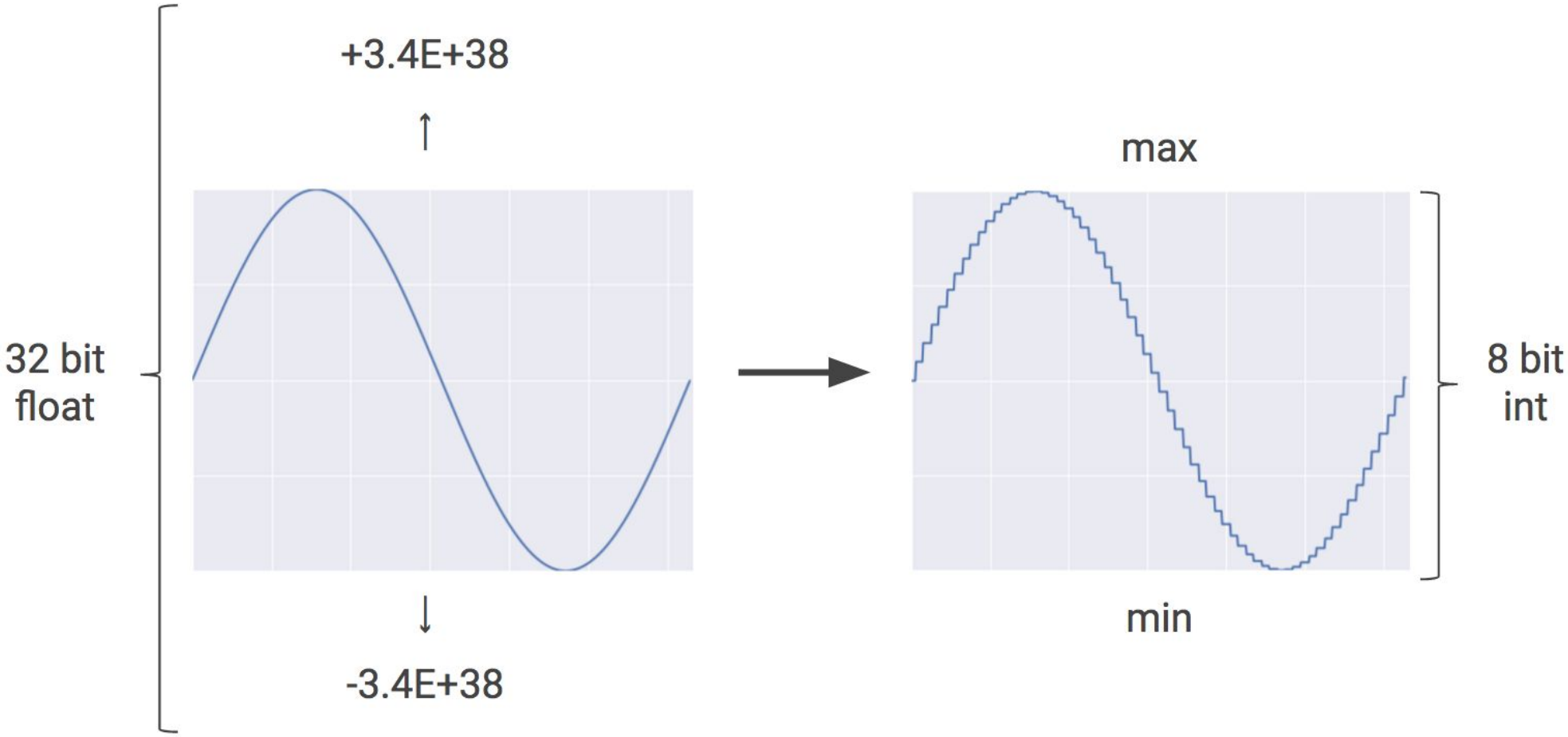
If an integer is small, Python uses the computer's 64 bits of memory to store the value.

If the integer is large, Python can represent the number using more bits (***assuming that there is room in the computer memory)

Big Integer Problems

In other languages, trying to calculate an integer bigger than the computer can accommodate in one memory location can lead to **overflow**.

Floats



Big floats

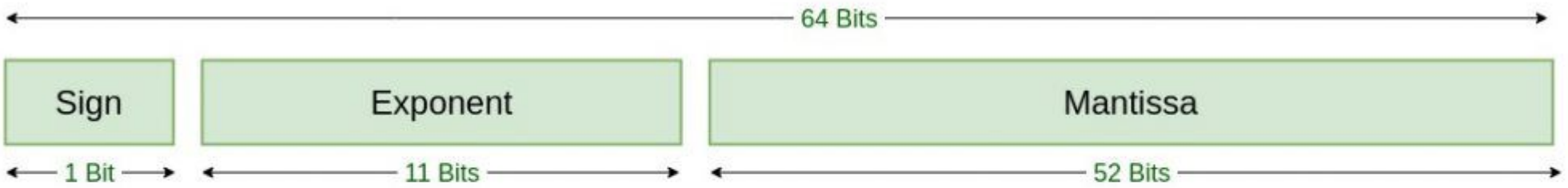
FLOAT data types can represent a much larger range of values than we would expect from 32 (or 64) bits.

Float Structure

The computer stores a FLOAT as a pair of integers.

Mantissa = digits in the float value

Exponent = keeps track of the decimal place



Double Precision
IEEE 754 Floating-Point Standard

Lightning Problem

Write a program that determines the distance to a lightning strike based on the time elapsed between the flash and the sound of thunder. The speed of sound is approximately 1100 ft/ sec and 1 mile is 5280 ft.

Lightning Problem

Write a program that determines the distance to a lightning strike based on the time elapsed between the flash and the sound of thunder. The speed of sound is approximately 1100 ft/ sec and 1 mile is 5280 ft.

Input: time elapsed,t

Process: t*1100/5280

Output: distance in miles

Calendar Problem

The Gregorian epact is the number of days between January 1st and the previous new moon. This value is used to figure out the date of Easter. It is calculated by these formulas (using int arithmetic):

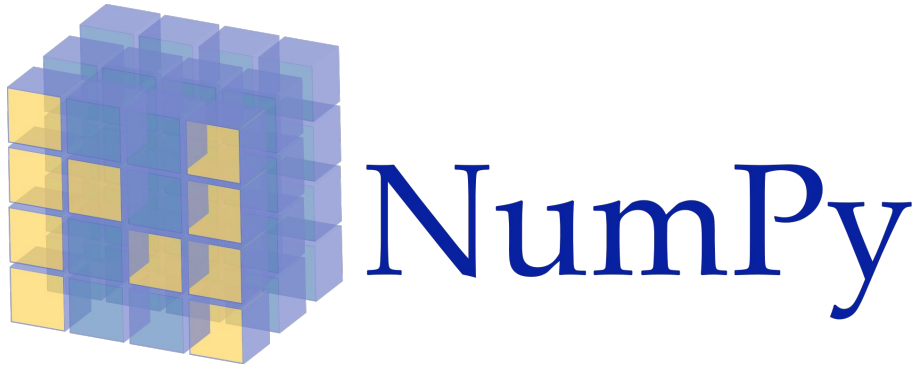
C = year/ /100

epact = (8 + (C/ /4)- C + ((80 + 13)/ /25) + 11(year%19))%30

Write a formula that prompts the user for a 4-digit year and then outputs the value of the epact.

NumPy Library

- The NumPy library is a very common Python library for working with numbers and matrices
- Used for data science, signal processing, machine learning



Conventions

```
import numpy as np
```

Then use np.NumPyfunction() to call specific NumPy functions

NumPy

- NumPy is useful for creating **arrays** and **matrices** in Python

```
import numpy as np
a = np.array([1, 2, 3])
print(a)
print()
b = np.array([[1,2],[3,4],[5,6]])
print(b)
```

NumPy

- NumPy is also useful for signal processing and plotting signals and graphs

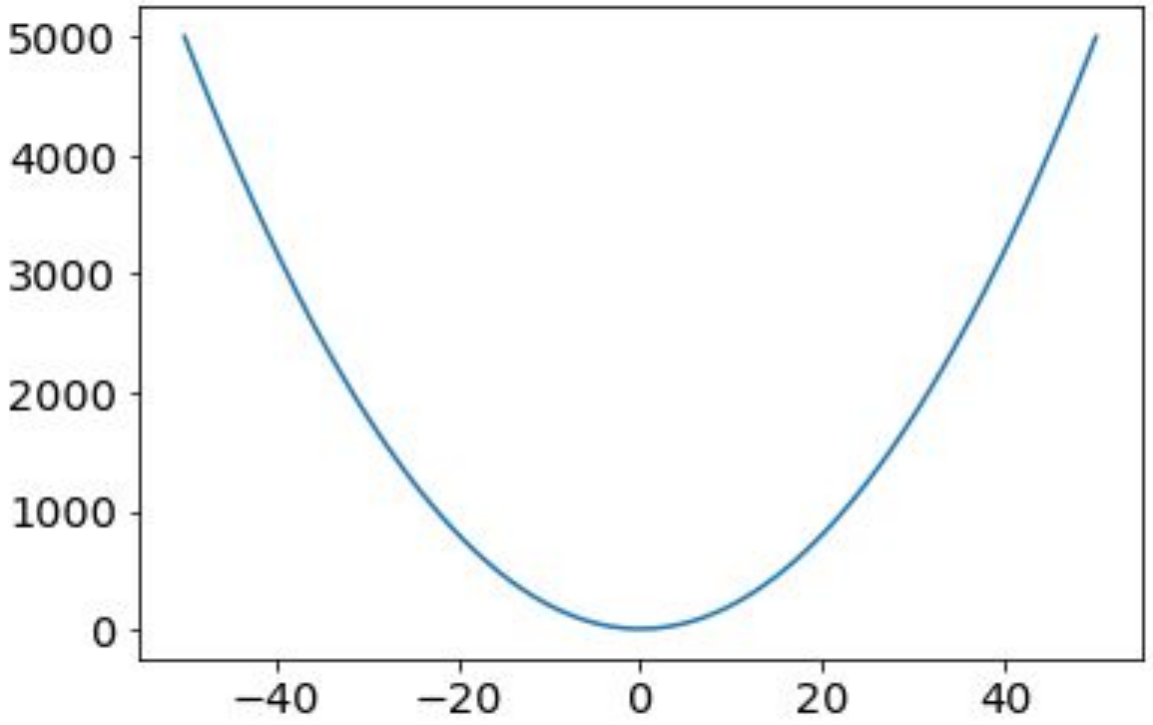
```
import numpy as np
import matplotlib.pyplot as plt


t = np.arange(100)
y = 2*t**2+4
print(y)
```

NumPy Problem

Modify the code to plot the function from a range of -50 to 50.

```
import numpy as np
import matplotlib.pyplot as plt
t = np.arange(100)
y = 2*t**2+4
plt.plot(t,y)
```



Coffee Shop

The Java coffee shop sells coffee at $10.50 a pound plus a fixed cost of $20 shipping. Write a program that calculates and plots the cost of 1 to 100 lbs of coffee.

The Python coffee shop sells coffee at $8.50 a pound plus a fixed cost of $150 shipping. Write a program that calculates and plots the cost of 1 to 100 lbs of coffee on the same plot.

Which shop should I go to if I want to buy 80 pounds of coffee?

Lab 1

Tomorrow: Tutorial time: Friday Jan 17, 8:35-11:35am MacOdrum Library 153

Posted: Today

Deadline: next Friday (Jan 24) 9:00am

Quiz 1

Quizzable topics:

Mostly will be focusing on **theory** from Chapter 1 and 2 Lectures.

Format: 10 Multiple Choice questions

Deadline: Sunday (Jan 19) at 11:55pm